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Open string under the modified Born-Infeld field

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Abstract

In this article we consider the two end-points of the string to be attached to D-brane with the different Born-Infeld field strength $\mathcal{F}$ and calculate the total momenta for the special case.

Keywords: Bloch vector.

1 INTRODUCTION

We consider a string ending on a Dp-brane, the bosonic part of the action is

$$S_B = \frac{1}{4\pi\alpha'} \int_{\Sigma} d^2\sigma \left[g^{\alpha\beta} G_{\mu\nu} \partial_\alpha X^\mu \partial_\beta X^\nu + \epsilon^{\alpha\beta} B_{\mu\nu} \partial_\alpha X^\mu \partial_\beta X^\nu \right] + \frac{1}{2\pi\alpha'} \oint_{\partial\Sigma} d\tau A_i(X) \partial_\tau X^i,$$

where A_i $(i = 0, 1, \cdots, p)$, is the $U(1)$ gauge field living on the Dp-brane [1]; [2]; [3]. The string background is

$$G_{\mu\nu} = \eta_{\mu\nu}, \quad \Phi = \text{constant}, \quad H = dB = 0.$$

Here we use the boundary condition of the action S_B so that we can get more specific equations of motion for a free field and the canonical momentum.

2 Equations of motion and the canonical momentum

Variation of the action yields the equations of motion for a free field

$$\left(\partial_\tau^2 - \partial_\sigma^2\right) X^\mu = 0 \tag{2.1}$$

and the following boundary conditions at $\sigma = 0$:

$$\begin{aligned} &\partial_\sigma X^i + \partial_\tau X^j \mathcal{F}_j^i = 0, && i, j = 0, 1, \cdots, p, \\ &X^a = x_0^a, && a = p+1, \cdots, 9, \end{aligned} \tag{2.2}$$

and at $\sigma = \pi$:

$$\partial_\sigma X^i + \partial_\tau X^j \mathcal{F}'^{\,i}_{\,j} = 0, \qquad i, j = 0, 1, \cdots, p. \tag{2.3}$$

Here

$$\mathcal{F} = B - F \qquad \text{and} \qquad \mathcal{F}' = B' - F'$$

are the modified Born-Infeld field strength and x_0^a, x_0^b are the location of the D-branes. Indices are raised and lowered by $\eta_{ij} = (-, +, \cdots, +)$.

The general solution of X^k to the equations

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of motion in (2.1) is [1]

$$X^k = x_0^k + \left(a_0^k\tau + b_0^k\sigma\right) + c_0^k\sigma\tau + d_0^k\left(\tau^2+\sigma^2\right) + \sum_{n\neq 0}\frac{e^{-in\tau}}{n}\left(ia_n^k\cos n\sigma + b_n^k\sin n\sigma\right) \quad (2.4)$$

and

$$X^a = x_0^a + b^a\sigma + \sum_{n\neq 0}\frac{e^{-in\tau}}{n}a_n^a\sin n\sigma,$$
for $a = p+1,\cdots,9,$

where $x_0^a + \pi b^a$ is the location of the D-brane to which the other end-point of the open string is attached.

Lemma 2.1. *The coefficients c_0^k and d_0^k in Eq. (2.4) are*

(a)

$$c_0^k = \sum_{n\in\mathbb{Z}}(-1)^n\left(in(b_n^l + a_n^j{\mathcal{F}'}^l_j) + \frac{1}{\pi}(b_n^j + a_n^k{\mathcal{F}'}^j_k){\mathcal{F}'}^l_j\right)({M'}^{-1})^k_l,$$

(b)

$$d_0^k = \frac{1}{2}\sum_{n\in\mathbb{Z}}(-1)^{n-1}\left(\frac{1}{\pi}(b_n^l + a_n^j{\mathcal{F}'}^l_j) + in(b_n^j + a_n^k{\mathcal{F}'}^j_k){\mathcal{F}'}^l_j\right)({M'}^{-1})^k_l,$$

where $M'_{ij} = \eta_{ij} - {\mathcal{F}'}^k_i{\mathcal{F}'}_{kj}$.

Proof. By (2.3) and (2.4) we have

$$\begin{aligned}
0 &= \partial_\sigma X^k + \partial_\tau X^j{\mathcal{F}'}^k_j\\
&= \partial_\sigma\Bigg(x_0^k + (a_0^k\tau + b_0^k\sigma) + c_0^k\sigma\tau + d_0^k(\tau^2+\sigma^2) + \sum_{n\neq 0}\frac{e^{-in\tau}}{n}(ia_n^k\cos n\sigma + b_n^k\sin n\sigma)\Bigg)\\
&\quad + \partial_\tau\Bigg(x_0^j + (a_0^j\tau + b_0^j\sigma) + c_0^j\sigma\tau + d_0^j\left(\tau^2+\sigma^2\right) + \sum_{n\neq 0}\frac{e^{-in\tau}}{n}(ia_n^j\cos n\sigma + b_n^j\sin n\sigma)\Bigg){\mathcal{F}'}^k_j\\
&= b_0^k + c_0^k\tau + 2d_0^k\sigma + \sum_{n\neq 0}\frac{e^{-in\tau}}{n}\left(-ina_n^k\sin n\sigma + nb_n^k\cos n\sigma\right)\\
&\quad + \Bigg(a_0^j + c_0^j\sigma + 2d_0^j\tau + \sum_{n\neq 0}\frac{-ine^{-in\tau}}{n} \times\left(ia_n^j\cos n\sigma + b_n^j\sin n\sigma\right)\Bigg){\mathcal{F}'}^k_j\\
&= b_0^k + a_0^j{\mathcal{F}'}^k_j + (c_0^k + 2d_0^j{\mathcal{F}'}^k_j)\tau + (2d_0^k + c_0^j{\mathcal{F}'}^k_j)\sigma\\
&\quad - \sum_{n\neq 0}e^{-in\tau}\left(i\sin n\sigma(a_n^k + b_n^j{\mathcal{F}'}^k_j) - \cos n\sigma(b_n^k + a_n^j{\mathcal{F}'}^k_j)\right)\\
&= (c_0^k + 2d_0^j{\mathcal{F}'}^k_j)\tau + (2d_0^k + c_0^j{\mathcal{F}'}^k_j)\sigma\\
&\quad - \sum_{n\in\mathbb{Z}}e^{-in\tau}\left(i\sin n\sigma(a_n^k + b_n^j{\mathcal{F}'}^k_j) - \cos n\sigma(b_n^k + a_n^j{\mathcal{F}'}^k_j)\right)
\end{aligned}$$

then, now since $\sigma = \pi$ and using the Taylor

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series, this identity can be written as

$$\begin{aligned}
&(c_0^k + 2d_0^j \mathcal{F'}_j^k)\tau + (2d_0^k + c_0^j \mathcal{F'}_j^k)\pi \\
&\quad + \sum_{n\in\mathbb{Z}} e^{-in\tau}(-1)^n (b_n^k + a_n^j \mathcal{F'}_j^k) \\
&= (c_0^k + 2d_0^j \mathcal{F'}_j^k)\tau + (2d_0^k + c_0^j \mathcal{F'}_j^k)\pi \\
&\quad + \sum_{n\in\mathbb{Z}} \left(\sum_{m=0}^{\infty} \frac{(-in\tau)^m}{m!} \right) (-1)^n (b_n^k + a_n^j \mathcal{F'}_j^k) \\
&= \left((2d_0^k + c_0^j \mathcal{F'}_j^k)\pi + \sum_{n\in\mathbb{Z}} (-1)^n (b_n^k + a_n^j \mathcal{F'}_j^k) \right) \\
&\quad + \left(c_0^k + 2d_0^j \mathcal{F'}_j^k \right. \\
&\quad \left. - i \sum_{n\in\mathbb{Z}} (-1)^n n (b_n^k + a_n^j \mathcal{F'}_j^k) \right) \tau \\
&\quad + \sum_{n\in\mathbb{Z}} \left(\sum_{m=2}^{\infty} \frac{(-in)^m}{m!} \right) (-1)^n (b_n^k + a_n^j \mathcal{F'}_j^k) \tau^m \\
&= 0.
\end{aligned}$$

Thus the above identical equation about τ shows that

$$(2d_0^k + c_0^j \mathcal{F'}_j^k)\pi + \sum_{n\in\mathbb{Z}} (-1)^n (b_n^k + a_n^j \mathcal{F'}_j^k) = 0, \tag{2.5}$$

$$c_0^k + 2d_0^j \mathcal{F'}_j^k - i \sum_{n\in\mathbb{Z}} (-1)^n n (b_n^k + a_n^j \mathcal{F'}_j^k) = 0, \tag{2.6}$$

and

$$\sum_{n\in\mathbb{Z}} n^m (-1)^n (b_n^k + a_n^j \mathcal{F'}_j^k) = 0 \quad \text{for } m \geq 2.$$

(a) From (2.5) we can easily obtain

$$\begin{aligned}
&2d_0^j \mathcal{F'}_j^k + c_0^l \mathcal{F'}_l^j \mathcal{F'}_j^k \\
&\qquad + \frac{1}{\pi} \sum_{n\in\mathbb{Z}} (-1)^n (b_n^j + a_n^l \mathcal{F'}_l^j) \mathcal{F'}_j^k = 0.
\end{aligned} \tag{2.7}$$

Subtracting (2.7) from (2.6) we get

$$\begin{aligned}
&c_0^k - i \sum_{n\in\mathbb{Z}} (-1)^n n (b_n^k + a_n^j \mathcal{F'}_j^k) \\
&\quad - c_0^l \mathcal{F'}_l^j \mathcal{F'}_j^k \\
&\quad - \frac{1}{\pi} \sum_{n\in\mathbb{Z}} (-1)^n (b_n^j + a_n^l \mathcal{F'}_l^j) \mathcal{F'}_j^k = 0
\end{aligned}$$

and

$$\begin{aligned}
&\sum_{n\in\mathbb{Z}} (-1)^n \left(in (b_n^k + a_n^j \mathcal{F'}_j^k) \right. \\
&\qquad \left. + \frac{1}{\pi} (b_n^j + a_n^l \mathcal{F'}_l^j) \mathcal{F'}_j^k \right) \\
&= c_0^k - c_0^l \mathcal{F'}_l^j \mathcal{F'}_j^k \\
&= c_0^l \eta_l^k - c_0^l \mathcal{F'}_l^j \mathcal{F'}_j^k \\
&= c_0^l (\eta_l^k - \mathcal{F'}_l^j \mathcal{F'}_j^k) \\
&= c_0^l M'^k_l
\end{aligned}$$

so

$$\begin{aligned}
c_0^l = &\sum_{n\in\mathbb{Z}} (-1)^n \left(in (b_n^k + a_n^j \mathcal{F'}_j^k) \right. \\
&\qquad \left. + \frac{1}{\pi} (b_n^j + a_n^l \mathcal{F'}_l^j) \mathcal{F'}_j^k \right) (M'^{-1})_k^l.
\end{aligned}$$

(b) In a similar manner, by (2.6) we have

$$\begin{aligned}
&c_0^j \mathcal{F'}_j^k + 2d_0^l \mathcal{F'}_l^j \mathcal{F'}_j^k \\
&\qquad - i \sum_{n\in\mathbb{Z}} (-1)^n n (b_n^j + a_n^l \mathcal{F'}_l^j) \mathcal{F'}_j^k = 0.
\end{aligned} \tag{2.8}$$

After dividing (2.5) by π, we subtract (2.8) from (2.5) and obtain

$$\begin{aligned}
&2d_0^k + \frac{1}{\pi} \sum_{n\in\mathbb{Z}} (-1)^n (b_n^k + a_n^j \mathcal{F'}_j^k) \\
&\quad - 2d_0^l \mathcal{F'}_l^j \mathcal{F'}_j^k \\
&\quad + i \sum_{n\in\mathbb{Z}} (-1)^n n (b_n^j + a_n^l \mathcal{F'}_l^j) \mathcal{F'}_j^k = 0
\end{aligned}$$

and

$$\begin{aligned}
&\frac{1}{2} \sum_{n\in\mathbb{Z}} (-1)^{n-1} \left(\frac{1}{\pi} (b_n^k + a_n^j \mathcal{F'}_j^k) \right. \\
&\qquad \left. + in (b_n^j + a_n^l \mathcal{F'}_l^j) \mathcal{F'}_j^k \right) \\
&= d_0^k - d_0^l \mathcal{F'}_l^j \mathcal{F'}_j^k \\
&= d_0^l \eta_l^k - d_0^l \mathcal{F'}_l^j \mathcal{F'}_j^k \\
&= d_0^l (\eta_l^k - \mathcal{F'}_l^j \mathcal{F'}_j^k) \\
&= d_0^l M'^k_l
\end{aligned}$$

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so

$$d_0^l = \frac{1}{2}\sum_{n\in\mathbb{Z}}(-1)^{n-1}\Bigg(\frac{1}{\pi}(b_n^k + a_n^j {\mathcal{F}'}^k_j) + in(b_n^j + a_n^l {\mathcal{F}'}^j_l){\mathcal{F}'}^k_j\Bigg)(M'^{-1})^l_k.$$

□

Remark 2.1. Let us consider the two end-points of the string to be attached to D-brane with the same $\mathcal{F}$ field. Then we can see that

$$b_n^k + a_n^j \mathcal{F}_j^k = 0, \qquad \text{for all } n$$

in [1]. Applying this fact to Lemma 2.1, we simply have $c_0^k = d_0^k = 0$, which equates the result obtained in [1].

Now the canonical momentum is given by

$$2\pi\alpha' P^k(\tau,\sigma) = \partial_\tau X^k + \partial_\sigma X^j\left(\frac{\mathcal{F}_j^k + {\mathcal{F}'}^k_j}{2}\right)$$

So by (2.4), we note that

$$\begin{aligned}
&2\pi\alpha' P^k(\tau,\sigma)\\
&= \partial_\tau\Big(x_0^k + a_0^k\tau + b_0^k\sigma + c_0^k\sigma\tau + d_0^k(\tau^2+\sigma^2)\\
&\qquad + \sum_{n\neq 0}\frac{e^{-in\tau}}{n}(ia_n^k\cos n\sigma + b_n^k\sin n\sigma)\Big)\\
&\quad + \partial_\sigma\Big(x_0^j + a_0^j\tau + b_0^j\sigma + c_0^j\sigma\tau + d_0^j(\tau^2+\sigma^2)\\
&\qquad + \sum_{n\neq 0}\frac{e^{-in\tau}}{n}(ia_n^j\cos n\sigma + b_n^j\sin n\sigma)\Big)\\
&\qquad \times\Big(\frac{\mathcal{F}_j^k + {\mathcal{F}'}^k_j}{2}\Big)\\
&= a_0^k + c_0^k\sigma + 2d_0^k\tau\\
&\quad - i\sum_{n\neq 0}e^{-in\tau}(ia_n^k\cos n\sigma + b_n^k\sin n\sigma)\\
&\quad + \Big(b_0^j + c_0^j\tau + 2d_0^j\sigma - \sum_{n\neq 0}e^{-in\tau}(ia_n^j\sin n\sigma\\
&\quad - b_n^j\cos n\sigma)\Big)\Big(\frac{\mathcal{F}_j^k + {\mathcal{F}'}^k_j}{2}\Big)\\
&= \Big(a_0^k + \frac{b_0^j(\mathcal{F}_j^k + {\mathcal{F}'}^k_j)}{2}\Big)\\
&\quad + \Big(c_0^k + d_0^j(\mathcal{F}_j^k + {\mathcal{F}'}^k_j)\Big)\sigma\\
&\quad + \Big(2d_0^k + \frac{c_0^j(\mathcal{F}_j^k + {\mathcal{F}'}^k_j)}{2}\Big)\tau\\
&\quad - \sum_{n\neq 0}e^{-in\tau}\Big\{i\Big(b_n^k + \frac{a_n^j(\mathcal{F}_j^k + {\mathcal{F}'}^k_j)}{2}\Big)\sin n\sigma\\
&\quad - \Big(a_n^k + \frac{b_n^j(\mathcal{F}_j^k + {\mathcal{F}'}^k_j)}{2}\Big)\cos n\sigma\Big\}\\
&= \Big(c_0^k + d_0^j(\mathcal{F}_j^k + {\mathcal{F}'}^k_j)\Big)\sigma\\
&\quad + \Big(2d_0^k + \frac{c_0^j(\mathcal{F}_j^k + {\mathcal{F}'}^k_j)}{2}\Big)\tau\\
&\quad - \sum_{n\in\mathbb{Z}}e^{-in\tau}\Big\{i\Big(b_n^k + \frac{a_n^j(\mathcal{F}_j^k + {\mathcal{F}'}^k_j)}{2}\Big)\sin n\sigma\\
&\quad - \Big(a_n^k + \frac{b_n^j(\mathcal{F}_j^k + {\mathcal{F}'}^k_j)}{2}\Big)\cos n\sigma\Big\}.
\end{aligned} \tag{2.9}$$

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Theorem 2.2. *If $\mathcal{F}' = -\mathcal{F}$, the total momenta*

$$P^k_{tot}(\tau) = \frac{\pi}{4\alpha'}c^k_0 + \frac{1}{\alpha'}d^k_0\tau + \frac{1}{2\alpha'}a^k_0 + \frac{1}{2\pi\alpha'}\sum_{n\neq 0}\frac{ie^{-in\tau}}{n}\left((-1)^n - 1\right)b^k_n,$$

where

$$c^k_0 = \frac{i}{2}\sum_{n\in\mathbb{Z}} n\left((1+(-1)^n)b^k_n + (1-(-1)^n)a^j_n\mathcal{F}^k_j\right),$$

$$d^k_0 = \frac{i}{4}\sum_{n\in\mathbb{Z}} n\left((1+(-1)^n)b^j_n\mathcal{F}^k_j + (1-(-1)^n)a^i_n\mathcal{F}^j_i\mathcal{F}^k_j\right) - \frac{1}{2\pi}\sum_{n\in\mathbb{Z}}(-1)^n\left(b^k_n - a^j_n\mathcal{F}^k_j\right).$$

Proof. By the condition $\mathcal{F}' = -\mathcal{F}$ and (2.9), we have

$$2\pi\alpha' P^k(\tau,\sigma) = c^k_0\sigma + 2d^k_0\tau - \sum_{n\in\mathbb{Z}} e^{-in\tau}\left(ib^k_n\sin n\sigma - a^k_n\cos n\sigma\right)$$

and so

$$\begin{aligned} P^k_{tot}(\tau) &= \int_0^\pi d\sigma P^k(\tau,\sigma) \\ &= \frac{1}{2\pi\alpha'}\int_0^\pi d\sigma\left(c^k_0\sigma + 2d^k_0\tau - \sum_{n\in\mathbb{Z}} e^{-in\tau}\left(ib^k_n\sin n\sigma - a^k_n\cos n\sigma\right)\right) \\ &= \frac{1}{2\pi\alpha'}\left(\frac{\pi^2}{2}c^k_0 + 2\pi d^k_0\tau + a^k_0\pi + \sum_{n\neq 0}\frac{ie^{-in\tau}}{n}\left((-1)^n - 1\right)b^k_n\right) \\ &= \frac{\pi}{4\alpha'}c^k_0 + \frac{1}{\alpha'}d^k_0\tau + \frac{1}{2\alpha'}a^k_0 + \frac{1}{2\pi\alpha'}\sum_{n\neq 0}\frac{ie^{-in\tau}}{n}\left((-1)^n - 1\right)b^k_n. \end{aligned}$$

And using the boundary condition (2.2) and Taylor series for τ we obtain

$$\sum_{n\in\mathbb{Z}}(b^k_n + a^j_n\mathcal{F}^k_j) = 0, \tag{2.10}$$

$$c^k_0 + 2d^j_0\mathcal{F}^k_j - i\sum_{n\in\mathbb{Z}} n(b^k_n + a^j_n\mathcal{F}^k_j) = 0, \tag{2.11}$$

and

$$\sum_{n\in\mathbb{Z}} n^m(b^k_n + a^j_n\mathcal{F}^k_j) = 0 \quad \text{for } m \geq 2.$$

Also applying the assumption $\mathcal{F}' = -\mathcal{F}$ to Eqs. (2.5) and (2.6), we have

$$(2d^k_0 - c^j_0\mathcal{F}^k_j)\pi + \sum_{n\in\mathbb{Z}}(-1)^n(b^k_n - a^j_n\mathcal{F}^k_j) = 0, \tag{2.12}$$

$$c^k_0 - 2d^j_0\mathcal{F}^k_j - i\sum_{n\in\mathbb{Z}}(-1)^n n(b^k_n - a^j_n\mathcal{F}^k_j) = 0. \tag{2.13}$$

Then by (2.11) and (2.13) we deduce that

$$2c^k_0 - i\sum_{n\in\mathbb{Z}} n(b^k_n + a^j_n\mathcal{F}^k_j) - i\sum_{n\in\mathbb{Z}}(-1)^n n(b^k_n - a^j_n\mathcal{F}^k_j) = 0$$

so

$$c^k_0 = \frac{i}{2}\sum_{n\in\mathbb{Z}} n\left((1+(-1)^n)b^k_n + (1-(-1)^n)a^j_n\mathcal{F}^k_j\right).$$

Finally, substituting the above c^k_0 into (2.12) we complete the proof. □

3 CONCLUSION

Here we focus on the coefficients c^k_0 and d^k_0 existing in the general solution of X^k explaining the equations of brane motion given by [1], i.e.,

$$X^k = x^k_0 + \left(a^k_0\tau + b^k_0\sigma\right) + c^k_0\sigma\tau + d^k_0\left(\tau^2 + \sigma^2\right) + \sum_{n\neq 0}\frac{e^{-in\tau}}{n}\left(ia^k_n\cos n\sigma + b^k_n\sin n\sigma\right)$$

and obtain coefficients value.

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