***Method Article***

**A Novel Method for Optimizing Fractional Grey Prediction Model**

**Abstract:** Aiming at the shortcoming that the FGM(1,1) model ignores the changes of the external environment and considers the grey action quantity as a fixed constant, the dynamic changing properties of the grey action quantity are studied, and the grey action quantity is simulated and predicted by using the DGM (1,1) model. On this basis, a new FGM(1,1,b) model with dynamic grey action quantity change with time is proposed, and the total primary energy consumption in the Middle East is taken as a numerical example to model and predict. The results show that this paper proposed FGM(1,1,b) model has higher prediction accuracy than the classical FGM(1,1) model, which verifies the practicability and effectiveness of the FGM(1,1,b) model. At the same time, it also provides relevant theoretical basis for the study of world energy development.

**Key words:** primary energy consumption; FGM(1,1) model; FGM(1,1,b) model; prediction accuracy; particle swarm optimization



## 1 Introduction

Energy is an important resource for human survival and development. The history of human social development is closely related to the history of human understanding and utilization of energy. The Middle East has always played a significant and far-reaching role in world economic politics and international relations with its rich energy resources. In 2014-2050, global energy demand will increase from 20.1 billion tons of standard coal to nearly 30 billion tons of standard coal, an average annual growth of about 1%, according to the Global Energy Review and Outlook issued by State Grid Energy Research Institute. Among them, the primary energy demand in the Middle East has increased by 60%, ranking the forefront in the world and gradually becoming the driving force for global energy demand growth. At the same time, according to the latest energy demand forecast released by Exxon Mobil in 2018, the energy demand in the Middle East will be 40% higher than in 2016 by 2040.With the rapid growth of energy demand in the Middle East, the energy export capacity of the Middle East has begun to weaken. Therefore, scientifically and reasonably predicting the total energy consumption in the Middle East will have important implications for the development of international relations and changes in the world's structure.

Ünler [1] proposed a prediction model based on particle swarm optimization (PSO) technology to predict the energy demand of Turkey, and to further verify the accuracy of the model, it is compared with the energy demand model based on ant colony optimization. Suganth et al. [2] summarized the energy demand forecasting models, including traditional time series, regression, econometrics, ARIMA, fuzzy logic and neural network and other models used to predict energy demand. Kumar et al. [3] respectively established the grey Markov model, the grey model of the rolling mechanism and the singular spectrum analysis, and used these three models to predict the consumption of crude oil, coal and electricity (public utilities) in India. Comparing the results with the predictions of the Indian Planning Commission, the results show that the three time series models have great potential in energy consumption prediction. Akay et al. [4] proposed a grey prediction method (GPRM) based on rolling mechanism to predict the total electricity consumption and industrial electricity consumption in Turkey, and compared it with the prediction results of the energy demand analysis model (MAED) adopted by Turkey's ministry of energy and natural resources (MENR). The results show that GPRM had higher prediction accuracy than MENR. He et al. [5] constructed the ADL-MIDAS model by using the mixed frequency data of quarterly GDP, quarterly value added and annual energy demand of various industries, and then selected the optimal Chinese energy demand forecasting model from different angles. The results show that the energy planning goals under the 13th Five-Year Plan are achievable. Barak et al. [6] used three different ARIMA-ANFIS models to predict Iran's annual energy consumption. In the first model, six different ANFIS are used to predict the nonlinear residuals. In model 2, the output of two ARIMA models and four characteristics are used as input for modeling. In mode 3, the model 2 is combined with the AdaBoost algorithm to carry out a diversified model combination. The prediction results show that the hybrid model is more accurate than the prediction of the single model. Marson et al. [7] used evolutionary algorithm and covariance matrix as a means of training neural network to make short-term predictions on Ireland's power demand, wind power generation and carbon dioxide concentration. The training results show that the neural network trained by the covariance matrix adaptive evolution strategy has the characteristics of fast convergence, high prediction accuracy and good robustness compared with other methods.

According to the above research, in the energy forecasting research, the main methods are traditional econometrics, time series, neural network, support vector machine and grey prediction. Among them, the grey prediction is widely used because of its simple calculation and less sample data. Grey predictions were first proposed by Chinese scholar Deng in the grey system theory [8] in the 1980s. Because of its superior ability to predict the "small sample, poor information" data sequence, it has become rapidly popular in academia and is widely used in various subject areas [9-11].Scholars have never stopped researching and improving the theory of grey systems. In summary, there are mainly improvements in the raw data sequence, improved initial conditions and improved model background values. However, in terms of model parameter optimization, the current research results are not many. Huang [12] studied the development coefficient $a$ through the DGM (1,1) model and proposed a new AGM (1,1) model with dynamic development coefficients. Chen [13] used the improved Euler's formula to obtain a new method for solving the parameters $a$ and $b$ , which improved the prediction accuracy of the model. Since the classical GM (1,1) model treats the grey action quantity $b$ as an invariant constant, the model considers the external disturbance to be stable. This will inevitably affect the prediction accuracy of the GM(1,1) model. In response to this problem, the literature [14] proposed a new grey action quantity optimization method, which uses $bt$ instead of the raw $b$ ; on this basis, the literature [15] uses $b_1+b_2k$ instead of $bt$ to further optimize the grey action quantity $b$ .Both methods optimize the grey action quantity and improve the accuracy of the model, but they all belong to the linear optimization method.

Therefore, based on the above research, this paper extends the method of model parameter improvement to the fractional FGM(1,1) model, and proposes a new nonlinear optimization method for grey action quantity. By dynamizing the grey action quantity $b$ of FGM(1,1), a new FGM(1,1,b) model is obtained. Finally, it is applied to the forecast of primary energy consumption in the Middle East, and compared with the classic FGM (1,1) model.

## 2. Prerequisite knowledge

### 2.1 Fractional order accumulation and inverse operators

**Definition 1.** Let $X^{(0)}=\left(x^{(0)}(1),x^{(0)}(2),...,x^{(0)}(n)\right)$ be a non-negative raw data sequence, and let the sequence $X^{(r)}=\left(x^{(r)}(1),x^{(r)}(2),\cdots,x^{(r)}(n)\right)(r\in R)$ be the *rth-order* accumulation generating operator of $X^{(0)}$ ($r$-$AGO$), where $x^{(r)}(k)=\sum_{i=1}^{k}x^{(r-1)}(i),k=1,2,...,n.$ $X^{(r)}$ is represented by the matrix $X^{(r)}=X^{(0)}A^r$ , where $A^r$ denotes an *rth-order* accumulation generation operator matrix ( $r$-$AGO$ ), and $A^r$ satisfies

$$A^r = \begin{pmatrix} \begin{bmatrix} r \\ 0 \end{bmatrix} & \begin{bmatrix} r \\ 1 \end{bmatrix} & \begin{bmatrix} r \\ 2 \end{bmatrix} & \cdots & \begin{bmatrix} r \\ n-1 \end{bmatrix} \\ 0 & \begin{bmatrix} r \\ 0 \end{bmatrix} & \begin{bmatrix} r \\ 1 \end{bmatrix} & \cdots & \begin{bmatrix} r \\ n-2 \end{bmatrix} \\ 0 & 0 & \begin{bmatrix} r \\ 0 \end{bmatrix} & \cdots & \begin{bmatrix} r \\ n-3 \end{bmatrix} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & \begin{bmatrix} r \\ 0 \end{bmatrix} \end{pmatrix}_{n\times n} . \quad (1)$$

Where, $\begin{bmatrix} r \\ i \end{bmatrix} = \frac{r(r+1)\cdots(r+i-1)}{i!} = \begin{pmatrix} r+i-1 \\ i \end{pmatrix} = \frac{(r+i-1)!}{i!(r-1)!}, \begin{bmatrix} 0 \\ i \end{bmatrix} = 0, \begin{bmatrix} 0 \\ 0 \end{bmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} = 1$.

In particular, when $r = 1$, a *1th-order* accumulation generation sequence $X^{(1)} = \left(x^{(1)}(1), x^{(1)}(2), \cdots, x^{(1)}(n)\right)$ can be obtained. Where $x^{(1)}(k) = \sum_{i=1}^{k} x^{(0)}(i), k = 1, 2, \cdots, n.$

$X^{(1)}$ is represented by the matrix $X^{(1)} = X^{(0)} A^1$, where $A^1$ represents an $1\text{-}AGO$ accumulation matrix, and

$$A^1 = \begin{pmatrix} 1 & 1 & 1 & \cdots & 1 \\ 0 & 1 & 1 & \cdots & 1 \\ 0 & 0 & 1 & \cdots & 1 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & 1 \end{pmatrix}_{n\times n} . \quad (2)$$

**Definition 2.** Let $x^{(r-1)}(k) = \sum_{i=1}^{k} x^{(r-1)}(i) - \sum_{i=1}^{k-1} x^{(r-1)}(i) = x^{(r)}(k) - x^{(r)}(k-1), k = 2, 3, ..., n$ be an *rth-order* inverse generation operator. Similarly, if $A^{-r}$ is used to represent the *rth-order* inverse generation operator ($r\text{-}IAGO$) matrix, then $X^{(0)} = X^{(r)} A^{-r}$ and $A^{-r}$ is

$$A^{-r} = \begin{pmatrix} \begin{bmatrix} -r \\ 0 \end{bmatrix} & \begin{bmatrix} -r \\ 1 \end{bmatrix} & \begin{bmatrix} -r \\ 2 \end{bmatrix} & \cdots & \begin{bmatrix} -r \\ n-1 \end{bmatrix} \\ 0 & \begin{bmatrix} -r \\ 0 \end{bmatrix} & \begin{bmatrix} -r \\ 1 \end{bmatrix} & \cdots & \begin{bmatrix} -r \\ n-2 \end{bmatrix} \\ 0 & 0 & \begin{bmatrix} -r \\ 0 \end{bmatrix} & \cdots & \begin{bmatrix} -r \\ n-3 \end{bmatrix} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & \begin{bmatrix} -r \\ 0 \end{bmatrix} \end{pmatrix}_{n\times n} . \quad (3)$$

Where,

$$\begin{bmatrix} -r \\ i \end{bmatrix} = \frac{-r(-r+1)\cdots(-r+i-1)}{i!} = (-1)\frac{ir(r-1)\cdots(r-i+1)}{i!} = (-1)^i \begin{pmatrix} r \\ i \end{pmatrix}, \begin{bmatrix} -r \\ i \end{bmatrix} = 0, i > r. \quad (4)$$

In particular, when $r = 1$, the $1\text{-}IAGO$ sequence can be represented as $x^{(0)}(k) = x^{(1)}(k) - x^{(1)}(k-1), k = 2, 3, ..., n$, and $X^{(0)}$ satisfies $X^{(0)} = X^{(1)} A^{-1}$, where $A^{-1}$ represents a $1\text{-}IAGO$ matrix, and

$$A^{-1} = \begin{pmatrix} 1 & -1 & 0 & \cdots & 0 \\ 0 & 1 & -1 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & 1 \end{pmatrix}_{n\times n} . \quad (5)$$

### 2.2 Fractional FGM (1,1) model

**Definition 1.** Let $X^{(0)}=\left(x^{(0)}(1),x^{(0)}(2),\cdots,x^{(0)}(n)\right)$ be the raw data sequence, and its *rth -order* accumulation generation sequence ( *r-AGO* ) be $X^{(r)}=\left(x^{(r)}(1),x^{(r)}(2),\cdots,x^{(r)}(n)\right)=X^{(0)}A^{r}$. The mean generated sequence of $X^{(r)}$ is

$$Z^{(r)}=\left(z^{(r)}(2),z^{(r)}(3),\cdots,z^{(r)}(n)\right). \tag{6}$$

In the equation(5), $z^{(r)}(k)=\frac{1}{2}\left(x^{(r)}(k)+x^{(r)}(k-1)\right),k=2,3,\cdots,n$ .Establishing *rth-order* grey differential equation

$$x^{(r-1)}(k)+az^{(r)}(k)=b,k=2,3,\cdots,n. \tag{7}$$

Correspondingly, the *rth-order* whitening differential equation is

$$\frac{dx^{(r)}(t)}{dt}+ax^{(r)}(t)=b. \tag{8}$$

In particular, when $r=1$, $x^{(r-1)}(k)+az^{(r)}(k)=b$ becomes a classic $x^{(0)}(k)+az^{(1)}(k)=b$ model, namely the GM (1, 1) model. Where $a$ is the development coefficient and $b$ is the grey action quantity. Let $\hat{u}=(a,b)^{T}$, according to the principle of least squares method

$$\hat{u}=\left(B_1^{T}B_1\right)^{-1}B_1^{T}Y_1, \tag{9}$$

where,

$$Y_1=\begin{pmatrix} x^{(r-1)}(2) \\ x^{(r-1)}(3) \\ \vdots \\ x^{(r-1)}(n) \end{pmatrix}, B_1=\begin{pmatrix} -z^{(r)}(2) & 1 \\ -z^{(r)}(3) & 1 \\ \vdots & \vdots \\ -z^{(r)}(n) & 1 \end{pmatrix}. \tag{10}$$

Let $\hat{x}^{(0)}(1)=x^{(0)}(1)$, solve the differential equation (7) and get the time response sequence as

$$\hat{x}^{(r)}(t+1)=\left(x^{(0)}(1)-\frac{b}{a}\right)e^{-at}+\frac{b}{a},t=1,2,\cdots,n-1,\cdots. \tag{11}$$

Obtained after discrete

$$\hat{x}^{(r)}(k+1)=\left(x^{(0)}(1)-\frac{b}{a}\right)e^{-ak}+\frac{b}{a},k=1,2,\cdots,n-1,\cdots. \tag{12}$$

The predicted value of $X^{(0)}$ after inverse generation operator( *r-IAGO* ) matrix is

$$\hat{X}^{(0)}=\hat{X}^{(r)}A^{-r}. \tag{13}$$

Where, $\hat{X}^{(0)}=\left(\hat{x}^{(0)}(1),\hat{x}^{(0)}(2),\cdots,\hat{x}^{(0)}(n)\right)$, $\hat{X}^{(r)}=\left(\hat{x}^{(r)}(1),\hat{x}^{(r)}(2),\cdots,\hat{x}^{(r)}(n)\right)$。

## 2.3 DGM (1,1) model

Let the non-negative raw data sequence $X^{(0)}$ be as described above, and the *1th-order* accumulation generation sequence ($1-AGO$) is

$$X^{(1)}=\left(x^{(1)}(1),x^{(1)}(2),\cdots,x^{(1)}(n)\right). \tag{14}$$

Where, $x^{(1)}(k)=\sum_{i=1}^{k}x^{(0)}(i)\ \ (k=1,2,\cdots,n)$。 Let sequence $X^{(0)}$ and $X^{(1)}$ be as described above, then call

$$\hat{x}^{(1)}(k+1)=\beta_1\hat{x}^{(1)}(k)+\beta_2, \tag{15}$$

a *1th-order* univariate discrete DGM (1,1) model, or a discrete form of the GM (1,1) model [17].

If $\hat{\beta}=(\beta_1,\beta_2)^T$ is a parameter column, and

$$Y_2=\begin{pmatrix} x^{(1)}(2) \\ x^{(1)}(3) \\ \vdots \\ x^{(1)}(n) \end{pmatrix},\ B_2=\begin{pmatrix} x^{(1)}(1) & 1 \\ x^{(1)}(2) & 1 \\ \vdots & \vdots \\ x^{(1)}(n-1) & 1 \end{pmatrix}. \tag{16}$$

Then the least squares estimation parameters $\hat{\beta}=(\beta_1,\beta_2)^T$ of the discrete grey prediction model $\hat{x}^{(1)}(k+1)=\beta_1\hat{x}^{(1)}(k)+\beta_2$ satisfies

$$\hat{\beta}=\left(B_2{}^T B_2\right)^{-1} B_2{}^T Y_2. \tag{17}$$

Let $\hat{x}^{(1)}(1)=x^{(0)}(1)$ be the recursive function

$$\hat{x}^{(1)}(k+1)=\beta_1^k\left(x^{(0)}(1)-\frac{\beta_2}{1-\beta_1}\right)+\frac{\beta_2}{1-\beta_1},k=1,2,\cdots n-1,\cdots. \tag{18}$$

Restore value is

$$\begin{aligned}\hat{x}^{(0)}(k+1)&=\hat{x}^{(1)}(k+1)-\hat{x}^{(1)}(k)\\&=(\beta_1-1)\left(x^{(0)}(1)-\frac{\beta_2}{1-\beta_1}\right)\beta_1^k,k=1,2,\cdots n-1,\cdots.\end{aligned} \tag{19}$$

# 3 Dynamic characteristics of grey action quantity and establishment of FGM(1,1,b) model

## 3.1 Dynamics of grey action quantity

From the grey differential equation $x^{(r-1)}(k)+az^{(r)}(k)=b,k=2,3,\cdots,n$ of the classical FGM(1,1) model, it can be seen that the classical FGM(1,1) model takes the grey action quantity $b$ as an invariant constant, ignores the influence of external changes on the system development, and models the external disturbances as invariant, and then realize the prediction. However, in the literature [18], it is proved that the raw sequence multiplied by constant $K$ not equal to zero to obtain a new sequence, the development coefficient of the new sequence is equal to the development coefficient of the raw sequence, and the grey action quantity of the new sequence is equal to $K$ times the grey action quantity of the raw sequence. The theorem shows that the grey action quantity has the property of changing with time. If the grey action quantity is regarded as a fixed constant for modeling and prediction, this will not conform to the law of system development, which will lead to errors in the model and affect the prediction accuracy of the model.

## 3.2 Establishment of FGM(1,1,b) model

Consider the grey differential equation $x^{(r-1)}(k)+az^{(r)}(k)=b$ of the FGM(1,1) model. When $k=2,3,\cdots,n$, the parameters $\hat{u}=(a,b)^T$ of the FGM(1,1) model can be estimated by the least squares method. Bringing the estimated parameter $a$ back to the grey differential equation $x^{(r-1)}(k)+az^{(r)}(k)=b$ of the FGM(1,1) model can be obtained.

$$k=2,b^{(0)}(1)=x^{(r-1)}(2)+az^{(r)}(2), \tag{20}$$

$$k=3,b^{(0)}(2)=x^{(r-1)}(3)+az^{(r)}(3), \tag{21}$$

$$\vdots$$

$$k=n,b^{(0)}(n-1)=x^{(r-1)}(n)+az^{(r)}(n). \tag{22}$$

The grey action quantity sequence $B=\left[b^{(0)}(1),b^{(0)}(2),\cdots,b^{(0)}(n-1)\right]$ is obtained by the above formula. This sequence was simulated and predicted using the DGM(1,1) model, and its recursive expression is

$$\hat{b}^{(1)}(t+1)=\beta_1^t\left(b^{(0)}(1)-\frac{\beta_2}{1-\beta_1}\right)+\frac{\beta_2}{1-\beta_1},t=1,2,\cdots n-1,\cdots. \tag{23}$$

Obtained after discrete

$$\hat{b}^{(1)}(k+1)=\beta_1^k\left(b^{(0)}(1)-\frac{\beta_2}{1-\beta_1}\right)+\frac{\beta_2}{1-\beta_1},k=1,2,\cdots n-1,\cdots. \tag{24}$$

The restored value is obtained from the discrete recursive expression

$$\hat{b}^{(0)}(k+1)=(\beta_1-1)\left(b^{(0)}(1)-\frac{\beta_2}{1-\beta_1}\right)\beta_1^k,k=1,2,\cdots n-1,\cdots. \tag{25}$$

In order to dynamically change the grey action quantity of the FGM(1,1) model, the $\hat{b}^{(0)}(k)$ $(k=1,2,\cdots,n,\cdots)$ series is used to replace the grey action quantity $b$ of the traditional FGM(1,1) model, and the FGM(1,1,b) model with dynamic change of grey action quantity is obtained .The time response sequence of the model is

$$\hat{x}^{(r)}(t+1)=\left(x^{(0)}(1)-\frac{(\beta_1-1)\left(b^{(0)}(1)-\frac{\beta_2}{1-\beta_1}\right)\beta_1^t}{a}\right)e^{-at}+\frac{(\beta_1-1)\left(b^{(0)}(1)-\frac{\beta_2}{1-\beta_1}\right)\beta_1^t}{a} \tag{26}$$

$$t=1,2,\cdots,n-1,\cdots.$$

Obtained after discrete

$$\hat{x}^{(r)}(k+1)=\left(x^{(0)}(1)-\frac{(\beta_1-1)\left(b^{(0)}(1)-\frac{\beta_2}{1-\beta_1}\right)\beta_1^k}{a}\right)e^{-ak}+\frac{(\beta_1-1)\left(b^{(0)}(1)-\frac{\beta_2}{1-\beta_1}\right)\beta_1^k}{a} \tag{27}$$

$$k=1,2,\cdots,n-1,\cdots.$$

The above formula(26) obtains the predicted value $\hat{X}^{(0)}$ of $X^{(0)}$ by inverse generation operator( $r$-$IAGO$ )matrix.

## 4 Determine the optimal order of the model

When using the fractional grey model for modeling prediction, we first need to determine the optimal order $r$ of the model, then perform the $rth$-$order$ accumulation summation on the raw data, and then solve the parameters $\hat{u}=(a,b)^T$ by the least squares method to obtain the time response. Sequence $\hat{x}^{(r)}(t+1), t=0,1,\cdots,n,\cdots$ is used for prediction. In order to solve the optimal order $r$ of the FGM(1,1) model and the FGM(1,1,b) model, the mathematical optimization model is established by using the mean absolute percentage error ( $MAPE_{fit}$ ) of the fitted data as the objective function. $r$ is the optimization parameter. Its form is as follows

$$\min_{r} MAPE_{fit} = \frac{1}{N}\sum_{k=1}^{N}\left|\frac{x^{(0)}(k)-\hat{x}^{(0)}(k)}{x^{(0)}(k)}\right| \times 100\%, \tag{28}$$

$$\begin{cases} r \in R, \\ k = 2,3,\cdots N, \\ \hat{x}^{(1)}(1) = x^{(0)}(1), \\ \hat{x}^{(0)}(k) = \hat{x}^{(1)}(k) - \hat{x}^{(1)}(k-1). \end{cases} \tag{29}$$

Where $N$ represents the number of data used to fit the modeling. Since the above equation (28) and (29) are nonlinear, direct solution is difficult. Therefore, the intelligent optimization algorithm -PSO is used to perform iterative optimization to solve the optimal order $r$ .

The Particle Swarm Optimization (PSO) algorithm was first proposed by Kennedy and Eberhart [19]. The algorithm is based on the simulation of the social activities of the flocks, and proposes a global random search algorithm based on swarm intelligence by simulating the behavior of the flocks interacting with each other. The specific algorithm steps are as follows.

***Step1:*** Initialize the population particle number $M$ , particle dimension $N$ , maximum iteration number $m_{max}$ , learning factor $\delta_1, \delta_2$ , inertia maximum weight $w_{max}$ , minimum weight $w_{min}$ , initial population particle maximum position $\xi_{max} = (\xi_{1,max}, \xi_{2,max}, \cdots, \xi_{N,max})$, minimum position $\xi_{min} = (\xi_{1,min}, \xi_{2,min}, \cdots, \xi_{N,min})$ , maximum speed $\zeta_{max} = (\zeta_{1,max}, \zeta_{2,max}, \cdots, \zeta_{N,max})$ , minimum speed $\zeta_{min} = (\zeta_{1,min}, \zeta_{2,min}, \cdots, \zeta_{N,min})$, Particle individual optimal position $pbest_i^1$ and optimal value $p_i^1$ and particle group global optimal position $gbest^1$ and optimal value $g^1$ ;

***Step2:*** Calculate the fitness value $MAPE_{fit}(r_i^m)$ of each particle in the particle group;

***Step 3:*** Compare each particle fitness value $MAPE_{fit}(r_i^m)$ with the individual extreme value $p_i^m$ and the particle group global optimal value $g^m$ ,respectively. If $MAPE_{fit}(r_i^m) < p_i^m$ ,update $p_i^m$ with $MAPE_{fit}(r_i^m)$ and replace the particle individual optimal position $pbest_i^m$ .If $MAPE_{fit}(r_i^m) < g^m$ , update $g^m$ with $MAPE_{fit}(r_i^m)$ and replace the global optimal position $gbest^m$ of the particle swarm;

***Step 4:*** Calculate the dynamic inertia weight $w$ and the iterative update speed value $\zeta$ and the position $\xi$ according to the following formula and perform boundary condition processing, where $rand(\ )$ is a random number between $[0,1]$;

$$\begin{aligned} & w = w_{max} - m\left(w_{max} - w_{min}\right)/m_{max}, \\ & \zeta_{i,j}^{m+1} = w\zeta_{i,j}^{m} + \delta_1 \times rand(\ )\left(pbest_{i,j}^{m} - \xi_{i,j}^{m}\right) + \\ & \qquad \delta_2 \times rand(\ )\left(gbest_{j}^{m} - \xi_{i,j}^{m}\right), \\ & \xi_{i,j}^{m+1} = \xi_{i,j}^{m} + \zeta_{i,j}^{m+1}, j = 1. \end{aligned} \tag{30}$$

***Step 5:*** Determine whether the termination condition is satisfied: if yes, the algorithm ends and outputs the optimization result; otherwise, it returns to Step 2.

# 5 Example analysis

## 5.1 Test criteria for the model

In order to further test the prediction accuracy of the model, this paper uses $MAPE_{fit}$, $MPAE_{pred}$ and $MAPE_{tol}$ as the evaluation indicators of the model, and compares the FGM (1,1,b) model with the FGM(1,1) model. Where $MAPE_{fit}$ is the mean absolute percent error of the model fit data, $MPAE_{pred}$ is the mean absolute percent error of the extrapolated predicted values, and $MAPE_{tol}$ is the total mean absolute percent error. The specific calculation formula is

$$MAPE_{fit} = \frac{1}{N}\sum_{k=1}^{N}\left|\frac{x^{(0)}(k) - \hat{x}^{(0)}(k)}{x^{(0)}(k)}\right| \times 100\%, \tag{31}$$

$$MAPE_{tol} = \frac{1}{n}\sum_{k=1}^{n}\left|\frac{x^{(0)}(k) - \hat{x}^{(0)}(k)}{x^{(0)}(k)}\right| \times 100\%, \tag{32}$$

$$MPAE_{pred} = \frac{1}{n-N}\sum_{k=N+1}^{n}\left|\frac{x^{(0)}(k) - \hat{x}^{(0)}(k)}{x^{(0)}(k)}\right| \times 100\%. \tag{33}$$

Where $N$ represents the number of modeling data samples and $n$ represents the total number of data samples.

## 5.2 Middle East Primary Energy Consumption Forecast

In order to verify the effectiveness and practicability of the above methods and models, this paper obtained the total primary energy consumption in the Middle East from 1981 to 1992 in the 2018 edition of "Energy Outlook" issued by BP as an example analysis data for fitting and prediction analysis. The specific data is shown in Table 1

Table 1: Total energy consumption in the Middle East, 1981 to 1992

(Million tones oil equivalent)

| Year | 1981 | 1982 | 1983 | 1984 | 1985 | 1986 |
|---|---|---|---|---|---|---|
| Primary Energy Consumption | 137.9 | 152.8 | 167.1 | 188.9 | 200.8 | 209.8 |
| Year | 1987 | 1988 | 1989 | 1990 | 1991 | 1992 |
| Primary Energy Consumption | 224.5 | 238.5 | 251.5 | 260.0 | 271.7 | 296.4 |

According to the data provided in Table 1 above, this paper selects the total energy consumption of the Middle East from 1981 to 1987 as the fitting data of the model, and uses the total energy consumption from 1988 to 1992 as the test data of the model. The FGM(1,1) model and the FGM(1,1,b) model proposed in this paper are established respectively. According to the above mathematical optimization model, the particle swarm optimization algorithm is used to determine the optimal order $r$ of each model. The fitting results and prediction accuracy of the two different models are compared and analyzed. The parameters of the FGM(1,1) model and the FGM(1,1,b) model are calculated and shown in Table 2

Table 2: Parameter calculation results of two models

| Model | Optimal order $r$ | Estimated parameter |
|---|---|---|
| FGM(1,1) | 0.0817 | $\hat{u}_1 = (a_1, b_1)^T = (0.0878, 39.4374)^T$ |
| FGM(1,1,b) | 0.7063 | $\hat{u}_2 = (a_2, b_2)^T = (-0.0073, 109.4364)^T$ |

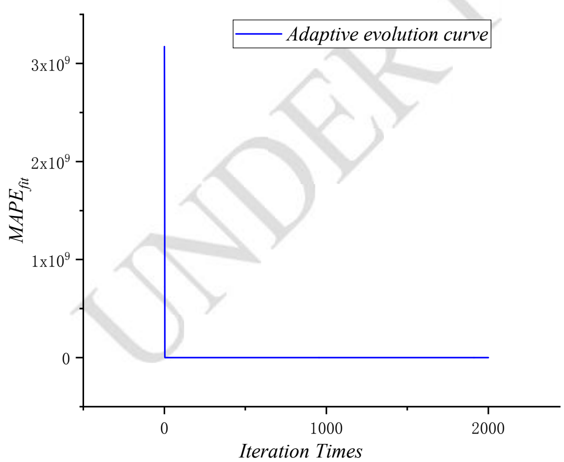


Fig.1 FGM(1,1) model PSO algorithm iterative process

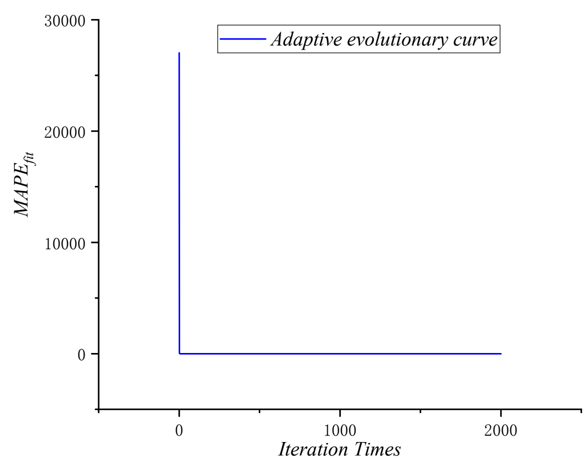


Fig.2 FGM(1,1,b) model PSO algorithm iterative process

Fig.1 and Fig.2 show the results of the iterative calculation of the FGM (1,1) model and the FGM (1,1,b) model using the PSO algorithm. In Fig.1, $MAPE_{fit}$ of the FGM(1,1) model converges to 0.7738, and the obtained optimal order $r = 0.0817$ .In Fig.2, the $MAPE_{fit}$ of the FGM(1,1,b) model converges to 0.6944, and the optimal order obtained is $r = 0.7063$ . The grey action quantity sequence $B = \left[ b^{(0)}(1), b^{(0)}(2), \cdots, b^{(0)}(n-1) \right]$ was calculated from the development coefficient $a$ , and the DGM(1,1) model was used to fit the sequence $B$ to describe the dynamic characteristics of the grey action $b$ with time. The parameters of the DGM(1,1) model are as follows

$$\hat{\beta} = (\beta_1, \beta_2)^T = (1.0038, 107.8878)^T .$$

Bring parameters $\hat{\beta} = (\beta_1, \beta_2)^T$ into equation (24) to get the recursive function of the restored DGM(1,1) model.

$$\hat{b}^{(0)}(k+1) = (1.0038-1)\left( b^{(0)}(1) - \frac{107.8878}{1-1.0038} \right) 1.0038^k, k = 1, 2, \cdots, n-2, \cdots.$$

Replace the grey action quantity $b$ in the FGM(1,1) model with $\hat{b}^{(0)}(k)$ , and obtain the FGM(1,1,b) model with the grey action quantity changing with time. The discrete time response sequence is

$$\hat{x}^{(1)}(k+1) = \left(1 - e^{-0.0073}\right) \left( x^{(0)}(1) + \frac{(1.0038-1)\left( b^{(0)}(2) - \dfrac{107.8878}{1-1.0038} \right) 1.0038^k}{0.0073} \right) e^{0.0073k}, \quad k = 1, 2, ..., n, \cdots.$$

Through the restoration time response sequence of the FGM(1,1) model and the FGM(1,1,b) model, the fitted and raw values of the two models and the relative errors are calculated, as shown in Table 3 below. The extrapolated predicted values and prediction errors of the two models are shown in Table 4 below.

Table 3: FGM (1,1) and FGM (1,1,b) model fitting and relative error comparison (Million tones oil equivalent)

| Year | Raw data | FGM(1,1) | Relative error(%) | FGM(1,1,b) | Relative error(%) |
|---|---|---|---|---|---|
| 1981 | 137.90 | 137.90 | 0.0000 | 137.90 | 0.0000 |
| 1982 | 152.80 | 152.80 | 0.0031 | 152.80 | 0.0003 |
| 1983 | 167.10 | 169.46 | 1.4096 | 167.09 | 0.0047 |
| 1984 | 188.90 | 185.16 | 1.9780 | 185.63 | 1.7306 |
| 1985 | 200.80 | 199.54 | 0.6282 | 200.61 | 0.0943 |
| 1986 | 209.80 | 212.56 | 1.3178 | 214.14 | 2.0681 |
| 1987 | 224.50 | 224.32 | 0.0814 | 226.68 | 0.9703 |
| $MAPE_{fit}$ | | **0.7738** | | **0.6944** | |

Table 4: Comparison of predict values and prediction errors of FGM(1,1) and FGM(1,1,b) models (Million tones oil equivalent)

| Year | Raw data | FGM(1,1) | Relative | FGM(1,1,b) | Relative |
|---|---|---|---|---|---|

| | | | error (%) | | error (%) |
|---|---|---|---|---|---|
| 1988 | 238.50 | 234.90 | 0.0814 | 226.68 | 0.9703 |
| 1989 | 251.50 | 244.40 | 1.5113 | 238.49 | 0.0031 |
| 1990 | 260.00 | 252.93 | 2.8224 | 249.76 | 0.6937 |
| 1991 | 271.70 | 260.58 | 2.7180 | 260.59 | 0.2266 |
| 1992 | 296.40 | 267.43 | 4.0923 | 271.08 | 0.2270 |
| $MPAE_{pred}$ | | **4.1768** | | **1.2484** | |
| $MAPE_{tol}$ | | **2.1917** | | **0.9252** | |

From the data in Table 3 above, the mean absolute percentage error ( $MAPE_{fit}$ ) of the FGM(1,1) model is 0.774%, and the mean absolute percentage error of the FGM(1,1,b) model ( $MAPE_{fit}$ ) is only 0.6955%, which is lower than the classic FGM (1,1) model. As can be known from Table 4 data, FGM (1,1) model extrapolation forecast values of mean absolute percentage errors ( $MPAE_{pred}$ ) and the total mean absolute percentage errors ( $MAPE_{tol}$ ) are 4.1837% and 1.2485% respectively, and FGM (1,1,b) model extrapolation forecast values of mean absolute percentage errors ( $MPAE_{pred}$ ) and the total average absolute percentage errors ( $MAPE_{tol}$ ) are 2.1947% and 0.9259%, respectively. They are significantly lower than the classic FGM (1,1) model.Fig.3 intuitively shows the fitting and prediction results of the two models.

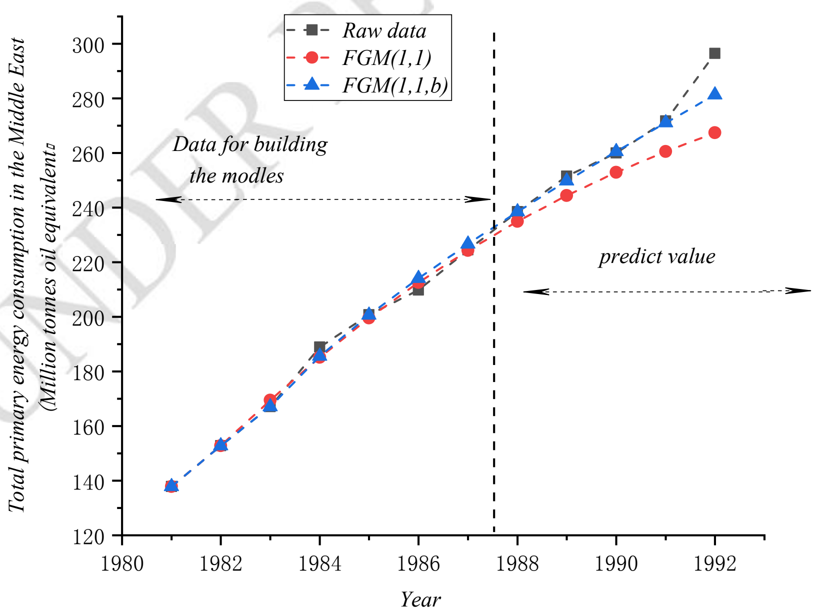


Fig. 3 Comparison of modeling results between FGM(1,1) model and FGM(1,1,b) model

It can be seen from Fig.3 above that the FGM(1,1,b) model proposed in this paper is better than the classical FGM(1,1) model. The validity and practicability of the FGM(1,1,b) model proposed in this paper are verified.

## 6 Conclusion

This paper proposes a FGM(1,1,b) model in which the grey action quantity can change dynamically with time. The grey action quantity sequence of FGM(1,1) model was fitted by DGM(1,1) model to make it dynamically change with time, which made up the defect of traditional FGM(1,1) model regarding grey action quantity as a constant, improved the prediction accuracy of FGM(1,1) model and extended the application range of FGM(1,1) model.

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