

COMPARATIVE STUDY OF FAILURE RATE OF BANK'S ATM: LOG NORMAL DISTRIBUTION APPRAOCH

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ABSTRACT

This research determined time to failure rate and number of successful transaction of selected banks in Nigeria, using Log normal distribution. Transformation technique was applied to the log-normal model to obtain a quadratic equation or polynomial regression that assisted in determining the parameters of the log-normal model. In addition, one-way ANOVA was used to test for equality of the average (or mean) time to failure rate and average number of successful service time of the banks. The research fitted the log-normal models of the banks with the help of SPSS 21 statistical software and the result showed that GT-Bank model has the highest variation of 90.3% for number of successful service time (t), while Fidelity bank model has the highest variation of 56.6% for time of failure rate. The one-way ANOVA result of the number of successful service time (min) showed a significant difference. The Tukey comparison tests showed that GT bank is significant at 5% and 10% from other banks. Hence, the number of successful service time (min) were not the same for all the five banks. However, the one-way ANOVA result of the banks in term of number of Time to Failure (t) (min) showed no significant difference among the five banks.

Key words: Failure rate and successful transaction, Log normal distribution, Transformation, polynomial regression, ANOVA, Tukey comparison tests

1.0 INTRODUCTION

Reliability of an equipment or machine is the probability that it will work and serve well for a specified period of time. This probability is modeled as a lifetime distribution.

Linear regression is a popular statistical tool that has been used successfully in many areas including survival analysis. In survival analysis, a log-transformation of the response variable converts a conventional linear model to an accelerated failure time model, which is an appealing alternative to the Cox (1972) proportional hazards model because of its direct interpretation (cf. Reid 1994).

According to Grambsch, and Therneut (2000), survival analysis deals with time to an event in system. An event can be death in biological system and failure in technical system. Often the time to an event is not known exactly but is known to fall in some interval, this phenomenon is called censoring which could be random or non-informative in analytical

approach. There are three main types of censoring, right, left and interval. If the event occur beyond the end of the study, then the data is right censored. Left censored data occurs when the event is observed, but the exact event time is unknown. Interval censoring means that individuals come in and out of observation and are missing. Most survival analytic method are designed for right censored observation.

The traditional regression methods are not equipped to handle censored data due to the fact that the time to event is restricted and is assumed to have a skewed distribution, and there is need to employ a statistical method that put into consideration the restriction caused by survival data.

One well known and widely applied method is the use of log-normal regression model. It is used to predict response variable or to estimate the mean of the response variable of the original scale for a new set of covariate values. (Haipeng and Zhengyuan. 2007).

In probability theory, a log-normal distribution is a continuous probability distribution of a random variable whose logarithm is normally distributed. Thus, if the random variable x is log-normally distributed, then $y = \ln(x)$ has a normal distribution. Alternatively, if y has a normal distribution, then the exponential function of y , $x = \exp(y)$, has a log-normal distribution. A random variable which is log-normally distributed takes only positive real values. The distribution is occasionally referred to as the Galton distribution.

The log-normal distribution is a statistical distribution of random variable that has a normally distributed logarithm. Log-normal distribution can model a random variable x , where $\log x$ is normally distributed. These distribution, under multiplication and division, are self-replicating. It is useful for modeling data that are skewed with low mean value and large variance. The log-normal distribution has been called the most commonly used life distribution model for any technology application. Stahel *et al.*, (2014).

Several studies have been done in the areas of log-normal distribution, log-normal regression, analysis of variance to test the equality of several mean. Some of such studies are the works of Kenneth (2011), Serfling (2002), Christopher *et al.* (2015), Mehta *et al.* (2017). Osborne, (2010), Rama, (2015), Akbar *et al.* (2016), Maria *et al.* (2016), Stefano and Toscani (2018), and many more.

However, failure of automated teller machines (ATM) in banks is rampant and frustrating. These has cause unnecessary delays in cash withdrawals as well as other activities cash may have been used for. This calls for measures to mitigate the failure rates of ATM and to do this, the time to failure rate needs to be ascertained first and consequently put under control. Hence the study seeks to analyze the time to failure rate and successful transaction of

different banks by fitting their log-normal model of successful transaction before failure of each ATM occurs, fitting a log-normal model of time to failure of automated teller machine of different banks, determining the time to failure rate and number of successful transaction in each bank, and determining the analysis of variance with log-normal data to test the equality of the mean (Average successful transaction) of the different banks.

Section 2 is the scope and limitation of study, section three and four are the research design and methodology respectively. Data analysis and interpretation of results, summary and conclusions are presented in section five, six and seven respectively.

2. SCOPE/ DELIMITATION OF THE STUDY

The study is carried out in five different banks in Port Harcourt ATM randomly selected by the use of simple random sampling technique. Twenty observations of time to failure and number of successful service time before failure were taken from each of the selected Automated Teller Machine (ATM). The nature of failure considered was out of cash and out of network or service and as such may not be extended to other source of failure. Hence the study only covers the following banks in Port Harcourt.

1. First Bank, East/West Road Rumuokoro
2. GT Bank, East/West Road Rumuokoro
3. UBA Bank, East/West Road, Port Harcourt
4. Ecobank, East/West Road Rumuokoro
5. Fidelity Bank, East/West Road Rumuokoro

3. Research Design

Primary data was collected from each of the banks. The number of successful transaction (y), successful service time (t) (min) and time to failure (t) (min) of five banks were obtained as shown below:

Table 3.1: DATA ON FIRST BANK SERVICE RECORD

Sample	No. of Successful Transaction (Y)	Successful Service Time (t) (min)	Time to Failure (t) (min)
1.	6	8	2
2.	10	14	3
3.	8	6	5
4.	12	18	10
5.	2	5	8
6.	5	4	2
7.	13	9	12
8.	4	12	2
9.	23	32	8
10.	9	6	22
11.	2	4	2

12.	11	20	32
13.	31	44	14
14.	29	38	19
15.	17	21	5
16.	14	38	33
17.	16	12	2
18.	20	46	28
19.	11	8	2
20.	13	12	8

Table 3.2: DATA ON GT BANK SERVICE RECORD

Sample	No. of successful Transaction (Y)	Successful Service Time (t) (min)	Time to Failure (t) (min)
1.	6	11	10
2.	10	16	5
3.	8	10	2
4.	15	26	3
5.	18	30	5
6.	12	22	2
7.	24	36	1
8.	18	24	1
9.	28	42	2
10.	3	4	6
11.	14	30	12
12.	19	46	12
13.	21	28	18
14.	34	46	8
15.	20	23	4
16.	27	49	32
17.	32	51	2
18.	13	27	4
19.	16	30	40
20.	17	28	7

Table3.3: DATA ON FIDELITY BANK SERVICE RECORD

Sample	No. of Successful Transaction (Y)	Successful Service Time (t) (min)	Time to Failure (t) (min)
1.	3	4	1
2.	6	8	2
3.	4	3	2
4.	11	9	12
5.	18	22	10
6.	8	15	3
7.	14	14	5
8.	19	22	14
9.	10	18	21
10.	13	24	11
11.	23	32	12
12.	25	37	2
13.	27	41	32
14.	12	20	24
15.	12	22	8
16.	26	41	34
17.	21	40	32
18.	28	52	38
19.	30	58	40
20.	15	28	12

Table 3.4: DATA ON ECOBANK SERVICE RECORD

Sample	No. of Successful Transaction (Y)	Successful Service Time (t) (min)	Time to Failure (t) (min)
1.	2	9	3
2.	4	12	4
3.	12	18	14
4.	6	8	2
5.	11	10	2
6.	17	13	1
7.	8	4	12
8.	22	33	11
9.	18	14	6
10.	28	36	19
11.	8	12	2
12.	24	29	21
13.	9	11	5
14.	30	27	4
15.	19	14	24
16.	16	18	13
17.	23	19	4
18.	35	48	22
19.	14	9	1
20.	17	8	9

Table 3.5: DATA ON UBA BANK SERVICE RECORD

Sample	No. of Successful Transaction (Y)	Successful Service Time (t) (min)	Time to Failure (t) (min)
1.	4	8	2
2.	12	5	1
3.	9	11	3
4.	18	24	20
5.	3	5	2
6.	12	13	1
7.	6	9	2
8.	11	7	10
9.	24	32	8
10.	22	12	2
11.	15	9	27
12.	14	12	4
13.	7	15	12
14.	9	8	6
15.	13	11	9
16.	19	28	5
17.	32	43	30
18.	26	34	5
19.	10	12	3
20.	18	33	14

4. Methodology: The Lognormal Distribution

Let x_1, x_2, \dots, x_n be independent positive random variable such that

$$Tn = \prod_{i=1}^n x_i \quad (1)$$

Then the log of their product is equivalent to the sum of their logs

$$\ln Tn = \sum_{i=1}^n \ln (x_i) \quad (2)$$

The following four assumptions are implicit in the use of the Log-normal distribution.

These are;

1. Stochastically independent
2. Normally distributed
3. Constant variance
4. Mean equal to zero.

Therefore, if $Z = \log(x)$ is normally distributed, then the distribution of x is called a log-normal distribution. The probability density function is given as;

$$f(x) = \frac{1}{t\delta\sqrt{2\pi}} \exp \frac{-[\ln(t)-\mu]^2}{2\delta^2}; \quad \delta^2\mu \in (-\infty, \infty) > 0, t \in (0, \infty) \quad (3)$$

where $x = t$

$$\text{Mean} = \exp(\mu + \delta^2/2)$$

$$\text{Variance} = \exp[(\delta^2) - 1] \exp[2\mu + \delta^2] \text{ and}$$

The cumulative density function is given as

$$\text{CDF} = \frac{1}{2} + \frac{1}{2} \exp \frac{[\ln(t)-\mu]}{\sqrt{2\delta}} \quad (4)$$

Let Equation (3) be written term of t , then

$$f(t) = \frac{1}{t\delta\sqrt{2\pi}} \exp \frac{-(\ln(t) - \mu)^2}{2\delta^2}$$

Taking natural logarithm to base e on both sides

$$\ln f(t) = \ln (t\delta\sqrt{2\pi})^{-1} - \frac{(\ln t - \mu)^2}{2\delta^2}$$

$$\ln f(t) = -\ln (t\delta\sqrt{2\pi}) - \frac{(\ln t - \mu)^2}{2\delta^2}$$

$$\ln f(t) = -\ln(t) - \ln(\delta\sqrt{2\pi}) - \frac{1}{2\delta^2} [(\ln t)^2 - 2\mu \ln(t) + \mu^2]$$

$$\ln f(t) = -\ln(t) - \ln(\delta\sqrt{2\pi}) - \frac{(\ln t)^2}{2\delta^2} + \frac{\mu}{\delta^2} + \frac{\mu}{\delta^2} \ln(t) - \frac{\mu^2}{2\delta^2}$$

Collect like terms

$$\ln f(t) = -(\delta\sqrt{2\pi}) - \frac{\mu^2}{2\delta^2} + \frac{\mu}{\delta^2} \ln(t) - \ln(t) - \frac{(\ln t)^2}{2\delta^2}$$

$$= -\left[\ln(\delta\sqrt{2\pi}) + \frac{\mu^2}{2\delta^2}\right] + \left(\frac{\mu}{\delta^2} - 1\right) \ln(t) - \frac{1}{2\delta^2} (\ln t)^2$$

$$\ln f(t) = \beta_0 + \beta_1 \ln(t) + \beta_2 (\ln(t))^2$$

$$y = \beta_0 + \beta_1 x + \beta_2 x^2 \quad (5)$$

where

$$y = \ln f(t)$$

$$x = \ln(t) \text{ and } x^2 = [\ln(t)]^2$$

Then, from Equation (5), obtain that

$$\beta_0 = -\left[\ln(\delta\sqrt{2\pi}) + \frac{\mu^2}{2\delta^2}\right]$$

$$\beta_1 = \left(\frac{\mu}{\delta^2} - 1\right)$$

$$\beta_2 = -\frac{1}{2\delta^2}$$

Equation (5) is a quadratic regression model or curvilinear model.

4.1 Parameter Estimation using Regression Techniques

A multiple linear regression model with K predictor variable (independent variables) x_1, x_2, \dots, x_k and a response variable (dependent variable) y was a generalization in Equation (5), then, the normal equation matrix can be written as

$$\begin{pmatrix} \beta_0 \\ \beta_1 \\ \beta_2 \end{pmatrix} = \begin{pmatrix} n & \sum x_1 & \sum x_2 \\ \sum x_1 & \sum x_1^2 & \sum x_1 x_2 \\ \sum x_2 & \sum x_1 x_2 & \sum x_2^2 \end{pmatrix} \begin{pmatrix} \sum y \\ \sum x_1 y \\ \sum x_2 y \end{pmatrix} \quad (6)$$

$$\hat{\beta} = (x'x)^{-1}(x'y)$$

where $x_1 = x = \ln(t)$; $x_2 = x^2 = [\ln(t)]^2$; $\hat{\beta}_0$, $\hat{\beta}_1$ and $\hat{\beta}_2$ are the parameter estimate

4.2 One – way ANOVA (Analysis of variance)

The ANOVA is used to measure the difference between variation amongst samples and variation within samples. It is a ratio of the variation between samples to the variation within sample which is based on the F-ratio. The model of the one-way ANOVA is

$$x_{ij} = \mu + x_i + e_{ij} \quad (7)$$

$$y_{ij} \sim N(N_y, \delta_y^2)$$

$$x_i \sim N(0, \delta_x^2)$$

$$e_i \sim N(0, \delta_e^2)$$

where

x_{ij} denote the jth observation from ith treatment

μ is the mean of the observation

x_i is fixed effects of the model

e_{ij} is the error term or the disturbance

4.2.1 Identifying sum of squares

$$\text{Total sum of squares TSS} = \sum_{i=1}^n \sum_{j=1}^n (x_{ij} - \bar{x})^2 \quad (8)$$

$$= \sum_{i=1}^k \sum_{j=1}^n x_{ij}^2 - \frac{T^2}{nk}$$

$$\text{Between sum of squares (BSS)} = \frac{1}{n} \sum_{i=1}^k T_i^2 - \frac{T^2}{nk} \quad (9)$$

$$\text{Within sum of squares (WSS)} = \frac{1}{k} \sum_{j=1}^n T_{.j}^2 - \frac{T^2}{nk} \quad (10)$$

4.2.2 One-Way ANOVA Table

Source of variation	Sum of squares	Degree of freedom	Mean square	F-ratio
Between samples (treatment)	BSS	$k - 1$	$MSB = \frac{BSS}{K-1}$	$\frac{MSB}{MSN}$
Within samples (Error)	WSS	$k (n - 1)$	$MSW = \frac{WSS}{K(n-1)}$	
Total	TSS	($nk - 1$)		

4.2.3 Hypothesis Test

H₀: $\mu_1 = \mu_2 = \mu_3 = \dots \mu_n$ (There is no significant difference in the mean successful transaction of the five different banks).

H₁: Not all the μ 's are equal, $i = 1, 2, \dots, n$ (There is a significant difference in the mean successful transaction of the five different banks).

4.2.4 Sample size

A sample is a subset of population unit selected for the purpose of drawing conclusion about the entire population unit. The sample size was obtained using the Yale formula;

$$n = \frac{N}{1 + Ne^2} = 100/5 = 20 \quad (11)$$

5. DATA ANALYSIS AND INTERPRETATIONS

In section four, Log-normal model parameters were derived for both number of successful service time (t) (min) and time to failure (t) (min). Thus, the parameters of the Log-normal model of five different banks were obtained in section 5.1 below with the help of SPSS 21 statistical software using data in Table 3.1 to Table 3.5 above.

5.1 Parameters Estimates of the Log-Normal Model of Five Banks, Using Regression Techniques

The parameters and R-squared of the five different banks for both number of successful service time (t) (min) and time to failure (t) (min) are in Appendix A and summarised in Table 5.1 below.

Table 5.1: Log-normal Models of Five Banks (Transformed models)

Banks	Log-normal Models		Remarks
	Time of failure (t)	Number of successful Service time (t)	
	Parameters estimates \pm Standard error (R^2) [Regr.ANOVA Values]	Parameters estimates \pm Standard error (R^2) [Regr.ANOVA Values]	
First Bank	$B_0=1.246\pm0.657$ (25.8%) [0.080] $B_1=0.875\pm0.791$ $B_2=-0.130\pm0.197$	$B_0=-0.906\pm1.245$ (67.6%) [0.000] $B_1=1.791\pm1.003$ $B_2=-0.193\pm0.189$	SST
GT-Bank	$B_0=3.091\pm0.346$ (10.0%) [0.409] $B_1=-0.564\pm0.422$ $B_2=-0.155\pm0.113$	$B_0=-0.860\pm0.657$ (90.3%) [0.000] $B_1=0.843\pm0.479$ $B_2=-0.100\pm0.085$	SST
Fidelity Bank	$B_0=1.515\pm0.334$ (56.6%) [0.001] $B_1=0.628\pm0.372$ $B_2=-0.051\pm0.091$	$B_0=3.233\pm1.285$ (89.1%) [0.000] $B_1=0.551\pm0.073$ $B_2=-0.066\pm0.087$	SST
Eco-Bank	$B_0=2.511\pm0.403$ (28.7%) [0.057] $B_1=-0.558\pm0.549$ $B_2=-0.258\pm0.160$	$B_0=1.971\pm1.985$ (47.8%) [0.004] $B_1=-0.383\pm1.483$ $B_2=-0.217\pm0.270$	SST
UBA Bank	$B_0=2.169\pm0.342$ (22.1%) [0.120] $B_1=0.054\pm0.451$ $B_2=-0.065\pm0.128$	$B_0=0.904\pm1.786$ (55.8%) [0.001] $B_1=0.517\pm1.372$ $B_2=-0.035\pm0.252$	SST

Footnote: SST- Successful Service Time

Recall, from Equation (5) that constants

$$\beta_0 = -\left[\ln(\sigma\sqrt{2\pi}) + \frac{\mu^2}{2\sigma^2}\right], \beta_1 = \left(\frac{\mu}{\sigma^2} - 1\right) \text{ and } \beta_2 = -\frac{1}{2\sigma^2}$$

Then, to determine the parameters of the Log-normal models (μ and σ^2)

$$\sigma^2 = -\frac{1}{2\beta_2} \text{ and } \mu = \sigma^2(\beta_1 + 1)$$

Table 5.2: Log-normal Models Parameters of Five Banks (Variance, standard deviation and Mean)

Banks	Log-normal Models	
	Time of failure (t)	Number of successful Service time (t)
	Parameters	Parameters
First Bank	$\sigma^2 = 3.85$ $\sigma = 1.96$ $\mu = 7.21$	$\sigma^2 = 2.59$ $\sigma = 1.61$ $\mu = 7.23$
GT-Bank	$\sigma^2 = 3.23$ $\sigma = 1.80$ $\mu = 1.41$	$\sigma^2 = 5.00$ $\sigma = 2.24$ $\mu = 9.22$
Fidelity Bank	$\sigma^2 = 9.80$ $\sigma = 3.13$ $\mu = 15.96$	$\sigma^2 = 7.58$ $\sigma = 2.75$ $\mu = 11.75$
Eco-Bank	$\sigma^2 = 1.94$ $\sigma = 1.39$ $\mu = 0.86$	$\sigma^2 = 2.30$ $\sigma = 1.52$ $\mu = 1.42$
UBA Bank	$\sigma^2 = 7.69$ $\sigma = 2.77$ $\mu = 8.11$	$\sigma^2 = 14.29$ $\sigma = 3.78$ $\mu = 21.67$

In Table 5.2, the Log-normal model of GT-Bank has the highest variation of 90.3% for number of successful service time (t), while the Log-normal model of Fidelity bank has the highest variation of 56.6% for time of failure rate.

The estimate Log-normal models are

$$f(t) = \frac{1}{t\sigma\sqrt{2\pi}} e^{\left(\frac{\ln t - \mu}{\sigma}\right)^2} = \frac{1}{2.24t\sqrt{2\pi}} e^{\left(\frac{\ln t - 9.22}{2.24}\right)^2} \text{ for number of successful}$$

service time (t)

$$f(t) = \frac{1}{t\sigma\sqrt{2\pi}} e^{\left(\frac{\ln t - \mu}{\sigma}\right)^2} = \frac{1}{3.13t\sqrt{2\pi}} e^{\left(\frac{\ln t - 15.96}{3.13}\right)^2} \text{ for time of failure rate}$$

5.3 One-Way ANOVA Successful Service Time (T) (Min) and Time to Failure (T) (Min) Between the Five Bank

The section is divided into two part, 1) one-way ANOVA successful service time (t) (min) and 2) one-way ANOVA time to failure (t) (min)

Table 5.3. One-Way ANOVA Successful Service Time (T) (Min) of the Five Banks

Successful Service Time (t) (min)

	Sum of Squares	Df	Mean Square	F	Sig. (p-value)
Between Groups	2486.340	4	621.585	3.587	0.009
Within Groups	16460.250	95	173.266		
Total	18946.590	99			

Footnote: the p-value is significant (sig.) at 1%, 5% and 10%

Table 5.3 showed the p-value of the one-way ANOVA is 0.009 which is less than the critical values of 0.05. This implies that there is significant difference among the five banks number of successful service time (t).

Therefore, the LSD and Tukey comparison tests were done to identify the bank that is significant as shown below.

Table 5.4: Multiple Comparison Test for Successful Service Time LSD Multiple Comparisons

Dependent Variable: Successful Service Time (t) (min)

LSD

(I) 1=First bank, 2=GT Bank, 3=Fidelity, 4=Ecobank, 5= UBA	(J) 1=First bank, 2=GT Bank, 3=Fidelity, 4=Ecobank, 5= UBA	Mean Difference (I- J)	Std. Error	Sig.	95% Confidence Interval	
					Lower Bound	Upper Bound
1.00	2.00	-11.10000*	4.16252	0.009	-19.3636	-2.8364
	3.00	-7.65000	4.16252	0.069	-15.9136	.6136
	4.00	.25000	4.16252	0.952	-8.0136	8.5136
	5.00	1.30000	4.16252	0.755	-6.9636	9.5636
2.00	1.00	11.10000*	4.16252	0.009	2.8364	19.3636
	3.00	3.45000	4.16252	0.409	-4.8136	11.7136
	4.00	11.35000*	4.16252	0.008	3.0864	19.6136
	5.00	12.40000*	4.16252	0.004	4.1364	20.6636
3.00	1.00	7.65000**	4.16252	0.069	-.6136	15.9136
	2.00	-3.45000	4.16252	0.409	-11.7136	4.8136
	4.00	7.90000**	4.16252	0.061	-.3636	16.1636
	5.00	8.95000*	4.16252	0.034	.6864	17.2136

4.00	1.00	- .25000	4.16252	0.952	-8.5136	8.0136
	2.00	-11.35000*	4.16252	0.008	-19.6136	-3.0864
	3.00	-7.90000**	4.16252	0.061	-16.1636	.3636
	5.00	1.05000	4.16252	0.801	-7.2136	9.3136
5.00	1.00	-1.30000	4.16252	0.755	-9.5636	6.9636
	2.00	-12.40000*	4.16252	0.004	-20.6636	-4.1364
	3.00	-8.95000*	4.16252	0.034	-17.2136	-.6864
	4.00	-1.05000	4.16252	0.801	-9.3136	7.2136

Footnote: *. The mean difference is significant at the 0.05 level and ** The mean difference is significant at the 0.10 level

Table 5.5: Multiple Comparison Test for Successful Service Time TUKEY HSD Multiple Comparisons

Dependent Variable: Successful Service Time (t) (min)

Tukey HSD

(I) 1=First bank, 2=GT Bank, 3=Fidelity, 4=Ecobank, 5= UBA	(J) 1=First bank, 2=GT Bank, 3=Fidelity, 4=Ecobank, 5= UBA	Mean Difference (I- J)	Std. Error	Sig.	95% Confidence Interval	
					Lower Bound	Upper Bound
1.00	2.00	-11.10000**	4.16252	0.067	-22.6754	.4754
	3.00	-7.65000	4.16252	0.358	-19.2254	3.9254
	4.00	.25000	4.16252	1.000	-11.3254	11.8254
	5.00	1.30000	4.16252	0.998	-10.2754	12.8754
2.00	1.00	11.10000**	4.16252	0.067	-.4754	22.6754
	3.00	3.45000	4.16252	0.921	-8.1254	15.0254
	4.00	11.35000**	4.16252	0.057	-.2254	22.9254
	5.00	12.40000*	4.16252	0.029	.8246	23.9754
3.00	1.00	7.65000	4.16252	0.358	-3.9254	19.2254
	2.00	-3.45000	4.16252	0.921	-15.0254	8.1254
	4.00	7.90000	4.16252	0.326	-3.6754	19.4754
	5.00	8.95000	4.16252	0.208	-2.6254	20.5254
4.00	1.00	-.25000	4.16252	1.000	-11.8254	11.3254
	2.00	-11.35000**	4.16252	0.057	-22.9254	.2254
	3.00	-7.90000	4.16252	0.326	-19.4754	3.6754
	5.00	1.05000	4.16252	0.999	-10.5254	12.6254
5.00	1.00	-1.30000	4.16252	0.998	-12.8754	10.2754
	2.00	-12.40000*	4.16252	0.029	-23.9754	-.8246
	3.00	-8.95000	4.16252	0.208	-20.5254	2.6254
	4.00	-1.05000	4.16252	0.999	-12.6254	10.5254

Footnote: *. The mean difference is significant at the 0.05 level and ** The mean difference is significant at the 0.10 level

Table 5.6: Means for groups in homogeneous subsets (TUKEY HSD Multiple Comparisons)
Successful Service Time (t) (min)

(I) 1=First bank, 2=GT Bank, 3=Fidelity, 4=Ecobank, 5= UBA	(J) 1=First bank, 2=GT Bank, 3=Fidelity, 4=Ecobank, 5= UBA	N	Subset for alpha = 0.05	
			1	2
TukeyHSD ^a	5.00	20	16.5500	
	4.00	20	17.6000	17.6000
	1.00	20	17.8500	17.8500
	3.00	20	25.5000	25.5000
	2.00	20		28.9500
	Sig.		0.208	0.057*

Means for groups in homogeneous subsets are displayed.

a. Uses Harmonic Mean Sample Size = 20.000.

The LSD and Tukey comparison tests in Tables 5.4 and 5.5 showed significant difference among the banks successful service time at 5% and 10%. Then, Tukey HSD mean for groups in homogeneous subsets showed that GT bank is not significant at 5% from others since its p-value 0.057. Hence, the number of successful service time (min) are not the same for all the five banks (or the number of successful service time (min) are the same for other banks except GT bank).

5.4 One-Way ANOVA Time to Failure (T) (Min) of the Five Banks

The section deals with one-way ANOVA time to failure (t) (min) of the banks

Table 5.7. One-Way ANOVA time to failure (t) (min) of the Five Banks

Time to Failure (t) (min)

	Sum of Squares	Df	Mean Square	F	Sig.
Between Groups	757.700	4	189.425	1.828	0.130
Within Groups	9845.050	95	103.632		
Total	10602.750	99			

Table 5.8: Means for groups in homogeneous subsets for Time to Failure (t) (min)

Tukey HSD

Banks	N	Subset for alpha = 0.05
		1
5.00	20	8.3000
2.00	20	8.8000
4.00	20	8.9500
1.00	20	10.9500
3.00	20	15.7500
Sig.		0.149

Means for groups in homogeneous subsets are displayed.

a. Uses Harmonic Mean Sample Size = 20.000.

Table 5.7 showed the p-value of the one-way ANOVA is 0.130 which is greater than the critical values of 0.05 (or 5%), implies that there is no significant difference among the five banks number of Time to Failure (t) (min). Tukey HSD means for groups in homogeneous subsets confirmed no significant difference among the banks time to failure rate, since the p-value of 0.149 which is greater than 5%. Hence, time to failure rate are the same for all the five banks.

6.1 SUMMARY

This research was aimed at determining the time of failure rate and number of successful transaction in five banks using log-normal models. Transformation technique was applied to the log-normal model to obtain a quadratic equation (or polynomial regression) that helped to determine the parameters of the log-normal model. In addition, a one way ANOVA was used to test the equality of the mean (or average) time of failure rate and mean number of successful transaction of the five banks.

6.2 CONCLUSION

The research fitted a log-normal models to the five different randomly selected banks. GT-Bank model has the highest variation of 90.3% for number of successful service time (t), while Fidelity bank model has the highest variation of 56.6% for time of failure rate.

The one-way ANOVA result of the number of successful service time (t) showed a significant difference among the banks. LSD and Tukey comparison tests showed a significant at 5% and 10%, then GT bank was significant at 10% from others banks. Hence, the number of successful service time (min) were not the same for all the five banks (or the number of successful service time (min) were the same for other banks except GT bank).

The one-way ANOVA result of the five banks of number of Time to Failure (t) (min) showed no significant difference among the banks. Tukey HSD means for groups in homogeneous subsets confirm no significant difference among the banks. Hence, time to failure rate are the same for all the five banks.

6.3 RECOMMENDATIONS

This analysis should be carried out using other reliability measures on all the banks in Nigeria.

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APPENDIX A

FIRST BANK SERVICE RECORD LOG-NORMAL MODEL Time to Failure (t) (min)

Model Summary

Model	R	R Square	Adjusted R Square	Std. Error of the Estimate
1	.508 ^a	.258	.170	.69972

a. Predictors: (Constant), LN(Xf)2, LN(Xf)

ANOVA^a

Model		Sum of Squares	df	Mean Square	F	Sig.
1	Regression	2.888	2	1.444	2.949	.080 ^b
	Residual	8.323	17	.490		
	Total	11.211	19			

a. Dependent Variable: LNY

b. Predictors: (Constant), LN(Xf)2, LN(Xf)

Coefficients^a

Model		Unstandardized Coefficients		Standardized Coefficients	t	Sig.
		B	Std. Error	Beta		
1	(Constant)	1.246	.657		1.898	.075
	LN(Xf)	.875	.791	1.175	1.106	.284
	LN(Xf)2	-.130	.197	-.701	-.659	.519

a. Dependent Variable: LNY

LOG-NORMAL MODEL FOR Successful Service Time (t) (min)

Variables Entered/Removed^a

Model	Variables Entered	Variables Removed	Method
1	LN(Xs)2, LN(Xs) ^b	.	Enter

a. Dependent Variable: LNY

b. All requested variables entered.

Model Summary

Model	R	R Square	Adjusted R Square	Std. Error of the Estimate
1	.822 ^a	.676	.638	.46230

a. Predictors: (Constant), LN(Xs)2, LN(Xs)

ANOVA^a

Model		Sum of Squares	df	Mean Square	F	Sig.
1	Regression	7.578	2	3.789	17.728	.000 ^b
	Residual	3.633	17	.214		
	Total	11.211	19			

a. Dependent Variable: LNY

b. Predictors: (Constant), LN(Xs)2, LN(Xs)

Coefficients^a

Model		Unstandardized Coefficients		Standardized Coefficients	t	Sig.
		B	Std. Error	Beta		
1	(Constant)	-.906	1.245		-.727	.477
	LN(Xs)	1.791	1.003	1.870	1.785	.092
	LN(Xs)2	-.193	.189	-1.069	-1.021	.322

a. Dependent Variable: LNY

GT BANK SERVICE RECORD

LOG-NORMAL MODEL Time to Failure (t) (min)

ANOVA^a

Model		Sum of Squares	df	Mean Square	F	Sig.
1	Regression	.657	2	.329	.944	.409 ^b
	Residual	5.922	17	.348		
	Total	6.579	19			

a. Dependent Variable: LNY

b. Predictors: (Constant), LN(Xf)2, LN(Xf)

Coefficients^a

Model		Unstandardized Coefficients		Standardized Coefficients	t	Sig.
		B	Std. Error	Beta		
1	(Constant)	3.091	.346		8.927	.000
	LN(Xf)	-.564	.422	-.999	-1.337	.199
	LN(Xf)2	-.155	.113	-1.023	-1.369	.189

a. Dependent Variable: LNY

LOG-NORMAL MODEL FOR Successful Service Time (t) (min)

ANOVA^a

Model		Sum of Squares	df	Mean Square	F	Sig.
1	Regression	5.939	2	2.969	78.775	.000 ^b
	Residual	.641	17	.038		
	Total	6.579	19			

a. Dependent Variable: LNY

b. Predictors: (Constant), LN(Xs)2, LN(Xs)

Coefficients^a

Model		Unstandardized Coefficients		Standardized Coefficients	t	Sig.
		B	Std. Error	Beta		
1	(Constant)	-.086	.657		-.131	.898
	LN(Xs)	.843	.479	.890	1.762	.096

	LN(Xs)2	-.010	.085	-.060	-.120	.906
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a. Dependent Variable: LNY

FIDELITY BANK SERVICE RECORD

LOG-NORMAL MODEL Time to Failure (t) (min)

Model Summary

Model	R	R Square	Adjusted R Square	Std. Error of the Estimate
1	.752 ^a	.566	.515	.45129

a. Predictors: (Constant), LN(Xf)2, LN(Xf)

ANOVA^a

Model		Sum of Squares	df	Mean Square	F	Sig.
1	Regression	4.515	2	2.257	11.083	.001 ^b
	Residual	3.462	17	.204		
	Total	7.977	19			

a. Dependent Variable: LNY

b. Predictors: (Constant), LN(Xf)2, LN(Xf)

Coefficients

Model		Unstandardized Coefficients		Standardized Coefficients	t	Sig.
		B	Std. Error	Beta		
	(Constant)	1.515	.334		4.534	.000
1	LN(Xf)	.628	.372	1.100	1.687	.110
	LN(Xf)2	-.051	.091	-.364	-.558	.584

a. Dependent Variable: LNY

LOG-NORMAL MODEL FOR Successful Service Time (t) (min)

ANOVA^a

Model		Sum of Squares	Df	Mean Square	F	Sig.
1	Regression	1168.352	2	584.176	69.255	.000 ^b
	Residual	143.398	17	8.435		
	Total	1311.750	19			

a. Dependent Variable: LNY

b. Predictors: (Constant), LN(Xs)2, LN(Xs)

Coefficients

Model		Unstandardized Coefficients		Standardized Coefficients	t	Sig.
		B	Std. Error	Beta		
	(Constant)	3.233	1.285		2.516	.022
1	LN(Xs)	.551	.073	1.026	7.512	.000
	LN(Xs)2	-.066	.087	-.105	-.765	.455

a. Dependent Variable: LNY

ECOBANK SERVICE RECORD

LOG-NORMAL MODEL Time to Failure (t) (min)

Model Summary

Model	R	R Square	Adjusted R Square	Std. Error of the Estimate
1	.535 ^a	.287	.203	.64163

a. Predictors: (Constant), LN(Xf)2, LN(Xf)

ANOVA^a

Model		Sum of Squares	df	Mean Square	F	Sig.
1	Regression	2.814	2	1.407	3.417	.057 ^b
	Residual	6.999	17	.412		
	Total	9.812	19			

a. Dependent Variable: LNY

b. Predictors: (Constant), LN(Xf)2, LN(Xf)

Coefficients

Model		Unstandardized Coefficients		Standardized Coefficients	t	Sig.
		B	Std. Error	Beta		
1	(Constant)	2.511	.403		6.236	.000
	LN(Xf)	-.558	.549	-.794	-1.016	.324
	LN(Xf)2	-.258	.160	-1.259	-1.612	.125

a. Dependent Variable: LNY

LOG-NORMAL MODEL FOR Successful Service Time (t) (min)

Model Summary

Model	R	R Square	Adjusted R Square	Std. Error of the Estimate
1	.691 ^a	.478	.416	.54898

a. Predictors: (Constant), LN(Xs)2, LN(Xs)

ANOVA^a

Model		Sum of Squares	df	Mean Square	F	Sig.
1	Regression	4.689	2	2.345	7.780	.004 ^b
	Residual	5.123	17	.301		
	Total	9.812	19			

a. Dependent Variable: LNY

b. Predictors: (Constant), LN(Xs)2, LN(Xs)

Coefficients

Model		Unstandardized Coefficients		Standardized Coefficients	t	Sig.
		B	Std. Error	Beta		
1	(Constant)	1.971	1.985		.993	.335
	LN(Xs)	-.383	1.483	-.325	-.258	.799
	LN(Xs)2	-.217	.270	-1.012	-.803	.433

a. Dependent Variable: LNY

UBA BANK SERVICE RECORD

LOG-NORMAL MODEL Time to Failure (t) (min)

ANOVA^a

Model		Sum of Squares	df	Mean Square	F	Sig.
1	Regression	1.575	2	.788	2.410	.120 ^b
	Residual	5.556	17	.327		
	Total	7.131	19			

a. Dependent Variable: LNY

b. Predictors: (Constant), LN(Xf)2, LN(Xf)

Coefficients

Model		Unstandardized Coefficients		Standardized Coefficients	t	Sig.
		B	Std. Error	Beta		
1	(Constant)	2.169	.342		6.341	.000
	LN(Xf)	.054	.451	.090	.119	.907
	LN(Xf)2	-.065	.128	-.383	-.508	.618

a. Dependent Variable: LNY

LOG-NORMAL MODEL FOR Successful Service Time (t) (min)

ANOVA^a

Model		Sum of Squares	df	Mean Square	F	Sig.
1	Regression	3.980	2	1.990	10.738	.001 ^b
	Residual	3.151	17	.185		
	Total	7.131	19			

a. Dependent Variable: LNY

b. Predictors: (Constant), LN(Xs)2, LN(Xs)

Coefficients^a

Model		Unstandardized Coefficients		Standardized Coefficients	t	Sig.
		B	Std. Error	Beta		
1	(Constant)	.904	1.786		.506	.619
	LN(Xs)	.517	1.372	.548	.377	.711
	LN(Xs)2	-.035	.252	-.200	-.137	.892

a. Dependent Variable: LNY

APPENDIX B

ANOVA

Successful Service Time (t) (min)

	Sum of Squares	Df	Mean Square	F	Sig.
Between Groups	2486.340	4	621.585	3.587	.009
Within Groups	16460.250	95	173.266		
Total	18946.590	99			

Successful Service Time (t) (min)

(I) 1=First bank, 2=GT Bank, 3=Fidelity, 4=Ecobank, 5=UBA	(J) 1=First bank, 2=GT Bank, 3=Fidelity, 4=Ecobank, 5=UBA	N	Subset for alpha = 0.05	
			1	2
Tukey HSD ^a	5.00	20	16.5500	
	4.00	20	17.6000	17.6000
	1.00	20	17.8500	17.8500
	3.00	20	25.5000	25.5000
	2.00	20		28.9500
	Sig.		.208	.057

Means for groups in homogeneous subsets are displayed.

a. Uses Harmonic Mean Sample Size = 20.000.

Failure Time Rate ANOVA

ANOVA

Time to Failure (t) (min)

	Sum of Squares	Df	Mean Square	F	Sig.
Between Groups	757.700	4	189.425	1.828	.130
Within Groups	9845.050	95	103.632		
Total	10602.750	99			

Time to Failure (t) (min)

Tukey HSD

VAR00010	N	Subset for alpha = 0.05	
		1	
5.00	20	8.3000	
2.00	20	8.8000	
4.00	20	8.9500	
1.00	20	10.9500	
3.00	20	15.7500	
Sig.		.149	

Means for groups in homogeneous subsets are displayed.

a. Uses Harmonic Mean Sample Size = 20.000.