Estimation of Stress Strength Reliability in Single component Models for different distributions.

ABSTRACT

This paper aims to estimate the stress-strength reliability parameter $R = P(Y < X)$, considering the two different cases of stress strength parameters, when the strength 'X' follows exponentiated inverse power Lindley distribution ,extended inverse Lindley and Stress 'Y' follows inverse power Lindley distribution and inverse Lindley distribution. The method of maximum likelihood estimation is used to obtain the reliability estimators. Illustrations are provided using R programming.

KEYWORDS: Inverse Lindley Distribution, Inverse Power Lindley Distribution, Extended Inverse Lindley Distribution, Exponentiated Inverse Lindley Distribution, Maximum likelihood estimator.

INTRODUCTION

The Lindley distribution (LD) was first introduced by D.V.Lindley [4]. The distribution is a mixture of the gamma distribution, with shape parameter 2 and scale parameter β and exponential distribution. Its probability distribution function (pdf) is given by

$$f(x;\beta)=\frac{\beta^2}{1+\beta^2}(1+x)e^{-\beta x} \quad ; \; x>0, \beta>0. \tag{1}$$

The corresponding cumulative distribution function (cdf) is given by:

$$F(x;\beta)=1-\left(1+\frac{\beta}{1+\beta}x\right)e^{-\beta x}. \tag{2}$$

Since Lindley distribution is only appropriate for modeling the data with monotonic increasing failure rate, its relevance may be restrained to the data that show non-monotonic shapes (bathtub and upside down bathtub) for their failure rates. Therefore, the LD has been extended to various ageing classes and introduced various generalized class of lifetime distribution based on Lindley distribution. H.Zakerzadeh and A.Dolati [12] introduced three parameters extension of the Lindley distribution. S.Nadarajah et al. [6], M.E.Ghitany et al. [2] proposed two parameter generalizations of the Lindley distribution, called as the generalized Lindley and power Lindley distributions. These distributions are generated using the exponentiation and power transformations to the Lindley distribution. F.Merovci [5] investigated transmuted Lindley and transmuted Lindley-geometric distributions respectively. The exponentiated power Lindley distribution was introduced by S.K.Ashour and M.A.Eltehiwy [1].

In the above cited reference, the authors mainly fixate on the estimation of increasing, decreasing and bathtub shaped failure rate. V.K.Sharma, S.K.Singh and U.Singh [9] proposed a lifetime model with upside-down bathtub shape hazard rate function that is efficient of modeling many real problems, for example failure of washing machines, survival of head and neck cancer patients, and survival of patients with breast cancer. Considering the fact that all inverse distribution acquire the upside-down bathtub shape for their hazard rates, V.K.Sharma, S.K.Singh,U.Singh and V.Agiwal [11], proffered an inverted version of the Lindley distribution that can be used to model the upside-down bathtub shape hazard rate data.

The inverse Lindley distribution take into account the inverse of a random variable with a Lindley distribution. If a random variable Y has a Lindley Distribution, then a random variable Y=1/X follows an inverse Lindley distribution with probability distribution function defined by

$$f(y;\beta)=\frac{\beta^2}{1+\beta^2}\left(\frac{1+x}{x^3}\right)e^{-\frac{\beta}{x}} \quad ; x>0, \beta>0. \tag{3}$$

The corresponding cumulative distribution function (cdf) of (3) is given in equation (4) as:

$$F(y;\beta)=\left(1+\frac{\beta}{1+\beta}\frac{1}{x}\right)e^{-\frac{\beta}{x}}. \tag{4}$$

In order to accomplish more flexible family of distributions, another generalization is the inverse power Lindley distribution suggested by Barco, Mazucheli and Janeiro [3] by considering the power transformation, $X=Y^{\frac{1}{\alpha}}$. Explicitly if a random variable Y follows Inverse Lindley Distribution, then the random variable $Z=Y^{\frac{1}{\alpha}}$ follows inverse power Lindley distribution with density and cumulative distribution functions defined respectively as

$$f(z;\alpha,\beta)=\frac{\alpha\beta^2}{1+\beta}\left(\frac{1+z^{\alpha}}{z^{2\alpha+1}}\right)e^{-\frac{\beta}{z^{\alpha}}} \qquad ;x>0,\alpha>0,\beta>0. \tag{5}$$

$$F(z;\alpha,\beta)=\left(1+\frac{\beta}{1+\beta}\frac{1}{z^{\alpha}}\right)e^{-\frac{\beta}{z^{\alpha}}}. \tag{6}$$

A new extension of inverse Lindley distribution was given by V.K.Sharma and Khandelwal [10], known as extended inverse Lindley distribution(EILD) which deals with more malleability with the effective shape parameter. Its probability density function (pdf) is given by:

$$f(x;\alpha,\beta)=\frac{\beta^{\alpha}}{(1+\beta)\Gamma\alpha}\frac{(1+x(\alpha-1))}{x^{\alpha+1}}e^{-\frac{\beta}{x}} \qquad ;x>0,\beta>0,\alpha>1. \tag{7}$$

The analogous cumulative distribution function (cdf) of (7) is given by:

$$F(x;\alpha,\theta)=pCDFInverseGamma(\alpha-1,\beta)+(1-p)CDFInverseGamma(\alpha,\beta) \qquad \alpha>1 \; ;\beta>0. \tag{8}$$

Where, $CDFInverseGamma(\alpha,\beta)=\frac{\Gamma(\beta/x,\alpha)}{\Gamma\alpha}$, $\Gamma(t,\alpha)=\int_t^{\infty}x^{\alpha-1}e^{-x}dx$ and $\Gamma\alpha=\int_0^{\infty}t^{(s-1)}e^{-t}dt$.

A new three parameter probability distribution introduced by Rameesa jan et.al [13] known as Exponentiated inverse power Lindley distributed (EIPLD). Its pdf and cdf is given by:

$$f(x)=\frac{\alpha\beta^2\gamma}{1+\beta}\left(\frac{1+x^{\alpha}}{x^{2\alpha+1}}\right)e^{-\frac{\beta\gamma}{x^{\alpha}}}\left(1+\frac{\beta}{1+\beta}\frac{1}{x^{\alpha}}\right)^{\gamma-1} \tag{9}$$

$$F(x)=\left[\left(1+\frac{\beta}{1+\beta}\frac{1}{x^{\alpha}}\right)e^{-\frac{\beta}{x^{\alpha}}}\right] \tag{10}$$

The stress strength parameter plays an important role in the reliability analysis. For example if X is the strength of a system which is subjected to stress Y, then the parameter $R=P(Y<X)$ measures the system performance and it is very common in the context of mechanical reliability of a system. Moreover, R provides the probability of a system failure, if the system fails whenever the applied stress is greater than its strength. Many authors developed the estimation procedures for estimating the stress–strength reliability from various lifetime models, see [7,8] and references cited therein.

In this paper, we have addressed the problem of estimating $R=P(Y<X)$ considering the two different cases for stress strength reliability

1) When stress follows inverse power Lindley distribution (IPLD) and strength follows exponentiated inverse power Lindley distribution (EIPLD).

2) When stress follows inverse Lindley distribution (ILD) and strength follows extended inverse Lindey distribution (EILD).

RELIABILITY AND ITS MAXIMUM LIKELIHOOD FUNCTION:

CASE 1: Let Y ~ IPLD(α,β_1) and X ~ EIPLD (α,β_2,γ) be independent random variables, Suppose that X represent the strength of a component exposed to Y stress, then the stress strength reliability(SSR) of this component is obtained as follows,

$$R= P(X>Y) = \int_0^\infty p(X > Y / Y = y) f_y(y) dy$$

$$= \int_0^\infty S_x(y) f_y(y) dy$$

$$= \int_0^\infty \left(1 + \frac{\beta_1}{1+\beta_1}\frac{1}{x^\alpha} e^{\frac{-\beta_1}{x^\alpha}}\right)^\gamma \frac{\alpha\beta_2^2}{1+\beta_2}\left(\frac{1+x^\alpha}{x^{2\alpha+1}}\right) e^{-\frac{\beta_2}{x^\alpha}} dx$$

$$= \frac{\alpha\beta_2^2}{1+\beta_2}\int_0^\infty \left(1+\frac{\beta_1}{1+\beta_1}\frac{1}{x^\alpha}\right)^\gamma \left(\frac{1+x^\alpha}{x^{2\alpha+1}}\right) e^{-\frac{\beta_1\gamma+\beta_2}{x^\alpha}} dx$$

$$= \frac{\alpha\beta_2^2}{1+\beta_2}\int_0^\infty \sum_{i=1}^{\infty} {}^\gamma C_i \left(\frac{\beta_1}{1+\beta_1}\right)^i \frac{1}{x^{i\alpha}} \left(\frac{1+x^\alpha}{x^{2\alpha+1}}\right) e^{-\frac{\beta_1\gamma+\beta_2}{x^\alpha}} dx$$

$$= \frac{\beta_2^2}{1+\beta_2}\sum_{i=1}^{\infty} {}^\gamma C_i \left(\frac{\beta_1}{1+\beta_1}\right)^i \left[\int_0^\infty t^{i+1} e^{-(\beta\gamma_1+\beta_2)} dt + \int_0^\infty t^i e^{-t(\beta_1\gamma+\beta_2)} dt\right]$$

$$R = \frac{\beta_2^2}{1+\beta_2}\sum_{i=1}^{\infty} {}^\gamma C_i \left(\frac{\beta_1}{1+\beta_1}\right)^i \frac{\Gamma(i+1)}{(\beta_1\gamma+\beta_2)^{i+1}}\left[\frac{i+1}{\beta_1\gamma+\beta_2}+1\right] \quad (1.1)$$

where R is independent of α

Suppose $x_1, x_2, ..., x_n$ is a random sample of size n_1 from EIPLD (α,β_1,γ) and $y_1, y_2, ... y_n$ is an independent random sample of size n_2 from IPL (α,β_2). The likelihood function l=l(θ) where $\theta = (\alpha,\beta_1,\beta_2,\gamma)$ based on the two independent random sample is given by:

$$l = \sum_{i=1}^{n_1} \ln[f_x(x_i)] + \sum_{j=1}^{n_2} \ln[f_y(y_j)]$$

$$l = n_1[\ln\alpha + 2\ln\beta_1 + \ln\gamma - \ln(1+\beta_1)] + \sum_{i=1}^{n_1}\ln(1+x_i^{\alpha}) - (2\alpha+1)\sum_{i=1}^{n_1}\ln x_i - \beta_1\gamma\sum_{i=1}^{n_1}\left(\frac{1}{x_i^{\alpha}}\right) + (\gamma-1)\sum_{i+1}^{n_1}\ln\left(1+\frac{\beta_1}{1+\beta_1}\frac{1}{x^\alpha}\right)$$

$$+ n_2[2\ln\beta_2 - \ln(1+\beta_2)] + \sum_{j=1}^{n_2}\ln(1+y_j) - 3\sum_{j=1}^{n_2}\ln(y_j) - \beta_2\sum_{j=1}^{n_2}\left(\frac{1}{y_j}\right)$$

The maximum likelihood estimator (MLE) $\hat{\theta}$ of θ is the solution of non-linear equations (1.2), (1.3), (1.4) (1.5)

$$\frac{\partial}{\partial\alpha} = \frac{n_1}{\alpha} + \sum_{i=1}^{n_1}\frac{x_i^\alpha \ln x_i}{1+x_i^\alpha} - 2\sum_{i=1}^{n_1}\ln x_i + \beta_1\gamma\sum_{i=1}^{n_1} x_i^{\alpha}\ln x_i + (\gamma-1)(1+\beta_1)\sum_{i=1}^{n_1}\frac{\beta_1}{1+2\beta_1}\ln\left(\frac{1}{x_i^\alpha}\right) \quad (1.2)$$

$$\frac{\partial l}{\partial \beta_1} = \frac{2n_1}{\beta_1} - \frac{n_1}{1+\beta_1} - \gamma\sum_{i=1}^{n_1}\left(\frac{1}{x_i{}^{\alpha}}\right) + (\gamma-1)\sum_{i=1}^{n_1}\frac{1}{1+\dfrac{\beta_1}{1+\beta_1}\dfrac{1}{x_i^{\alpha}}}\frac{\partial}{\partial x}\left(1+\frac{\beta_1}{1+\beta_1}\frac{1}{x_i^{\alpha}}\right) \qquad (1.3)$$

$$\frac{\partial l}{\partial \beta_1} = \frac{2n_1 + n_1\beta_1}{\beta_1(1+\beta_1)} - \gamma\sum_{i=1}^{n_1}\frac{1}{x^{\alpha}} + (\gamma-1)\sum_{i=1}^{n_1}\frac{x_i^{\alpha}}{(1+\beta_1)x_i^{\alpha}+\beta_1}\left(\frac{1}{x_i^{\alpha}}\frac{1}{(1+\beta_1)}\right) = 0$$

$$\frac{\partial l}{\partial \beta_2} = \frac{2n_2}{\beta_2} - \frac{n_2}{1+\beta_2} - \sum_{j=1}^{n_2}\left(\frac{1}{y_j}\right)$$

(1.4)

$$= \frac{2n_2 + 2n_2\beta_2 - n_2\beta_2}{\beta_2(1+\beta_2)} - \sum_{j=1}^{n_2}\frac{1}{y_j} = 0$$

$$= -\beta_2{}^2\sum_{i=1}^{n_2}\frac{1}{y_j} - \beta_2\left[\sum_{j=1}^{n_2}\frac{1}{y_j} - n_2\right] + 2n_2 = 0$$

$$\hat{\beta}_2 = \beta_2(\hat{\alpha}) = -\frac{\left(\sum_{j=1}^{n_2}\frac{1}{y_j} - n_2\right) + \sqrt{\left(\sum_{j=1}^{n_2}\frac{1}{y_j} - n_2\right)^2 + 8n_2\sum_{j=1}^{n_2}\frac{1}{y_j}}}{2\sum_{j=1}^{n_2}\left(\frac{1}{y_j}\right)}$$

$$\frac{\partial l}{\partial \gamma} = \frac{n_1}{\gamma} - \beta_1\sum_{i=1}^{n_1}\frac{1}{x_i^{\alpha}} + \sum_{i=1}^{n_1}\ln\left(1+\frac{\beta_1}{1+\beta_1}\frac{1}{x_i^{\alpha}}\right) \qquad (1.5)$$

$$\hat{\gamma} = \gamma(\hat{\alpha}) = \frac{n_1}{\beta_1\sum_{i=1}^{n_1}\frac{1}{x_i^{\alpha}} + \sum_{i=1}^{n_1}\ln\left(1+\frac{\beta_1}{1+\beta_1}\frac{1}{x_i^{\alpha}}\right)}$$

Where $\hat{\alpha}$ is the solution of non linear equation :

$$G(\alpha) = \frac{n_1}{\alpha} + \sum_{i=1}^{n_1}\frac{x_i^{\alpha}\ln x_i}{1+x_i^{\alpha}} - 2\sum_{i=1}^{n_1}\ln x_i + \beta_1(\hat{\alpha})\sum_{i=1}^{n_1}x_i{}^{\alpha}\ln x_i + (\gamma(\hat{\alpha})-1)(1+\beta_1(\hat{\alpha}))\sum_{i=1}^{n_1}\frac{\beta_1(\hat{\alpha})}{1+2\beta_1(\hat{\alpha})}\ln\left(\frac{1}{x_i^{\alpha}}\right)$$

Case 2: Let X ~ EILD(α, β_1) and Y ~ ILD(β_2) be independent random variables, Suppose that X represent the strength of a component exposed to Y stress, then the Stress Strength Reliability (SSR) of this component is obtained as follows,

R= P(X>Y) = $\int_0^{\infty} p(X > Y / Y = y) f_y(y)dy$

R = $\int_0^{\infty}\left(\int_0^{x} g(y)dy\right)f(x)dx$

$$= \int_0^{\infty}\left(\int_0^{x}\frac{\beta_2{}^2}{1+\beta_2}\left(\frac{1+y}{y^3}\right)e^{-\frac{\beta_2}{y}}dy\right)\frac{\beta_1^{\alpha}e^{-\frac{\beta_1}{x}}\left[x(\alpha-1)+1\right]}{1+\beta_1 x^{\alpha+1}\Gamma\alpha}dx$$

$$= \frac{\beta_1^{\alpha}}{1+\beta_1}\frac{1}{\Gamma\alpha}\int_0^{\infty}\left(1+\frac{\beta_2}{1+\beta_2}\frac{1}{x}\right)\frac{e^{-\frac{1}{x}(\beta_1+\beta_2)}}{x^{\alpha+1}}\left[x(\alpha-1)+1\right]dx$$

$$= \frac{\beta_1^{\alpha}}{1+\beta_1}\frac{1}{\Gamma\alpha}\left[(\alpha-1)\int_0^{\infty}\frac{e^{-(\beta_1+\beta_2)\frac{1}{x}}}{x^{\alpha}}dx+\int_0^{\infty}\frac{e^{-(\beta_1+\beta_2)\frac{1}{x}}}{x^{\alpha+1}}dx+(\alpha-1)\frac{\beta_2}{1+\beta_2}\int_0^{\infty}\frac{e^{-(\beta_1+\beta_2)\frac{1}{x}}}{x^{\alpha+1}}dx+\frac{\beta_2}{1+\beta_2}\int_0^{\infty}\frac{e^{-(\beta_1+\beta_2)\frac{1}{x}}}{x^{\alpha+2}}dx\right]$$

$$= \frac{\beta_1^{\alpha}}{1+\beta_1}\frac{1}{\Gamma\alpha}\left[\frac{(\alpha-1)\Gamma(\alpha-1)}{(\beta_1+\beta_2)^{\alpha-1}}+\frac{\Gamma\alpha}{(\beta_1+\beta_2)^{\alpha}}+\frac{(\alpha-1)\beta_2}{1+\beta_2}\left(\frac{\Gamma\alpha}{(\beta_1+\beta_2)^{\alpha}}\right)+\frac{\beta_2}{(1+\beta_2)}\frac{\Gamma(\alpha+1)}{(\beta_1+\beta_2)^{\alpha+1}}\right]$$

$$R= \frac{\beta_1^{\alpha}}{(1+\beta)_1(\beta_1+\beta_2)^{\alpha+1}}\left[\frac{(1+\beta_2)(\beta_1+\beta_2)^2+(1+\beta_2)(\beta_1+\beta_2)+(\alpha\beta_2-\beta_2)(\beta_1+\beta_2)+\alpha\beta_2}{(1+\beta_2)}\right] \qquad (2.1)$$

Suppose $x_1, x_2, ..., x_n$ is a random sample of size n_1 from EILD (α, β_1) and $y_1, y_2, ...y_n$ is an independent random sample of size n_2 from ILD (β_2). The likelihood function $l = l\ (\theta)$ where $\theta = (\alpha, \beta_1, \beta_2)$ based on the two independent random sample is given by:

$$l = \sum_{i=1}^{n_1}\ln\left[f_x(x_i)\right]+\sum_{j=1}^{n_2}\ln\left[f_y(y_j)\right]$$

The maximum likelihood estimator (MLE) $\hat{\alpha}, \hat{\beta}_1, \hat{\beta}_2$ of α, β_1, β_2 is the solution of non-linear equations (2.2), (2.3) , (2.4)

$$\frac{\partial}{\partial\alpha} = n_1\log\beta_1+\sum_{i=1}^{n_1}\frac{x_i}{(x_i(\alpha-1)+1)}-\sum_{i=1}^{n_1}\log x_i-\frac{n}{\Gamma\alpha}\frac{\partial}{\partial x}\Gamma\alpha \qquad (2.2)$$

$$\frac{\partial}{\partial\beta_1}l = n_1\left(\frac{\alpha}{\beta_1}-\frac{1}{1+\beta_1}\right)-\sum_{i=1}^{n_1}\frac{1}{x_i} \qquad (2.3)$$

$$\frac{\partial l}{\partial\beta_1} = \frac{n_1\alpha}{\beta_1}-\frac{n_1}{1+\beta_1}-\sum_{x=1}^{n_1}\left(\frac{1}{x_j}\right)=0$$

$$= \frac{n_1\alpha+n_1\alpha\beta_1-n_1\beta_1}{\beta_1(1+\beta_1)}-\sum_{i=1}^{n_1}\frac{1}{x_i}=0$$

$$= -\beta_1^2\sum_{i=1}^{n_1}\frac{1}{x_i}-\beta_1\left[\sum_{i=1}^{n_1}\frac{1}{x_i}-n_1\alpha+n_1\right]+n_1\alpha = 0$$

$$\hat{\beta}_1 = \beta_1(\hat{\alpha}) = -\frac{\left(\sum_{i=1}^{n_1}\frac{1}{x_i}-n_1\alpha+n_1\right)+\sqrt{\left(\sum_{i=1}^{n_1}\frac{1}{x_i}-n_1\alpha\right)^2+4n_1\alpha\sum_{i=1}^{n_1}\frac{1}{x_i}}}{2\sum_{i=1}^{n_1}\left(\frac{1}{x_i}\right)}$$

Now, $$\frac{\partial l}{\partial\beta_2} = \frac{2n_2}{\beta_2}-\frac{n_2}{1+\beta_2}-\sum_{j=1}^{n_2}\left(\frac{1}{y_j}\right) \qquad (2.4)$$

$$= -\beta_2{}^2 \sum_{i=1}^{n_2} \frac{1}{y_j} - \beta_2\left[\sum_{j=1}^{n_2}\frac{1}{y_j} - n_2\right] + 2n_2 = 0$$

$$\hat{\beta}_2 = \beta_2(\hat{\alpha}) = -\frac{\left(\sum_{j=1}^{n_2}\frac{1}{y_j}-n_2\right)+\sqrt{\left(\sum_{j=1}^{n_2}\frac{1}{y_j}-n_2\right)^2+8n_2\sum_{j=1}^{n_2}\frac{1}{y_j}}}{2\sum_{j=1}^{n_2}\left(\frac{1}{y_j}\right)}$$

Where $\hat{\alpha}$ is the solution of non linear equation :

$$G(\alpha) = n_1 \log \beta_1(\hat{\alpha}) + \sum_{i=1}^{n_1}\frac{x_i}{\left(x_i(\alpha-1)+1\right)} - \sum_{i=1}^{n_1}\log x_i - \frac{n}{\Gamma\alpha}\frac{\partial}{\partial x}\Gamma\alpha$$

Table 1: Stress Strength reliability when stress follows IPLD and strength follows EIPLD

$\gamma = 1$								
		β_1						
		0.1	1.5	1.9	2.6	2.8	3.5	4.2
β_2	0.2	0.2581	0.0151	0.0097	0.0053	0.0045	0.0029	0.0020
	1.3	0.0828	0.0964	0.0764	0.0528	0.0479	0.0351	0.0268
	2.5	0.0425	0.1004	0.0879	0.0690	0.0645	0.0514	0.0417
	4.6	0.0222	0.0809	0.0766	0.0673	0.0646	0.0559	0.0484
$\gamma = 2$								
		β_1						
		0.1	1.5	1.9	2.6	2.8	3.5	4.2
β_2	0.2	0.3004	0.0078	0.0049	0.0026	0.0022	0.0014	0.00099
	1.3	0.1493	0.0812	0.0591	0.0369	0.0328	0.0227	0.0166
	2.5	0.0807	0.1085	0.0867	0.0605	0.0550	0.0406	0.0310
	4.6	0.0432	0.1078	0.0943	0.0739	0.0690	0.0549	0.0444
$\gamma = 3$								
		β_1						
		0.1	1.5	1.9	2.6	2.8	3.5	4.2
β_2	0.2	0.2960	0.0052	0.0032	0.0017	0.0015	0.0009	0.0006
	1.3	0.2035	0.0666	0.0466	0.0279	0.0245	0.0166	0.0119
	2.5	0.1152	0.1019	0.0773	0.0507	0.0455	0.0323	0.0241
	4.6	0.0631	0.1156	0.0959	0.0700	0.0643	0.0488	0.0381

Table 2: Stress Strength reliability when stress follows ILD and strength follws EILD

β_2 =0.1											
		α									
		1.01	1.1	1.5	2.5	2.8	3.2	3.8	4.5	5.3	6.2
β_1	0.2	0.8891	0.8762	0.8165	0.6635	0.6191	0.5623	0.4831	0.4008	0.3204	0.2463
	0.3	0.9379	0.9300	0.8925	0.7886	0.7562	0.7130	0.6492	0.5777	0.5016	0.4242
	0.5	0.9726	0.9687	0.9498	0.8939	0.8754	0.8499	0.8104	0.7631	0.7087	0.6484

	0.8	0.9879	0.9859	0.9763	0.9473	0.9374	0.9235	0.9014	0.8740	0.8411	0.8027
	1.2	0.9943	0.9931	0.9878	0.9717	0.9661	0.9583	0.9457	0.9299	0.9105	0.8873
	2.5	0.9986	0.9982	0.9962	0.9905	0.9886	0.9859	0.9815	0.9760	0.9693	0.9610
β_2 =0.4											
α											
β_1		1.01	1.1	1.5	2.5	2.8	3.2	3.8	4.5	5.3	6.2
	0.2	0.5725	0.5357	0.3941	0.1721	0.1326	0.0930	0.0539	0.0282	0.0132	0.0055
	0.3	0.6914	0.6596	0.5299	0.2913	0.2408	0.1858	0.1245	0.0771	0.0439	0.0230
	0.5	0.8187	0.7955	0.6954	0.4788	0.4247	0.3602	0.2790	0.2049	0.1423	0.0933
	0.8	0.9013	0.8857	0.8165	0.6499	0.6035	0.5450	0.4647	0.3827	0.3039	0.2322
	1.2	0.9461	0.9358	0.8889	0.7695	0.7341	0.6880	0.6213	0.5485	0.4725	0.3966
	2.5	0.9843	0.9797	0.9584	0.9019	0.8844	0.8607	0.8248	0.7829	0.7352	0.6826
β_2 =1.5											
α											
β_1		1.01	1.1	1.5	2.5	2.8	3.2	3.8	4.5	5.3	6.2
	0.2	0.2939	0.2492	0.1187	0.0177	0.0099	0.0046	0.0014	0.0003	0.0001	0.0000
	0.3	0.3958	0.3458	0.1884	0.0395	0.0245	0.0129	0.0049	0.0016	0.0004	0.0001
	0.5	0.5439	0.4919	0.3125	0.0969	0.0676	0.0417	0.0200	0.0084	0.0031	0.0010
	0.8	0.6825	0.6341	0.4549	0.1923	0.1476	0.1032	0.0600	0.0315	0.0150	0.0064
	1.2	0.7876	0.7460	0.5836	0.3086	0.2535	0.1945	0.1298	0.0804	0.0461	0.0244
	2.5	0.9167	0.8902	0.7800	0.5537	0.4981	0.4318	0.3472	0.2680	0.1982	0.1403

Fig. 1.1: Graphical Overview of Stress-Strength Reliability Expression (1.1) for gamma=1 and different values of beta1 and beta2.

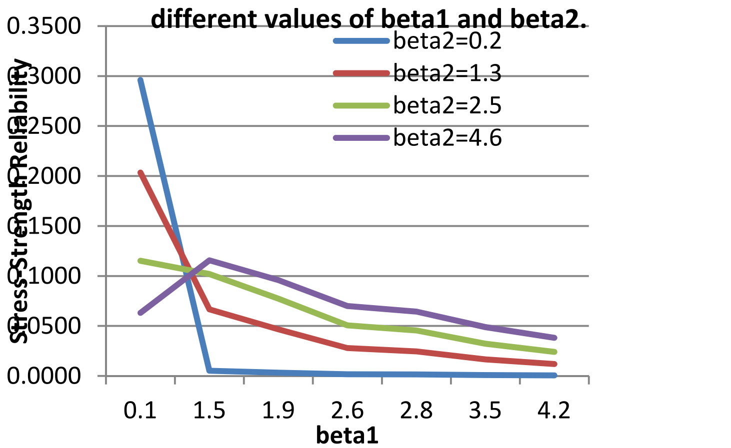


Fig. 1.2: Graphical Overview of Stress-Strength Reliability Expression (1.1) for gamma=2 and different values of beta1 and beta2.

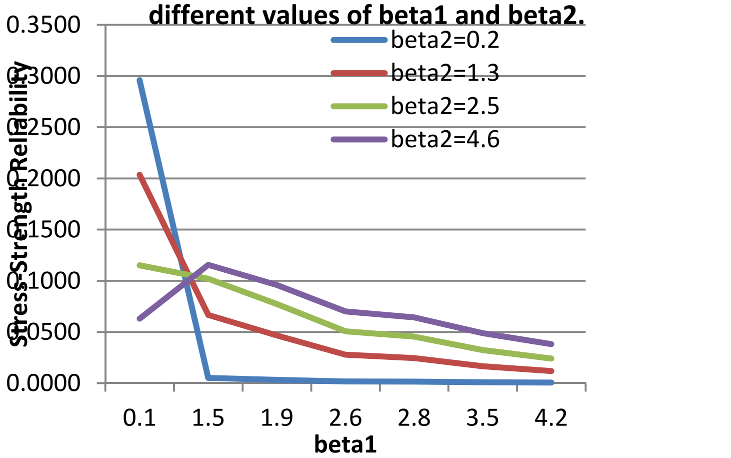


Fig. 1.3: Graphical Overview of Stress-Strength Reliability Expression (1.1) for gamma=3 and different values of beta1 and beta2.

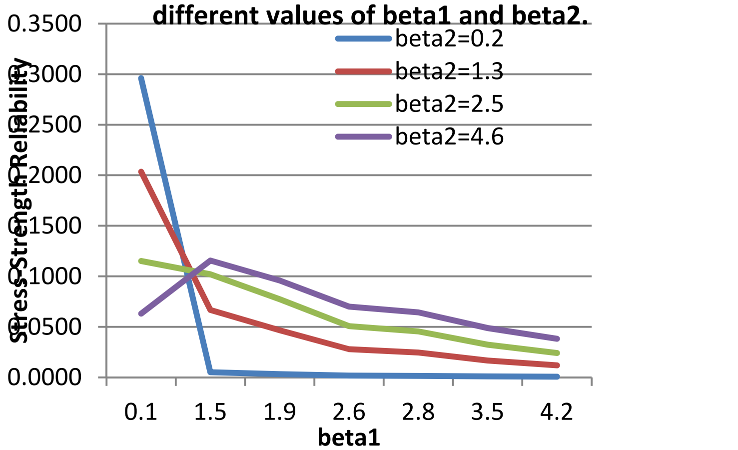
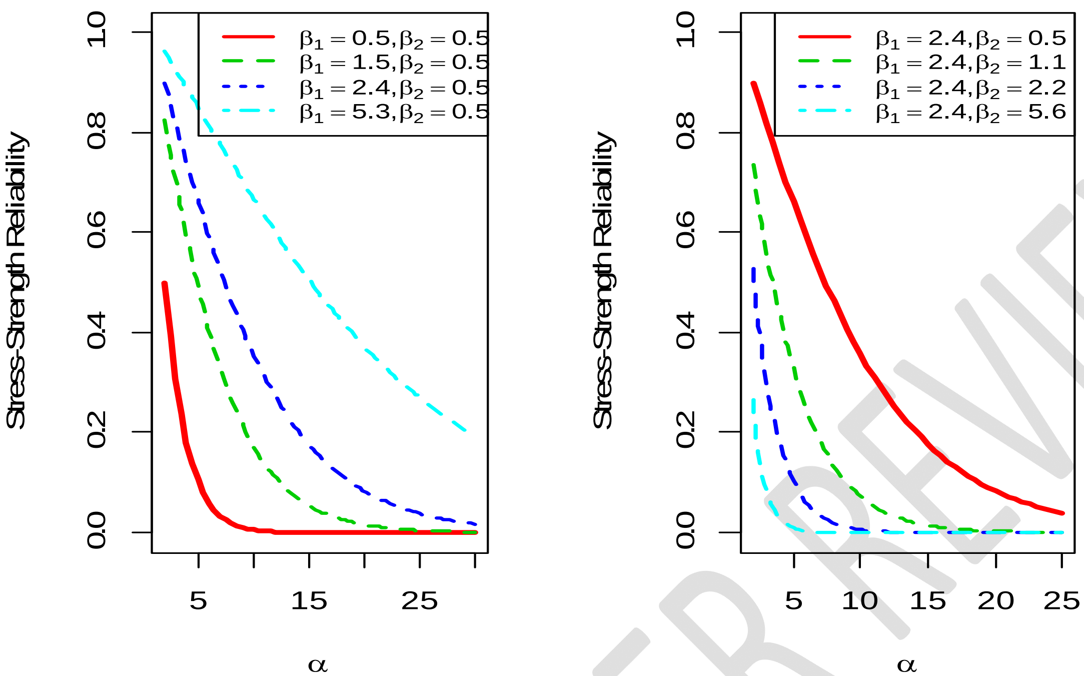


Fig.2: Graphical Overview of Stress Strength Reliability for different Values of Parameters



CONCLUSION:

In this paper, we have studied the stress strength reliability considering the two different cases of stress strength parameters. When strength X ~ EIPLD (α, β_2, γ) and stress Y ~ IPL (α, β_1), it was observed that with increase in the value of strength parameter γ with fixed parameters (β_1, β_2), the stress strength reliability increases. However it is seen that, with increase in the value of stress parameter β_2, the stress strength reliability decreases keeping (γ, β_1) fixed (table 1). Further the graphical overview of Stress strength reliability for $\gamma_1, \gamma_2, \gamma_3$ for different values of (β_1, β_2) are shown in fig. (1.1, 1.2, 1.3) respectively. Also, when the strength X ~ EILD(α, β_1) and Stress Y ~ ILD(β_2), it was found that as the value of stress parameter β_2 increases, keeping the strength parameters fixed (α, β_1), the stress strength parameter decreases. While as, with increase in the value of strength parameter β_1, the stress strength reliability increases, keeping α, β_2 fixed (table 2). Hence we conclude that with decrease in the value of stress parameter and increase in value of strength parameter, reliability of single component system increases resulting in efficiency of system model.

REFERENCES

[1]S.K.Ashour and M.A.Eltehiwy, "Exponentiated power Lindley distribution", Journal of Advanced Research 6, 895–905, 2015

[2] M.E.Ghitany, D. K. Al-Mutairi and S. M. Aboukhamseen, "Estimation of the reliability of a stress–strength system from power Lindley distributions," Communications in Statistics – Simulation and Computation (2013).

[3] Kelly Vanessa Parede Barco, Josmar Mazucheli & Vanderly Janeiro, The inverse power Lindley distribution, Communications in Statistics - Simulation and Computation (2016)
[4] D. V. Lindley, Fiducial distributions and Bayes theorem,, Journal of the Royal Statistical Society 20,102-107, 1958
[5] F.Merovci, "Transmuted Lindley distribution", Int. J. Open Problems Compt. Math. 6, 63–7, 2013.
[6] S.Nadarajah, H.Bakouch and R.Tahmasbi," A generalized Lindley distribution", Sankhya B - Applied and Interdisciplinary Statistics, 73, 331–359, 2011.
[7] V.K.Sharma, S.K.Singh, U.Singh and V.Agiwal, "The inverse Lindley distribution: A stress reliability model with application to head and neck cancer data", Journal of Industrial and Production Engineering 32, 162–173,2015
[8] G.R.Srinivasa ,"Estimation of reliability in multicomponent stress–strength model based on generalized exponential distribution," Colombian Journal of Statistics, 35, 67–76 ,2012.
[9] V.K.Sharma, S.K. Singh, and U.Singh, "A new upside-down bathtub shaped hazard rate model for survival data analysis", Applied Mathematics and Computation 239 , 242-253.,2014.
[10] V.K. Sharma, Pragya Khandelwal, "On the extension of inverse Lindley distribution", Journal of Data Science 15, 205-220,2017
[11] V.K. Sharma, S.K. Singh, and U.Singh, and V.Agiwal ,"The inverse lindley distribution: A stress strength reliability model", doi:10.1080/21681015.2015.1025901

[12] H.Zakerzadeh and A. Dolati, "A generalised Lindley distribution", Journal of math extension, 3, 13–25,2009
[13] J. Rameesa ,N.Bashir ,T.R.Jan,P.Bilal , " Exponentiated Inverse Power Lindley Distribution and its Application", Pakistan journal of statistics and operations research,2018.(to appear).