

Mathematical Modeling to Estimation Leaf Area of *Pimenta dioica* from Linear Dimensions

ABSTRACT

The objective of this work was to obtain regression equations and to indicate the most appropriate from different mathematical models for the estimation of the leaf area of Allspice (*Pimenta dioica*) by non-destructive method. 500 leaves of plants located in the municipality of São Mateus, North of Espírito Santo State, Brazil, were collected, 400 of which were used to adjust the equations and 100 for validation. The length (L) along the main midrib, the maximum width (W), the product of the length with the width (LW) and the observed leaf area (OLA) were measured from all leaves. We fitted models of linear equations of first degree, quadratic and power, where OLA was the dependent variable in function of L, W and LW. From the 100 sheets intended for validation, and using the adjusted equations for each mathematical model, the estimated leaf area (ELA) was obtained. Subsequently, a simple linear regression was fitted for each model of the proposed equation in which ELA was the dependent variable and OLA the independent variable. The means obtained from ELA and OLA were compared by Student's t test 5% probability. The mean absolute error (MAE), the root mean square error (RMSE) and Willmott's index d were also determined. The best fit had as selection criterion the non-significance of the comparative means of ELA and OLA, MAE and RMSE values closer to zero and value of the coefficient of determination coefficient (R^2) close to one. Thus, the power model ($ELA = 0.7605(LW)^{0.9926}$ and $R^2 = 0.9764$) based on the product of length and width (LW) is the most appropriate for estimating the leaf area of *Pimenta dioica*.

Keywords: Pimenta dioica; non-destructive method; leaf dimension.

1. INTRODUCTION

Allspice (*Pimenta dioica*) is a species widely used in cooking as a spice and seasoning. In Central America and the Caribbean, its use is present in folk medicine, where its leaves are used in the treatment of diabetes, hypertension, obesity, influenza, cold, fever, digestive tract problems, abdominal pain and menstrual cramps [1].

The leaf is a fundamental organ of plants, so the determination of the leaf area allows studying factors such as photosynthesis, evapotranspiration, plant growth and development, light interception and responses to stimuli such as irrigation and fertilization [2]. Thus, there are direct and indirect methods to determine the leaf area of a plant. The direct methods mostly consist of techniques that involve the destruction of the leaves, necessitating equipment that is not always accessible [3]. Indirect methods, on the other hand, are non-destructive, simple, fast and do not require specific equipment [4].

Among indirect methods, the estimation of leaf area through mathematical equation models has been widely used. These models are based on the relation between linear dimensions of the leaves as the length and the width or the interaction of both measures, and these models can be used in the field, under different stages of development and growth of plants, non-destructive manner, low cost and high degree of precision [5].

Mathematical models for estimating leaf area have already been developed for various crops, such as *Cucumis melo* L. [6], *Crambe abyssinica* [7], *Jatropha curcas* [4], *Passiflora* spp. [8], *Coffea canephora* [9], *Vitis vinífera* L. [10], *Passiflora mucronata* [11], *Annona cherimila* Mill. [12], *Crotalaria juncea* [5], *Litchi chinensis* Sonn. [13], and *Prunus armeniaca* L. [14].

Different species present distinct leaf anatomy requiring specific models that estimate the leaf area for each of them. In this sense, the objective of this study was to generate regression equations and test from mathematical models estimating the leaf area of *Pimenta dioica* from linear dimensions of the leaves, of form non-destructively.

2. MATERIAL AND METHODS

The study was conducted with the species *Pimenta dioica*, known as Allspice, whose leaves were collected in the municipality of São Mateus, North of the State of Espírito Santo, Brazil, located at 18°40'36" south latitude and 39°51'35" east longitude. The region is characterized by tropical AW (tropical humid) climate, with rainfall in summer and dry winter according to the classification of köppen [15].

500 leaves of 12 different plants were counted, and in each sampled plant, leaves were collected at all stages of development at the four cardinal points, which did not present damage or attack of diseases or pests, as suggested by Oliveira et al. [13]. The leaves were packed in plastic bags and transported to the Plant Improvement Laboratory of the University of Espírito Santo (CEUNES).

The length (L) along the main midrib and the maximum width (W) of the leaf blade were measured in cm with ruler graduated in millimeters (Fig. 1). Subsequently, the multiplication of the length along the main midrib and the maximum width of the leaf blade (LW), in cm^2 , were determined. The observed leaf area (OLA, in cm^2) was determined through images of the scanned leaves, with the aid of a Photosmart C4280[®] scanner. The images were saved in the formed TIFF (Tag Image File Format) with 75 dpi and processed with the help of ImageJ[®] software [16].

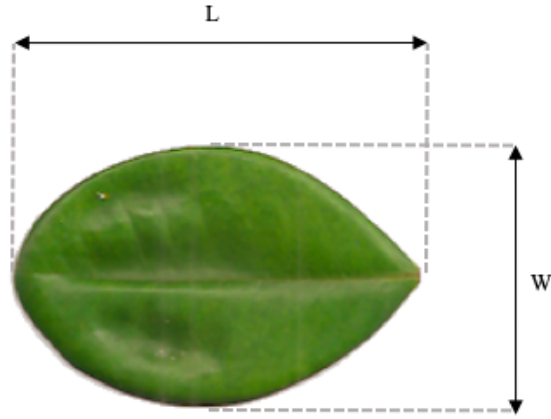


Fig. 1. Representation of the length (L) along the midrib and the maximum width (W) of leaves of *Pimenta dioica*

Among the 500 leaves, two groups, one with 400 leaves and the other with 100 leaves, were randomly selected, which were used, respectively, to adjust the modeling and for validation. In both groups, for the characteristics L, W, LW and OLA, the descriptive statistic was determined obtaining the minimum, maximum, mean, amplitude and coefficient of variation (CV) values.

For modeling, OLA was used as a dependent variable in function of L, W and LW as independent variables, being tested the models linear of first degree represented by $ELA = \hat{\beta}_0 + \hat{\beta}_1 x$, quadratic represented by $ELA = \hat{\beta}_0 + \hat{\beta}_1 x + \hat{\beta}_2 x^2$ and power represented by $ELA = \hat{\beta}_0 x^{\hat{\beta}_1}$, totaling nine equations for the estimation of the leaf area of *Pimenta dioica*.

For the validation the values of L, W and LW of the 100 sheets sampled for this purpose were substituted in the proposed equations for modeling, thus obtaining the estimated leaf area (ELA), in cm^2 . A simple linear regression was then fitted ($ELA = \hat{\beta}_0 + \hat{\beta}_1 x$) and their respective determination coefficient (R^2) for each modeling equation, where ELA was used as a dependent variable in function of OLA.

It was also determined the mean absolute error (MAE), the root mean square error (RMSE) and Willmott's index d [17], for all nine equations estimated by expressions 1, 2 and 3.

$$MAE = \frac{1}{n-1} \sum_{i=1}^n |ELA - OLA| \quad (1)$$

$$RMSE = \sqrt{\frac{1}{n-1} \sum_{i=1}^n (ELA - OLA)^2} \quad (2)$$

$$d = 1 - \left[\frac{\sum_{i=1}^n (ELA_i - OLA_i)^2}{\sum_{i=1}^n (|ELA_i - \overline{OLA}| + |OLA_i - \overline{OLA}|)^2} \right] \quad (3)$$

In which: ELA, are the estimated values of leaf area; OLA, are the observed values of leaf area; \overline{OLA} is the average of the leaf area values observed and n, is the number of sheets sampled for validation, being 100 in the present study.

The criteria used to select the best model that estimated the leaf area of the *Pimenta dioica* in function of L, W and LW were: coefficients of determination (R^2) closest to one; MAE and RMSE close to zero and index d closer to one. Statistical analyzes were performed using software R [18], with scripts developed for the ExpDes.pt version 1.2 data package [19].

3. RESULTS AND DISCUSSION

The high variability found in leaf size, verified by the amplitude and high coefficient of variation (CV) of the length (L), the width (W), product of the length with the width (LW) and the observed leaf area (OLA) (Table 1), is important for the establishment of mathematical models, since it allows to verify the use of sheets of different sizes in the generation of the equations. According to Pezzini et al. [20] this high variability is desired since it indicates the presence of small, medium and large leaves in the sample, resulting in more precise mathematical models that estimate the leaf area with better precision of the different phases of the development of the plant. Thus, the adjustment of equations from models should be from a high range of leaf sizes collected in different parts of the plant [21, 5]

In relation to the leaves used for the validation, the values of the characteristics studied were between the range of the values used for the establishment of the mathematical models, since the extrapolation of the data may lead to mis estimate of the leaf area [22].

Table 1. Descriptive statistics with value minimum, maximum, average, amplitude and coefficient of variation (CV) of the variables: length (L); width (W); product of the length and width (LW) of leaves of *Pimenta dioica*

Variable	Unit	Minimum	Maximum	Average	Amplitude	CV (%)
400 leaves were used for modeling						
L	cm	3.00	14.00	8.44	11.00	23.31
W	cm	2.10	8.70	5.03	6.60	21.84
LW	cm ²	6.72	117.45	44.33	110.73	41.84
OLA	cm ²	4.65	87.43	32.77	82.78	41.91
100 leaves for validation						
L	cm	5.00	13.50	8.65	8.50	20.53
W	cm	2.70	7.30	4.98	4.60	20.86
LW	cm ²	13.50	98.55	44.58	85.05	38.64
OLA	cm ²	9.87	71.47	32.68	61.60	38.65

Among the nine equations adjusted for the different models tested, it was observed that the LW based equations as independent variable presented better prediction in the leaf area estimation, and the three models proposed in this study (linear first degree, quadratic and power) presented similar behavior in relation to the coefficient of determination (R^2). Thus, the models linear first degree ($ELA = 0.342431 + 0.731690(LW)$ and $R^2 = 0.9763$), quadratic ($ELA = 0.9932763 + 0.7004475(LW) + 0.0003180(LW)^2$ and $R^2 = 0.9764$) and power ($ELA =$

$0.7605(LW)^{0.9926}$ and $R^2 = 0.9764$) are the most adequate in the estimation of leaf area of *Pimenta dioica*, taking as base only the coefficient of determination.

However, R^2 should not only be used as a criterion for choosing the best fit of the equation, since its application may lead to an inaccurate estimation of the leaf area, thus requiring the use of adequate techniques to validate the data [23]. Thus, in the present study, the mean absolute error (MAE), the root mean square error (RMSE) and the Willmott d index should be taken into account in the selection of the model that best estimates the leaf area of *Pimenta dioica*.

Table 2. Equation with linear adjustment of first degree, quadratic and power and its respective coefficient of determination (R^2) using the observed leaf area (OLA) as dependent variable, in function of length (L), width (W), product of length with width (LW) of leaves *Pimenta dioica*

Model	Equation	R^2
Linear	$ELA = -22.763 + 6.577 (L)$	0.8881
Linear	$ELA = -26.4029 + 11.7638 (W)$	0.8854
Linear	$ELA = 0.342431 + 0.731690(LW)$	0.9763
Quadratic	$ELA = -3.12129 + 1.66112 (L) + 0.29095(L)^2$	0.8991
Quadratic	$ELA = -8.1467 + 4.2357(W) + 0.7399(W)^2$	0.8931
Quadratic	$ELA = 0.9932763 + 0.7004475(LW) + 0.0003180(LW)^2$	0.9764
Power	$ELA = 0.7386(L)^{1.7610}$	0.8989
Power	$ELA = 1.6298(W)^{1.8363}$	0.8908
Power	$ELA = 0.7605(LW)^{0.9926}$	0.9764

In the validation of the equations from 100 leaves destined randomly for this purpose (Fig. 2) the model using LW was more accurate in the estimation of the leaf area of *Pimenta dioica*. This model presented higher R^2 values in the modeling and validation equation. The MAE and RMSE in the LW based models were smaller than in the equations obtained only when L or W was used individually, and this same behavior was observed by Carvalho et al. [5] studying the leaf area estimation for *Crotalaria juncea*, indicating greater precision in the models that use LW as independent variable. However, the power model stood out, being slightly superior to the linear models of first degree and quadratic confirming by the validation with lower values of MAE and RMSE and index d closest to one.

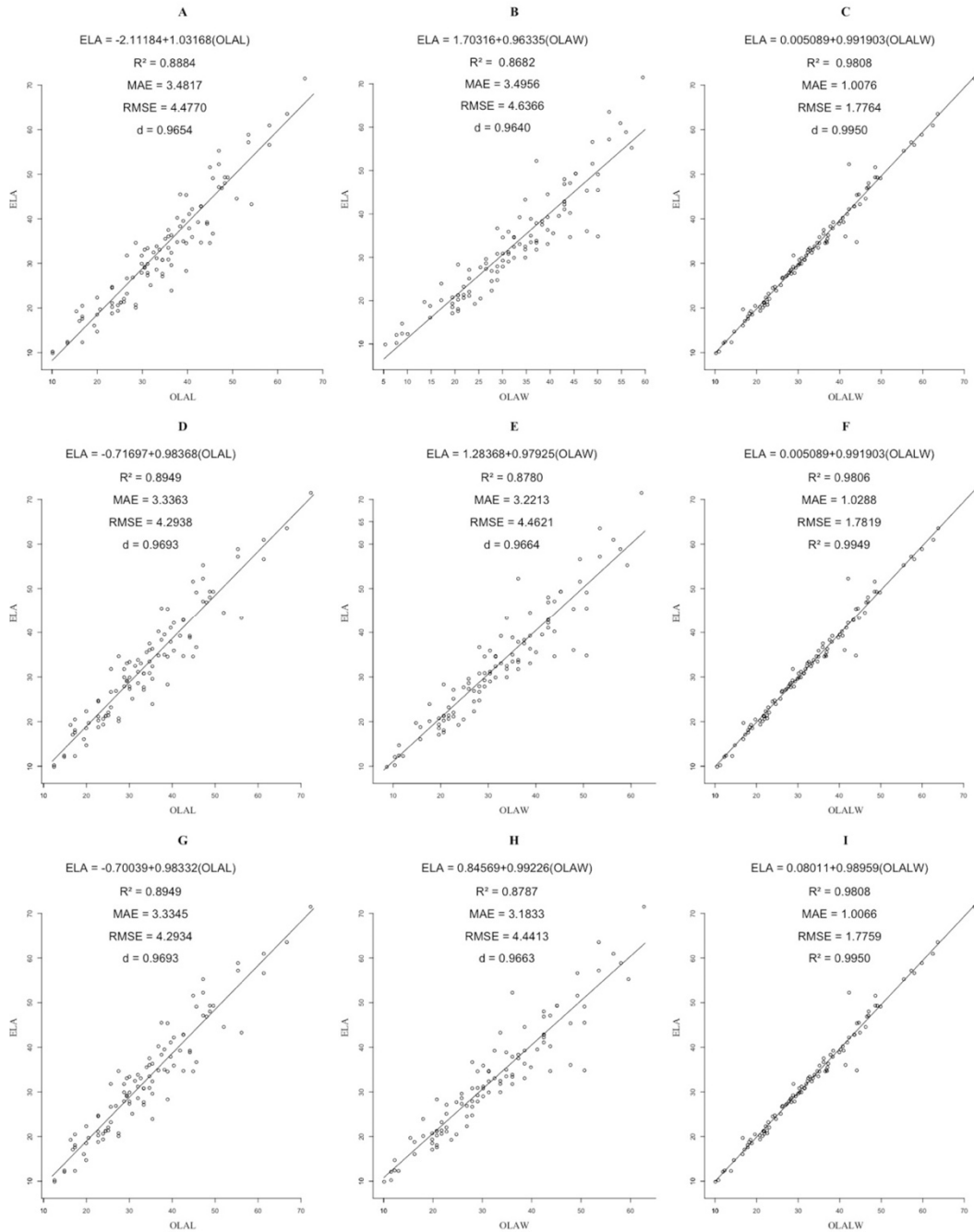


Fig. 2. First degree linear adjustment validation equation and its respective determination coefficient (R^2) using the estimated leaf area (ELA) as the dependent variable obtained by first degree linear modeling equations (A, B and C), quadratic (D, E and F) and power (G, H and I), as a function of leaf length observed (OLAL), width (OLAW) and length product with width (OLALW) e erro absoluto do erro (EAM), raiz quadrada médio do erro (RQME) e índice d Willmott of *Pimenta dioica* leaf

Models obtained with only a linear dimension have been reported as more accurate for some species as *Cucumis melo* L. [6], *Crambe abyssinica* [7] and *Passiflora mucronata* [11]. However, these models in most cases show tendentious estimates of leaf area, especially for small and large leaves [23]. In this way, the combination of measurements presents better adjustment in the determination of the leaf area of *Pimenta dioica*, being more efficient than models based on a single linear measure.

Therefore, the measurement of leaf area of *Pimenta dioica* should be performed based on two non-destructive measurements by the power model equation represented by $ELA = 0.7605(LW)^{0.9926}$ (Fig. 3) where, LW is the length multiplication along the central midrib and the largest leaf width. Studies carried out for several species such as *Jatropha curcas* [4], *Passiflora spp.*[8], *Vitis vinífera* L. [10], *Coffea canephora* [9], *Crotalaria juncea* [5], *Litchi chinensis* Sonn. [13], *Prunus armeniaca* L. [14] and *Annona cherimila* Mill. [12] indicated that the product of the combination of length and width is more predictive in estimating the leaf area of these species.

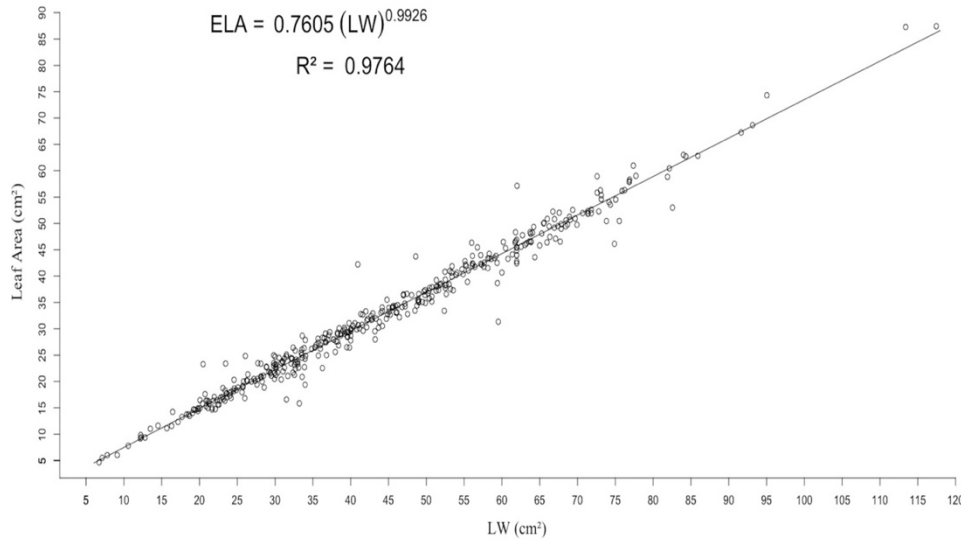


Fig. 3. Equation of power model and determination coefficient (R^2), using the foliar area observed (OLA) as dependent variable, in function of the product of the length and width (LW) of leaves *Pimenta dioica*

4. CONCLUSION

The power model ($ELA = 0.7605(LW)^{0.9926}$ and $R^2 = 0.9764$) based on the product of length and width (LW) is the most indicated, being possible to determine with high degree the accuracy, in a safe way, without the necessity of the destruction of the leaves and with cheap equipment the estimation of the leaf area of *Pimenta dioica*.

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