Original Research Article

Oblique Propagation of Nonlinear Solitary Waves in Magnetized Plasma with Nonextensive Electrons

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ABSTRACT

In this paper, the properties of obliquely propagating nonlinear solitary waves have been investigated theoretically under the effect of magnetic field. The plasma system is considered to be consisting of nonextensively distributed electrons and stationary ions. The nonlinear Korteweg-de-Vries (KdV) equation and its solution have been derived by using reductive perturbation method. Effect of various parameters such as electron nonextensivity, external magnetic field and obliqueness on the properties of solitary waves is investigated numerically. A critical value of nonextensivity is found at which solitary structures transit from negative to positive potential. The numerical results are interpreted graphically. The results may be useful for understanding the wave propagation in laboratory and space plasmas where magnetic field is present.

Keywords: Magnetized plasma, q-nonextensive distribution, reductive perturbation method, nonlinear waves and soliton

1. INTRODUCTION

The nonlinear wave structures have provided a fascinating research field for plasma physics community due to their importance in explaining various laboratory, space and astrophysical atmosphere [1-3]. Nonlinear structures like solitons, shock waves, double layers etc. are observed both in space and laboratory. Out of them, solitons have become a main source of interest for researchers from across the globe owing to their rich physical insight underlying the various nonlinear phenomena. Solitons are stable nonlinear entities that arise due to delicate balance of nonlinearity and dispersion. Nonlinear wave structures in various plasma models and compositions have been investigated for the last half century, both theoretically and observationally [4-8]. There exists a strong magnetic field on the surface of fast rotating neutron stars and in the pulsar magnetosphere [9-10] which has a significant impact on the nonlinear wave propagation. Considering this, an immense interest has been developed in researchers to study nonlinear propagation of ion-acoustic waves in magnetized plasmas [11-16]. The nonlinear propagation of the electron-acoustic (EA) waves in magnetized plasma has been considered by Dubouloz et al [17]. They reported that the electric field spectrum produced by an Electron-acoustic solitary wave (EASW) is not significantly modified by the presence of a magnetic field. Mace and Hellberg [18] studied the influence of the magnetic field on the features of the weakly nonlinear electron-acoustic waves in magnetized plasma. They have predicted the existence of negative potential structures in both magnetized and unmagnetized cases. Devanandhan et al [19] have investigated electron-acoustic solitary waves (EASWs) in two component magnetized plasma and predicted negative solitary potential structures. They further showed that with the increase in magnetic field, the soliton electric field amplitude increases while the soliton width and pulse duration decreases. The properties of small amplitude wave in magnetized plasma are investigated by Pakzad and Javidan [20]. It was found that both rarefactive (negative amplitude) and compressive (positive amplitude) solitons can be propagated and soilton profiles became narrower in stronger magnetic fields.

The deviations of electron populations from their thermodynamic equilibrium have been reported by many space plasma observations. A nonextensive distribution is the most generalized distribution to study the linear and nonlinear properties of solitary waves in different plasma systems, where the non-equilibrium stationary states exist. The nonextensive statistical mechanics has gathered immense attention over the last two decades. This mechanics is based on the deviations of Boltzmann-Gibbs-Shannon (B-G-S) entropy measures first recognized by Renyi [21] subsequently proposed by Tsallis [22]. The Maxwellian distribution in Boltzmann–Gibbs statistics is valid universally for the macroscopic ergodic equilibrium systems. While for systems having long-range interactions, the complete description of the features becomes inadequate with Maxwellian distribution. The parameter q that underpins the generalized entropy of Tsallis. Further, q is associated to the underlying dynamics of the system and measures the amount of its nonextensivity. Generalized entropy of whole is greater (smaller) than the entropies of subsequent parts if q<1 i.e. superextensivity (q>1 i.e. subextensivity). The nonextensive statistics has found applications in a large quantity of astrophysical and cosmological atmospheres such as steller polytropes [23], the solar neutrino problem [24], peculiar velocity distributions of galaxies [25] and systems with long range interactions and also fractal-like space-times. Different types of waves, viz. ion acoustic (IA) waves, electron-acoustic (EA) waves, or dust-acoustic (DA) waves, in nonextensive plasmas are investigated by many researchers considering one or two components to be nonextensive [26-34]. In the present investigation, we aim at studying the obliquely propagating solitary waves in magnetized plasma system with nonextensive distributed electrons. The paper is organized as follows: in Sec. 2, the basic equations governing the plasma dynamics and their analysis are given. In Sec. 3, we present the numerical analysis and discussion of the results. Finally, we conclude the paper in Sec. 4.

2. BASIC EQUATIONS AND NONLINEAR ANALYSIS

Let us consider the homogeneous magnetized plasma containing q-nonextensive electrons and stationary ions. The external static magnetic field is assumed to point in the z-direction i.e. $B = B_0\hat{z}$. The dynamics of the propagation of waves in such magnetized plasma is governed by the following set of normalized equations:

$$\frac{\partial n}{\partial t} + \nabla.(nu) = 0 \qquad (1)$$

$$\frac{\partial u}{\partial t} = (u.\nabla)u = -\nabla\phi - \omega_0(u \times \hat{z}) - \frac{5}{3}\frac{\sigma}{n^{1/3}}\nabla n \qquad (2)$$

$$\nabla^2\phi = n_e - n \qquad (3)$$

where *n* and *u* are the ion number density and ion fluid velocity normalized to equilibrium plasma density n_0 and ion acoustic speed $C_s = (T_e/m)^{1/2}$, T_e is the electron temperature and m is the mass of positively charged ions, respectively. ϕ is the electrostatic wave potential normalized to T_e/e, where e is the magnitude of electron charge and $\sigma = T_i/T_e$ with T_i being the ion temperature. In this plasma model, ion plasma period $\omega_p^{-1} = (m/4\pi n_o e^2)^{1/2}$, the Debye length $\lambda_D = (T_e/4\pi n_o e^2)^{1/2}$ and ion cyclotron frequency is given by $\omega_c = (eB_0/m\omega_p)$. The number density of electron fluid with nonextensive distribution is given by:

$$n_e = \left(1+(q-1)\phi\right)^{\frac{(q+1)}{2(q-1)}} \tag{4}$$

where q is the nonextensivity parameter. The electron distribution reduces to the well-known Maxwell Boltzmann distribution for the extensive limiting case q approaches to 1 [34]. In transformations given by Gardner and Morikawa [35] put $\alpha=1/2$ the stretched coordinates becomes $\xi=\varepsilon^{1/2}(l_x x+l_y y+l_z z-v_0 t)$, $\tau=\varepsilon^{3/2}t$. Here v_0 is the linear phase velocity and ε is a small parameter. l_x, l_y, l_z are the direction cosines of the wave vector with respect to the x, y and z axes respectively. The perturbed quantities are expanded in power series of ε as follows:

$$\begin{aligned}
n &= 1+\varepsilon n^{(1)}+\varepsilon^2 n^{(2)}+\varepsilon^3 n^{(3)}+\ldots\ldots\ldots\ldots \\
u_{x,y} &= 0+\varepsilon^{\frac{3}{2}}u_{x,y}^{(1)}+\varepsilon^2 u_{x,y}^{(2)}+\varepsilon^{\frac{5}{2}}u_{x,y}^{(3)}+\ldots\ldots\ldots \\
u_z &= 0+\varepsilon u_z^{(1)}+\varepsilon^2 u_z^{(2)}+\varepsilon^3 u_z^{(3)}+\ldots\ldots\ldots\ldots \\
\phi &= 0+\varepsilon\phi^{(1)}+\varepsilon^2\phi^{(2)}+\varepsilon^3\phi^{(3)}+\ldots\ldots\ldots\ldots
\end{aligned} \tag{5}$$

Now using the number density of electron fluid given by equation (4), stretching coordinates ξ and τ and the expansions (5) into (1)-(3). Comparing the coefficients of lowest order of ε i.e. $\varepsilon^{3/2}$, we get the linear dispersion relation which is given by the following expression.

$$v_0^2 = \frac{l_z^2}{c_1}\left[1+\frac{5}{3}\sigma c_1\right] \tag{6}$$

where $c_1=(q+1)/2$ and the phase velocity depends upon the ion to electron temperature ratio σ, the strength of nonextensivity q and obliqueness of propagation γ. Mathematical relation (6) shows that phase velocity increases with ion to electron temperature ratio σ and decreases with non-extensive parameter (q) for all ranges of q. The q-dependence of phase velocity occurs from the factor c_1 in the expression (6). Similar kind of behavior has been observed by Sahoo et al [36] and Akhtar et al [28] in their respective researches.

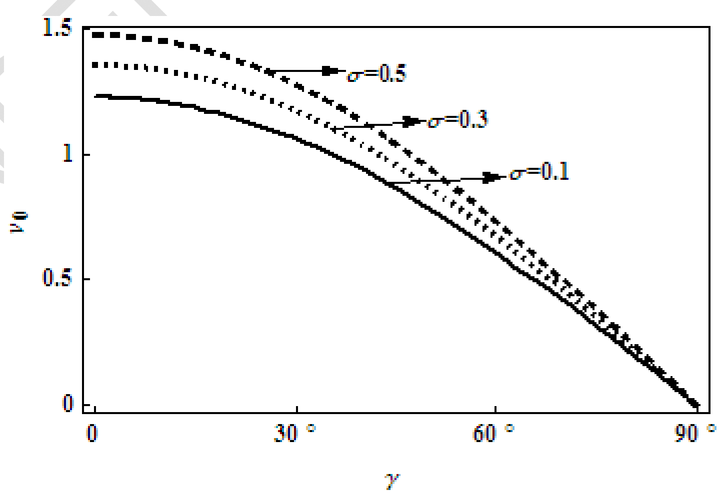

Fig.1. Variation of wave phase velocity (v_0) with angle of propagation (γ) for σ= 0.10 (Solid Line), 0.30 (Dotted Line), 0.50 (Dashed Line) with q= 0.5.

Figure 1 shows the typical variation of the phase velocity v_0 with respect to angle of propagation γ for three different values of ion to electron temperature ratio σ. It is observed that wave phase velocity decreases with angle between the direction of the wave propagation vector k and the external magnetic field B_0. The decrease of v_0 with γ also becomes clear from the expression (6) where $v_0 \propto \sqrt{\cos\gamma}$ and becomes zero for $\gamma = 90^0$. This decreasing trend of v_0 with γ is similar to that observed by Misra and Wang [37]. It is also clear from the figure that phase velocity increases with increase in temperature ratio σ. Further, the wave phase velocity is found to be independent of the magnetic field strength and decreases with nonextensivity q.

Going to the next higher order of ε i.e. ε^2 and by doing algebraic manipulations, we get the following Korteweg-de Vries (KdV) equation (7) in which we have replaced $\phi^{(1)}$ with ϕ for simplicity.

$$\frac{\partial \phi}{\partial \tau} + A\phi\frac{\partial \phi}{\partial \xi} + B\frac{\partial^3 \phi}{\partial \xi^3} = 0 \qquad (7)$$

where A is non-linear and B is dispersion coefficients and are given as:

$$A = l_z\sqrt{c_1}\sqrt{1+\frac{5}{3}\sigma c_1}\left[\frac{3}{2} - \left[\frac{5\sigma}{18} + \frac{c_2}{c_1^3}\right]\frac{c_1}{\left[1+\frac{5}{3}\sigma c_1\right]}\right] \qquad (8)$$

$$B = \frac{1}{2}\frac{l_z}{c_1\sqrt{c_1}\sqrt{1+\frac{5}{3}\sigma c_1}}\left[1 + \left[\frac{1-l_z^2}{\omega_0^2}\right]\left[1+\frac{5}{3}\sigma c_1\right]^2\right] \qquad (9)$$

The stationary solitary wave solution of Eq. (7) is directly given by

$$\phi = \phi_0\left[\sec h\left(\frac{\eta}{\delta}\right)\right]^2 \qquad (10)$$

where the amplitude ϕ_0 and width δ of the soliton are given by $\phi_0 = 3u_0/A$ and $\delta = (4B/u_0)^{1/2}$ and here $\eta = \xi - u_0\tau$. From the expressions of A and B (Eqns. (8) and (9)), it is found that the amplitude of the soliton does not depend on the external magnetic field but depends on the ion and electron temperature ratio σ. On the other hand, the width of the soliton depends on the strength of external magnetic field.

3. RESULTS AND DISCUSSION

We have investigated the effects of q-nonextensive electrons, magnetic field and angle of propagation on the nonlinear wave propagation of solitary waves in magnetized plasma. To describe the nonlinear propagation of the waves, we have derived a KdV equation (7) and obtained solitary wave solution (10). Depending upon the value of nonlinear coefficient A, the solitary wave might be associated with positive or negative potentials. Equation (8) indicates that A is dependent on parameters q, σ, $l_z = \cos(\gamma)$ which define the nature of solitary waves.

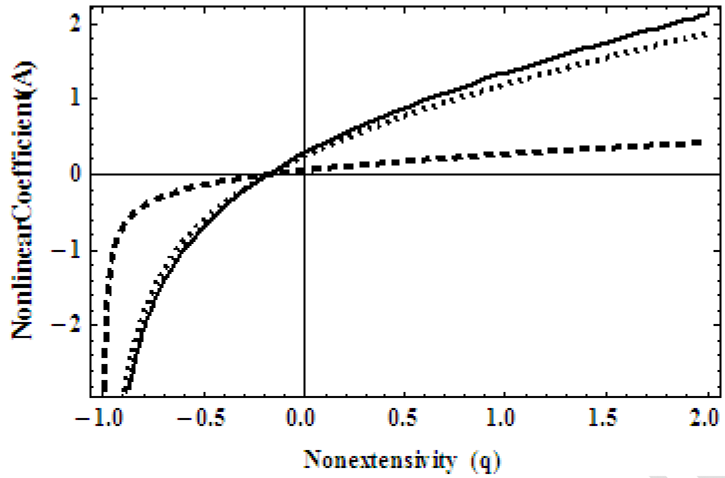

Fig.2. Variation of nonlinear coefficient (A) with nonextensivity q for $\gamma = 0^0$ (Solid Line), 40^0 (Dotted Line), 80^0 (Dashed Line) with $\sigma = 0.50$.

The nonlinear coefficient (A) as a function of nonextensivity (q) is displayed in Figure 2 for three different values of angle γ. A transition from negative to positive potential structures results with increase in non-extensive parameter (q). A negative critical value q_c is obtained for a fixed set of parametric values. We observe that at $q > q_c$, positive (hump shape or commonly known as compressive soliton) solitary waves exist, whereas at $q < q_c$, negative (dip shape or rarefactive solitons) solitary waves exist. It may be further mentioned that the critical value q_c remains same for all values of γ. From Eq. (8), it can be seen that magnitude of external magnetic field (B_0) has no influence on the nonlinear coefficient A. In order to investigate the effect of nonextensive parameter q, solitary wave profiles for $0<q<2$ and $-1<q<0$ have been displayed in Figures 3 and 4 respectively. As q = 1 corresponds to Maxwellian behavior, hence increase in nonextensivity leads to change in the amplitude of

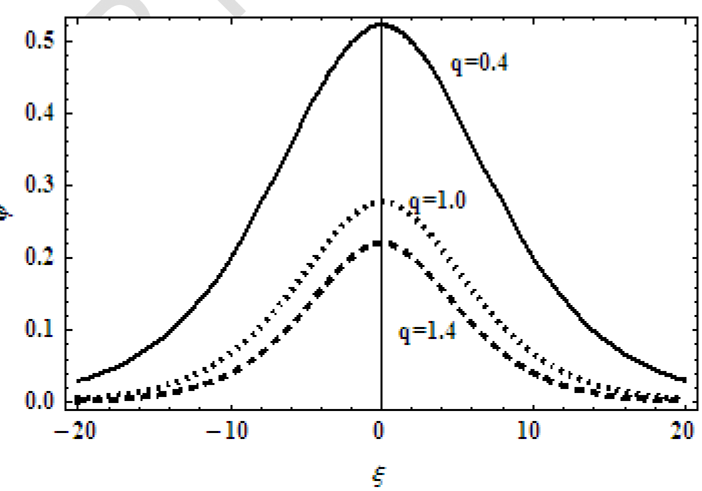

Fig. 3. Variation of soliton solution ϕ as a function of ξ for three different values of q= 0.4 (Solid Line), 1.0 (Dotted Line) and 1.4 (Dashed Line) with $\gamma = 30^0$, $\sigma = 0.20$, $\omega_0 = 0.40$ and $u_0 = 0.10$.

the solitary structures. It can be interpreted that towards the higher value of nonextensivity from the Maxwellian i.e. q=1, peak amplitude decreases while it increases while it increases

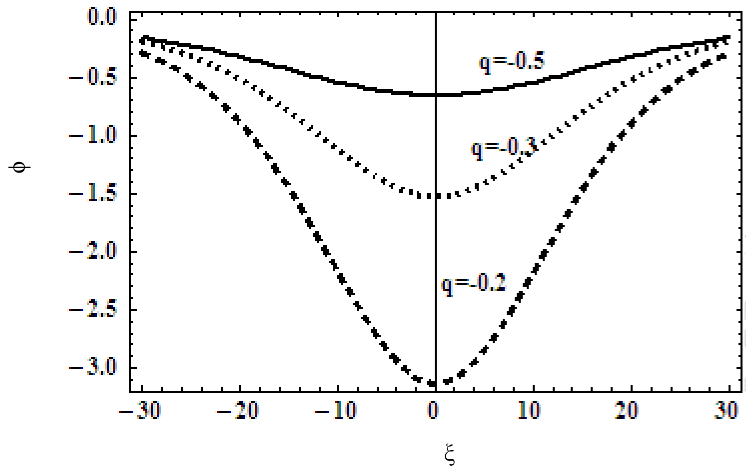

Fig.4. Variation of soliton solution ϕ as a function of ξ for different values of q = - 0.5 (Solid Line), - 0.3 (Dotted Line) and - 0.2 (Dashed Line) with γ = 60^0, σ = 0.20, ω_0 = 0.40 and u_0 = 0.10.

towards lower nonextensivity side (q=0.5). In other words, the amplitude of the positive (negative) KdV soliton is seen to decrease with the increasing values of nonextensivity. This behavior of soliton amplitude is similar to that observed by Pakzad and Javidan [20], Akhtar et al [28] and Sahoo et al [36]. It is observed that width of both the solitons decreases with increase in q, which are in agreement with the results of Pakzad and Javidan [20] and Akhtar et al [28] but contrary to Sahoo et al [36].

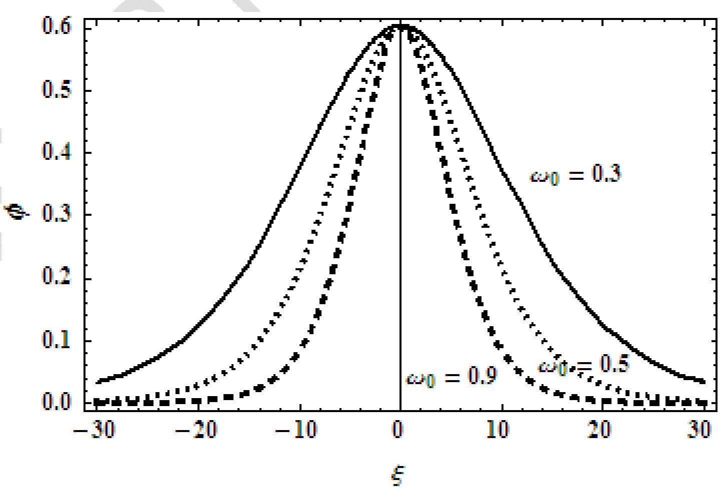

Fig.5. Variation of soliton solution ϕ as a function of ξ for different values of ω_0 = 0.30 (Solid Line), 0.50 (Dotted Line) and 0.90 (Dashed Line) with q = 0.5, γ =50^0, σ = 0.20 and u_0 = 0.10.

The ion cyclotron frequency (ω_0) characterizes the influence of the magnetic field strength on the properties of solitons. Figure 5 shows the variation of the soliton solution ϕ with parameter ξ for three different values of ion cyclotron frequency ω_0. It has been seen that the amplitude remains same while the width of the soliton decreases on applying stronger magnetic field. It is due to the fact that the radius of particle circular motion is reduced due to the greater magnetic field. From expression of amplitude, it is also evident that amplitude of the solitary waves does not influenced by the intensity of the external magnetic field B_0. The width of the soliton is influenced by the ion cyclotron frequency through the coefficient B in the expression of width. Thus the effect of external magnetic field is to make the soliton spiky, which is in agreement with the results obtained by Sahoo et al [36] and Misra and Wang [37]. A similar kind of behavior is observed for the range of nonextensivity $-1<q<0$ (not shown).

Further, the angle of propagation (γ) in the presence of magnetic field also plays an important role in soliton mechanism. In order to understand this behavior, a plot of soliton width δ as a function of ω_0 has been given in figure 6 taking three different values of γ and other parameters are given in caption. From the figure 6, it becomes clear that $\gamma=0^0$ has no effect on the width of soliton as indicated by a straight line (solid line). However, for $\gamma\neq0^0$, the behavior changes and magnetic field start playing its role. Dotted ($\gamma= 40^0$) and dashed lines ($\gamma= 80^0$) show that width starts decreasing with the application of magnetic field. This behavior sets in early for larger γ. Similar result was predicted by Pakzad and Javidan [20]. It is further observed that ion to electron temperature ratio (σ) has stronger influence on the soliton structures. It is evident that higher magnitudes of σ cause significant reduction in the amplitude of the solitary waves. But the soliton width increases with increase in σ, a result

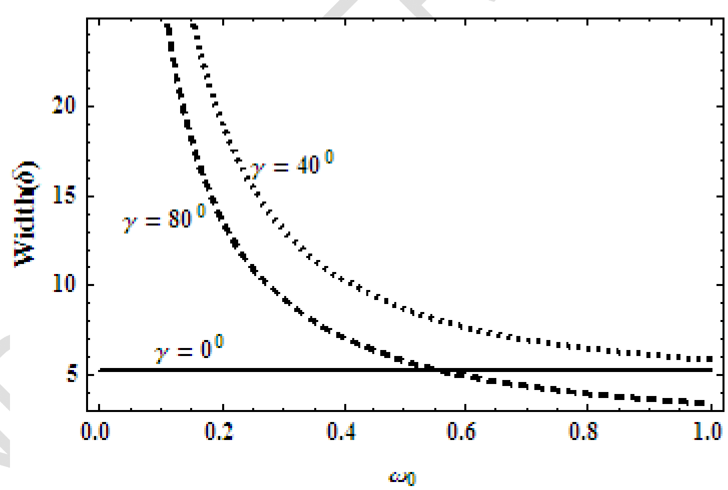

Fig.6. Width of soliton (δ) as a function of ω_0 for different values of $\gamma = 0^0$ (Solid Line), 40^0 (Dotted Line) and 80^0 (Dashed Line) with $\sigma = 0.20$, $\omega_0 = 0.40$ and $u_0 = 0.10$.

which is in agreement with Akhtar et al [28].Figure 7 presents soliton width δ as a function of angle of propagation (γ) with respect to magnetic field with three different values of ion to electron temperature ratio σ. This figure clearly shows that for the lower limit (0^0-50^0) of angle of propagation, the width of the positive (negative) KdV soliton increases. But for the higher limits (50^0-90^0) of angle of propagation, width of both the solitons decreases. Also, it is clear from this figure that as γ approaches to 90^0, the width of the solitary waves approaches to zero. Similar results were obtained by Haider et al [38] and Pakzad and Javidan [20].

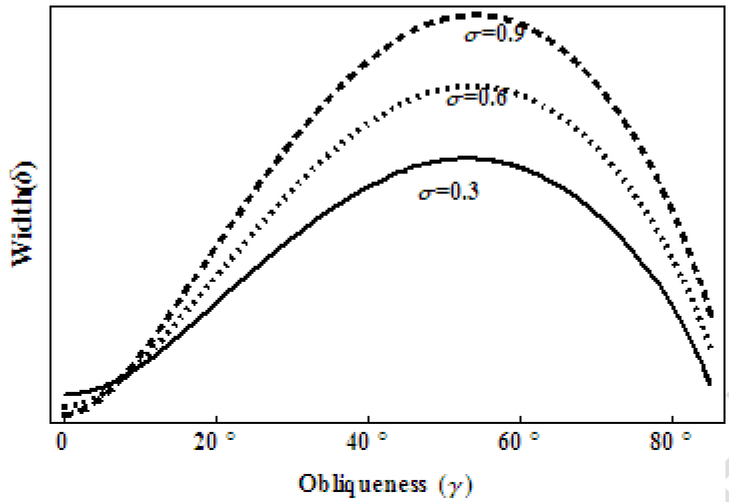

Fig.7. Variation of soliton width δ as a function of γ for different values of σ = 0.30 (Solid Line), 0.60 (Dotted Line) and 0.90 (Dashed Line) with q = 0.5, ω_0 = 0.40 and u_0 = 0.10.

To study the effects of the angle of propagation of the wave amplitude of soliton wave profile has been plotted as a function of γ in Figure 8. For the positive nonextensivity *0<q<2,* the amplitude of the compressive soliton increases with the increase in obliqueness of the wave propagation γ. An opposite trend has been observed for the negative nonextensivity i.e amplitude decreases with the increase in γ (not shown here). It may be mentioned that our results are similar to Pakzad and Javidan [20] for the range *-1<q<0*, however, for the range *0<q<2,*, these results show an opposite trend. Pakzad and Javidan [20] showed that amplitude of the solitary waves decreases with increase in angle of propagation for both the ranges *-1<q<0*, and *0<q<2*.

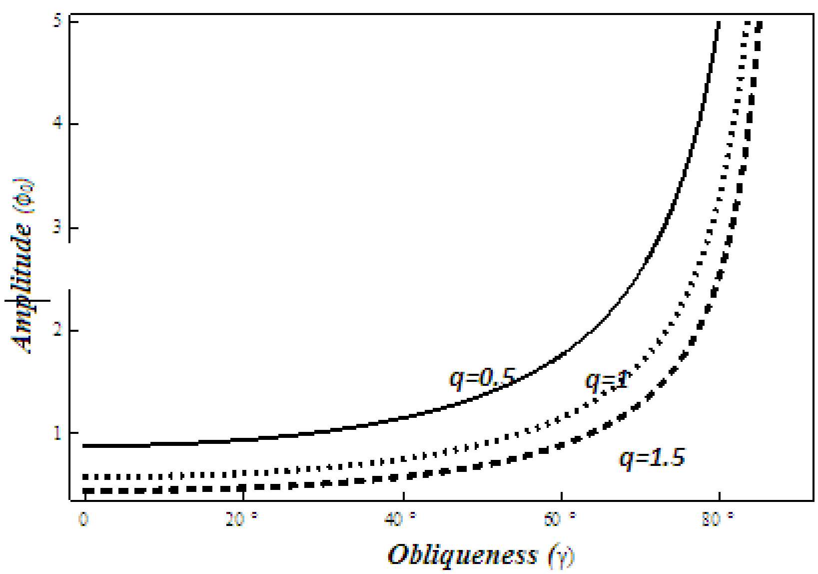

Fig.8. Variation of soliton amplitude $\phi_0$as a function of γ for different values of q = 0.5 (Solid Line), 1.0 (Dotted Line) and 1.5 (Dashed Line) with σ = 0.50, ω_0 = 0.40 and u_0 = 0.30.

. 4. CONCLUSION

The q-nonextensive electrons, strength of magnetic field, ion to electron temperature ratio and obliqueness of wave propagation significantly change the solitary structures. In nut shell, our main findings are summarized below:

(i) Phase velocity of solitary wave decreases with increase in the q-nonextensive parameter and angle of propagation but increases with ion to electron temperature ratio.

(ii) There exists a critical value q_c for a fixed set of parametric values. For $q > q_c$, positive (hump shape) solitary waves exist, whereas at $q < q_c$, negative (dip shape) solitary waves exist.

(iii) The amplitude of the positive (negative) KdV soliton is seen to decrease with the increasing values of nonextensivity. It is also observed that width of both kind of solitons decreases with increase in q.

(iv) The amplitude of solitary structures is independent of the applied magnetic field however the width decreases with magnetic field.

(v) For the lower limit ($0^0 – 50^0$) of angle of propagation, the width of the positive (negative) KdV soliton increases. But for the higher limits ($50^0 – 90^0$) of angle of propagation, width decreases.

(vi) The amplitude of the positive KdV soliton increases with the increase in obliqueness of the wave propagation γ. On the other hand, the amplitude of the negative KdV soliton decreases with the increase in γ.

The present investigation may help us to understanding the study of nonlinear waves in astrophysical plasmas. The comparison of the obtained results with other particle distributions can help us to find better knowledge in plasma physics.

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