PROFIT ANALYSIS OF NON IDENTICAL REPAIRABLE UNITS SUBJECT TO TWO PHASE REPAIR OF SYSTEM

Abstract: The paper deals with profit analysis of three non-identical units A, B and C in which either Unit A or one of the units B and C should work for successful functioning of the system. Two types are repairman are available in the system viz. Ordinary and Expert repairman. The expert repairman is called only when system breaks down. Unit A gets priority for repair and is repaired by expert while as Unit B and C are repaired by ordinary repairman if the system doesn't fail totally. The failure time distributions of units-A, B and C are taken as exponential. The distribution of time to repair of units is assumed to be general.

Keywords- Mean Sojourn time, Availability Analysis, Expected Number of Visits by Regular and Expert Repairman, Profit Analysis of system.

I. INTRODUCTION

Several studies on profit analysis of repairable redundant system model have been done in the past. Kapur and Kapur worked on non-identical redundant system under repair and preventive maintenance [6]. Agnihotri and Satsangi analysed dissimilar units with priority based repair and inspection of system model [1]. However, Garage and Kumar investigated complex model under different failures and repair conditions [3]. Gaver studied failure time and availability analysis of a parallel system [4]. Further, Gupta Mahi and Sharma analysed standby system with interrelated repair and failure times [5]. Dhillon discussed availability analysis of a system with standby and commom cause failures [2]. Recently Yusuf and Bala worked on stochastic modeling of parallel units under different failure conditions [7]. In most of the cases the authors assume the independent life times of the units in analyzing the redundant system models. But, in many realistic situations we observe that the rate of failure of an operating unit increases if its redundant unit working in parallel has already failed. This type of situations is visualized in many cases.

II. ASSUMPTIONS AND SYSTEM DESCRIPTION

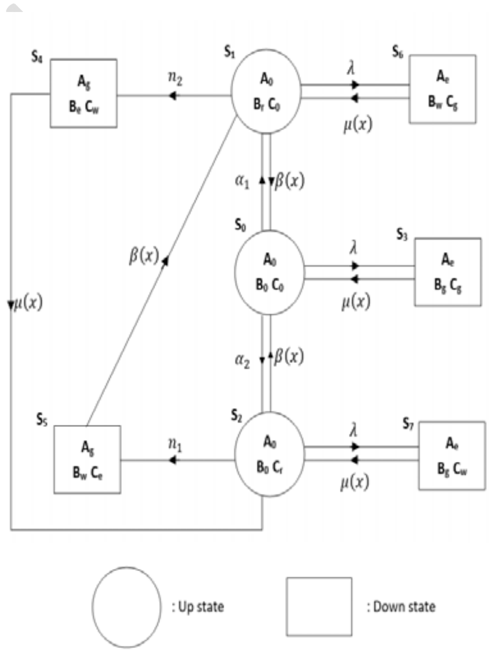
- The system comprises of three non-identical units A, B and C in which either Unit A or one of the units Band C should work for successful functioning of the system.
- There are two types of repairman available in the system: ordinary and expert repairman. The expert repairman is called only when system breaks down
- Two types of repair facility is available to repair failed unit in which A gets priority for repair and is repaired by expert and B and C are repaired by ordinary repairman if the system doesn't fail totally.

- The failure time distributions of unit-A, B and C are taken exponential while as repair time distribution is assumed to be general.

III. NOTATIONS

λ : Failure rate of unit-A.
α_1 : Constant failure rate of unit-B when unit-C is good
α_2 : Constant failure rate of unit-C when unit-B is good.
n_1: Constant failure rate of unit-B when unit-C has failed.
n_2: Constant failure rate of unit-C when unit-B has failed.
$\beta(.)$: Repair distribution of unit-B and C by regular repairman.
$\mu(.)$: Distribution of repair of unit-A, B and C by expert repairman
Ψ_i : Mean sojourn time in state S_i.
$A_0, B_0, C_0 / A_g, B_g, C_g$: Unit-A,B,C is in operative and normal (N) mode / good.
$B_r, C_r / B_w, C_w$: Unit-B,C is in failure (F) mode and under repair / waits for repair by regular repairman.by regular repairman.
A_e, B_e, C_e : Unit-A, B, C is in failure (F) mode and under repair / waits for repair by expert repairman.

TRANSITION DIAGRAM



IV. TRANSITION PROBABILITIES

The various transition probabilities using simple calculations, $Q_{ij}(t) = P[X_{n+1} = j, T_{n+1} - T_n \leq t | X_n = i]$ are obtained. Taking the limit as $t \to \infty$, steady state probabilities is obtained as

$$p_{01} = \frac{\alpha_1}{\alpha_1+\alpha_2+\lambda}$$
$$p_{02} = \frac{\alpha_2}{\alpha_1+\alpha_2+\lambda}$$
$$p_{03} = \frac{\lambda}{\alpha_1+\alpha_2+\lambda}$$
$$p_{10} = \tilde{\beta}(\lambda + n_2)$$
$$p_{14} = \frac{n_2}{\lambda+n_2}[1 - \tilde{\beta}(\lambda + n_2)]$$
$$p_{16} = \frac{\lambda}{\lambda+n_2}[1 - \tilde{\beta}(\lambda + n_2)]$$
$$p_{20} = \tilde{\beta}(\lambda + n_1)$$
$$p_{25} = \frac{n_1}{\lambda+n_1}[1 - \tilde{\beta}(\lambda + n_1)]$$
$$p_{27} = \frac{\lambda}{\lambda+n_1}[1 - \tilde{\beta}(\lambda + n_1)]$$
$$p_{30} = p_{42} = p_{51} = p_{61} = p_{72} = 1$$

From these probabilities, the following condition holds:

$$p_{01} + p_{02} + p_{03} = 1;$$
$$p_{10} + p_{14} + p_{16} = 1;$$
$$p_{20} + p_{25} + p_{27} = 1;$$
$$p_{30} = p_{42} = p_{51} = p_{61} = p_{72} = 1$$

V. MEAN SOJOURN TIME

The mean sojourn time Ψ_i in state S_i is defined as $\Psi_i = E[T_i] = \int_0^\infty P(T_i > t)dt$ are calculated

$$\Psi_0 = \int_0^\infty e^{-(\alpha_1+\alpha_2+\lambda)t}\, dt = \frac{1}{(\alpha_1+\alpha_2+\lambda)}$$

$\Psi_1 = \int_0^\infty e^{-(\lambda+n_2)t}\bar{\beta}(t)dt = \frac{1}{(\lambda+n_2)}[1 - \tilde{\beta}(\lambda+n_2)]$

$\Psi_2 = \int_0^\infty e^{-(\lambda+n_1)t}\bar{\beta}(t)dt = \frac{1}{(\lambda+n_1)}[1 - \tilde{\beta}(\lambda+n_1)]$

$\Psi_3 = \int_0^\infty \bar{\mu}(t)\,dt$
$\Psi_4 = \int_0^\infty \bar{\mu}(t)\,dt$
$\Psi_5 = \int_0^\infty \bar{\mu}(t)\,dt$
$\Psi_6 = \int_0^\infty \bar{\mu}(t)\,dt$
$\Psi_7 = \int_0^\infty \bar{\mu}(t)\,dt$

VII. AVAILABILITY ANALYSIS

Simple probabilistic technique are used to find recurrence relations among availabilities. The availability of the system in steady state will be given by

$A_0 = \lim_{t\to\infty} A_0(t) = \lim_{s\to 0} s\,A_0^*(s) = N_2(0)/D_2'(0)$

where

$N_2(0) = [(1-p_{16})(1-p_{27}) - p_{25}p_{14}]\Psi_0 + [p_{01}(1-p_{27}) + p_{02}p_{25}]\,\Psi_1 + [p_{02}(1-p_{16}) + p_{01}p_{14}]\Psi_2$

and

$D_2'(0) = [(1-p_{16})(1-p_{27}) - p_{25}p_{14}](\Psi_0 + p_{03}\Psi_3) + [p_{01}(1-p_{27}) + p_{02}p_{25}]\Psi_1 + [p_{02}(1-p_{16}) + p_{01}p_{14}]\Psi_2 + [\,p_{01}p_{14}(1-p_{27}) + p_{14}p_{25}p_{02}]\Psi_4 + [p_{02}p_{25}(1-p_{16}) + p_{14}p_{25}p_{01}]\Psi_5 + [p_{01}p_{16}(1-p_{27}) + p_{16}p_{25}p_{02}]\Psi_6 + [p_{14}p_{27}p_{01} + p_{02}p_{27}(1-p_{16})]\Psi_7$

And the mean up time during (0, t] is

$\mu_{up} = \int_0^t A_0(u)du$

VIII. BUSY PERIOD ANALYSIS FOR REGULAR REPAIRMAN

Using simple probabilistic technique and solving the equations, the busy period of repairman in steady state is given by

$B_0 = \lim_{t\to\infty} B_0(t) = \lim_{s\to 0} sB_0^*(s) = \frac{N_3(0)}{D_2'(0)}$

$N_3(0) = [p_{01}(1-p_{27}) + p_{02}p_{25}]\Psi_1 + [p_{02}(1-p_{16}) + p_{01}p_{14}]\Psi_2$

and expected duration in (0,t]

and $D_2'(0)$ is same as in the case of availability.

$\mu_{br}(t) = \int_0^t B_0(u)du$, so that $\mu_{br}^* = B_0^*/s$

IX. BUSY PERIOD ANALYSIS FOR EXPERT REPAIRMAN

Using simple probabilistic technique and solving the equations, the busy period analysis of expert repairman in steady state is given by

$B_0^E = \lim_{t\to\infty} B_0^{E*}(t) = lim_{s\to 0}\, sB_0^{E*}(s) = \frac{N_4(0)}{D_2'(0)}$

where

$N_4(0) = p_{03}[(1-p_{16})(1-p_{27}) - p_{25}p_{14}]\Psi_3 + [p_{01}p_{14}(1-p_{27}) + p_{02}p_{25}p_{14}]\Psi_4 + [p_{02}p_{25}(1-p_{16}) + p_{01}p_{14}p_{25}]\Psi_5 + [p_{01}p_{16}(1-p_{27}) + p_{02}p_{25}p_{16}]\Psi_6 + [p_{02}p_{27}(1-p_{16}) + p_{01}p_{14}p_{27}]\Psi_7$

and $D_2'(0)$ is same as in the case of availability

and the expected duration in (0,t] is given as

$\mu_{be}(t) = \int_0^t B_0^E(u)du$

X. EXPECTED NUMBER OF VISITS BY REGULAR REPAIRMAN

Simple probabilistic techniques are used and solving the equations, the number of visits per unit time in steady state is given by

$V_0(0) = \lim_{t\to\infty} \frac{V_0(t)}{t} = \frac{N_5(0)}{D_2'(0)}$

where
$N_5(0) = (p_{01} + p_{02})[(1 - p_{16})(1 - p_{27}) - p_{25}p_{14}] + [\, p_{01}\,(1 - p_{27}) + p_{02}p_{25}\,]$
$(1 - p_{10}) + [p_{02}(1 - p_{16}) + p_{01}p_{14}](1 - p_{20})$

Now the expected visits by the regular repairman in (0,t]

$\mu_{vr}(t) = \int_0^t V_0(u)du$

X1, EXPECTED NUMBER OF VISITS BY EXPERT REPAIRMAN

Using the definition of $V_i^E(t)$, the recursive relations among $V_i^E(t)$ can be easily developed.

Number of visits in steady state by repairman, is given by

$V_0(0) = \lim_{t\to\infty} \frac{V_0(t)}{t} = \frac{N_6(0)}{D_2'(0)}$

where,

$N_6(0) = p_{03}[(1 - p_{16})(1 - p_{27}) - p_{25}p_{14}] + p_{01}[(1 - p_{27}) + p_{02}p_{25}](1 - p_{10}) + [p_{02}(1 - p_{16}) + p_{01}p_{14}](1 - p_{20})$

$D_2'(0)$ is same as availability analysis

Now the expected number of visits by the expert repairman in (0,t]

$\mu_{ve}(t) = \int_0^t V_0^E(u)du$

XII. PROFIT ANALYSIS

Therefore, profit analysis of the system can be given as:

$$\mathrm{P}_1 = \mathrm{K}_0\mathrm{A}_0 - \mathrm{K}_1\mathrm{B}_0 - \mathrm{K}_2 B_0^E - \mathrm{K}_3\mathrm{V}_0 - \mathrm{K}_4 V_0^E$$

where

$\mathrm{K}_0, \mathrm{K}_1, \mathrm{K}_2, \mathrm{K}_3, \mathrm{K}_4$ = cost associated per unit time for which regular and expert repairman is available, busy and visits of repairman in system respectively.

XIII. STUDY OF SYSTEM BEHAVIOUR

The behavior of availability and profit analysis of the system is studied

TABLE-1: Effect of α_1 and fixed parameters α_2, λ, γ_1, n_1 and n_2 on Availability.

α_1	Availability		
	$\alpha_2 = 0.50, \lambda = 0.34$, $\gamma_1 = 0.08, n_1 = 0.57$, $n_2 = 0.63$,	$\alpha_2 = 0.55, \lambda = 0.37$, $\gamma_1 = 0.07, n_1 = 0.52$, $n_2 = 0.65$,	$\alpha_2 = 0.60, \lambda = 0.41$, $\gamma_1 = 0.09, n_1 = 0.53$, $n_2 = 0.67$,
0.1	0.732143	0.718196	0.693035
0.2	0.729272	0.715739	0.690012
0.3	0.726475	0.713335	0.687058
0.4	0.723747	0.710984	0.684187
0.5	0.721086	0.708683	0.681395
0.6	0.718491	0.706431	0.67868
0.7	0.715958	0.704227	0.676037
0.8	0.713486	0.702068	0.673466
0.9	0.711071	0.699954	0.670962
1.0	0.708713	0.697882	0.668522

TABLE 2: Effect of γ_1 and fixed parameters α_1, $\alpha_2, \lambda, n_1, n_2$ on Availability

γ_1	Availability		
	$\alpha_1 = 0.95, \alpha_2 = 0.40$, $\lambda = 0.38, n_1 = 0.30$, $n_2 = 0.42$,	$\alpha_1 = 0.90, \alpha_2 = 0.43$, $\lambda = 0.32, n_1 = 0.38$, $n_2 = 0.58$,	$\alpha_1 = 0.99, \alpha_2 = 0.52$, $\lambda = 0.34, n_1 = 0.41$, $n_2 = 0.53$,
0.1	0.724638	0.757576	0.746269
0.2	0.72606	0.758407	0.747547
0.3	0.727356	0.759165	0.748686
0.4	0.728454	0.759815	0.749642
0.5	0.729333	0.760343	0.750407
0.6	0.730004	0.760754	0.750998
0.7	0.730493	0.761062	0.751442
0.8	0.730831	0.761283	0.751765
0.9	0.731052	0.761436	0.751995
1.0	0.731181	0.761536	0.752152

TABLE-3: Effect of α_1 and fixed parameters $\alpha_2, \lambda, \gamma_1, n_1, n_2, k_0, k_1, k_2, k_3, k_4$ on Profit.

α_1	Profit		
	$\alpha_2 = 0.31, \lambda = 0.20,$ $\gamma_1 = 0.09, n_1 = 0.37,$ $n_2 = 0.02, k_0 = 1000$ $k_1 = 500, k_2 = 470,$ $k_3 = 300, k_4 = 270$	$\alpha_2 = 0.30, \lambda = 0.18,$ $\gamma_1 = 0.08, n_1 = 0.32,$ $n_2 = 0.03, k_0 = 990$ $k_1 = 400, k_2 = 350,$ $k_3 = 320, k_4 = 250$	$\alpha_2 = 0.28, \lambda = 0.14,$ $\gamma_1 = 0.06, n_1 = 0.22,$ $n_2 = 0.13, k_0 = 950$ $k_1 = 420, k_2 = 380,$ $k_3 = 300, k_4 = 220$
0.1	485.549	528.069	567.779
0.2	426.797	466.676	503.385
0.3	369.586	406.694	440.064
0.4	313.828	348.052	377.774
0.5	259.454	290.692	316.481
0.6	206.402	234.566	256.157
0.7	154.621	179.63	196.777
0.8	104.06	125.843	138.316
0.9	54.6748	73.1687	80.7533
1.0	6.42292	21.5704	24.0666

CONCLUSION:

In Table 1, Availability w.r.t. α_1 and fixed values of parameter α_2, λ, γ_1, n_1, n_2 is studied. Also in Table 3, Profit w.r.t. α_1 and fixed values of parameters $\alpha_2, \lambda, \gamma_1, n_1, n_2, k_0, k_1, k_2, k_3, k_4$ is calculated and in both cases it is analysed that Availability and Profit analysis of the system decreases w.r.t. α_1 (failure rate) keeping other parameters fixed. In Table 2, Availability γ_1 w.r.t. and fixed values of parameter$\alpha_1, \alpha_2, \lambda, n_1, n_2$ is computed. Also In Table 4, Profit w.r.t. γ_1 and fixed values of parameter $\alpha_1, \alpha_2, \lambda, n_1, n_2, k_0, k_1, k_2, k_3, k_4$ is studied and It is seen that Availability and Profit analysis of the system inc oth be sys rat inc uni inc

TABLE-4: Effect of γ_1 and fixed parameters $\alpha_1, \alpha_2, \lambda, n_1, n_2, k_0, k_1, k_2, k_3, k_4$ on Profit

γ_1	Profit		
	$\alpha_1 = 0.48, \alpha_2 = 0.54,$ $\lambda = 0.11, n_1 = 0.24,$ $n_2 = 0.13, k_0 = 1000$ $k_1 = 420, k_2 = 200,$ $k_3 = 300, k_4 = 320$	$\alpha_1 = 0.46, \alpha_2 = 0.52,$ $\lambda = 0.12, n_1 = 0.25,$ $n_2 = 0.14, k_0 = 980$ $k_1 = 400, k_2 = 250,$ $k_3 = 290, k_4 = 310$	$\alpha_1 = 0.44, \alpha_2 = 0.50,$ $\lambda = 0.16, n_1 = 0.39,$ $n_2 = 0.18, k_0 = 970$ $k_1 = 390, k_2 = 280,$ $k_3 = 295, k_4 = 330$
0.1	3.96394	36.25	6.55174
0.2	172.844	180.677	112.459
0.3	264.886	260.275	177.319
0.4	316.422	304.823	217.228
0.5	345.256	329.426	241.494
0.6	361.255	342.648	255.975
0.7	370.024	349.445	264.423
0.8	374.74	352.672	269.229
0.9	377.204	353.964	271.891

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