Mathematical modeling of typhoid fever disease incorporating unprotected humans in the spread dynamics.

Julia Wanjiku Karunditu

Catholic University of East Arica

Department of Mathematics and Actuarial Science,

Box 62157-00200, Nairobi Kenya.

DR George Kimathi

Catholic university of Eastern Africa,

Department of Mathematics and Actuarial science,

Box 62157-00200, Nairobi-Kenya.

DR Shaibu Osman

Catholic university of Eastern Africa,

Department of Mathematics and Actuarial science,

Box 62157-00200, Nairobi-Kenya.

Abstract

A deterministic mathematical model of typhoid fever incorporating unprotected humans is formulated in this study and employed to study local and global stability of equilibrium points. The model incorporating Susceptible, unprotected, Infectious and Recovered humans which are analyzed mathematically and also result into a system of ordinary differential equations which are used for interpretations and comparison to the qualitative solutions in studying the spread dynamics of typhoid fever. Jacobian matrix was considered in the study of local stability of disease free equilibrium point and Castillo-Chavez approach used to determine global stability of disease free equilibrium point. Lyapunov function was used to study global stability of endemic equilibrium point. Both equilibrium points (DFE and EE) were found to be local and globally asymptotically stable. This means that the disease will be dependent on numbers of unprotected humans and other factors who contributes positively to the transmission dynamics.

.**Key words**: Basic reproduction number; Invariant region; Positivity of solution; Mathematical model; Unprotected humans; Disease Free Equilibrium; Endemic equilibrium point; Global stability.

1 Introduction

Typhoid fever is an endemic infectious disease that can be classified as enteritis diseases, and it is caused by presence of bacterium called *Salmonella Typhi* in the human body [15]. The disease is a common infectious disease in human beings and it's transmitted through food and water contaminated with faeces and urine of an infected person [3, 11, 12 and 17].The disease is endemic in developing countries where it continuously causes illness and death. This is brought about by unsafe water supply, poor food hygiene and also wanting environmental sanitation. Incubation period is 7 to 14 days [3, 17]. General symptoms and effects of typhoid are the following; headache, stomachache, Joint ache, backache, muscle pain, loss of appetite, vomiting, diarrhea, rashes and fever. According to World Health Organization an estimated 17 million illness cases of typhoid fever are reported per year worldwide resulting to 0.6 million deaths annually [4,5]. A lot of research has so far been done the last two decades, different researchers have come up with different mathematical models for instance[1,2,8]on other diseases among others and [3,5,6,7,1112,15,17] for typhoid fever . Authors in [6] model considers four compartments: susceptible, infective, asymptomatic carries and recovered. According to their model they investigated that infectious humans was responsive to treatment as well changes in levels of carries. Treatment was also very effective on reduction of typhoid cases when effect of carries was not much and treatment not effective if effect of carriers was high.

Authors in [7] developed a PSITR model, which consisted of five compartments: vaccinated, Susceptible, Infective, treated and recovered. There is an extension to the work done by author [11 and 12].According to the author their model does not considers recovered compartment as an important key in the spread dynamics of typhoid fever and had put into consideration that all the treated humans recovered then with time loses immunity. They laid their interest in vaccination against typhoid and according to their model findings typhoid disease spread largely depends on contact rate with the infectious human .Therefore this lead to their suggestion that this typhoid burden can be reduced by reducing the effective contact rate with infectious humans .Further author in [11 and 12] incorporated protection against typhoid using differential equation, it was a SPIT model which represent susceptible, protected, infected and finally treated. Their model was based on assumption that once treated no more disease. They found out that with successful protection, the infection decreases over time; however with low protection infection is high and will persist in the population.

In reference to [11,12] the author considered global stability of equilibrium points of typhoid fever model with protection, in their SPIT model they found out the disease transmission can be kept minimal or manageable when protection is involved. Motivated by this work by the author [11 and 12], who disregarded the unprotected humans in the spread dynamics, we considered a SEIR model incorporating the unprotected humans in the spread dynamics instead of protection basing our argument that the unprotected humans are key in the spread dynamics and contributes in spreading the infection when they interact with other population subclasses and also the environment.

2 Description and model formulation

We formulated a deterministic model for spread dynamics of typhoid fever that considers human population at time t. The model framework is divided into four compartments as follows.

Susceptible (S), Unprotected (E), Infective (I) and Recovered (R).The model has the following procedure $S \to E \to I \to R \to S$.

We have used the following parameters in our model.(i) μ is the natural death rate (ii) α is the disease induced death rate. (iii) Λ Human recruitment rate (birth). (iv) β Disease interaction rate. (v) Ω unprotected symptoms showing rate (vi) γ Infective recovery rate and finally (vii) δ this is the rate at which recovered humans loses temporary immunity obtained through treatment and get the disease back again.

Since we are dealing with a population; all population compartments is positive $\forall t > 0$ in the feasible region $\varphi = \{S,\ E,\ I,\ R\} \in \varphi \subset R^4_+$. It can be shown that all the solutions are bounded in φ , $\forall t > 0$ such that $0 \le N \le \frac{\Lambda}{\mu}$. This makes the model to be epidemiologically well posed in the region φ and is justified to be analyzed.

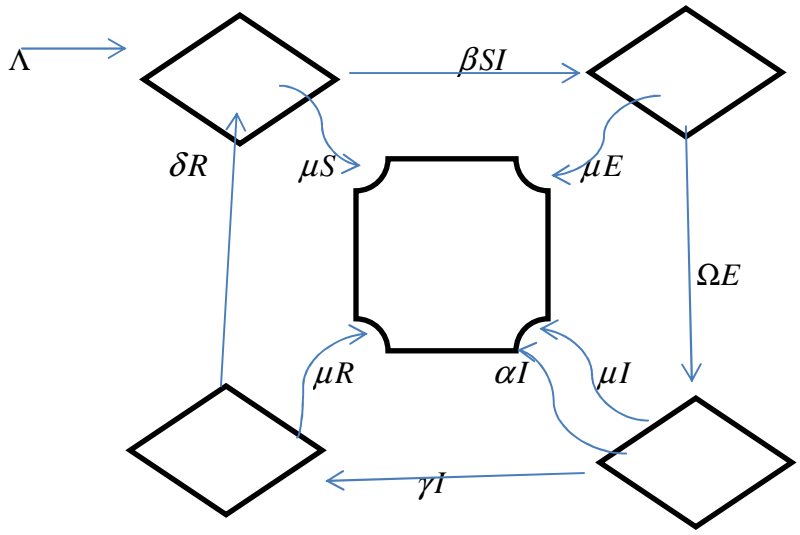


Fig 1. Compartmental diagram for an SEIR model of spread of typhoid fever with unprotected human compartment.

From the above model the transition between compartments can now be expressed into four non-linear differential equations defined as follows [20]

$$
\begin{aligned}
i)\ &\frac{dS}{dt} = \Lambda + \delta R - \beta SI - \mu S \\
ii)\ &\frac{dE}{dt} = \beta SI - \Omega E - \mu E \\
iii)\ &\frac{dI}{dt} = \Omega E - \gamma I - \alpha I - \mu I \\
iv)\ &\frac{dR}{dt} = \gamma I - \delta R - \mu R
\end{aligned}
\qquad (1)
$$

Whereby $N(t) = S(t) + E(t) + I(t) + R(t)$

The model equations are non- linear and they describe the following. (i) Describes dynamics of susceptible humans. (ii) Describes dynamics of unprotected humans. (iii) Describes dynamics of infected humans. (iv) Describes dynamics of recovered humans.

3 Disease free equilibrium point (DFE)

At disease free equilibrium point given by $(E_1^{\cdot}) = (S^* E^* I^* R^*)$; there is no disease in the population which implies absence of infective, unprotected and recovered humans. In our model is obtained by setting dynamical system of equations in 1 to zero as done by authors in [18]. At this point E=I=R=0, therefore $(E^* = 0, I^* = 0 and R^* = 0)$ while S $\neq$ 0 in all the differential equations.

Using the first equation of system 1, equating it to zero and making S the subject gives

$$\Lambda + \delta R - \beta SI - \mu S = 0.$$
$$\Lambda - \mu S = 0$$
$$S^{\cdot} = \frac{\Lambda}{\mu}$$

The disease free equilibrium becomes $(S^* E^* I^* R^*) = \left(\frac{\Lambda}{\mu}, 0, 0, 0\right)$

4. Basic reproductive number

The basic reproductive number (R_0) which is average number of secondary infections caused by one infectious individual introduced in a completely susceptible population is obtained using next generation matrix according to [9, 16, 20] as indicated here using the infected compartment E and I, their rate of change equations and considering the partial derivatives of m and n with respect to E and I leading to square matrices F and V respectively described as

$$F = \begin{pmatrix} 0 & 0 \\ \beta S & 0 \end{pmatrix} \text{ and } V = \begin{pmatrix} (\Omega + \mu) & -\Omega \\ 0 & (\alpha + \mu + \gamma) \end{pmatrix}$$

Finding inverse of V and multiplying it with F

$$V^{-1} = \begin{pmatrix} \frac{1}{(\Omega + \mu)} & \frac{\Omega}{(\Omega + \mu)(\alpha + \mu + \gamma)} \\ 0 & \frac{1}{(\alpha + \mu + \gamma)} \end{pmatrix}$$

$$FV^{-1}=\begin{pmatrix} 0 & 0 \\ \dfrac{\beta S}{(\Omega+\mu)} & \dfrac{\beta S\Omega}{(\Omega+\mu)(\alpha+\mu+\gamma)} \end{pmatrix}$$

Introducing Eigen values and solving the determinant gives two Eigen values as follows $\lambda = 0$ and $\lambda=\dfrac{\beta S\Omega}{(\Omega+\mu)(\alpha+\mu+\gamma)}$. The most dominant eigenvalue is $\lambda=\dfrac{\beta S\Omega}{(\Omega+\mu)(\alpha+\mu+\gamma)}$ which forms our basic reproductive number. At disease free equilibrium $R_0=\dfrac{\beta\Lambda\Omega}{\mu(\Omega+\mu)(\alpha+\mu+\gamma)}$

Theorem 2

Disease free equilibrium is locally asymptotically stable if less than unity and unstable if greater than unity.

Proof

Basic reproductive number is $R_0=\dfrac{\beta\Lambda\Omega}{\mu(\Omega+\mu)(\alpha+\mu+\gamma)}$

At Disease Free Equilibrium $R<1$ hence $\dfrac{\beta\Lambda\Omega}{\mu(\Omega+\mu)(\alpha+\mu+\gamma)}<1$

Making Ω the subject

$$\Omega<\frac{\mu^2(\alpha+\mu+\gamma)}{\beta\Lambda-u(\alpha+\mu+\gamma)}.$$

Therefore if $\Omega<\dfrac{\mu^2(\alpha+\mu+\gamma)}{\beta\Lambda-u(\alpha+\mu+\gamma)}$, disease free equilibrium will be locally stable.

Lemma 1

If $R_0>1$ then it follows that $\dfrac{\beta\Lambda\Omega}{\mu(\Omega+\mu)(\alpha+\mu+\gamma)}>1$. This implies that $\Omega>\dfrac{\mu^2(\alpha+\mu+\gamma)}{\beta\Lambda-u(\alpha+\mu+\gamma)}$ which means that DFE is locally asymptotically unstable.

5. Endemic equilibrium point (EE).

At Endemic equilibrium point (E_2^*) disease exists. The variables are all nonzero, evaluating the state variables of equations of the system 2, the endemic equilibrium points of dynamical systems given as follows according to authors [12, 14].

$$S^{**} = \frac{(\Omega+\mu)(\gamma+\alpha+\mu)}{\Omega\beta}$$

$$E^{**} = \frac{(\gamma+\alpha+\mu)(\delta+\mu)\{\Lambda\Omega\beta - \mu\{(\Omega+\mu)(\gamma+\alpha+\mu)\}}{\beta\Omega\{(\delta+\mu)(\Omega+\mu)(\gamma+\alpha+\mu)\} - \gamma\Omega\delta}$$

$$I^{**} = \frac{(\delta+\mu)}{\beta} \bullet \{\frac{\Lambda\Omega\beta - \mu\{(\Omega+\mu)(\gamma+\alpha+\mu)\}}{\{(\delta+\mu)(\Omega+\mu)(\gamma+\alpha+\mu)\} - \gamma\Omega\delta}$$

$$R^{**} = \frac{1}{\beta}\{\frac{\{\gamma\Lambda\Omega\beta - \gamma\mu\{(\Omega+\mu)(\gamma+\alpha+\mu)\}}{\{(\delta+\mu)(\Omega+\mu)(\gamma+\alpha+\mu)\} - \gamma\Omega\delta}$$

5 Global Stability of the disease free equilibrium

This stability is obtained by using Castillo-Chavez et al (2002) approach, whereby the model is first rewritten as follows according to [9, 10 and 12]

$$\frac{dM}{dt} = H(M,N) \text{ and } \frac{dN}{dt} = G(M,N),\ G(M,0) = 0$$

Where $M = (S)$ and $N = (E,I)$ with components of $M \in R$ denoting the susceptible population and components of $N \in R^2$ denotes the unprotected (E) and infected (I).

$E^*_1 = G(M^*,0)$ Represent disease free equilibrium.

The following are conditions.) i) $\frac{dM}{dt} = H(M,0), M^*$ is globally asymptotically stable (GAS) ii) $G(M,N) = AN - \bar{G}(M,N), \bar{G}(M,0) = 0$ for $(M,N) \in \varphi$

Where $G(M,N)$ $A = D_N G(W^*,0)$ is M- matrix (the off diagonal elements of A are non-negative) and φ is the region where the model lies.

H (M, N) $= (\Lambda + \delta R - \beta SI - \mu S)$ and G (M, N) $= \begin{pmatrix} \beta SI - \Omega E - \mu E \\ \Omega E - \gamma I - \alpha I - \mu I \end{pmatrix}$

At disease free equilibrium (DFE); E=0, I=0 and R=0 while φ =R^4

Further $\frac{dM}{dt} = H(M, O)$ which is also equal to $\Lambda + \delta R - \mu S$.

M*=(S*, 0, 0, 0) is globally stable hence condition 1 satisfied.

For condition 2

$$\bar{G}(M,N) = \begin{pmatrix} \beta SI \\ 0 \end{pmatrix} and A = \begin{pmatrix} -a-\lambda & \beta S^* \\ \Omega & -b-\lambda \end{pmatrix}.$$

$$\text{AN-}\bar{G}\,(M, N) = \begin{pmatrix} (-a-\lambda)E + \beta S^* I - \beta SI \\ \Omega E + (-b-\lambda)I - 0 \end{pmatrix}$$

$$= \begin{pmatrix} (-a-\lambda)E + \beta IS \\ \Omega E + (-b-\lambda)I \end{pmatrix} \qquad (9)$$

Replacing a and b, the equation 9 become

$\begin{pmatrix} \beta SI - \Omega E - \mu E \\ \Omega E - \gamma I - \alpha I - \mu I \end{pmatrix}$ This also gives $G(M,N)$

Hence $G(M,N)$ is satisfied and proofed to be globally asymptotically stable

7. Global stability of endemic equilibrium

Proof: By the use of lyapunov function defined by LaSalle [1976] and also in [13, 16, and 18 and19], we have determined global stability as follows

$$L(S^{**}, E^{**}, I^{**}, R^{**}) = (S - S^{**} - S^{**}\ln(\frac{S^{**}}{S}) + \left(E - E^{**} - E^{**}\ln(\frac{E^{**}}{E}\right) + I - I^{**} - I^{**}\ln(\frac{I^{**}}{I}) + R - R^{**} - R^{**}\ln(\frac{R^{**}}{R})$$

computing the derivative of L along the solutions of the system is directly:

$$\frac{dL}{dt} = \left(\frac{s-s^{**}}{s}\right)\frac{dS}{dt} + \left(\frac{E-E^{**}}{E}\right)\frac{dE}{dt} + \left(\frac{I-I^{**}}{I}\right)\frac{dI}{dt} + \left(\frac{R-R^{**}}{R}\right)\frac{dR}{dt} \qquad (10)$$

Substituting the equations of system 1 in equation 10, the equation becomes

$$\frac{dL}{dt} = \left[\left(\frac{s-s^{**}}{s}\right)\Lambda + \delta R - (\beta I + \mu)S\right] + \left[\left(\frac{E-E^{**}}{E}\right)\beta SI - (\Omega+\mu)E\right] + \left[\left(\frac{I-I^{**}}{I}\right)\Omega E - (\gamma+\alpha+\mu)I\right] + \left[\left(\frac{R-R^{**}}{R}\right)\gamma I - (\delta+\mu)R\right] \qquad (11)$$

Expanding equation 11, it produces

$$\frac{dL}{dt}=\Lambda+\delta R-(\beta I+\mu)S-\Lambda\frac{S^{**}}{S}-\delta R\frac{S^{**}}{S}+(\beta I+\mu)S^{**}+\beta SI-(\Omega+\mu)E-\beta SI\frac{E^{**}}{E}+$$

$$(\Omega+\mu)E^{**}+\Omega E-(\gamma+\alpha+\mu)I-\Omega E\frac{I^{**}}{I}+(\gamma+\alpha+\mu)I^{**}+\gamma I-(\delta+\mu)R-\gamma I\frac{R^{**}}{R}+(\delta+\mu)R^{**}$$

Further simplification result to

$$\frac{dL}{dt}=\left[\Lambda+\delta R+(\beta I+\mu)S^{**}+\beta SI+(\Omega+\mu)E^{**}+\Omega E+(\gamma+\alpha+\mu)I^{**}+\gamma I+(\delta+\mu)R^{**}\right]$$

$$-\left[(\beta I+\mu)S+\Lambda\frac{S^{**}}{S}+\delta R\frac{S^{**}}{S}+(\Omega+\mu)E+\beta SI\frac{E^{**}}{E}+(\gamma+\alpha+\mu)I+\Omega E\frac{I^{**}}{I}+(\delta+\mu)R+\gamma I\frac{R^{**}}{R}\right]$$

From equation it's clear that ; $\frac{dL}{dt}=A-B$ .Where A are the positive terms and B are the negative ones, such that;

$$A=\Lambda+\delta R+(\beta I+\mu)S^{**}+\beta SI+(\Omega+\mu)E^{**}+\Omega E+(\gamma+\alpha+\mu)I^{**}+\gamma I+(\delta+\mu)R^{**}$$

$$B=(\beta I+\mu)S+\Lambda\frac{S^{**}}{S}+\delta R\frac{S^{**}}{S}+(\Omega+\mu)E+\beta SI\frac{E^{**}}{E}+(\gamma+\alpha+\mu)I+\Omega E\frac{I^{**}}{I}+(\delta+\mu)R+\gamma I\frac{R^{**}}{R}$$

If $A<B$ then $\frac{dL}{dt}\le 0$

$\frac{dL}{dt}=0$ Only if $S=S^{**},E=E^{**},I=I^{**},R=R^{**}$

The largest invariant set in $\{(S,\ E,\ I,\ R)\in\varphi:\frac{dL}{dt}=0\}$ is a singleton E_2^*,where E_2^* is the endemic equilibrium. Therefore, the endemic equilibrium is globally asymptotically stable in the invariant region φ if $A<B$.

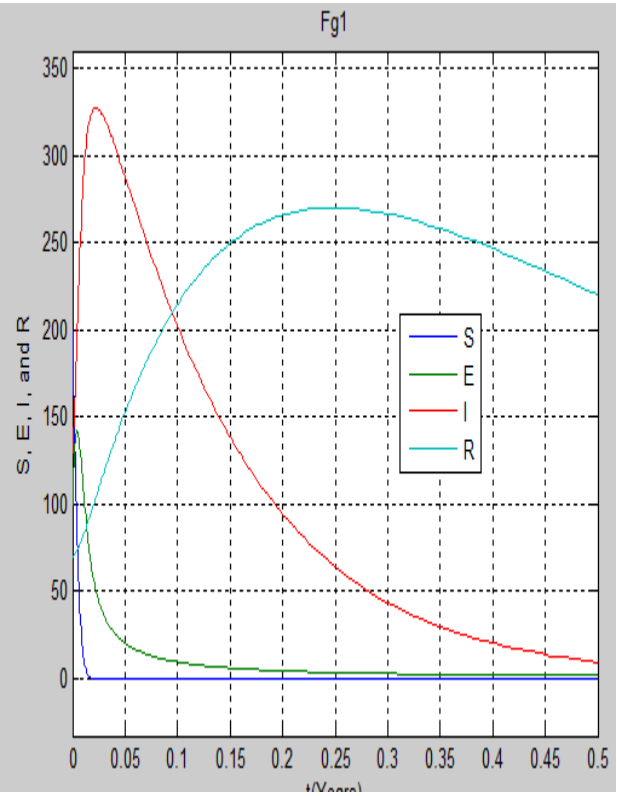
6 Discussion.

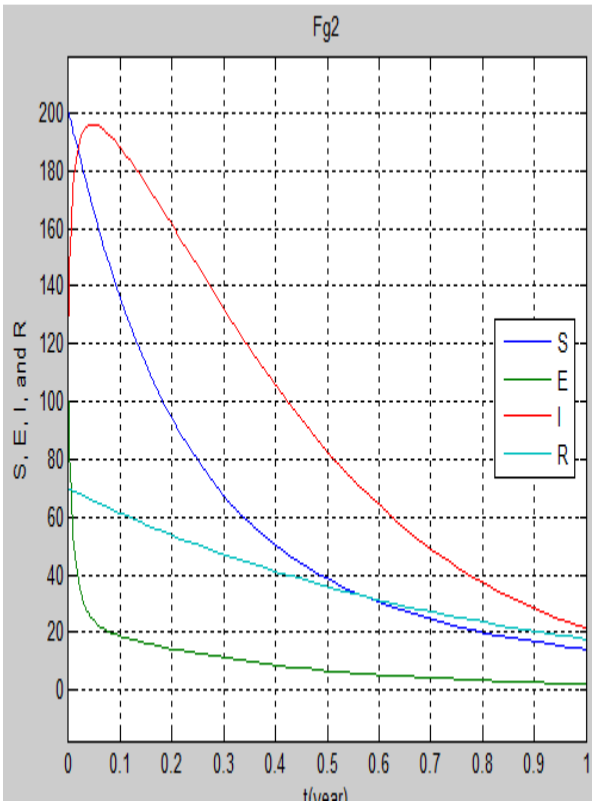
In this research we modeled unprotected human compartment in the spread dynamics of typhoid fever in human. Global stability of both equilibrium point (Disease Free Equilibrium point and Endemic Equilibrium point) was carried out. From our findings of stability analysis of equilibrium points is stable when $R_0<1$ and unstable when $R_0>1$. This shows that unprotected humans have great impact in the spread dynamics and need to be considered amongst other protective factors if typhoid fever is to be effectively eradicated from human population. Our model clearly portrays direct variation between the unprotected and infected compartment.

From our finding if $\Omega<\frac{\mu^2(\alpha+\mu+\gamma)}{\beta\Lambda-u(\alpha+\mu+\gamma)}$, there disease equilibrium will be stable and typhoid disease will not have a hand in the population. However if $\Omega>\frac{\mu^2(\alpha+\mu+\gamma)}{\beta\Lambda-\mu(\alpha+\mu+\gamma)}$;then disease will be dependent on the unprotected humans and other prevailing circumstances.

6. Results

Numerical results of this mode is in form of graphs. Some parameters are preset while others are obtained from other authors for example [6].A Matlab software (odesolve) was employed to run the test and graphs below are obtained as shown.





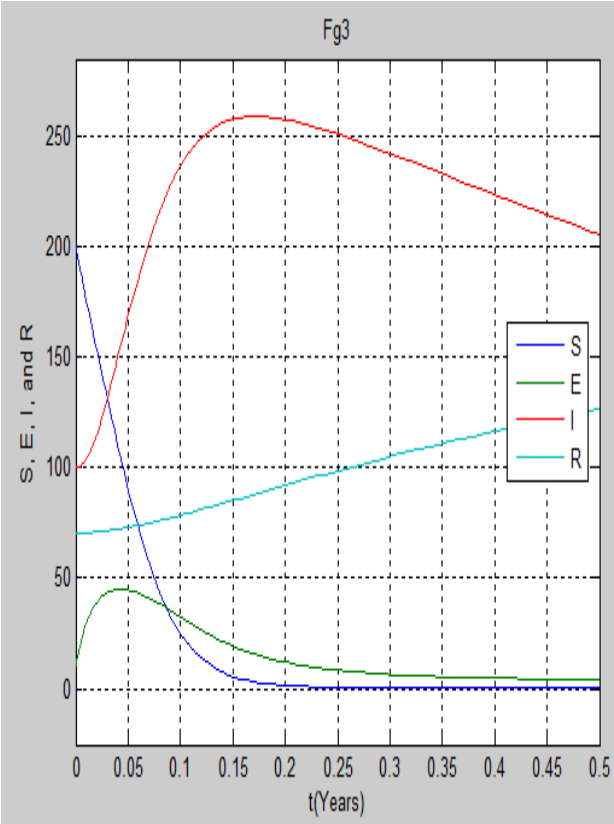
For Fg 1 parameters $\Lambda = 750, \delta = 0.125, \beta = 0.0125, \mu = 0.15, \Omega = 9.25, \gamma = 0.625, \alpha = 1.503$

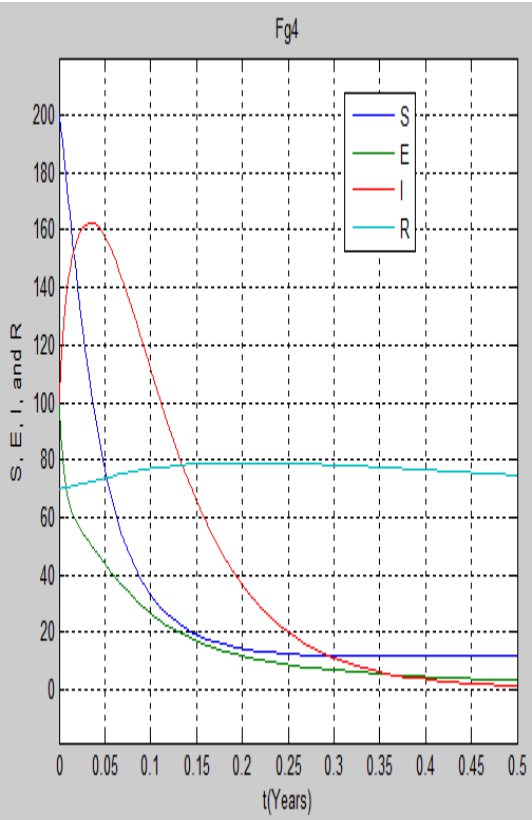
$S^{**} = 200, E^{**} = 25, I^{**} = 130, R^{**} = 70$

For Fg 2 parameters

$\Lambda = 0.75, \delta = 0.0125, \beta = 0.0125, \mu = 1.5, \Omega = 0.000295, \gamma = 0.0625, \alpha = 1.503$

$S^{**} = 200, E^{**} = 100, I^{**} = 130, R^{**} = 70$





For Fg 3 parameters $\Lambda = 0.75, \delta = 0.125, \beta = 0.125, \mu = 0.15, \Omega = 0.925, \gamma = 0.625, \alpha = 0.1503$

$S^{**} = 200, E^{**} = 10, I^{**} = 100, R^{**} = 70$

For Fg4 $\Lambda = 0.075, \delta = 0.0125, \beta = 1.25, \mu = 1.5, \Omega = 0.295, \gamma = 6.25, \alpha = 0.1503$

$S^{**} = 200, E^{**} = 100, I^{**} = 100, R^{**} = 70$

The parameters vary within a range of 0 to 1.5.The initial conditions of endemic equilibrium of the unprotected differ so as to realize the effect of the unprotected in the spread dynamics as shown in. Fg1 and Fg3 and also in Fg2 and Fg4.Generally increase or decrease in the unprotected compartment causes a change in the direct curve.

7. Discussion.

In this research we modeled unprotected human compartment in the spread dynamics of typhoid fever in human. Global stability of both equilibrium point (Disease Free Equilibrium point and Endemic Equilibrium point) was carried out. From our findings of stability analysis of equilibrium points is stable when $R_0 < 1$ and unstable when $R_0 > 1$. This shows that unprotected humans have great impact in the spread dynamics and need to be considered amongst other

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8. Conclusion and recommendation

We conclude that there is direct variation relationship between the unprotected and infectious compartment, because increasing the unprotected with 10% cause increase in infective; therefore the unprotected humans contribute significantly to the spread dynamics of typhoid fever disease. Therefore we recommend policy makers in health sectors to incorporate protection measures to avoid the disease prevailing in the population. Deaths due to typhoid will be low if the unprotected group is managed properly by providing periodic oral protection doses or vaccines at birth.

Competing interest

No competing interest exists from the author.

REFERENCES

1. Shaibu Osman, Oluwole Daniel Makinde and David Mwangi Theuri. Stability analysis and modeling of listeriosis dynamics in human and animal population.Global journal of pure and Applied Mathematics.14(1):115-137,2018c.
2. Oluwole Daniel Makinde. A domain decomposition approach to a sir epidemic model with constant vaccination strategy. Applied mathematics and computation 184 (2):842-848, 2007.
3. WHO Background paper on vaccination against typhoid fever using new generation vaccines-presented at the SAGE November 2007 meeting. WHO 2007:11.
4. Ivanoff BN, Levine MM, Lambert fever: Present statuses Bulletin of the world heal organisation1994; 72 (6):957-971.
5. Adetunde, I.A, 2008. Mathematical model of spread of typhoid fever, World journal of applied Science and technology vol.3 No.2 (2011).10-12.
6. Moffat N.Chamuchi, Johana K.Sigee, Jeconiah A.Okello and James M.Okwoyo. SII_CR Model and Simulation of the Effects of Carriers on the Transmission Dynamics of Typhoid Fever in KISII Town Kenya.Vol.2 No.3 2014.
7. Peter O. J, Ibrahim M.O, Akinduko O.B Rabiu M. on Mathematics Model For the control of Typhoid Fever.''ISOR Journal of Mathematics (ISOR-JM) Vol.13, Issue 4 Ver.II (Jul-Aug.2017), pp. 60-66.
8. Shaibu Osman, oluwale Daniel Makinde and David Mwangi theuri. Mathematical modeling of transmission dynamics of Anthrax in human and animal population. Mathematical Theory and Modelling, 2018b.

9. Ochoche J.M and Gweryina R.I. on mathematical model on measles with vaccination and two phases of infectiousness. ISSN 2278-5728,p-ISSN:2319-765X Vol. 10 Issue 1VerIV(Feb)2014,pp95-105.
10. Castillo-chavez, Feng .z and Hung W. on computation of basic reproductive number and its role on global stability.2002 Vol.125, IMA p229-250.
11. Nthiiri J.K. on global stability of equilibrium points of typhoid fever model with protection;21(5):1-6 2017;Article no.BJMSCS.32690 ISSN:2231-0851
12. Nthiiri J.K, Lawi G.O, Akinyi C.O on Mathematical modeling of typhoid fever disease in co-orporating protection against infection BJMCS 23325 (2016)P(1-10).
13. Soufiane Elkhaiar and Abdelilah Kaddar on stability analysis of a SEIR with treatment. Article ID 101266 Vol.1 (2017), doi: 10.11131/2017/101266pp. (1-16).
14. Kalu A, Agwi I.A, Agbanyim Akuagwuon N. on Mathematical Analysis of the Endemic Equilibrium Dynamics of The transmission dynamics of Tuberculosis on International Journal of Scientific and Technology research volume 2, issue 12, December 2013.
15. Mushsyabasa S.,Bhunu C.P. and NgarakanaG on mathematical analysis of a typhoid model with carriers, direct and indirect disease transmissions .ISSN 0973-9424,Vol 7 No1(January 2013)pp79-90.
16. Leopard C.Mpande,Damian Kajunguri,Emmanuel A.Mpolya on Modelling and stabilityanalysis for measles metapopulation model with vaccination. ISSN 2328-5605 and 2328-5613 Vol.4,No 6,2015,pp,431-444.
17. Muhammad A.K., Muhammad P., Saeed I. on Mathematical Analysis of Typhoid Model with Saturated Incidence Rate.Vol.7 2017, 92D25 no 2 p 65-78.
18. LaSalle J.S. (1976).on stability of dynamical systems.CBMS-NSFregional conference series in applied mathematics Vol.25 SIAM.
19. Hongbin Guo and MichaelY.LI on global stability in a mathematical model of tuberculosis. Article in Canadian applied mathematics quarterly Vol.4 number 2 summers 2006.
20. Stephen Edward,Kitengeso Raymond E.,Kiria Gabriel T.,Felician n.,Mwema G.,Mafarasa A. on mathematical model for control and elimination of the transmission dynamics of measles.2015;4(6):396-408.ISSN 2328-5605 doi 10.11648/j.acm.20150406.12