Original Research Article

A Two-Stage Group Sampling Plan Based on Truncated Life Tests for an Odd Generalized Exponential Log-Logistic Distribution

Abstract: In this article, a time truncated life test based on two-stage group acceptance sampling plan is proposed for lifetime of an item follows odd generalized exponential log-logistic distribution. The ability about the lot acceptance can be made in the first or second stage according to the number of failures from each group. The optimal parameters for the proposed plan are determined such that both producer's as well as consumer's risks are contented simultaneously for the specified unreliability when group size and test duration are specified. The efficiency of the proposed sampling plan is evaluated in terms of average sample number with the existing sampling plan. The results are explained with the help of industrial example.

Keywords: Odd generalized exponential log-logistic distribution (OGELLD); average sample number; producer's and consumer's risk.

1. Introduction

Acceptance sampling plan is an inspecting methodology in statistical quality control or reliability tests which are most commonly used in the field of industries, bio-medical sciences and business to manage the product reliability and to make the decision of accepting or rejecting the product by the consumer. Currently, in industrial environment it is necessary to produce high quality products with the help of modern statistical quality control (SQC) techniques. These SQC techniques are very needful for any manufacturing process for improving the quality of the product, since the application of SQC tools will reduce the variability in the manufacturing process and manufactured products. According to Montgomery (2009), one can assess the quality of the product in terms of its performance, durability, features, serviceability, aesthetics, quality, reliability and its conformance to standards collectively and they are termed as the dimensions of the quality. Therefore, we can say that in selecting among competing products and services, the quality has become the most significant factor for consumer's satisfaction.

Several authors have investigated the acceptance sampling plans based on truncated life tests for various lifetime distributions, which are available in the literature of acceptance sampling for example Epstein (1954), Sobel and Tischendrof (1959), Gupta and Groll (1961), Gupta (1962), Fertig and Mann (1980), Kantam and Rosaiah (1998), Kantam *et al.* (2001).

Generally in acceptance sampling plans for a truncated life test are frequently used to obtain the sample size from a lot under consideration. For a usual sampling plan a single item will be tested in a tester. However, in practical situations there may be a tester in which multiple items can be installed at the same time, since it reduces the testing time and cost. The acceptance sampling plan under this type of testers will be called a group acceptance sampling plan (GASP). The inspection of multiple items simultaneously can be made easy to the experimenter for testing. Two stage group acceptance sampling is the extension of GASP which involves two groups. The GASP is more advantageous than the conventional sampling plans in terms of minimum inspection, so that the considerable testing time and cost can be reduced which was proposed by Aslam *et al.* (2009). The advantage of two stage group sampling plan is that it reduces the average sample number (ASN) as compared to the GASP.

The group acceptance sampling plans will be useful for sudden death testing. A group acceptance sampling plans based on truncated life tests for various distribution were discussed by Balasooriya (1995), Pascual and Meeker (1998), Wu *et al*. (2001), Jun *et al*. (2006), Aslam and Jun (2009a, 2009b) and Rao (2009, 2010).

Aslam *et al.* (2010) proposed a two stage group sampling plan for the Weibull distribution, which improved the results given in Aslam and Jun (2009b) in terms of the ASN. A two stage group sampling plan for Burr Type X based on percentiles was proposed by Aslam *et al.* (2013). Rao (2013) formulated a two-stage group sampling plan based on truncated life tests for a Marshall-Olkin extended exponential distribution. Rao *et al*. (2014) developed a two-stage group sampling plan for exponentiated Fréchet distribution. Azam *et al.* (2015) proposed a two stage group sampling plan for half normal percentiles. Anburajan and Ramaswamy (2015) developed a two stage group acceptance sampling plans based on truncated life tests using log-logistic and gamma distributions. Ramaswamy and Jayasri (2015) developed time truncated two stage group sampling plan for various distributions. Rao and Rao (2016) proposed a two stage group sampling plan based on life tests for half logistic distribution. By exploring the literature on two stage group sampling plan, we are interested to develop a two stage group sampling plan for the truncated life test when the lifetime of a product follows the odd generalized exponential log-logistic distribution (OGELLD). The probability density function (pdf) and cumulative distribution function (cdf) of OGELLD respectively are given as follows:

$$f(\mathrm{t};\sigma,\lambda,\theta,\gamma)=\frac{\gamma\theta}{\lambda\sigma}\left(t/\sigma\right)^{\theta-1}\left[1-e^{\frac{-1}{\lambda}(t/\sigma)^{\theta}}\right]^{\gamma-1}e^{-\frac{1}{\lambda}(t/\sigma)^{\theta}},\ t>0,\ \sigma,\lambda,\theta,\gamma>0 \quad (1)$$

$$F(\mathrm{t};\sigma,\lambda,\theta,\gamma)=\left[1-e^{-\frac{1}{\lambda}(t/\sigma)^{\theta}}\right]^{\gamma},t>0,\ \sigma,\lambda,\theta,\gamma>0.$$

(2)

where σ, λ are the scale parameters and θ,γ are the shape parameters. The 100q-th percentile of the OGELLD is given as:

$$t_q=\sigma\eta_q,\quad \text{where}\quad \eta_q=\left[-\lambda\ln(1-q^{1/\gamma})\right]^{1/\theta}. \quad (3)$$

In this article, we propose a two stage group acceptance sampling plan for truncated life tests when the life time of the product is assumed to follow an OGELLD. We construct the tables for finding the number of groups required for each stage of the proposed plan so as to minimize the average sample number under the constraints of satisfying the producer's and consumer's risks simultaneously. The design of two-stage plan is discussed in Section 2. The comparison with the single-stage group sampling plan is presented in Section 3. Methodology is illustrated with industrial application in Section 4 and some conclusions are given in Section 5.

2. Two-stage group acceptance sampling plan

Aslam *et al.* (2012) proposed the two-stage group sampling plan. The operating procedure of the two stage group sampling (Aslam *et al.* 2012) plan is explained below.

First stage:

- Extract the first random sample of size n_1 from a lot submitted for inspection.
- Randomly allocate *r* items to each of the g_1 groups (or testers) so that $n_1=rg_1$ and set them on test for the test time t_0.
- Accept the lot if the total number of failures from each group is smaller than or equal to c_1.
- Truncate the test and reject the lot as soon as the number of failures in any group is larger than c_2 before t_0. Otherwise, go to the second stage.

Second stage:

- Extract a second random sample of size n_2 from the same lot.
- Randomly allocate *r* items to each of g_2 groups, so that $n_2=rg_2$ and put them on test for t_0.

- Accept the lot if the number of failures in each group is less than or equal to c_1. Otherwise, truncate the test and reject the lot if the number of failures in any group is greater than c_1 before t_0.

A two-stage group sampling plan is the generalization of the many sampling plans. The proposed two stage group sampling plan with $c_1 = c_2$ reduces to a (single-stage) group sampling plan. The major design parameters of the present plan will be the number of groups in each of two stages. The acceptance numbers c_1 and c_2 can be determined as well, but it is found that the plan with c_1=0*,* c_2=1 minimizes the ASN. The two-stage sampling plan with c_1=0*,* c_2=1 can be practically useful because lower acceptance numbers are often preferred by the customers. The lot acceptance probability from the first stage in the two-stage group sampling plan is given by

$$P_a^{(1)} = \sum_{i=0}^{c_1} \binom{rg_1}{i} p^i (1-p)^{rg_1 - i} \tag{4}$$

where p is the probability that an item in a group fails by time t_0.

It would be convenient to determine the termination time t_0 as a multiple of the specified percentile t_q^0 such that $t_0 = \delta_q^0 t_q^0$ for a constant δ_q^0. Then, the probability of a failure occurs during the termination time t_0 denoted by $p = F(t_0; \sigma, \lambda, \theta, \gamma)$ *i.e.,*

$$p = \left[1 - \exp\left\{-\frac{1}{\lambda}\left(\frac{t_0}{\sigma}\right)^{\theta}\right\}\right]^{\gamma} = \left[1 - \exp\left\{-\frac{1}{\lambda}\left(\frac{\eta \delta_q^0}{\left(t_q / t_q^0\right)}\right)^{\theta}\right\}\right]^{\gamma} \tag{5}$$

The lot rejection probability from the first stage is obtained by

$$P_r^{(1)} = 1 - \sum_{i=0}^{c_2} \binom{rg_1}{i} p^i (1-p)^{rg_1 - i} \tag{6}$$

The lot acceptance probability from the second stage will be

$$P_a^{(2)} = \left[1 - \left(p_a^{(1)} + p_r^{(1)}\right)\right]\left[\sum_{i=0}^{c_1} \binom{rg_2}{i} p^i (1-p)^{rg_2 - i}\right] \tag{7}$$

Therefore, the lot acceptance probability in the proposed two-stage group sampling plan is given by

$$L(p) = P_a^{(1)} + P_a^{(2)} \tag{8}$$

For the case of $c_1 = 0,\ c_2 = 1,$ the acceptance probability of Equation (8) reduces to

$$L(p) = (1-p)^{rg_1} + rg_1 p\left(1-p\right)^{rg_1-1}\left(1-p\right)^{rg_2} \tag{9}$$

When the quality level based on the percentile ratio t_q/t_q^0 between the true percentile t_q and targeted percentile t_q^0, the two-point approach of finding the design parameters is to determine the minimum number of groups g_1 and g_2, to satisfy the following two inequalities

$$L\left(p/\left(t_q/t_q^0\right) = \delta_1\right) \le \beta \tag{10}$$

$$L\left(p/\left(t_q/t_q^0\right) = \delta_2\right) \ge 1-\alpha$$

(11)

where, δ_1 is the percentile ratio at the consumer's risk and δ_2 is the percentile ratio at the producer's risk. In this study, the ratio δ_1 is taken as 1.

Let p_1 and p_2 are the failure probabilities corresponding to producer's and consumer's risks, respectively.

where $$p_1 = \left[1-\exp\left\{-\frac{1}{\lambda}\left(\frac{\eta\delta_q^0}{\left(t_q/t_q^0\right)}\right)^{\theta}\right\}\right]^{\gamma} \ and\ p_2 = \left[1-\exp\left\{-\frac{1}{\lambda}\left(\eta\delta_q^0\right)^{\theta}\right\}\right]^{\gamma} \tag{12}$$

There exist a multiple solutions of design parameters satisfying equations (10) and (11), so we need to select them to minimize the ASN for our two-stage group sampling plan. The ASN for the two-stage sampling plan is obtained by

$$\text{ASN} = rg_1 + rg_2\left(1-P_a^{(1)}-P_r^{(1)}\right) \tag{13}$$

where $P_a^{(1)}$ and $P_r^{(1)}$ are evaluated at $p = p_2$. Therefore, the design parameters for the proposed two-stage group sampling plan can be obtained by the solution from the following optimization when p_2 is specified:

$$\text{Minimize ASN} = rg_1 + rg_2\left(1-P_a^{(1)}-P_r^{(1)}\right) \tag{14a}$$

Subject to constraints

$$L\left(p_1\right) \le \beta \tag{14b}$$

$$L\left(p_2\right) \ge 1-\alpha \tag{14c}$$

$$1 \le g_2 \le g_1 \tag{14d}$$

$$0 \le c_1 < c_2 \tag{14e}$$

g_1, g_2, c_1 and c_2 are integers (14f)

The constraint (14d) is specified because it may not be desirable if the number of groups in the second stage is larger than that in the first stage.

Therefore, the design parameters of the proposed plan g_1, g_2 are determined for a given α and β, δ_1 the percentile ratio, at the consumer's risk and the percentile ratio, δ_2, at the producer's risk, such that it is minimized $\mathrm{ASN}(p_2)$ and inequalities (14b) and (14c) are satisfied simultaneously for specified values of parameters, λ, θ and γ, termination ratio δ_q^0 and the number of testers, *r*. The minimum number of groups required for the two-stage group acceptance sampling plan for different parametric combinations are $\lambda = 0.5, 1.5, 2.0$ and $\theta = \gamma = 1.5, 2.0$ according to percentile ratios $t_q / t_q^0 = 4, 6, 8, 10$, when $r = 3, 5$ are estimated for 50th percentiles for a specified four levels of the consumer's risks $\beta = 0.25, 0.10, 0.05, 0.01$ which are presented in Tables 1 to 3. For the data under consideration given in Section 4. The plan parameters obtained for the estimated values of λ, θ, γ *i.e.,* $\hat{\lambda} = 0.28, \hat{\theta} = 0.63$ and $\hat{\gamma} = 11.19$ are presented in Table 4. As mentioned earlier, (c_1, c_2) were determined as (0, 1) in all cases and δ_q^0 =1.

It is observed from the Tables 1 to 4 that the other parameters are remains same, the number of groups required decrease as the group size increases from $r = 3$ to 5 and also the ASN increases marginally. It is also indicates that large group size requires smaller and helps us to have a quick decisions with minimum experimental cost and saving the time. Another interesting observation from tables is the number of groups are not influenced by change in shape parameters.

3. Comparisons with single-stage group sampling plans

In order to know the usefulness of this plan, we consider a single-stage group sampling plan (having group size *r*) and table is prepared accordingly. As mentioned earlier, this will be the case when $c_1 = c_2 = c$ in the two-stage group sampling plan. The lot acceptance probability (operating characteristic) under this plan will be given by

$$OC = P_a = \sum_{i=0}^{c} \binom{rg}{i} p^i \left(1-p\right)^{rg-i} \tag{15}$$

where g is the number of groups required. Obviously, the ASN is obtained by r times g.

Similarly as in the two-stage group sampling plan, a table for determining the number of groups and acceptance number required are obtained according to the value of the specified unreliability at a given consumer's risk. Table 5 shows the minimum numbers of groups required for the single stage group sampling plan with c = 0 or c = 1 when $r = 3, 5$ according to values of percentile ratios δ_2=4, 6, 8, 10 and δ_1=1 at β = 0.25, 0.10, 0.05 and 0.01 with known and estimated parameters.

We noticed from the Table 5 that there is increase in the groups when the acceptance number changes from $c = 0$ to $c = 1$, also we observe that the number of groups required becomes lesser when group size increase from $r = 3$ to $r = 5$. Moreover, the proposed two stage group acceptance sampling plan provides better results than single stage plan in terms of ASN and OC values. Comparing Tables 1 to 6, we noticed that ASN for the single stage sampling plan with $c = 0$ is smaller than that of the two stage group sampling plan with $c_1 = 0, c_2 = 1$. Finally, if we consider the OC and ASN values at the same time, the two stage group acceptance sampling plans is much better than the single stage group sampling plan.

4. Industrial Applications

In this section, we use a real data set to show that the OGELLD can be a suitable model. The data set consists of 25 observations on runoff amounts at Jug Bridge, Maryland, which was originally given by Folks and Chhikara (1978). To be self-contained, this data set is reproduced as follows:

0.17, 0.23, 0.33, 0.39, 0.39, 0.40, 0.45, 0.52, 0.56, 0.59, 0.64, 0.66, 0.70, 0.76, 0.77, 0.78, 0.9 5, 0.97, 1.02, 1.12, 1.19, 1.24, 1.59,1.74, 2.92.

Using exploratory data analysis and then goodness-of-fit, we show a rough indication of the goodness off it for our model by plotting the superimposed for the data shows that the OGELLD is a good fit and also it is emphasized with Q-Q plot, displayed in Figure 1. The maximum likelihood estimates of the parameters of OGELLD for runoff amounts are $\hat{\lambda} = 0.2824, \hat{\theta} = 0.6339, \hat{\gamma} = 11.1941$ and the Kolmogorov-Smirnov test and found that the maximum distance between the data and the fitted of the OGELLD is 0.0673 with p-value is

0.9999. Therefore, the parameters of OGELLD provides reasonable fits for the runoff amounts.

Suppose that an experimenter would like to use the proposed two stage sampling plan to establish the true unknown 50thpercentile lifetime for the product is at least 2 hours and experiment will be stopped after 2 hours. Further, suppose that in the laboratory the experimenter has facility to install 3 items on a tester. This information leads to $\delta_q^0 = 1$. Let β = 0.05 and $t_q / t_q^0 = 4$ with $\alpha = 0.05$ for this experiment. The above data is well fitted to the parameters of OGELLD with $\hat{\lambda} = 0.2824, \hat{\theta} = 0.6339, \hat{\gamma} = 11.1941$. So, from Table 4, the two-stage acceptance sampling plan parameters at $r = 3$ are $g_1 = 2, g_2 = 1, c_1 = 0$ and $c_2 = 1$. The two-stage acceptance sampling plan is implemented as follows: randomly select 6 items and distribute 3 items into each of 2 testers and accept the product if no failure from each tester in 2 hours and reject the product if more than 1 failure from any tester before 2 hours. If one failure is observed from any tester then go to the second stage, then randomly select another 3 items from the lot and distribute 3 items into each tester. If the total number of failure items from the two-stage testing within 2 hours for each stage is less than one then the lot is accepted; otherwise, the lot is rejected. The probability of acceptance for this plan is 99.27% and ASN is 6.25.

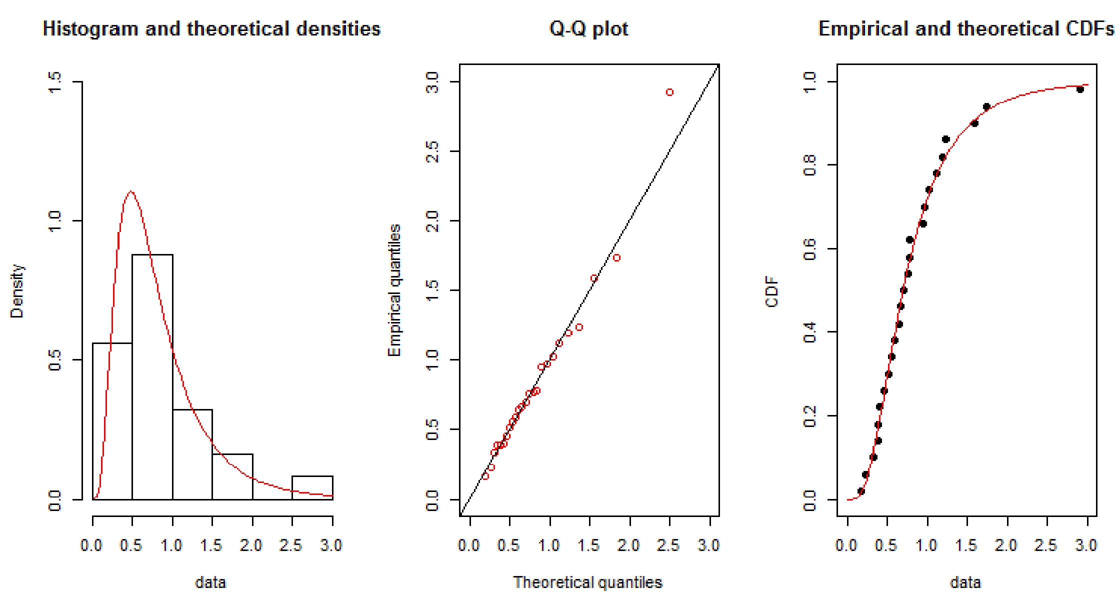

Figure 1: Histogram with estimated pdf, cdf and Q-Q plot

5. Conclusion

In this article, a two-stage grouped acceptance sampling plan is developed when the lifetime of a product follow odd generalized exponential log-logistic distribution with known and estimated parameters. The plan parameters like the number of groups in each stage were determined so as to minimize the ASN subject to satisfying the consumer's as well as producer's risk simultaneously. Tables for the plan parameters are constructed under various combinations such as group sizes, producer's and consumer's risk and so on. An industrial example has been presented to illustrate the applications of the proposed two-stage group acceptance sampling plan. We made the comparison between the proposed two-stage group acceptance sampling and single stage acceptance sampling plan. We observed from the tables that the number of groups required decrease as the group size increases from $r = 3$ to 5 and also the ASN increases marginally, sample size decreases as the group size increases, which indicates that a larger group size may be more economical and it reduces the experimental time and cost. We proposed two-stage group acceptance sampling plan, since it performs much better in terms of the average sample number (ASN) and the operating characteristics than in single-stage group acceptance sampling plan.

Table 1: The minimum number of groups required and ASN in the two-stage sampling plan for $\lambda = \theta = \gamma = 2$ for OGELLD with 50th percentile.

		$\delta = 0.5, r = 3$				$\delta = 1.0, r = 3$				$\delta = 0.5, r = 5$				$\delta = 1.0, r = 5$			
β	t_q/t_q^0	g1	g2	ASN	L(p2)	g1	g2	ASN	L(p2)	g1	g2	ASN	L(p2)	g1	g2	ASN	L(p2)
0.25	4	12	1	36.04	0.9999	1	1	3.05	0.9996	7	1	35.06	0.9999	1	1	5.13	0.9990
	6	12	1	36.01	1.0000	1	1	3.01	1.0000	7	1	35.01	1.0000	1	1	5.03	1.0000
	8	12	1	36.00	1.0000	1	1	3.00	1.0000	7	1	35.00	1.0000	1	1	5.01	1.0000
	10	12	1	36.00	1.0000	1	1	3.00	1.0000	7	1	35.00	1.0000	1	1	5.00	1.0000
0.10	4	18	1	54.06	0.9998	2	1	6.10	0.9990	10	1	50.09	0.9998	1	1	5.13	0.9990
	6	18	1	54.01	1.0000	2	1	6.02	1.0000	10	1	50.02	1.0000	1	1	5.03	1.0000
	8	18	1	54.00	1.0000	2	1	6.01	1.0000	10	1	50.01	1.0000	1	1	5.01	1.0000
	10	18	1	54.00	1.0000	2	1	6.00	1.0000	10	1	50.00	1.0000	1	1	5.00	1.0000
0.05	4	22	1	66.07	0.9997	2	1	6.10	0.9990	13	1	65.11	0.9997	1	1	5.13	0.9990
	6	22	1	66.01	1.0000	2	1	6.02	1.0000	13	1	65.02	1.0000	1	1	5.03	1.0000
	8	22	1	66.00	1.0000	2	1	6.01	1.0000	13	1	65.01	1.0000	1	1	5.01	1.0000
	10	22	1	66.00	1.0000	2	1	6.00	1.0000	13	1	65.00	1.0000	1	1	5.00	1.0000
0.01	4	30	1	90.09	0.9995	3	1	9.14	0.9982	18	1	90.16	0.9994	2	1	10.26	0.9973
	6	30	1	90.02	1.0000	3	1	9.03	0.9999	18	1	90.03	1.0000	2	1	10.06	0.9999
	8	30	1	90.01	1.0000	3	1	9.01	1.0000	18	1	90.01	1.0000	2	1	10.02	1.0000

	10	30	1	90.00	1.0000	3	1	9.00	1.0000	18	1	90.00	1.0000	2	1	10.01	1.0000

Table 2: The minimum number of groups required and ASN in the two-stage sampling plan for $\lambda = 2,\ \theta = 1.5$ and $\gamma = 1.5$ for OGELLD with 50th percentile.

		$\delta = 0.5, r = 3$				$\delta = 1.0, r = 3$				$\delta = 0.5, r = 5$				$\delta = 1.0, r = 5$			
β	t_q / t_q^0	g1	g2	ASN	L(p2)	g1	g2	ASN	L(p2)	g1	g2	ASN	L(p2)	g1	g2	ASN	L(p2)
0.25	4	5	1	15.35	0.9892	1	1	3.33	0.9826	3	1	15.59	0.9871	1	1	5.85	0.9540
	6	5	1	15.16	0.9981	1	1	3.15	0.9968	3	1	15.26	0.9977	1	1	5.39	0.9910
	8	5	1	15.08	0.9995	1	1	3.08	0.9991	3	1	15.14	0.9994	1	1	5.22	0.9973
	10	5	1	15.05	0.9998	1	1	3.05	0.9996	3	1	15.09	0.9998	1	1	5.13	0.9990
0.10	4	7	1	21.47	0.9809	2	1	6.59	0.9560	4	1	20.75	0.9799	1	1	5.85	0.9540
	6	7	1	21.21	0.9966	2	1	6.28	0.9914	4	1	20.34	0.9964	1	1	5.39	0.9910
	8	7	1	21.12	0.9990	2	1	6.15	0.9975	4	1	20.18	0.9990	1	1	5.22	0.9973
	10	7	1	21.07	0.9996	2	1	6.10	0.9990	4	1	20.11	0.9996	1	1	5.13	0.9990
0.05	4	9	1	27.57	0.9709	2	1	6.59	0.9560	5	1	25.90	0.9713	1	1	5.85	0.9540
	6	9	1	27.27	0.9947	2	1	6.28	0.9914	5	1	25.42	0.9948	1	1	5.39	0.9910
	8	9	1	27.15	0.9985	2	1	6.15	0.9975	5	1	25.23	0.9985	1	1	5.22	0.9973
	10	9	1	27.09	0.9994	2	1	6.10	0.9990	5	1	25.14	0.9994	1	1	5.13	0.9990
0.01	4	12	1	36.70	0.9528	--	--	--	--	7	1	36.15	0.9510	--	--	--	--
	6	12	1	36.35	0.9911	3	1	9.39	0.9842	7	1	35.56	0.9907	2	1	10.72	0.9769
	8	12	1	36.19	0.9974	3	1	9.22	0.9953	7	1	35.31	0.9973	2	1	10.41	0.9930
	10	12	1	36.12	0.9990	3	1	9.14	0.9982	7	1	35.20	0.9990	2	1	10.26	0.9973

Table 3: The minimum number of groups required and ASN in the two-stage sampling plan for $\lambda = 0.5,\ \theta = 1.5$ and $\gamma = 1.5$ for OGELLD with 50th percentile.

		$\delta = 0.5, r = 3$				$\delta = 1.0, r = 3$				$\delta = 0.5, r = 5$				$\delta = 1.0, r = 5$			
β	t_q / t_q^0	g1	g2	ASN	L(p2)	g1	g2	ASN	L(p2)	g1	g2	ASN	L(p2)	g1	g2	ASN	L(p2)
0.25	4	5	1	15.35	0.9892	1	1	3.3314	0.9826	3	1	3.33	0.9871	1	1	5.85	0.9540
	6	5	1	15.16	0.9981	1	1	3.1456	0.9968	3	1	3.15	0.9977	1	1	5.39	0.9910
	8	5	1	15.08	0.9995	1	1	3.0788	0.9991	3	1	3.08	0.9994	1	1	5.22	0.9973
	10	5	1	15.05	0.9998	1	1	3.0485	0.9996	3	1	3.05	0.9998	1	1	5.13	0.9990
0.10	4	7	1	21.47	0.9809	2	1	6.5865	0.9560	4	1	6.59	0.9799	1	1	5.85	0.9540
	6	7	1	21.21	0.9966	2	1	6.2767	0.9914	4	1	6.28	0.9964	1	1	5.39	0.9910
	8	7	1	21.12	0.9990	2	1	6.1534	0.9975	4	1	6.15	0.9990	1	1	5.22	0.9973
	10	7	1	21.07	0.9996	2	1	6.0954	0.9990	4	1	6.10	0.9996	1	1	5.13	0.9990
0.05	4	9	1	27.57	0.9709	2	1	6.5865	0.9560	5	1	6.59	0.9713	1	1	5.85	0.9540
	6	9	1	27.27	0.9947	2	1	6.2767	0.9914	5	1	6.28	0.9948	1	1	5.39	0.9910
	8	9	1	27.15	0.9985	2	1	6.1534	0.9975	5	1	6.15	0.9985	1	1	5.22	0.9973
	10	9	1	27.09	0.9994	2	1	6.0954	0.9990	5	1	6.10	0.9994	1	1	5.13	0.9990
0.01	4	12	1	36.70	0.9528	--	--	--	--	7	1	--	0.9510	--	--	--	--
	6	12	1	36.35	0.9911	3	1	9.3946	0.9842	7	1	9.39	0.9907	2	1	10.72	0.9769
	8	12	1	36.19	0.9974	3	1	9.2240	0.9953	7	1	9.22	0.9973	2	1	10.41	0.9930
	10	12	1	36.12	0.9990	3	1	9.1407	0.9982	7	1	9.14	0.9990	2	1	10.26	0.9973

Table 4: The minimum number of groups required and ASN in the two-stage sampling plan for $\hat{\lambda}=0.28,\ \hat{\theta}=0.63$ and $\hat{\gamma}=11.19$ for OGELLD with 50th percentile.

β	t_q/t_q^0	$\delta=0.5, r=3$				$\delta=1.0, r=3$				$\delta=0.5, r=5$				$\delta=1.0, r=5$			
		g1	g2	ASN	L(p2)	g1	g2	ASN	L(p2)	g1	g2	ASN	L(p2)	g1	g2	ASN	L(p2)
0.25	4	6	1	18.04	0.9999	1	1	3.13	0.9972	3	1	15.06	0.9999	1	1	5.36	0.9922
	6	6	1	18.01	1.0000	1	1	3.03	0.9999	3	1	15.01	1.0000	1	1	5.07	0.9997
	8	6	1	18.00	1.0000	1	1	3.01	1.0000	3	1	15.00	1.0000	1	1	5.02	1.0000
	10	6	1	18.00	1.0000	1	1	3.00	1.0000	3	1	15.00	1.0000	1	1	5.01	1.0000
0.10	4	9	1	27.06	0.9997	2	1	6.26	0.9927	5	1	25.10	0.9997	1	1	5.36	0.9922
	6	9	1	27.01	1.0000	2	1	6.05	0.9997	5	1	25.01	1.0000	1	1	5.07	0.9997
	8	9	1	27.00	1.0000	2	1	6.01	1.0000	5	1	25.00	1.0000	1	1	5.02	1.0000
	10	9	1	27.00	1.0000	2	1	6.00	1.0000	5	1	25.00	1.0000	1	1	5.01	1.0000
0.05	4	11	1	33.08	0.9996	2	1	6.26	0.9927	6	1	30.12	0.9996	1	1	5.36	0.9922
	6	11	1	33.01	1.0000	2	1	6.05	0.9997	6	1	30.02	1.0000	1	1	5.07	0.9997
	8	11	1	33.00	1.0000	2	1	6.01	1.0000	6	1	30.00	1.0000	1	1	5.02	1.0000
	10	11	1	33.00	1.0000	2	1	6.00	1.0000	6	1	30.00	1.0000	1	1	5.01	1.0000
0.01	4	15	1	45.10	0.9993	3	1	9.37	0.9864	9	1	45.17	0.9992	2	1	10.67	0.9800
	6	15	1	45.01	1.0000	3	1	9.08	0.9995	9	1	45.02	1.0000	2	1	10.14	0.9992
	8	15	1	45.00	1.0000	3	1	9.02	1.0000	9	1	45.01	1.0000	2	1	10.04	0.9999
	10	15	1	45.00	1.0000	3	1	9.01	1.0000	9	1	45.00	1.0000	2	1	10.01	1.0000

Table 5: The number of groups in the single-stage sampling plan for OGELLD with 50th percentile.

β	t_q/t_q^0	$\lambda=\theta=\gamma=2,\ r=3$						$\lambda=\theta=\gamma=2,\ r=5$					
		Single with $c=0$			Single with $c=1$			Single with $c=0$			Single with $c=1$		
		g	ASN	OC	g	ASN	OC	g	ASN	OC	g	ASN	OC
	4	7	21	0.9924	13	39	0.9999	4	20	0.9928	8	40	0.9999
	6	7	21	0.9985	13	39	1.0000	4	20	0.9986	8	40	1.0000
	8	7	21	0.9995	13	39	1.0000	4	20	0.9995	8	40	1.0000
0.25	10	7	21	0.9998	13	39	1.0000	4	20	0.9998	8	40	1.0000
	4	11	33	0.9882	19	57	0.9998	7	35	0.9874	11	55	0.9998
	6	11	33	0.9976	19	57	1.0000	7	35	0.9975	11	55	1.0000
	8	11	33	0.9992	19	57	1.0000	7	35	0.9992	11	55	1.0000
0.10	10	11	33	0.9997	19	57	1.0000	7	35	0.9997	11	55	1.0000
	4	14	42	0.9849	22	66	0.9997	9	45	0.9839	14	70	0.9997
	6	14	42	0.9970	22	66	1.0000	9	45	0.9968	14	70	1.0000
	8	14	42	0.9990	22	66	1.0000	9	45	0.9990	14	70	1.0000
0.05	10	14	42	0.9996	22	66	1.0000	9	45	0.9996	14	70	1.0000
	4	22	66	0.9764	31	93	0.9995	13	65	0.9768	19	95	0.9994
	6	22	66	0.9953	31	93	1.0000	13	65	0.9953	19	95	1.0000
	8	22	66	0.9985	31	93	1.0000	13	65	0.9985	19	95	1.0000
0.01	10	22	66	0.9994	31	93	1.0000	13	65	0.9994	19	95	1.0000
		$\lambda=2,\theta=\gamma=1.5,\ r=3$						$\lambda=2,\theta=\gamma=1.5,\ r=5$					
	4	-	-	-	6	18	0.9889	-	-	-	4	20	0.9864
	6	3	9	0.9678	6	18	0.9981	2	10	0.9643	4	20	0.9976
	8	3	9	0.9829	6	18	0.9995	2	10	0.9810	4	20	0.9993
0.25	10	3	9	0.9896	6	18	0.9998	2	10	0.9884	4	20	0.9997

	4	-	-	-	8	24	0.9808	-	-	-	5	25	0.9792
	6	-	-	-	8	24	0.9965	-	-	-	5	25	0.9963
	8	5	15	0.9717	8	24	0.9990	3	15	0.9717	5	25	0.9989
0.10	10	5	15	0.9827	8	24	0.9996	3	15	0.9827	5	25	0.9996
	4	-	-	-	10	30	0.9707	-	-	-	6	30	0.9707
	6	-	-	-	10	30	0.9946	-	-	-	6	30	0.9946
	8	6	18	0.9661	10	30	0.9985	4	20	0.9624	6	30	0.9985
0.05	10	6	18	0.9793	10	30	0.9994	4	20	0.9770	6	30	0.9994
	4	-	-	-	13	39	0.9527	-	-	-	8	40	0.9504
	6	-	-	-	13	39	0.9911	-	-	-	8	40	0.9906
	8	-	-	-	13	39	0.9974	-	-	-	8	40	0.9973
0.01	10	9	27	0.9691	13	39	0.9990	6	30	0.9657	8	40	0.9990

Table 6: The number of groups in the single-stage sampling plan for OGELLD with 50th percentile (Continue)

		$\lambda = 0.5, \theta = \gamma = 1.5,\ r = 3$						$\lambda = 0.5, \theta = \gamma = 1.5,\ r = 5$					
		Single with $c=0$			Single with $c=1$			Single with $c=0$			Single with $c=1$		
β	t_q/t_q^0	g	ASN	OC	g	ASN	OC	g	ASN	OC	g	ASN	OC
	4	-	-	-	6	18	0.9889	-	-	-	4	20	0.9864
	6	3	9	0.9678	6	18	0.9981	2	10	0.9643	4	20	0.9976
	8	3	9	0.9829	6	18	0.9995	2	10	0.9810	4	20	0.9993
0.25	10	3	9	0.9896	6	18	0.9998	2	10	0.9884	4	20	0.9997
	4	-	-	-	8	24	0.9808	-	-	-	5	25	0.9792
	6	-	-	-	8	24	0.9965	-	-	-	5	25	0.9963
	8	5	15	0.9717	8	24	0.9990	3	15	0.9717	5	25	0.9989
0.10	10	5	15	0.9827	8	24	0.9996	3	15	0.9827	5	25	0.9996
	4	-	-	-	10	30	0.9707	-	-	-	6	30	0.9707
	6	-	-	-	10	30	0.9946	-	-	-	6	30	0.9946
	8	6	18	0.9661	10	30	0.9985	4	20	0.9624	6	30	0.9985
0.05	10	6	18	0.9793	10	30	0.9994	4	20	0.9770	6	30	0.9994
	4	-	-	-	13	39	0.9527	-	-	-	8	40	0.9504
	6	-	-	-	13	39	0.9911	-	-	-	8	40	0.9906
	8	-	-	-	13	39	0.9974	-	-	-	8	40	0.9973
0.01	10	9	27	0.9691	13	39	0.9990	6	30	0.9657	8	40	0.9990
		$\hat{\lambda} = 0.28, \hat{\theta} = 0.63, \hat{\gamma} = 11.19$						$\hat{\lambda} = 0.28, \hat{\theta} = 0.63, \hat{\gamma} = 11.19$					
	4	4	12	0.9904	7	21	0.9999	2	10	0.9920	4	20	0.9999
	6	4	12	0.9987	7	21	1.0000	2	10	0.9989	4	20	1.0000
	8	4	12	0.9997	7	21	1.0000	2	10	0.9998	4	20	1.0000
0.25	10	4	12	0.9999	7	21	1.0000	2	10	0.9999	4	20	1.0000
	4	6	18	0.9857	10	30	0.9997	4	20	0.9841	6	30	0.9997
	6	6	18	0.9981	10	30	1.0000	4	20	0.9979	6	30	1.0000
	8	6	18	0.9996	10	30	1.0000	4	20	0.9996	6	30	1.0000
0.10	10	6	18	0.9999	10	30	1.0000	4	20	0.9999	6	30	1.0000
	4	7	21	0.9833	11	33	0.9997	5	25	0.98020	7	35	0.9996
	6	7	21	0.9978	11	33	1.0000	5	25	0.9974	7	35	1.0000
	8	7	21	0.9995	11	33	1.0000	5	25	0.9994	7	35	1.0000
0.05	10	7	21	0.9999	11	33	1.0000	5	25	0.9998	7	35	1.0000
	4	11	33	0.9739	16	48	0.9993	7	35	0.9724	10	50	0.9992
	6	11	33	0.9965	16	48	1.0000	7	35	0.9963	10	50	1.0000
	8	11	33	0.9993	16	48	1.0000	7	35	0.9992	10	50	1.0000
0.01	10	11	33	0.9998	16	48	1.0000	7	35	0.9998	10	50	1.0000

The cells with hyphens (-) indicate that parameters cannot be found to satisfy the conditions.

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