On the significance of the Fields' Energy-momentum Tensor

Abstract

This work discusses the significance of the energy-momentum tensor of physical fields. The Noether theorem shows how this tensor can be derived from the Lagrangian density of a given field. This work proves that the energy-momentum tensor can also be used for a consistency test of a field theory. The results show that the Dirac Lagrangian of a spin-1/2 massive particle yields consistent results. On the other hand, problems exist with the present structure of quantum electrodynamics, and with quantum fields of massive particles that are described by a second order differential equation. All problematic results are confirmed by an independent analysis.

Keywords: The Energy-Momentum Tensor, The Lagrangian Density, Classical and Quantum Fields, Consistency Test.

1 Introduction

The following example illustrates the significance of the energy-momentum tensor as an element of a coherent structure of a physical theory. In the 1960s, Shockley and James presented a paradox where a stationary system of a charge and a magnet has a non-zero electromagnetic linear momentum [1]. They coined the term "hidden momentum" for a description of the missing momentum. Momentum is a fundamental element of physics and the somewhat mysterious "hidden momentum" concept indicates an unsettled problem. Soon after the publication of this paradox, Coleman and Van Vleck provided a general proof showing that the system's total linear momentum must be balanced [2]. Their analysis relies on general conservation properties of the energy-momentum tensor. Later, Comay analyzed the energy-momentum tensor of the Shockley and James system [3]. This analysis proves that an explicit mechanical linear momentum exists in the system. In particular, if a nonvanishing pressure gradient exists along a closed loop of current then effects related to the energy-momentum tensor yield a nonzero mechanical linear momentum. This mechanical momentum balances the electromagnetic linear momentum, and also supports the validity of the Coleman and Van Vleck general analysis. Thus, the Shockley and James paradox is an example that demonstrates the crucial role of the energy-momentum tensor as an element of theoretical physics.

The main objective of this work is to show that relations between the Lagrangian density of a physical field theory and its energy-momentum tensor can be used as a tool for a consistency examination of this theory. This is a new feature of the energy-momentum tensor, and the validity of the results is confirmed by an independent analysis.

Greek indices run from 0 to 3, and Latin indices run from 1 to 3. Units where $\hbar = c = 1$ are used. This unit system requires one dimension, and the unit of length

$[L]$ is used. Most expressions are written in the standard notation of relativistic covariant quantities. The metric is diagonal and its entries are (1,-1,-1,-1). The second section describes some fundamental principles that are used below. The third section presents properties of the energy-momentum tensor. The fourth section discusses electromagnetic fields. The Dirac field of a massive spin-1/2 particle is discussed in the fifth section. Fields that are described by a second order equation are analyzed in the sixth section. The seventh section contains a discussion of some aspects of the results, and the last section presents a summary of this work.

2 Relevant Principles

The following principles are used in the analysis which is carried out in this work.

P.1 The variational principle and its associated Lagrangian density are regarded as fundamental elements of the present structure of field theory. The following quotations that are taken from textbooks support this statement. "All field theories used in current theories of elementary particles have Lagrangians of this form" (see [4], p. 300). Another support of this approach states that the variational principle is "the foundation on which virtually all modern theories are predicated" (see [5], p. 353).

P.2 Wigner's analysis of the unitary representations of the inhomogeneous Lorentz group proves that a quantum particle is characterized by mass and spin. A massless particle has two components of helicity [6–8]. It means that a quantum theory must provide a consistent expression for angular momentum. Furthermore, physical particles belong to one of the following categories: massive particles whose 4-momentum is time-like and massless particles that have a null 4-momentum, where $E^2 - p^2 = 0$.

P.3 The correspondence between nonrelativistic quantum mechanics and classical physics is discussed in textbooks (see [9], pp. 25-27, 137, 138; [10], pp. 19-21). It turns out that similar relationships hold between other physical theories. The following quotation which is found in a well-known textbook, indicates the significance of the correspondence between quantum theories. "First, some good news: quantum field theory is based on the same quantum mechanics that was invented by Schroedinger, Heisenberg, Pauli, Born, and others in 1925-26, and has been used ever since in atomic, molecular, nuclear and condensed matter physics" (see [4], p. 49). A general discussion of correspondence between physical theories is presented in the literature (see [11], pp. 1-6).

These principles provide constraints that apply to the acceptability of a physical theory. They are denoted by P.n, where n denotes the specific principle. Other principles that are not mentioned above are also used, and they are mentioned below at appropriate places.

3 Properties of the Energy-Momentum Tensor

The variational principle P.1 guarantees the existence of many important properties of physical equations. This work examines the relevance of this principle to the construction of the energy-momentum tensor of some field theories.

Density has the dimension $[L^{-3}]$ and energy has the dimension $[L^{-1}]$. Therefore, energy density has the dimension $[L^{-4}]$. Furthermore, density is the 0-component of a 4-current (see [12], p. 75), and energy is the 0-component of the energy-momentum 4-vector (see [12], p. 29). Therefore, energy density is the component T^{00} of a second rank tensor $T^{\mu\nu}$, whose dimension is $[L^{-4}]$. This tensor is called the energy-momentum tensor. Entries of this tensor have a physical meaning (see [12], pp. 82-85). For example, the four entries $T^{\mu 0}$ represent energy-momentum density, the four entries

$T^{0\nu}$ represent energy 4-current, and for each i, the row $T^{i\nu}$ represents momentum 4-current.

Consider the energy-momentum tensor of an elementary massive quantum particle in its rest frame. Isotropy of space means that entries that depend on a spatial direction must vanish. In particular, momentum-related entries and energy 3-current entries must vanish. (This conclusion also holds for an elementary quantum particle whose spin does not vanish. Indeed, spin is an axial vector whereas 3-momentum and 3-current are polar vectors. Hence, spin cannot affect this conclusion.) It follows that the required tensor takes the following form

$$T^{\mu\nu} = \begin{pmatrix} m\rho & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}, \qquad (1)$$

where m denotes the particle's mass and ρ denotes its density. The tensor (1) is symmetric in the particle's rest frame. Hence, it is symmetric in all frames (see [13], p. 77).

As stated above, the Wigner analysis of the inhomogeneous Lorentz group P.2 shows that a quantum particle is characterized by mass and spin. Hence, the theory's structure must provide a well-defined expression for angular momentum. It turns out that a symmetric energy-momentum tensor is required for this end (see [12], pp. 82-85). This outcome also applies to fields that represent a massless particle, like the photon, which has no rest frame.

The Noether theorem shows that a Lagrangian density $\mathcal{L}(\psi, \psi_{,\mu})$, which does not depend explicitly on space-time coordinates, yields a consistent expression for the energy-momentum tensor. This tensor takes the form

$$T^{\mu\nu} = \frac{\partial \mathcal{L}}{\partial \psi_{,\nu}} g^{\mu\alpha} \psi_{,\alpha} - g^{\mu\nu} \mathcal{L} \qquad (2)$$

(see [12], p. 83; [14], p. 310). The tensor (2) satisfies energy-momentum conservation

$$T^{\mu\nu}_{,\nu} = 0. \qquad (3)$$

The definition (2) is consistent with the required dimension of $T^{\mu\nu}$. Indeed, in units where $\hbar = 1$ the action S is dimensionless. Hence, the definition

$$S = \int \mathcal{L}\, d^4x \tag{4}$$

together with $c = 1$ prove that the dimension of the Lagrangian density and of its energy-momentum tensor (2) is $[L^{-4}]$.

The correspondence principle P.3 indicates that the energy-momentum tensor of a classical body is relevant to an analysis of the energy momentum tensor of quantum fields. In the rest frame of a classical macroscopic body, this tensor takes the form

$$T^{\mu\nu} = \begin{pmatrix} \epsilon & 0 & 0 & 0 \\ 0 & p & 0 & 0 \\ 0 & 0 & p & 0 \\ 0 & 0 & 0 & p \end{pmatrix}, \tag{5}$$

where ϵ denotes energy density and p denotes pressure (see [12], p. 92).

The following discussion examines the energy-momentum tensor (2) that is derived from the Lagrangian density of specific fields.

4 Maxwellian Fields

Textbooks on Maxwellian electrodynamics show the standard derivation of the electromagnetic energy momentum tensor. Here the Lagrangian density of free electromagnetic fields (see [12], p. 86; [15] p. 601)

$$\mathcal{L}_{EM} = -\frac{1}{16\pi} F^{\mu\nu} F_{\mu\nu} \tag{6}$$

is used. Evidently, this expression is a Lorentz scalar which does not explicitly depend on the space-time coordinates. The result of the calculation is the following non-symmetric tensor (see [12], p. 86)

$$T^{\mu\nu} = -\frac{1}{4\pi} \frac{\partial A_\lambda}{\partial x_\mu} F^{\nu\lambda} + \frac{1}{16\pi} g^{\mu\nu} F_{\alpha\beta} F^{\alpha\beta} \tag{7}$$

This non-symmetric result means that something is wrong with the derivation of (7). Indeed, the calculation begins with the Lorentz scalar Lagrangian density (6), which does not depend explicitly on the space-time coordinates. In this case, the Noether theorem says that angular momentum should be conserved (see [16], pp. 18, 19). Obviously, if a theory conserves angular momentum then it must provide a consistent expression for this quantity. However, section 3 shows that a consistent definition of angular momentum requires a *symmetric* energy-momentum tensor. Hence, the non-symmetric result (7) means that something is wrong with its derivation.

The following lines show the validity of this conclusion. Consider the radiation fields of a plane electromagnetic wave. Its fields are perpendicular to the direction of motion and to each other (see [12], p. 120; [15] p. 271). The well-known formula of energy density of electrodynamics (see [12], p. 81; [15] p. 237)

$$E_{fields} = (E^2 + B^2)/8\pi \tag{8}$$

means that radiation fields carry a positive amount of energy. In the literature the particle associated with radiation fields is called *real photon*.

Remembering this point, let us examine the Rutherford scattering of an electron by another charged particle (see [17], pp. 186-191). This process is determined by bound fields of the particles. The 4-momentum transfer q^μ is the difference between the 4-momentum of the incoming electron and that of the outgoing electron. q^μ is a spacelike 4-vector (see [17], p. 190). Therefore, the Wigner analysis of the inhomogeneous Lorentz group P.2 shows that no genuine particle is exchanged between the colliding particles. This conclusion is recognized by the general community, and the 4-momentum transfer q^μ is called *virtual photon* (see e.g. [5], p. 65). It means that radiation fields and bound fields do not represent the same physical entity.

The following examination of the hydrogen atom bound fields proves the same result. The electronic states of this atom are documented in textbooks. A selection rule for a one photon transition requires $l' = l \pm 1$, where l, l' denote the initial and the final spatial angular momentum of the atom (see [9], p. 264). Two results are derived from this rule:

1. The photon's angular momentum is unity. This value is documented in this authorized report [18]. (The photon is a massless particle and its angular momentum is called helicity.)

2. The atomic angular momentum is determined by its electronic state. It means that the electromagnetic bound fields of the atom make a null contribution to its angular momentum.

Comparing properties of radiation fields with those of bound fields, one concludes that the different value of angular momentum and requirement P.2 mean that these fields are different physical objects. Furthermore, the spacelike property of the 4-momentum transfer of a Rutherford scattering means that no genuine physical particle is associated with bound fields.

Using this result, let us derive the energy-momentum tensor of radiation fields. The formal form of the result is the apparently nonsymmetric tensor (7). Textbooks show how (7) can be corrected (see [12], pp. 86, 87). Here one adds to (7) the following term

$$W^{\mu\nu} = \frac{1}{4\pi}\frac{\partial A^\mu}{\partial x^\lambda}F^{\nu\lambda} = \frac{1}{4\pi}\frac{\partial}{\partial x^\lambda}(A^\mu F^{\nu\lambda}). \tag{9}$$

and obtains the symmetric tensor

$$T^{\mu\nu} = \frac{1}{4\pi}\,(-F^{\mu\lambda}F^{\nu}_{\lambda} + \frac{1}{4}\,g^{\mu\nu}F^{\alpha\beta}F_{\alpha\beta}). \tag{10}$$

A direct calculation proves that the tensor (9) vanishes for radiation fields [19]. It means that a symmetric energy-momentum tensor is directly obtained for radiation fields.

It is shown above that bound fields are not regarded as independent physical objects in standard calculations of Rutherford scattering and of atomic states. The previous calculation of the energy-momentum tensor provides another argument that supports the claim that electrodynamics that is based on the variational principle yields consistent results, provided bound fields are not treated as free dynamical variables.

5 The Dirac Fields

Let us apply the ordinary construction of the energy-momentum tensor (2) to a Dirac field. The Lagrangian density of a free Dirac field is (see [14], p. 52; [16], p. 54)

$$\mathcal{L} = \bar{\psi}(i\gamma^\mu \partial_\mu - m)\psi. \tag{11}$$

Here the dimension of the expression inside the brackets is $[L^{-1}]$. For this reason the dimension of a Dirac function ψ is $[L^{-3/2}]$. Using the general formula for the energy-momentum tensor (2), one finds that the tensor of this field is

$$T^{\mu\nu} = \bar{\psi} i\gamma^\nu g^{\mu\alpha}\partial_\alpha \psi - g^{\mu\nu}\mathcal{L}. \tag{12}$$

Let us use the definition $\bar{\psi} \equiv \psi^\dagger \gamma^0$ and examine entries of (12). Section 3 shows that the entry T^{00} is energy density. Here the expression (12) corresponds to the Legendre transformation which casts the mechanical Lagrangian to the Hamiltonian (see [20], p. 131; [21], p. 337). This transformation removes terms that are proportional to the first order time-derivative of the coordinates. The result is

$$T^{00} = \psi^\dagger(-i\boldsymbol{\alpha} \cdot \boldsymbol{\nabla} + \beta m)\psi, \tag{13}$$

where $\gamma^0 \boldsymbol{\gamma} = \boldsymbol{\alpha}$. The expression inside the brackets of (13) is the Dirac Hamiltonian of a free particle (see [22], p. 11), and $\psi^\dagger\psi$ is the Dirac density. Hence, (13) represents energy density, in accordance with section 3.

Let us see the form of off-diagonal entries of (12) where $\mu \neq \nu$.

1. If $\mu = k > 0$ and $\nu = 0$ then $\gamma^0\gamma^0 = 1$ and (12) is

$$T^{k0} = \psi^\dagger(-i\partial_k)\psi. \tag{14}$$

Therefore T^{k0} is the momentum density, in accordance with section 3.

2. If $\mu = 0$ and $\nu = k > 0$ then (12) is

$$T^{0k} = \psi^\dagger(i\alpha_k\partial_0)\psi. \tag{15}$$

Here the Dirac α_k is the kth component of the velocity operator (see [22], p. 11). Hence, T^{0k} is the energy current. The relativistic 3-momentum is $\boldsymbol{p} = E\boldsymbol{v}$, where E is the energy and $\boldsymbol{v}$ is the 3-velocity. It means that $T^{\nu 0} = T^{0\nu}$. This relation is consistent with the symmetry of this tensor, in accordance with section 3.

Other components of the energy-momentum tensor of the Dirac field can be calculated analogously. This section proves that an application of the standard construction of the energy-momentum tensor (2) to the Dirac field yields quantities that are consistent with physical requirements.

6 Fields of Second Order Quantum Equations

The $[L^{-4}]$ dimension of the Lagrangian density of a quantum field and a second order quantum differential equation mean that the dimension of this field is $[L^{-1}]$. For this reason, this kind of quantum theory is intrinsically different from the Dirac theory of a massive quantum particle, where the dimension of the field's function is $[L^{-3/2}]$.

An example of a Lagrangian density of a second order quantum theory of a massive particle is

$$\mathcal{L} = g^{\mu\nu}\phi^*_{,\mu}\phi_{,\nu} - m^2\phi^*\phi + OT, \tag{16}$$

where OT denotes other terms. In the case of the Klein-Gordon (KG) field, OT is null (see [4], p. 21; [16], p. 38). The Lagrangian (16) describes correctly the appropriate expression for the Higgs boson (see [14], p. 715). An analogous expression holds for the electroweak $W^{\pm}$, Z bosons (see [23], p. 518) and for the Proca theory of a massive photon (see [15], pp. 597-601). The mass dependence of the Lagrangian density (16) is expressed only by its second term.

Let us utilize the standard construction of the energy-momentum tensor (2) and find an expression for energy density T^{00} of these fields. The mass-dependent term of (16) contains no derivatives. Therefore, in the corresponding energy-momentum tensor, this term takes the same form with the opposite sign

$$T^{00} = m^2\phi^*\phi + ... \tag{17}$$

This expression for energy density depends *quadratically* on mass. Therefore, it is inconsistent with the standard form of the energy-momentum tensor (1) as well as with the celebrated relativistic expression $E = mc^2$, which depend *linearly* on mass.

For this reason, the structure of the energy-momentum tensor of a second order Lagrangian density demonstrates an inherent inconsistency of the associated theory.

7 Discussion

The following arguments support the validity of the problematic issues that are obtained above from an analysis of the energy-momentum tensor.

It is interesting to note that the symmetric energy-momentum tensor of electromagnetic fields (10) can be directly obtained from a Lagrangian density, if entries of the metric tensor $g^{\mu\nu}$ are regarded as the dynamical variables (see [12], pp. 290-293). The following argument indicates that this issue does not settle the problem.

In principle, the Lagrangian density is used for a derivation of the equations of motion of dynamical variables, called the Euler-Lagrange equations (see e.g. [4], p. 300). Other quantities can also be derived from it. In the case where gravitational fields are regarded as dynamical variables, the symmetric energy-momentum tensor of electromagnetic fields (10) is obtained. By contrast, if the electromagnetic quantities are treated as the system's dynamical variables then the unacceptable non-symmetric expression (7) is obtained! Hence, the following problem arises: *Why contradictory results are obtained from two legitimate procedures?*

This problem indicates that something is wrong with the derivation of the incorrect electromagnetic energy-momentum tensor (7). Evidently, a further analysis of this issue can only clarify the root of the problem. The discussion of section 4 shows that there are very good reasons for the removal of bound fields from the analysis. This action also yields a symmetric energy-momentum tensor of the electromagnetic fields.

The following quotations support the foregoing claim that the present structure of Maxwellian electrodynamics contains an unsettled problem. Thus, one respectable textbook examines the electromagnetic 4-potential and states: "The fact that A^0 vanishes in all Lorentz frames shows vividly that A^μ cannot be a four vector" (see [4], p. 251). By contrast, other respectable textbooks treat A^μ as a genuine 4-vector (see e.g. [12], pp. 47-49; [15], pp. 572-578). These contradictory statements indicate that an unsettled problem still exists in quantum electrodynamics (QED). For a further discussion of this problem, see [24, 25].

Let us examine the theory of KG particles that was introduced by Pauli and Weisskopf [26]. Their Hamiltonian of an electrically charged KG particle contains terms that depend *quadratically* on the electric charge and on the 4-potential as well (see eq. (37.a) therein). This property violates Maxwellian electrodynamics, where the interaction strength is proportional to the electric charge and depends linearly on the 4-potential.

The Proca field is a modification of Maxwellian electrodynamics. It is an example of a Lagrangian density that contains an additional term which is proportional to m^2, and the particle is called "a massive photon" (see [15], pp. 597-601). This is a special example of the m^2 discrepancy which is discussed in section 6. The m^2 problem of the Proca Lagrangian has already been pointed out in the literature [27]. It turns out that the experimental side strongly disapproves the Proca idea of a massive photon. Thus, the experimental upper limit of the photon mass is smaller than the electron mass by the amazing amount of 24 orders of magnitude [18].

Unsettled theoretical problems exist with the $W^{\pm}, Z$ and the Higgs particles, whose Lagrangian density contains a problematic term which is proportional to m^2. For example, these particles have a mass $m > 0$, but in spite of the fact that these particles have been proposed more than half a century ago, textbooks still do not show a consistent expression for their density in particular and for their 4-current in general. By contrast, a consistent expression for density of a Dirac particle has been found about one month after the publication of the Dirac equation [28, 29].

It is interesting to note that the negative attributes of second order quantum equations that are derived above are consistent with Dirac's lifelong objection to these equations (see [30], pp. 1-8).

8 Conclusions

This work points out the significance of the energy-momentum tensor of physical fields. For example, the Shockley-James hidden momentum enigma [1] has been settled by means of this tensor, which provides explicit expressions that show the required momentum balance [3].

The present work indicates that this tensor can also be used for a consistency test of field theories. Specific discussions prove that the present form of the Lagrangian density of many fields contains erroneous elements. The analysis shows that this new application of the energy-momentum tensor yields correct results, and each problematic field theory contains other kinds of inconsistencies. The Dirac field is problem-free, whereas problems exist with QED, and with second order quantum equations of massive particles.

COMPETING INTERESTS DISCLAIMER:

Authors have declared that no competing interests exist. The products used for this research are commonly and predominantly use products in our area of research and country. There is absolutely no conflict of interest between the authors and producers of the products because we do not intend to use these products as an avenue for any litigation but for the advancement of knowledge. Also, the research was not funded by the producing company rather it was funded by personal efforts of the authors.

References

[1] W. Shockley and R. P. James, Phys. Rev. Lett., **18**, 876 (1967).

[2] S. Coleman and J. H. Van Vleck, Phys. Rev., **171**, 1370 (1968).

[3] E. Comay, Am. J. Phys., **64**, 1028 (1996).

[4] Weinberg S. The Quantum Theory of Fields, Vol. I. Cambridge University Press, Cambridge; 1995.

[5] D. Griffiths, *Introduction to Elementary Particles*, 2nd edition, Weinheim, Wiley-VCH, 2008.

[6] Wigner, E. (1939). On Unitary Representations of the Inhomogeneous Lorentz Group. Ann. Math. 40, 149-204.

[7] Schweber S. S. (1964). An Introduction to Relativistic Quantum Field Theory. New York: Harper & Row. (See pp. 44-53.)

[8] Sternberg, S. (1994). Group Theory and Physics. Cambridge: Cambridge University Press. (See pp. 143-150.)

[9] Schiff, L. I. (1955). Quantum Mechanics. New York: McGraw-Hill.

[10] Landau, L. D., & Lifshitz, M. E. Quantum Mechanics (Pergamon, London, 1959)

[11] Rohrlich F. Classical Charged Particle. World Scientific, New Jersey; 2007.

[12] Landau, L. D., & Lifshitz, M. E. (2005). The Classical Theory of Fields. Amsterdam: Elsevier.

[13] R. Adler, M. Bazin and M. Schiffer, Introduction to General Relativity (McGraw-Hill, New York, 1965).

[14] Peskin ME, Schroeder DV. An Introduction to Quantum Field Theory. Addison-Wesley: Reading Mass; 1995.

[15] Jackson, J. D. (1975). Classical Electrodynamics. New York: John Wiley.

[16] Bjorken J. D. and Drell S. D. Relativistic Quantum Fields. McGraw-Hill, New York; 1965.

[17] D. H. Perkins, *Introduction to High Energy Physics*, Menlo Park CA, Addison-Wesley, 1987.

[18] Patrignani C. et al. (Particle Data Group) (2016). and 2017 update. Chin. Phys. C, 40, 100001.

[19] E. Comay, A Consistent Construction of the Electromagnetic Energy-Momentum Tensor, OALib, **5**, 1 (2018).
`https://www.scirp.org/journal/PaperInformation.aspx?PaperID=82391`

[20] Landau, L. D., & Lifshitz, M. E. Mechanics (Pergamon, Oxford, 1960)

[21] H. Goldstein, C. Poole & J. Safko, Classical Mechanics, 3rd edition (Addison Wesley, San Francisco, 2002).

[22] J. D. Bjorken and S.D. Drell, *Relativistic Quantum Mechanics* (McGraw-Hill, New York, 1964).

[23] G. Sterman *An Introduction to Quantum Field Theory* (Cambridge University Press, Cambridge, 1993).

[24] E. Comay, Acta Phys. Pol. A, **133**, 1294 (2018).
http://przyrbwn.icm.edu.pl/APP/PDF/133/app133z5p26.pdf

[25] E. Comay, OALib, **5** (11), 1 (2018).
https://www.scirp.org/journal/PaperInformation.aspx?PaperID=88593.

[26] W. Pauli and V. Weisskopf, Helv. Phys. Acta, **7**, 709 (1934). English translation: A. I. Miller *Early Quantum Electrodynamics*, Cambridge, University Press, 1994. pp. 188-205.

[27] E. Comay, Nuovo Cim. **B113**, 733 (1998).

[28] P. A. M. Dirac, Proc. Roy. Soc. Lond. **A117**, 610 (1928).

[29] C. G. Darwin, Proc. Roy. Soc. Lond. **A118**, 654 (1928).

[30] P. A. M. Dirac, *Mathematical Foundations of Quantum Theory*, Ed. A. R. Marlow (Academic, New York, 1978).