

Review Article

On the role of squared neutron number in reducing nuclear binding energy in the light of Electromagnetic, Weak and Nuclear gravitational constants – A Review

Abstract: With reference to our recently proposed three virtual atomic gravitational constants and nuclear elementary charge, close to stable mass numbers, it is possible to show that, squared neutron number plays a major role in reducing nuclear binding energy. In this context, Z=30 onwards, ‘inverse of the strong coupling constant’, can be inferred as a representation of the maximum strength of nuclear interaction and 10.09 MeV can be considered as a characteristic nuclear binding energy coefficient. Coulombic energy coefficient being 0.695 MeV, semi empirical mass formula - volume, surface, asymmetric and pairing energy coefficients can be shown to be 15.29 MeV, 15.29 MeV, 23.16 MeV and 10.09 MeV respectively. Volume and Surface energy terms can be represented with $(A-A^{2/3}-1)*15.29$ MeV. With reference to nuclear potential of 1.162 MeV and coulombic energy coefficient, close to stable mass numbers, nuclear binding energy can be fitted with two simple terms having an effective binding energy coefficient of $[10.09-(1.162+0.695)/2] = 9.16$ MeV. With further study, semi empirical mass formula can be simplified.

Keywords: three virtual atomic gravitational constants; nuclear elementary charge; nuclear stability; binding energy; squared neutron number;

24

251. Introduction

26Objective of this paper is to review, simplify and establish the concepts proposed in our recent 27papers and conference proceedings [1,2,3,4] pertaining to nuclear stability and binding energy 28connected with three virtual atomic gravitational constants.

29The most desirable cases of any unified description are:

- 30
- 31 a) To implement gravity in microscopic physics and to estimate the magnitude of the
32 Newtonian gravitational constant (G_N).
 - 33 b) To simplify the complicated issues of known physics. (Understanding nuclear stability,
34 nuclear binding energy, nuclear charge radii and neutron life time etc.)
 - 35 c) To predict new effects, arising from a combination of the fields inherent in the unified
36 description. (Understanding strong coupling constant, Fermi’s weak coupling constant and
37 radiation constants etc.)
 - 38 d) To develop a model of microscopic quantum gravity.

39 1.1 History of the three atomic gravitational constants

- 40
- 41 (1) Since 1974, K. Tennakone, Abdus Salam, C. Sivaram, K.P.Sinha, Dj. Sijacki, Y. Ne’eman,

J.J. Perng, J. Strathdee, Usha Raut, V. de Sabbata, E. Recami, T.R. Mongan, Robert Oldershaw and S.G. Fedosin like many scientists proposed the existence of ‘Nuclear’ or ‘strong’ gravitational constant with a magnitude approximately (10^{35} to 10^{39}) times the Newtonian gravitational constant. In this context, one can see a detailed discussion by F. Akinto and Farida Tahir in their arXiv preprint [5].

- (2) In 2010, 2011 and 2012, in a series of papers, we proposed the existence of ‘electromagnetic’ gravitational constant [6,7,8]. In 2016 Franck Delplace also proposed its existence [9].
- (3) In 2013, Roberto Onofrio proposed the existence of ‘weak’ gravitational constant [10].

1.2 To estimate the Newtonian gravitational constant in a theoretical approach

According to Rosi et al [11]: There is no definitive relationship indeed between G_N and the other fundamental constants and no theoretical prediction for its value to test the experimental results. Improving the knowledge of G_N has not only a pure metrological interest, but is also important for the key role that this fundamental constant plays in theories of gravitation, cosmology, particle physics, astrophysics, and geophysical models.

To estimate the value of G_N in a theoretical approach, we would like to suggest the following points.

- (1) Interaction constants are connected both with global phenomena of physics and with phenomena at small distances, such as quantum gravity. Therefore, the search for relations among the constants of the four types of interactions is important, relevant and necessary. At present, there exist no basic formulae or mechanisms using by which one can develop at least models with ad hoc relations. In a unified approach, one can see a great initiative taken by J. E. Brandenburg [12]. It would be important to consider in detail such theories as microscopic quantum gravity and a combination of the fields inherent in the unified description of the four interactions.
- (2) As there is a large gap in between nuclear and Planck scales, with currently believed notion of unification paradigm, it seems impossible to implement gravity in atomic, nuclear and particle physics.
- (3) G_N is a man created empirical constant and is having no physical existence. Clearly speaking, it is not real but virtual. For understanding the secrets of large scale gravitational effects, scientists consider it as a physical constant.
- (4) In the same way, each atomic interaction can be allowed to have its own virtual gravitational constant.
- (5) With a combined study of the four gravitational constants, their magnitudes can be refined for a better fit and understanding of the nature.

1 **1.3 Four basic semi empirical reference relations**

2
3 With reference to our recent publications and conference presentations [1-4],[6-8] & [13-20], we
4 propose the following set of four semi empirical ‘reference’ relations. Let,

5
6 Electromagnetic gravitational constant = G_e

7 Nuclear gravitational constant = G_s

8 Weak gravitational constant = G_w

9

$$\frac{m_p}{m_e} \cong 2\pi \sqrt{\frac{4\pi\epsilon_0 G_e m_e^2}{e^2}} \cong \left(\frac{G_e m_e^2}{\hbar c} \right) \left(\frac{G_s m_p^2}{\hbar c} \right) \quad (1)$$

10
11
12 $\hbar c \cong \left(\frac{m_p}{m_e} \right)^2 \left(G_e^2 G_N \right)^{1/3} m_p^2 \quad (2)$

13
14 $G_F \cong \left[\left(G_e m_p^2 \right)^2 \left(G_N m_p^2 \right) \right]^{\frac{1}{3}} \left(\frac{2G_s m_p}{c^2} \right)^2 \cong \frac{4G_w \hbar^2}{c^2} \quad (3)$

15
16 $\frac{G_w}{G_N} \cong \left(\frac{m_p}{m_e} \right)^{10} \quad (4)$

17Based on relation (1), magnitudes of (G_e, G_s) can be estimated. Based on relation (2), magnitude of
18 G_N can be estimated. Based on relation (3), magnitudes of (G_F, G_w) can be estimated [10, 21]. Again,
19 based on relation (4), G_N can be estimated. Estimated values seem to be:

20

$$\boxed{\begin{aligned} G_e &\cong 2.374335 \times 10^{37} \text{ m}^3 \text{kg}^{-1} \text{sec}^{-2} \\ G_s &\cong 3.329561 \times 10^{28} \text{ m}^3 \text{kg}^{-1} \text{sec}^{-2} \\ G_w &\cong 2.909745 \times 10^{22} \text{ m}^3 \text{kg}^{-1} \text{sec}^{-2} \\ G_N &\cong 6.679855 \times 10^{-11} \text{ m}^3 \text{kg}^{-1} \text{sec}^{-2} \\ G_F &\cong 1.44021 \times 10^{-62} \text{ J.m}^3 \end{aligned}}$$

21
22Even though our approach is speculative, role played by the four gravitational constants seems to be
23fairly natural. This kind of approach may help in producing a variety of such relations by using
24which in near future, an absolute set of relations can be developed. Proceeding further, estimated
25absolute theoretical value of G_N can be considered as a standard reference for future experiments. In
26a verifiable approach we have developed many interesting relations and we are working on deriving
27them from basic principles.

28

29 **302 Three simple assumptions pertaining to nuclear physics**

30

31 We propose the following three assumptions.

1
2) There exists a strong elementary charge in such a way that,

$$4 \quad \frac{e_s}{e} \cong \left(\frac{G_s m_p^2}{\hbar c} \right) \cong \left[\frac{G_s m_e^2}{(G_e^2 G_N)^{1/3} m_p^2} \right] \quad (5)$$

$$6 \quad \frac{e_s^2}{e^2} \cong \left(\frac{G_s m_p^2}{\hbar c} \right)^2 \cong \left(\frac{G_s m_p^3}{G_e m_e^3} \right) \quad (6)$$

$$8 \quad \frac{e_s G_s}{e G_w} \cong \left(\frac{m_p}{m_e} \right)^2 \quad (7)$$

10) 2) Strong coupling constant [20] can be expressed with,

$$12 \quad \alpha_s \cong \left(\frac{e}{e_s} \right)^2 \cong \left(\frac{\hbar c}{G_s m_p^2} \right)^2 \cong \left(\frac{G_e m_e^3}{G_s m_p^3} \right) \quad (8)$$

14) 3) Nuclear charge radius can be addressed with,

$$16 \quad R_0 \cong \frac{2G_s m_p}{c^2} \quad (9)$$

17) Based on relations (5) to (9),

| |
|---|
| $e_s \cong 2.9463591e$ |
| $\alpha_s \cong 0.1151937$ |
| $\frac{1}{\alpha_s} \cong 8.681032$ |
| $R_0 \cong 1.23929 \times 10^{-15} \text{ m}$ |

20) 3 Understanding proton-neutron stability with three atomic gravitational constants

22) Let,

$$23 \quad \left. \begin{aligned} s &\cong \left\{ \left(\frac{e_s}{m_p} \right) \div \left(\frac{e}{m_e} \right) \right\} \cong 0.001605 \\ &\cong \frac{G_s m_p m_e}{\hbar c} \cong \frac{\hbar c}{G_e m_e^2} \cong \frac{G_s^2}{G_e G_w} \end{aligned} \right\} \quad (10)$$

24) 25) Nuclear beta stability line can be addressed with a relation of the form [relation 8 of ref.22],

$$\begin{aligned} A_s &\equiv 2Z + s(2Z)^2 \equiv 2Z + (4s)Z^2 \\ &\equiv 2Z + kZ^2 \equiv Z(2 + kZ) \end{aligned} \quad (11)$$

where $(4s) \equiv k \equiv 0.0064185$

By considering a factor like $[2 \pm \sqrt{k}]$, likely possible range of A_s can be addressed with,

$$\left. \begin{aligned} & (A_s)_{lower}^{upper} \cong Z[(2 \pm 0.08) + kZ] \\ & \rightarrow \left\{ \begin{aligned} & (A_s)_{lower} \cong Z(1.92 + kZ) \\ & (A_s)_{mean} \cong Z(2.0 + kZ) \\ & (A_s)_{upper} \cong Z(2.08 + kZ) \end{aligned} \right. \end{aligned} \right\} \quad (12)$$

Interesting point to be noted is that, for $Z=112, 113$ and 114 , estimated lower stable mass numbers are $296, 299$ and 302 respectively. Corresponding neutron numbers are $184, 186$ and 188 . These neutron numbers are very close to the currently believed shell closure at $N=184$. It needs further study [23]. See table 1.

Table-1: Likely possible range of A_s for Z=5 to 118

| Proton number | $(A_s)_{lower}$ | $(A_s)_{mean}$ | $(A_s)_{upper}$ |
|---------------|-----------------|----------------|-----------------|
| 5 | 10 | 10 | 11 |
| 6 | 12 | 12 | 13 |
| 7 | 14 | 14 | 15 |
| 8 | 16 | 16 | 17 |
| 9 | 18 | 19 | 19 |
| 10 | 20 | 21 | 21 |
| 11 | 22 | 23 | 24 |
| 12 | 24 | 25 | 26 |
| 13 | 26 | 27 | 28 |
| 14 | 28 | 29 | 30 |
| 15 | 30 | 31 | 33 |
| 16 | 32 | 34 | 35 |
| 17 | 34 | 36 | 37 |
| 18 | 37 | 38 | 40 |
| 19 | 39 | 40 | 42 |
| 20 | 41 | 43 | 44 |
| 21 | 43 | 45 | 47 |
| 22 | 45 | 47 | 49 |
| 23 | 48 | 49 | 51 |
| 24 | 50 | 52 | 54 |
| 25 | 52 | 54 | 56 |
| 26 | 54 | 56 | 58 |
| 27 | 57 | 59 | 61 |
| 28 | 59 | 61 | 63 |
| 29 | 61 | 63 | 66 |
| 30 | 63 | 66 | 68 |
| 31 | 66 | 68 | 71 |
| 32 | 68 | 71 | 73 |
| 33 | 70 | 73 | 76 |
| 34 | 73 | 75 | 78 |
| 35 | 75 | 78 | 81 |

| | | | |
|----|-----|-----|-----|
| 36 | 77 | 80 | 83 |
| 37 | 80 | 83 | 86 |
| 38 | 82 | 85 | 88 |
| 39 | 85 | 88 | 91 |
| 40 | 87 | 90 | 93 |
| 41 | 90 | 93 | 96 |
| 42 | 92 | 95 | 99 |
| 43 | 94 | 98 | 101 |
| 44 | 97 | 100 | 104 |
| 45 | 99 | 103 | 107 |
| 46 | 102 | 106 | 109 |
| 47 | 104 | 108 | 112 |
| 48 | 107 | 111 | 115 |
| 49 | 109 | 113 | 117 |
| 50 | 112 | 116 | 120 |
| 51 | 115 | 119 | 123 |
| 52 | 117 | 121 | 126 |
| 53 | 120 | 124 | 128 |
| 54 | 122 | 127 | 131 |
| 55 | 125 | 129 | 134 |
| 56 | 128 | 132 | 137 |
| 57 | 130 | 135 | 139 |
| 58 | 133 | 138 | 142 |
| 59 | 136 | 140 | 145 |
| 60 | 138 | 143 | 148 |
| 61 | 141 | 146 | 151 |
| 62 | 144 | 149 | 154 |
| 63 | 146 | 151 | 157 |
| 64 | 149 | 154 | 159 |
| 65 | 152 | 157 | 162 |
| 66 | 155 | 160 | 165 |
| 67 | 157 | 163 | 168 |
| 68 | 160 | 166 | 171 |
| 69 | 163 | 169 | 174 |
| 70 | 166 | 171 | 177 |
| 71 | 169 | 174 | 180 |
| 72 | 172 | 177 | 183 |
| 73 | 174 | 180 | 186 |
| 74 | 177 | 183 | 189 |
| 75 | 180 | 186 | 192 |
| 76 | 183 | 189 | 195 |
| 77 | 186 | 192 | 198 |
| 78 | 189 | 195 | 201 |
| 79 | 192 | 198 | 204 |
| 80 | 195 | 201 | 207 |
| 81 | 198 | 204 | 211 |
| 82 | 201 | 207 | 214 |
| 83 | 204 | 210 | 217 |
| 84 | 207 | 213 | 220 |
| 85 | 210 | 216 | 223 |
| 86 | 213 | 219 | 226 |
| 87 | 216 | 223 | 230 |
| 88 | 219 | 226 | 233 |
| 89 | 222 | 229 | 236 |
| 90 | 225 | 232 | 239 |
| 91 | 228 | 235 | 242 |
| 92 | 231 | 238 | 246 |
| 93 | 234 | 242 | 249 |
| 94 | 237 | 245 | 252 |

| | | | |
|-----|-----|-----|-----|
| 95 | 240 | 248 | 256 |
| 96 | 243 | 251 | 259 |
| 97 | 247 | 254 | 262 |
| 98 | 250 | 258 | 265 |
| 99 | 253 | 261 | 269 |
| 100 | 256 | 264 | 272 |
| 101 | 259 | 267 | 276 |
| 102 | 263 | 271 | 279 |
| 103 | 266 | 274 | 282 |
| 104 | 269 | 277 | 286 |
| 105 | 272 | 281 | 289 |
| 106 | 276 | 284 | 293 |
| 107 | 279 | 287 | 296 |
| 108 | 282 | 291 | 300 |
| 109 | 286 | 294 | 303 |
| 110 | 289 | 298 | 306 |
| 111 | 292 | 301 | 310 |
| 112 | 296 | 305 | 313 |
| 113 | 299 | 308 | 317 |
| 114 | 302 | 311 | 321 |
| 115 | 306 | 315 | 324 |
| 116 | 309 | 318 | 328 |
| 117 | 312 | 322 | 331 |
| 118 | 316 | 325 | 335 |

4 Unified energy coefficients of Semi empirical mass formula (SEMF)

Let,

A characteristic nuclear binding energy coefficient be expressed as,

$$B_0 \cong \frac{e^2}{8\pi\varepsilon_0(G_s m_p/c^2)} \cong \left(\frac{1}{\alpha_s}\right) \left(\frac{e^2}{4\pi\varepsilon_0 R_0}\right) \cong 10.09 \text{ MeV} \quad (13)$$

With reference to a new factor of the form, $\ln\left(\frac{e^2}{4\pi\varepsilon_0 G_s m_p m_e}\right) \cong 1.515$,

(1) Volume or surface energy coefficient can be expressed as $a_v \cong a_s \cong 1.515 * 10.09 \cong 15.29 \text{ MeV}$.

(2) Asymmetric energy coefficient can be expressed as, $a_a \cong 1.515 a_v \cong 1.515 a_s \cong 1.515 * 15.29 \cong 23.16 \text{ MeV}$.

(3) Pairing energy coefficient can be expressed as, $a_p \cong B_0 \cong 10.09 \text{ MeV}$.

(4) 10.09 MeV, 15.29 MeV and 23.16 MeV seem to follow a geometric series with a geometric ratio, 1.515.

(5) For ($Z \geq 10$), by considering coulombic energy coefficient as $a_c \cong 0.695 \text{ MeV}$, nuclear binding energy [22,24,25] can be estimated with,

$$B_A \cong \left\{ \left[(A - A^{2/3} - 1) * 15.29 \right] - \left[\frac{Z^2}{A^{1/3}} * 0.695 \right] - \frac{(A - 2Z)^2}{A} * 23.16 \pm \left[\frac{10.09}{\sqrt{A}} \right] \right\} \text{ MeV} \quad (14)$$

1 Data estimated with relation (14) can be compared with the standard relation [22],

2

$$B_A \cong \left\{ (A * 15.78) - (A^{2/3} * 18.34) - \frac{Z(Z-1)}{A^{1/3}} * 0.71 - \frac{(A-2Z)^2}{A} * 23.21 \pm \frac{12.0}{\sqrt{A}} \right\} \text{MeV} \quad (15)$$

3 See Table 2 for the isotopic binding energy of Z=50 estimated with relation (14) compared with
4 relation (15).

| Table 2 : Isotopic binding energy of Z = 50 | | | | | |
|---|-------------|----------------|--|---|-------------|
| Proton number | Mass number | Neutron number | Estimated Binding energy (MeV) [Relation (14)] | SEMF binding energy (MeV) [Relation (15)] | Error (MeV) |
| 50 | 100 | 50 | 810.97 | 809.31 | -1.66 |
| 50 | 101 | 51 | 824.07 | 822.27 | -1.80 |
| 50 | 102 | 52 | 838.72 | 837.17 | -1.55 |
| 50 | 103 | 53 | 850.92 | 849.24 | -1.69 |
| 50 | 104 | 54 | 864.69 | 863.24 | -1.45 |
| 50 | 105 | 55 | 876.05 | 874.47 | -1.58 |
| 50 | 106 | 56 | 888.97 | 887.63 | -1.34 |
| 50 | 107 | 57 | 899.54 | 898.07 | -1.47 |
| 50 | 108 | 58 | 911.67 | 910.44 | -1.23 |
| 50 | 109 | 59 | 921.48 | 920.13 | -1.35 |
| 50 | 110 | 60 | 932.87 | 931.76 | -1.11 |
| 50 | 111 | 61 | 941.97 | 940.74 | -1.24 |
| 50 | 112 | 62 | 952.65 | 951.65 | -1.00 |
| 50 | 113 | 63 | 961.08 | 959.96 | -1.12 |
| 50 | 114 | 64 | 971.08 | 970.20 | -0.88 |
| 50 | 115 | 65 | 978.87 | 977.88 | -1.00 |
| 50 | 116 | 66 | 988.24 | 987.48 | -0.76 |
| 50 | 117 | 67 | 995.42 | 994.55 | -0.87 |
| 50 | 118 | 68 | 1004.18 | 1003.55 | -0.64 |
| 50 | 119 | 69 | 1010.80 | 1010.05 | -0.75 |
| 50 | 120 | 70 | 1018.98 | 1018.47 | -0.51 |
| 50 | 121 | 71 | 1025.05 | 1024.43 | -0.62 |
| 50 | 122 | 72 | 1032.69 | 1032.30 | -0.38 |
| 50 | 123 | 73 | 1038.24 | 1037.75 | -0.49 |
| 50 | 124 | 74 | 1045.35 | 1045.10 | -0.25 |
| 50 | 125 | 75 | 1050.41 | 1050.05 | -0.36 |
| 50 | 126 | 76 | 1057.03 | 1056.91 | -0.12 |
| 50 | 127 | 77 | 1061.62 | 1061.39 | -0.23 |
| 50 | 128 | 78 | 1067.76 | 1067.77 | 0.01 |
| 50 | 129 | 79 | 1071.90 | 1071.81 | -0.09 |
| 50 | 130 | 80 | 1077.59 | 1077.74 | 0.15 |
| 50 | 131 | 81 | 1081.30 | 1081.35 | 0.05 |
| 50 | 132 | 82 | 1086.57 | 1086.85 | 0.28 |
| 50 | 133 | 83 | 1089.87 | 1090.06 | 0.19 |
| 50 | 134 | 84 | 1094.72 | 1095.15 | 0.43 |
| 50 | 135 | 85 | 1097.63 | 1097.96 | 0.33 |
| 50 | 136 | 86 | 1102.09 | 1102.66 | 0.57 |
| 50 | 137 | 87 | 1104.63 | 1105.11 | 0.48 |
| 50 | 138 | 88 | 1108.71 | 1109.42 | 0.71 |

| | | | | | |
|----|-----|-----|---------|---------|------|
| 50 | 139 | 89 | 1110.89 | 1111.52 | 0.62 |
| 50 | 140 | 90 | 1114.61 | 1115.47 | 0.86 |
| 50 | 141 | 91 | 1116.45 | 1117.23 | 0.77 |
| 50 | 142 | 92 | 1119.83 | 1120.84 | 1.01 |
| 50 | 143 | 93 | 1121.34 | 1122.27 | 0.93 |
| 50 | 144 | 94 | 1124.38 | 1125.55 | 1.16 |
| 50 | 145 | 95 | 1125.59 | 1126.67 | 1.08 |
| 50 | 146 | 96 | 1128.31 | 1129.63 | 1.32 |
| 50 | 147 | 97 | 1129.21 | 1130.45 | 1.24 |
| 50 | 148 | 98 | 1131.63 | 1133.10 | 1.47 |
| 50 | 149 | 99 | 1132.24 | 1133.64 | 1.40 |
| 50 | 150 | 100 | 1134.37 | 1136.00 | 1.63 |

4.1 Observations pertaining to Term1 to Term2 of relation (14):

- (1) Ratio of (Term1-Term2)/10.09 MeV is a straight line and slope is practically constant for $Z = 10$ to 100 . See Figure 2.
- (2) With further study, Term1 and Term2 can be unified into a single term.

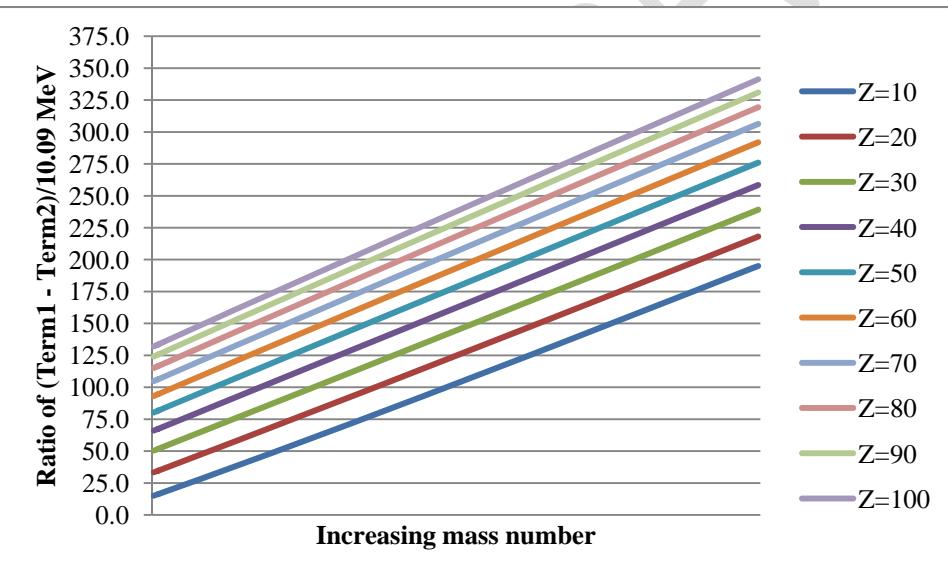


Figure 1: Ratio of (Term1-Term2)/10.09 MeV for $Z=10$ to 100

5 Understanding nuclear binding energy with single and unified energy coefficient

A. New integrated model

Based on the new integrated model proposed by N. Ghahramany et al [26,27],

$$B(Z, N) = \left\{ A - \left(\frac{(N^2 - Z^2) + \delta(N - Z)}{3Z} + 3 \right) \right\} \frac{m_n c^2}{\gamma} \quad (16)$$

where, γ = Adjusting coefficient \approx (90 to 100).

1 if $N \neq Z$, $\delta(N-Z) = 0$ and if $N = Z$, $\delta(N-Z) = 1$.

2
3 Readers are encouraged to see references there in [26, 27] for derivation part. Point to be noted is
4 that, close to the beta stability line, $\left[\frac{N^2 - Z^2}{3Z} \right]$ takes care of the combined effects of coulombic and
5 asymmetric effects. Point to be noted here is that, nuclear binding energy can be addressed with
6 a single energy coefficient.

7 **B. Our unified approach**

8 Interesting points to be noted are:

- 9
10 1) $Z \geq 30$ seems to represent a characteristic reference number in understanding nuclear binding
11 of light and heavy atomic nuclides.
12 2) With reference to electromagnetic interaction and based on proton number,
13
14 a) For $Z \geq 30$, maximum strength of nuclear binding energy can be addressed with
15 $\beta \equiv (1/\alpha_s) \approx 8.68$.
16 b) For $Z < 30$, strength of nuclear binding energy can be addressed with,

17

$$\beta \approx \left(\frac{Z}{30} \right)^{\sqrt{k}} \left(\frac{1}{\alpha_s} \right) \approx \left(\frac{Z}{30} \right)^{0.08} \times 8.68. \quad (17)$$

- 18 3) Close to stable mass numbers, mass number helps in increasing binding energy and squared
19 neutron number aids in reducing the binding energy.
20 4) There exists a single and unified binding energy coefficient and it can be chosen to fall in
21 between,

22

$$\left. \begin{aligned} B_{\text{effective}} &\approx \frac{e_s^2}{8\pi\varepsilon_0(G_s m_p/c^2)} - \frac{3}{5} \left(\frac{e^2}{4\pi\varepsilon_0 R_0} \right) \approx 8.928 \text{ MeV} \\ B_{\text{effective}} &\approx \frac{e_s^2}{8\pi\varepsilon_0(G_s m_p/c^2)} - \left(\frac{e^2}{4\pi\varepsilon_0 R_0} \right) \approx 9.395 \text{ MeV} \end{aligned} \right\} \quad (18)$$

23
24 where $\frac{3}{5} \left(\frac{e^2}{4\pi\varepsilon_0 R_0} \right) \approx 0.695 \text{ MeV}$ and $\left(\frac{e^2}{4\pi\varepsilon_0 R_0} \right) \approx 1.162 \text{ MeV}$ can be considered as repulsive nuclear
25 binding energy coefficients. To fit the data we consider,

26

$$B_{\text{effective}} \approx \frac{8.928 + 9.395}{2} \approx 9.16 \quad (19)$$

27
28 Based on the above relations and **close to the stable mass numbers of** ($Z \approx 2$ to 118), with a
29 common energy coefficient of 9.16 MeV, we would like to suggest the following two terms for
30 fitting and understanding nuclear binding energy.

31

1 First term helps in **increasing** the binding energy and can be considered as,
 2

$$T_1 \approx \eta \times A \times 9.16 \text{ MeV}$$

3 where $\begin{cases} \eta \approx \left(\frac{Z}{30}\right)^{0.08} & \text{for } Z < 30 \\ \eta \approx 1 & \text{for } Z \geq 30 \end{cases}$ (20)

4 Second term helps in **decreasing** the binding energy and can be considered as,
 5

$$T_2 \approx \eta \left[\left(\left(\frac{k}{\ln(30)} \right) N^2 \right) + \frac{1}{2} \right] \times 9.16 \text{ MeV}$$

$$\approx \eta \left[(0.00189 N^2) + \frac{1}{2} \right] \times 9.16 \text{ MeV}$$
 (21)

6 Considering light atomic nuclides, we have introduced the numerical factor $\frac{1}{2}$. It needs further
 7 study. Thus, close to stable mass numbers, binding energy can be fitted with,
 8

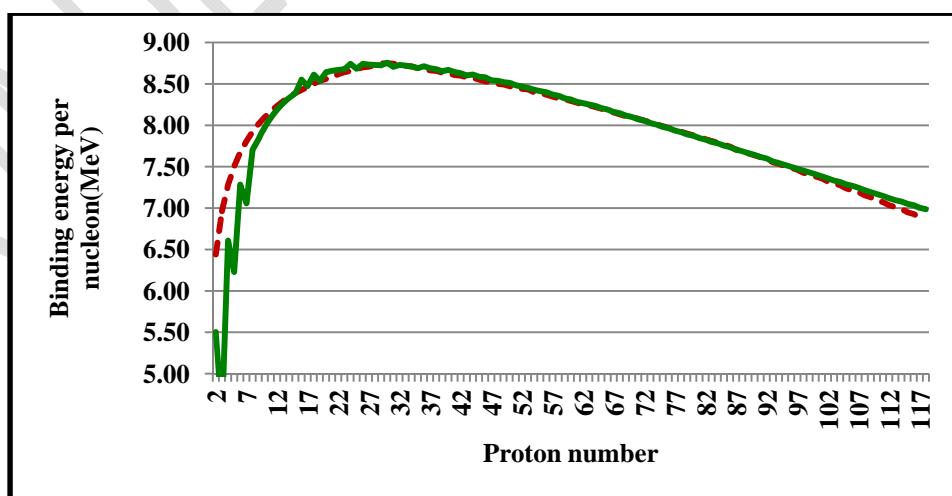
$$(B_A) \approx T_1 - T_2$$

$$\approx \eta \left\{ A - \left[(0.00189 N^2) + \frac{1}{2} \right] \right\} 9.16 \text{ MeV}$$

$$\approx \left(\frac{Z}{30} \right)^{0.08} \left\{ A - \left[(0.00189 N^2) + \frac{1}{2} \right] \right\} 9.16 \text{ MeV} \quad (\text{for } Z < 30)$$

$$\approx \left\{ A - \left[(0.00189 N^2) + \frac{1}{2} \right] \right\} 9.16 \text{ MeV} \quad (\text{for } Z \geq 30)$$
 (22)

9 See Figure 2. Dashed red curve plotted with relations (17) to (22) can be compared with the
 10 green curve plotted with the standard semi empirical mass formula (SEMF). For medium and
 11 heavy atomic nuclides, fit is excellent. It seems that some correction is required for light and
 12 super heavy atoms. See Table 3 for the estimated data close to stable mass numbers.
 13



18 **Figure 2: Binding energy per nucleon close to stable mass numbers of $Z = 2$ to 118**
 19

Table 3: Estimated nuclear binding energy close to stable mass numbers of Z = 2 to 118

| Proton number | Estimated mass number close to stable mass number | Neutron number | Estimated Binding energy (MeV) | SEMF binding energy | Error (MeV) |
|---------------|---|----------------|--------------------------------|---------------------|-------------|
| 2 | 4 | 2 | 25.76 | 22.01 | -3.75 |
| 3 | 6 | 3 | 41.77 | 26.88 | -14.90 |
| 4 | 8 | 4 | 58.24 | 52.86 | -5.37 |
| 5 | 10 | 5 | 75.02 | 62.29 | -12.74 |
| 6 | 12 | 6 | 92.07 | 87.39 | -4.67 |
| 7 | 14 | 7 | 109.32 | 98.81 | -10.51 |
| 8 | 16 | 8 | 126.74 | 123.25 | -3.49 |
| 9 | 19 | 10 | 152.33 | 148.85 | -3.48 |
| 10 | 21 | 11 | 170.06 | 167.52 | -2.55 |
| 11 | 23 | 12 | 187.91 | 186.14 | -1.76 |
| 12 | 25 | 13 | 205.84 | 204.72 | -1.13 |
| 13 | 27 | 14 | 223.86 | 223.22 | -0.64 |
| 14 | 29 | 15 | 241.96 | 241.65 | -0.31 |
| 15 | 31 | 16 | 260.12 | 259.98 | -0.14 |
| 16 | 34 | 18 | 286.48 | 290.77 | 4.29 |
| 17 | 36 | 19 | 304.77 | 305.06 | 0.29 |
| 18 | 38 | 20 | 323.11 | 327.23 | 4.12 |
| 19 | 40 | 21 | 341.49 | 341.47 | -0.01 |
| 20 | 43 | 23 | 368.02 | 371.57 | 3.55 |
| 21 | 45 | 24 | 386.47 | 389.59 | 3.12 |
| 22 | 47 | 25 | 404.96 | 407.47 | 2.51 |
| 23 | 49 | 26 | 423.47 | 425.20 | 1.72 |
| 24 | 52 | 28 | 450.08 | 454.57 | 4.50 |
| 25 | 54 | 29 | 468.63 | 468.89 | 0.25 |
| 26 | 56 | 30 | 487.21 | 489.58 | 2.37 |
| 27 | 59 | 32 | 513.81 | 515.20 | 1.40 |
| 28 | 61 | 33 | 532.40 | 532.52 | 0.11 |
| 29 | 63 | 34 | 551.02 | 549.67 | -1.35 |
| 30 | 66 | 36 | 577.57 | 577.93 | 0.35 |
| 31 | 68 | 37 | 594.63 | 591.98 | -2.65 |
| 32 | 71 | 39 | 619.48 | 619.81 | 0.32 |
| 33 | 73 | 40 | 636.44 | 636.62 | 0.19 |
| 34 | 75 | 41 | 653.36 | 653.27 | -0.09 |
| 35 | 78 | 43 | 677.93 | 677.88 | -0.05 |
| 36 | 80 | 44 | 694.75 | 697.05 | 2.30 |
| 37 | 83 | 46 | 719.11 | 721.32 | 2.20 |
| 38 | 85 | 47 | 735.83 | 737.59 | 1.76 |
| 39 | 88 | 49 | 759.99 | 761.58 | 1.59 |
| 40 | 90 | 50 | 776.59 | 780.20 | 3.60 |
| 41 | 93 | 52 | 800.55 | 803.88 | 3.33 |
| 42 | 95 | 53 | 817.05 | 819.75 | 2.70 |
| 43 | 98 | 55 | 840.80 | 843.16 | 2.37 |
| 44 | 100 | 56 | 857.20 | 861.24 | 4.05 |
| 45 | 103 | 58 | 880.74 | 884.37 | 3.63 |
| 46 | 106 | 60 | 904.14 | 909.61 | 5.47 |
| 47 | 108 | 61 | 920.36 | 922.70 | 2.34 |

| | | | | | |
|-----|-----|-----|---------|---------|-------|
| 48 | 111 | 63 | 943.56 | 947.65 | 4.09 |
| 49 | 113 | 64 | 959.68 | 962.85 | 3.17 |
| 50 | 116 | 66 | 982.66 | 987.48 | 4.81 |
| 51 | 119 | 68 | 1005.51 | 1009.66 | 4.15 |
| 52 | 121 | 69 | 1021.46 | 1024.59 | 3.13 |
| 53 | 123 | 70 | 1037.38 | 1039.35 | 1.97 |
| 54 | 127 | 73 | 1066.60 | 1070.45 | 3.85 |
| 55 | 129 | 74 | 1082.38 | 1085.10 | 2.72 |
| 56 | 132 | 76 | 1104.67 | 1108.72 | 4.05 |
| 57 | 135 | 78 | 1126.83 | 1130.06 | 3.23 |
| 58 | 138 | 80 | 1148.84 | 1153.27 | 4.43 |
| 59 | 140 | 81 | 1164.38 | 1165.55 | 1.17 |
| 60 | 143 | 83 | 1186.19 | 1188.52 | 2.33 |
| 61 | 146 | 85 | 1207.86 | 1209.31 | 1.45 |
| 62 | 149 | 87 | 1229.39 | 1231.90 | 2.51 |
| 63 | 151 | 88 | 1244.69 | 1245.86 | 1.18 |
| 64 | 154 | 90 | 1266.01 | 1268.20 | 2.19 |
| 65 | 157 | 92 | 1287.20 | 1288.44 | 1.25 |
| 66 | 160 | 94 | 1308.24 | 1310.44 | 2.19 |
| 67 | 163 | 96 | 1329.15 | 1330.38 | 1.22 |
| 68 | 166 | 98 | 1349.93 | 1352.04 | 2.12 |
| 69 | 169 | 100 | 1370.56 | 1371.69 | 1.13 |
| 70 | 171 | 101 | 1385.40 | 1385.09 | -0.32 |
| 71 | 174 | 103 | 1405.83 | 1404.54 | -1.29 |
| 72 | 177 | 105 | 1426.12 | 1425.66 | -0.45 |
| 73 | 180 | 107 | 1446.27 | 1444.84 | -1.43 |
| 74 | 183 | 109 | 1466.28 | 1465.66 | -0.61 |
| 75 | 186 | 111 | 1486.15 | 1484.57 | -1.58 |
| 76 | 189 | 113 | 1505.88 | 1505.10 | -0.79 |
| 77 | 192 | 115 | 1525.48 | 1523.74 | -1.74 |
| 78 | 195 | 117 | 1544.94 | 1543.98 | -0.95 |
| 79 | 198 | 119 | 1564.25 | 1562.37 | -1.88 |
| 80 | 201 | 121 | 1583.44 | 1582.34 | -1.10 |
| 81 | 204 | 123 | 1602.48 | 1600.48 | -2.00 |
| 82 | 207 | 125 | 1621.38 | 1620.17 | -1.21 |
| 83 | 210 | 127 | 1640.15 | 1638.07 | -2.08 |
| 84 | 213 | 129 | 1658.77 | 1657.49 | -1.28 |
| 85 | 216 | 131 | 1677.26 | 1675.15 | -2.11 |
| 86 | 219 | 133 | 1695.61 | 1694.32 | -1.30 |
| 87 | 223 | 136 | 1718.30 | 1718.61 | 0.30 |
| 88 | 226 | 138 | 1736.31 | 1737.47 | 1.16 |
| 89 | 229 | 140 | 1754.17 | 1754.62 | 0.45 |
| 90 | 232 | 142 | 1771.90 | 1773.24 | 1.34 |
| 91 | 235 | 144 | 1789.49 | 1790.16 | 0.67 |
| 92 | 238 | 146 | 1806.94 | 1808.53 | 1.59 |
| 93 | 242 | 149 | 1828.28 | 1830.19 | 1.90 |
| 94 | 245 | 151 | 1845.39 | 1848.29 | 2.90 |
| 95 | 248 | 153 | 1862.36 | 1864.75 | 2.40 |
| 96 | 251 | 155 | 1879.18 | 1882.62 | 3.44 |
| 97 | 254 | 157 | 1895.88 | 1898.87 | 2.99 |
| 98 | 258 | 160 | 1916.07 | 1922.72 | 6.65 |
| 99 | 261 | 162 | 1932.42 | 1938.72 | 6.31 |
| 100 | 264 | 164 | 1948.62 | 1956.10 | 7.48 |
| 101 | 267 | 166 | 1964.69 | 1971.90 | 7.20 |

| | | | | | |
|-----|-----|-----|---------|---------|-------|
| 102 | 271 | 169 | 1983.96 | 1993.59 | 9.64 |
| 103 | 274 | 171 | 1999.68 | 2009.17 | 9.49 |
| 104 | 277 | 173 | 2015.26 | 2026.08 | 10.82 |
| 105 | 281 | 176 | 2033.80 | 2047.28 | 13.48 |
| 106 | 284 | 178 | 2049.04 | 2063.96 | 14.92 |
| 107 | 287 | 180 | 2064.14 | 2079.10 | 14.96 |
| 108 | 291 | 183 | 2081.95 | 2099.83 | 17.88 |
| 109 | 294 | 185 | 2096.71 | 2114.76 | 18.06 |
| 110 | 298 | 188 | 2114.00 | 2136.55 | 22.55 |
| 111 | 301 | 190 | 2128.41 | 2151.27 | 22.86 |
| 112 | 305 | 193 | 2145.18 | 2171.33 | 26.15 |
| 113 | 308 | 195 | 2159.24 | 2185.85 | 26.60 |
| 114 | 311 | 197 | 2173.17 | 2201.63 | 28.46 |
| 115 | 315 | 200 | 2189.21 | 2221.27 | 32.05 |
| 116 | 318 | 202 | 2202.79 | 2236.83 | 34.04 |
| 117 | 322 | 205 | 2218.32 | 2254.82 | 36.49 |
| 118 | 325 | 207 | 2231.56 | 2270.17 | 38.61 |

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2 We are working on applying the proposed relations (17) to (22) to ($A \ll A_s$) and ($A \gg A_s$). See
3 Tables 4 to 10 for the estimated isotopic binding energy of $Z = 20, 28, 40, 50, 66, 82$ and 100
4 respectively. It needs further study.
5

| Table 4 : Isotopic binding energy of Z=20 | | | | | |
|---|-------------|----------------|--------------------------------|---------------------|-------------|
| Proton number | Mass number | Neutron number | Estimated Binding energy (MeV) | SEMF binding energy | Error (MeV) |
| 20 | 40 | 20 | 343.58 | 339.70 | -3.88 |
| 20 | 41 | 21 | 351.76 | 350.10 | -1.65 |
| 20 | 42 | 22 | 359.91 | 363.19 | 3.28 |
| 20 | 43 | 23 | 368.02 | 371.57 | 3.55 |
| 20 | 44 | 24 | 376.10 | 382.69 | 6.59 |
| 20 | 45 | 25 | 384.15 | 389.32 | 5.18 |
| 20 | 46 | 26 | 392.16 | 398.73 | 6.57 |
| 20 | 47 | 27 | 400.14 | 403.85 | 3.70 |
| 20 | 48 | 28 | 408.09 | 411.76 | 3.67 |
| 20 | 49 | 29 | 416.00 | 415.54 | -0.46 |
| 20 | 50 | 30 | 423.88 | 422.13 | -1.75 |

| Table 5 : Isotopic binding energy of Z=28 | | | | | |
|---|-------------|----------------|--------------------------------|---------------------|-------------|
| Proton number | Mass number | Neutron number | Estimated Binding energy (MeV) | SEMF binding energy | Error (MeV) |
| 28 | 58 | 30 | 508.33 | 501.75 | -6.58 |
| 28 | 59 | 31 | 516.39 | 511.65 | -4.74 |
| 28 | 60 | 32 | 524.41 | 523.97 | -0.44 |
| 28 | 61 | 33 | 532.40 | 532.52 | 0.11 |
| 28 | 62 | 34 | 540.36 | 543.50 | 3.13 |
| 28 | 63 | 35 | 548.28 | 550.82 | 2.53 |
| 28 | 64 | 36 | 556.17 | 560.58 | 4.41 |

| | | | | | |
|----|----|----|--------|--------|-------|
| 28 | 65 | 37 | 564.03 | 566.79 | 2.76 |
| 28 | 66 | 38 | 571.85 | 575.45 | 3.60 |
| 28 | 67 | 39 | 579.63 | 580.65 | 1.01 |
| 28 | 68 | 40 | 587.38 | 588.30 | 0.91 |
| 28 | 69 | 41 | 595.10 | 592.57 | -2.53 |
| 28 | 70 | 42 | 602.78 | 599.30 | -3.48 |
| 28 | 71 | 43 | 610.43 | 602.74 | -7.70 |
| 28 | 72 | 44 | 618.05 | 608.62 | -9.43 |

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| Table 6 : Isotopic binding energy of Z=40 | | | | | |
|---|-------------|----------------|--------------------------------|---------------------|-------------|
| Proton number | Mass number | Neutron number | Estimated Binding energy (MeV) | SEMF binding energy | Error (MeV) |
| 40 | 86 | 46 | 746.59 | 740.40 | -6.19 |
| 40 | 87 | 47 | 754.15 | 749.73 | -4.41 |
| 40 | 88 | 48 | 761.66 | 761.18 | -0.48 |
| 40 | 89 | 49 | 769.15 | 769.63 | 0.48 |
| 40 | 90 | 50 | 776.59 | 780.20 | 3.60 |
| 40 | 91 | 51 | 784.01 | 787.83 | 3.82 |
| 40 | 92 | 52 | 791.39 | 797.57 | 6.19 |
| 40 | 93 | 53 | 798.73 | 804.43 | 5.70 |
| 40 | 94 | 54 | 806.04 | 813.41 | 7.37 |
| 40 | 95 | 55 | 813.32 | 819.55 | 6.23 |
| 40 | 96 | 56 | 820.56 | 827.80 | 7.24 |
| 40 | 97 | 57 | 827.76 | 833.27 | 5.50 |
| 40 | 98 | 58 | 834.94 | 840.84 | 5.91 |
| 40 | 99 | 59 | 842.07 | 845.67 | 3.60 |
| 40 | 100 | 60 | 849.18 | 852.61 | 3.44 |
| 40 | 101 | 61 | 856.24 | 856.85 | 0.61 |
| 40 | 102 | 62 | 863.28 | 863.18 | -0.09 |
| 40 | 103 | 63 | 870.28 | 866.86 | -3.41 |
| 40 | 104 | 64 | 877.24 | 872.63 | -4.61 |
| 40 | 105 | 65 | 884.17 | 875.78 | -8.39 |
| 40 | 106 | 66 | 891.06 | 881.02 | -10.05 |

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| Table 7 : Isotopic binding energy of Z=50 | | | | | |
|---|-------------|----------------|--------------------------------|---------------------|-------------|
| Proton number | Mass number | Neutron number | Estimated Binding energy (MeV) | SEMF binding energy | Error (MeV) |
| 50 | 110 | 60 | 940.78 | 931.76 | -9.02 |
| 50 | 111 | 61 | 947.84 | 940.74 | -7.11 |
| 50 | 112 | 62 | 954.88 | 951.65 | -3.23 |
| 50 | 113 | 63 | 961.88 | 959.96 | -1.92 |
| 50 | 114 | 64 | 968.84 | 970.20 | 1.36 |
| 50 | 115 | 65 | 975.77 | 977.88 | 2.11 |
| 50 | 116 | 66 | 982.66 | 987.48 | 4.81 |
| 50 | 117 | 67 | 989.52 | 994.55 | 5.03 |
| 50 | 118 | 68 | 996.35 | 1003.55 | 7.20 |
| 50 | 119 | 69 | 1003.14 | 1010.05 | 6.91 |
| 50 | 120 | 70 | 1009.90 | 1018.47 | 8.57 |

| | | | | | |
|----|-----|----|---------|---------|-------|
| 50 | 121 | 71 | 1016.62 | 1024.43 | 7.81 |
| 50 | 122 | 72 | 1023.31 | 1032.30 | 9.00 |
| 50 | 123 | 73 | 1029.96 | 1037.75 | 7.79 |
| 50 | 124 | 74 | 1036.58 | 1045.10 | 8.52 |
| 50 | 125 | 75 | 1043.16 | 1050.05 | 6.89 |
| 50 | 126 | 76 | 1049.71 | 1056.91 | 7.19 |
| 50 | 127 | 77 | 1056.23 | 1061.39 | 5.16 |
| 50 | 128 | 78 | 1062.71 | 1067.77 | 5.07 |
| 50 | 129 | 79 | 1069.15 | 1071.81 | 2.66 |
| 50 | 130 | 80 | 1075.56 | 1077.74 | 2.18 |
| 50 | 131 | 81 | 1081.94 | 1081.35 | -0.59 |
| 50 | 132 | 82 | 1088.28 | 1086.85 | -1.43 |
| 50 | 133 | 83 | 1094.59 | 1090.06 | -4.53 |
| 50 | 134 | 84 | 1100.86 | 1095.15 | -5.72 |
| 50 | 135 | 85 | 1107.10 | 1097.96 | -9.14 |

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| Table 8 : Isotopic binding energy of Z=66 | | | | | |
|---|-------------|----------------|--------------------------------|---------------------|-------------|
| Proton number | Mass number | Neutron number | Estimated Binding energy (MeV) | SEMF binding energy | Error (MeV) |
| 66 | 154 | 88 | 1272.17 | 1262.97 | -9.20 |
| 66 | 155 | 89 | 1278.26 | 1270.46 | -7.81 |
| 66 | 156 | 90 | 1284.33 | 1279.65 | -4.68 |
| 66 | 157 | 91 | 1290.36 | 1286.71 | -3.65 |
| 66 | 158 | 92 | 1296.36 | 1295.47 | -0.89 |
| 66 | 159 | 93 | 1302.32 | 1302.10 | -0.21 |
| 66 | 160 | 94 | 1308.24 | 1310.44 | 2.19 |
| 66 | 161 | 95 | 1314.14 | 1316.67 | 2.54 |
| 66 | 162 | 96 | 1319.99 | 1324.60 | 4.61 |
| 66 | 163 | 97 | 1325.82 | 1330.45 | 4.63 |
| 66 | 164 | 98 | 1331.61 | 1337.98 | 6.38 |
| 66 | 165 | 99 | 1337.36 | 1343.46 | 6.10 |
| 66 | 166 | 100 | 1343.08 | 1350.61 | 7.53 |
| 66 | 167 | 101 | 1348.76 | 1355.73 | 6.96 |
| 66 | 168 | 102 | 1354.41 | 1362.52 | 8.10 |
| 66 | 169 | 103 | 1360.03 | 1367.28 | 7.25 |
| 66 | 170 | 104 | 1365.61 | 1373.72 | 8.11 |
| 66 | 171 | 105 | 1371.16 | 1378.15 | 7.00 |
| 66 | 172 | 106 | 1376.67 | 1384.25 | 7.58 |
| 66 | 173 | 107 | 1382.15 | 1388.36 | 6.21 |
| 66 | 174 | 108 | 1387.59 | 1394.13 | 6.54 |
| 66 | 175 | 109 | 1393.00 | 1397.92 | 4.93 |
| 66 | 176 | 110 | 1398.37 | 1403.38 | 5.01 |
| 66 | 177 | 111 | 1403.71 | 1406.87 | 3.16 |
| 66 | 178 | 112 | 1409.01 | 1412.02 | 3.01 |
| 66 | 179 | 113 | 1414.28 | 1415.22 | 0.94 |
| 66 | 180 | 114 | 1419.52 | 1420.07 | 0.56 |
| 66 | 181 | 115 | 1424.72 | 1422.99 | -1.72 |
| 66 | 182 | 116 | 1429.88 | 1427.56 | -2.33 |
| 66 | 183 | 117 | 1435.02 | 1430.21 | -4.81 |
| 66 | 184 | 118 | 1440.11 | 1434.49 | -5.62 |

| | | | | | |
|----|-----|-----|---------|---------|-------|
| 66 | 185 | 119 | 1445.17 | 1436.88 | -8.30 |
| 66 | 186 | 120 | 1450.20 | 1440.90 | -9.31 |

| Table 9 : Isotopic binding energy of Z=82 | | | | | |
|---|-------------|----------------|--------------------------------|---------------------|-------------|
| Proton number | Mass number | Neutron number | Estimated Binding energy (MeV) | SEMF binding energy | Error (MeV) |
| 82 | 202 | 120 | 1596.76 | 1587.37 | -9.39 |
| 82 | 203 | 121 | 1601.76 | 1593.56 | -8.20 |
| 82 | 204 | 122 | 1606.71 | 1601.28 | -5.44 |
| 82 | 205 | 123 | 1611.64 | 1607.16 | -4.47 |
| 82 | 206 | 124 | 1616.53 | 1614.58 | -1.95 |
| 82 | 207 | 125 | 1621.38 | 1620.17 | -1.21 |
| 82 | 208 | 126 | 1626.20 | 1627.29 | 1.08 |
| 82 | 209 | 127 | 1630.99 | 1632.59 | 1.60 |
| 82 | 210 | 128 | 1635.74 | 1639.42 | 3.68 |
| 82 | 211 | 129 | 1640.45 | 1644.45 | 3.99 |
| 82 | 212 | 130 | 1645.14 | 1650.99 | 5.86 |
| 82 | 213 | 131 | 1649.78 | 1655.75 | 5.97 |
| 82 | 214 | 132 | 1654.40 | 1662.03 | 7.63 |
| 82 | 215 | 133 | 1658.97 | 1666.53 | 7.55 |
| 82 | 216 | 134 | 1663.52 | 1672.53 | 9.01 |
| 82 | 217 | 135 | 1668.03 | 1676.78 | 8.75 |
| 82 | 218 | 136 | 1672.50 | 1682.52 | 10.02 |
| 82 | 219 | 137 | 1676.94 | 1686.52 | 9.58 |
| 82 | 220 | 138 | 1681.35 | 1692.02 | 10.67 |
| 82 | 221 | 139 | 1685.72 | 1695.77 | 10.06 |
| 82 | 222 | 140 | 1690.05 | 1701.03 | 10.97 |
| 82 | 223 | 141 | 1694.35 | 1704.55 | 10.19 |
| 82 | 224 | 142 | 1698.62 | 1709.56 | 10.94 |
| 82 | 225 | 143 | 1702.85 | 1712.86 | 10.00 |
| 82 | 226 | 144 | 1707.05 | 1717.64 | 10.59 |
| 82 | 227 | 145 | 1711.21 | 1720.71 | 9.49 |
| 82 | 228 | 146 | 1715.34 | 1725.26 | 9.92 |
| 82 | 229 | 147 | 1719.44 | 1728.12 | 8.68 |
| 82 | 230 | 148 | 1723.50 | 1732.46 | 8.96 |
| 82 | 231 | 149 | 1727.52 | 1735.10 | 7.58 |
| 82 | 232 | 150 | 1731.51 | 1739.22 | 7.71 |
| 82 | 233 | 151 | 1735.47 | 1741.67 | 6.20 |
| 82 | 234 | 152 | 1739.39 | 1745.58 | 6.19 |
| 82 | 235 | 153 | 1743.28 | 1747.82 | 4.54 |
| 82 | 236 | 154 | 1747.13 | 1751.53 | 4.40 |
| 82 | 237 | 155 | 1750.94 | 1753.58 | 2.63 |
| 82 | 238 | 156 | 1754.73 | 1757.08 | 2.36 |
| 82 | 239 | 157 | 1758.48 | 1758.94 | 0.47 |
| 82 | 240 | 158 | 1762.19 | 1762.26 | 0.07 |
| 82 | 241 | 159 | 1765.87 | 1763.93 | -1.94 |
| 82 | 242 | 160 | 1769.51 | 1767.06 | -2.45 |
| 82 | 243 | 161 | 1773.12 | 1768.55 | -4.57 |
| 82 | 244 | 162 | 1776.70 | 1771.49 | -5.20 |
| 82 | 245 | 163 | 1780.24 | 1772.81 | -7.43 |

| | | | | | |
|----|-----|-----|---------|---------|--------|
| 82 | 246 | 164 | 1783.74 | 1775.57 | -8.17 |
| 82 | 247 | 165 | 1787.22 | 1776.72 | -10.50 |

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| Table 10 : Isotopic binding energy of Z = 100 | | | | | |
|---|-------------|----------------|--------------------------------|---------------------|-------------|
| Proton number | Mass number | Neutron number | Estimated Binding energy (MeV) | SEMF binding energy | Error (MeV) |
| 100 | 256 | 156 | 1919.61 | 1909.69 | -9.92 |
| 100 | 257 | 157 | 1923.36 | 1915.13 | -8.22 |
| 100 | 258 | 158 | 1927.07 | 1921.96 | -5.11 |
| 100 | 259 | 159 | 1930.75 | 1927.18 | -3.57 |
| 100 | 260 | 160 | 1934.39 | 1933.78 | -0.61 |
| 100 | 261 | 161 | 1938.00 | 1938.78 | 0.78 |
| 100 | 262 | 162 | 1941.58 | 1945.16 | 3.58 |
| 100 | 263 | 163 | 1945.12 | 1949.94 | 4.82 |
| 100 | 264 | 164 | 1948.62 | 1956.10 | 7.48 |
| 100 | 265 | 165 | 1952.10 | 1960.68 | 8.58 |

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5 **6 Understanding neutron life time with electromagnetic and weak gravitational constants**

6 One of the key objectives of any unified description is to simplify or eliminate the complicated
7 issues of known physics. In this context, in a quantitative approach, we noticed that,
8 electromagnetic and weak gravitational constants play a crucial role in understanding and
9 estimating neutron life time [28,29]. The following strange relation can be given some
10 consideration.

$$13 \quad t_n \cong \left(\frac{G_e}{G_w} \right) \left(\frac{G_e m_n^2}{(m_n - m_p)c^3} \right) \cong \left(\frac{G_e^2 m_n^2}{G_w (m_n - m_p)c^3} \right) \cong 874.94 \text{ sec} \quad (23)$$

14 Plausible point to be noted is that, relativistic mass of neutron seems to play a crucial role in
15 understanding the increasing neutron life time. It can be understood with,

$$18 \quad t_n \propto \frac{m_n^2}{[1 - (v^2/c^2)]} \quad \text{and} \quad t_n \cong \frac{874.94 \text{ sec}}{[1 - (v^2/c^2)]} \quad (24)$$

19 **7 Nuclear charge radii**

20 As per the current literature [30], nuclear charge radii can be expressed with the following
21 formulae.

$$24 \quad R_c \cong \left\{ 1 + \left[0.015 \left(\frac{N - (N/Z)}{Z} \right) \right] \right\} Z^{1/3} \times 1.245 \text{ fm} \quad (25)$$

$$R_c \approx \left\{ 1 - 0.349 \left(\frac{N-Z}{N} \right) \right\} N^{1/3} \times 1.262 \text{ fm} \quad (26)$$

$$R_c \approx \left\{ 1 - \left[0.182 \left(\frac{N-Z}{A} \right) \right] + \frac{1.652}{A} \right\} A^{1/3} \times 0.966 \text{ fm} \quad (27)$$

Our earlier proposed relation [29] is,

$$R_{(Z,A)} \approx \left\{ Z^{1/3} + \left(\sqrt{Z(A-Z)} \right)^{1/3} \right\} \left(\frac{G_s m_p}{c^2} \right) \quad (28)$$

Based on these relations and by considering the charge radii of stable atomic nuclides, R_0 and G_s can be fitted.

8 Results and Discussion

Based on the data presented in tables 2 to 10, we would like to suggest that,

(1) Semi empirical mass formula is having 5 energy terms and 5 different energy coefficients. Those 5 energy coefficients are no way connected with unification paradigm. This can be considered as a major drawback of traditional nuclear binding energy estimation scheme.

(2) From the proposed relations, our recent and earlier works and from Ghahramany's integrated nuclear model [25, 26], it is very clear to say that, nuclear binding energy can be understood with a single unified energy coefficient.

(3) Close to stable mass numbers, squared neutron number plays a major role in reducing major part of nuclear binding energy. It can be confirmed from Tables 3 to 10.

(4) The ratio, $4s \approx k \approx \frac{4G_s m_p m_e}{\hbar c} \approx \frac{4\hbar c}{G_e m_e^2} \approx \frac{4G_s^2}{G_e G_w} \approx 0.0064185$ seems to play a very interesting role in estimating neutron-proton stability and estimating the major reduction part of nuclear binding energy. Hence it can be validated as a characteristic result oriented number.

(5) We are working on understanding the physics connected with $Z = 30$.

(6) With further study, based on the relations (17) to (22), binding energy for $(A \ll A_s)$ and $(A \gg A_s)$, can be understood and semi empirical mass formula can be modified into a much more simple form.

8. Conclusion

- 1) Understanding nuclear binding energy with a single energy coefficient and two simple terms in terms of fundamental interactions is a very challenging task. In this context, we tried our level best in presenting a very simple and effective two term semi empirical formula with one unique energy coefficient. It needs further study.
- 2) Current unification paradigm is failing in developing a 'practical unification procedure'. Even though our approach is speculative, role played by the four gravitational constants seems to be fairly natural. This kind of approach may help in producing a variety of such

- relations by using which in near future, an absolute set of relations can be developed. Proceeding further, estimated absolute theoretical value of G_n can be considered as a standard reference for future experiments.
- 3) By implementing four such gravitational constants in String theory models, it may be possible to explore the hidden unified physics. With further study, a practical model of materialistic quantum gravity can be developed and magnitude of the Newtonian gravitational constant can be estimated in a theoretical approach bound to Fermi scale.

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