PROFIT ANALYSIS OF NON IDENTICAL REPAIRABLE UNITS SUBJECT TO TWO PHASE REPAIR OF SYSTEM

Abstract: The paper deals with the profit analysis of three non-identical units A, B, and C in which either Unit A or one of the units B and C should work for the successful functioning of the system. Two types of repairman are available in the system viz. Ordinary and Expert repairman. The expert repairman is called only when the system breaks down. Unit A gets priority for repair and is repaired by expert repairman while as Unit B and C are repaired by ordinary repairman if the system doesn't fail totally. The failure time distribution of unit-A, B and C are taken as exponential. The distribution of time to repair of units are assumed to be general.

Keywords- Mean Sojourn time, Availability Analysis, Expected Number of Visits by Regular and Expert Repairman, Profit Analysis of system.

I. INTRODUCTION

Several studies on profit analysis of repairable redundant system model have been done in the past. Kumar and Kadyan worked on Profit analysis of a system of non-identical units with degradation and replacement [6]. However, Yusuf and Bala investigated Stochastic modeling of a two unit parallel system under two types of failures [3]. Gupta et.al studied two unit standby system with correlated failure and repair times [5]. Sureria, and Anand put forth the concept of cost benefit analysis of a computer system with priority to software replacement over hardware repair [7]. Further, Mahmoud, and Moshref worked on a two unit cold standby system considering hardware, human error failures and preventive maintenance [4]. Navas et.al, discussed reliability analysis in railway repairable systems [2]. Recently Wu-Lin Chen analyzed system reliability analysis of retrial machine repair systems and a single server of working breakdown and recovery policy [1]. In most of the case, the authors assume the independent lifetimes of the units in analyzing the redundant system models. But, in many realistic situations, we observe that the rate of failure of an operating unit increases if its redundant unit working in parallel has already failed. This type of situations is visualized in many cases. So keeping the above fact in view, the aim of a present paper is to analyze a three non-identical unit complex system arranged in such a way that the system failure occurs only if either unit-A or both the units B and C fail totally.

II. ASSUMPTIONS AND SYSTEM DESCRIPTION

- The system comprises of three non-identical units A, B and C in which either Unit A or one of the units B and C should work for the successful functioning of the system.

- There are two types of repairman available in the system: ordinary and expert repairman. The expert repairman is called only when a system breaks down.
- Two types of repair facility are available to repair failed unit in which A gets priority for repair and is repaired by expert and B and C are repaired by ordinary repairman if the system doesn't fail totally.
- The failure time distributions of unit-A, B and C are taken exponential while as repair time distribution is assumed to be general.

III. NOTATIONS

λ : Failure rate of unit-A.

α_1 : Constant failure rate of unit-B when unit-C is good

α_2 : Constant failure rate of unit-C when unit-B is good.

n_1: Constant failure rate of unit-B when unit-C has failed.

n_2: Constant failure rate of unit-C when unit-B has failed.

$\beta(.)$: Repair distribution of unit-B and C by regular repairman.

$\mu(.)$: Distribution of repair of unit-A, B and C by expert repairman

Ψ_i : Mean sojourn time in state S_i.

$A_0, B_0, C_0 / A_g, B_g, C_g$: Unit-A, B,C is in operative and normal (N) mode / good.

$B_r, C_r / B_w, C_w$: Unit-B,C is in failure (F) mode and under repair / waits for repair by regular repairman.by regular repairman.

A_e, B_e, C_e : Unit-A, B, C is in failure (F) mode and under repair / waits for repair by expert repairman.

The possible states of the system are:

$S_0 = [A_0, B_0, C_0]$ $S_1 = [A_0, B_r, C_0]$

$S_2 = [A_0, B_0, C_r]$ $S_3 = [A_e, B_g, C_g]$

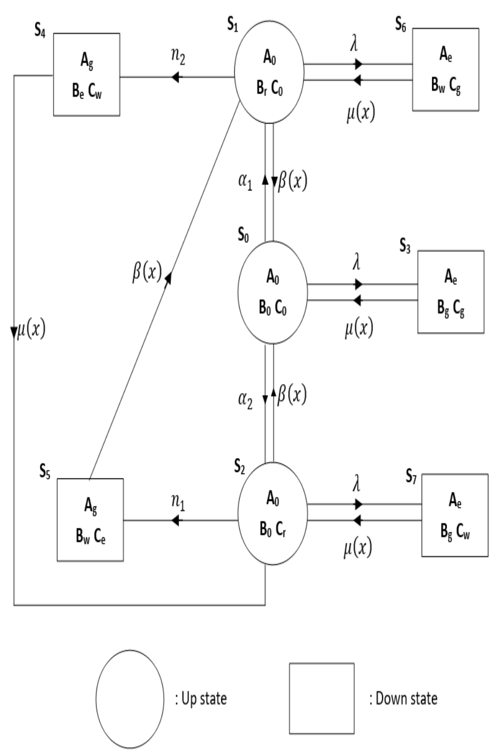
$S_4 = [A_g, B_e, C_w]$

$S_5 = [A_g, B_w, C_e]$

$S_6 = [A_e, B_w, C_g]$

$S_7 = [A_e, B_g, C_w]$

TRANSITION DIAGRAM



The states S_0, S_1 and S_2 are up states while S_3, S_4, S_5, S_6 and S_7 are down states. Initially, all the states are in the operating condition and the system failure occurs if either unit A fails or both B and C fails completely. Upon failure of system unit A gets priority over B and C. Further, all the seven states are regenerative states.

IV. TRANSITION PROBABILITIES

The various transition probabilities using simple calculations, $Q_{ij}(t) = P[X_{n+1} = j, T_{n+1} - T_n \leq t | X_n = i]$ are obtained. Taking the limit as $t \to \infty$, steady state probabilities is obtained as

$$p_{01} = \frac{\alpha_1}{\alpha_1+\alpha_2+\lambda} \qquad p_{02} = \frac{\alpha_2}{\alpha_1+\alpha_2+\lambda}$$

$$p_{03} = \frac{\lambda}{\alpha_1+\alpha_2+\lambda} \qquad p_{10} = \beta^*(\lambda+n_2)$$

$$p_{14} = \frac{n_2}{\lambda+n_2}[1-\beta^*(\lambda+n_2)]$$

$$p_{16} = \frac{\lambda}{\lambda+n_2}[1-\beta^*(\lambda+n_2)]$$

$$p_{20} = \beta^*(\lambda+n_1)$$

$$p_{23} = \frac{n_1}{\lambda+n_1}[1-\beta^*(\lambda+n_1)]$$

$$p_{27} = \frac{\lambda}{\lambda+n_1}[1-\beta^*(\lambda+n_1)]$$

$$p_{30} = p_{42} = p_{31} = p_{61} = p_{72} = 1$$

From these probabilities, the following condition holds:

$$p_{01} + p_{02} + p_{03} = 1,$$
$$p_{10} + p_{14} + p_{16} = 1,$$
$$p_{20} + p_{23} + p_{27} = 1,$$
$$p_{30} = p_{41} = p_{51} = p_{61} = p_{72} = 1$$

V. MEAN SOJOURN TIME

The mean sojourn time Ψ_i in state S_i is defined as $\Psi_i = E[T_i] = \int_0^\infty P(T_i > t)\,dt$ are calculated

$$\Psi_0 = \int_0^\infty e^{-(\alpha_1+\alpha_2+\lambda)t}\,dt = \frac{1}{(\alpha_1+\alpha_2+\lambda)}$$

$$\Psi_1 = \int_0^\infty e^{-(\lambda+n_2)t}\bar{\beta}(t)\,dt = \frac{1}{(\lambda+n_2)}[1-\beta^*(\lambda+n_2)]$$

$$\Psi_2 = \int_0^\infty e^{-(\lambda+n_1)t}\bar{\beta}(t)\,dt = \frac{1}{(\lambda+n_1)}[1-\beta^*(\lambda+n_1)]$$

$$\Psi_3 = \Psi_4 = \Psi_5 = \int_0^\infty \bar{g}(t)\,dt$$
$$\Psi_6 = \Psi_7 = \int_0^\infty \bar{g}(t)\,dt$$

VII. AVAILABILITY ANALYSIS

Simple probabilistic techniques are used to find recurrence relations among availabilities. The availability of the system in a steady state will be given by

$$A_0 = \lim_{t\to\infty} A_0(t) = \lim_{s\to 0} s\,A_0^*(s) = N_2(0)/D_2'(0)$$

Where

$$N_2(0) = [(1-p_{16})(1-p_{27}) - p_{23}p_{14}]\Psi_0 + [p_{01}(1-p_{27}) + p_{02}p_{23}]\Psi_1 + [p_{02}(1-p_{16}) + p_{01}p_{14}]\Psi_2$$

$$D_2'(0) = [(1-p_{16})(1-p_{27}) - p_{23}p_{14}](\Psi_0 + p_{03}\Psi_3) + [p_{01}(1-p_{27}) + p_{02}p_{23}]\Psi_1 + [p_{02}(1-p_{16}) + p_{01}p_{14}]\Psi_2 + [p_{01}p_{14}(1-p_{27}) + p_{14}p_{23}p_{02}]\Psi_4 + [p_{02}p_{23}(1-p_{16}) + p_{14}p_{23}p_{01}]\Psi_5 + [p_{01}p_{16}(1-p_{27}) + p_{16}p_{23}p_{02}]\Psi_6 + [p_{14}p_{27}p_{01} + p_{02}p_{27}(1-p_{16})]\Psi_7$$

And the mean up time during (0, t] is

$$\mu_{up} = \int_0^t A_0(u)\,du$$

VIII. BUSY PERIOD ANALYSIS FOR REGULAR REPAIRMAN

Using simple probabilistic technique and solving the equations, the busy period of repairman in a steady state is given by

$$B_0 = \lim_{t\to\infty} B_0(t) = \lim_{s\to 0} s B_0^*(s) = \frac{N_3(0)}{D_2'(0)}$$

$$N_3(0) = [p_{01}(1-p_{27}) + p_{02}p_{23}]\Psi_1 + [p_{02}(1-p_{16}) + p_{01}p_{14}]\Psi_2$$

and expected duration in (0,t]

and $D_2'(0)$ is same as in the case of availability.

$\mu_{br}(t) = \int_0^t B_0(u)du$, so that $\mu_{br}^{*} = B_0^{*}/s$

IX. BUSY PERIOD ANALYSIS FOR EXPERT REPAIRMAN

Using simple probabilistic technique and solving the equations, the busy period analysis of expert repairman in a steady state is given by

$$B_0^E = \lim_{t\to\infty} B_0^{E*}(t)$$
$$= \lim_{s\to 0} sB_0^{E*}(s) = \frac{N_4(0)}{D_2'(0)}$$

Where

$N_4(0) =$
$p_{03}[(1-p_{16})(1-p_{27}) -$
$p_{28}p_{14}]\Psi_3 + [p_{01}p_{14}(1-p_{27}) +$
$p_{02}p_{28}p_{14}]\Psi_4 +$
$[p_{02}p_{28}(1-p_{16}) +$
$p_{01}p_{14}p_{28}]\Psi_5 + [p_{01}p_{16}(1-p_{27}) +$
$p_{02}p_{28}p_{16}]\Psi_6 + [p_{02}p_{27}(1-p_{16}) +$
$p_{01}p_{14}p_{27}]\Psi_7$

and $D_2'(0)$ is same as in the case of availability

and the expected duration in (0,t] is given as

$\mu_{be}(t) = \int_0^t B_0^E(u)du$

X. EXPECTED NUMBER OF VISITS BY REGULAR REPAIRMAN

Simple probabilistic techniques are used and solving the equations, the number of visits per unit time in a steady state is given by

$$V_0(0) = \lim_{t\to\infty} \frac{V_0(t)}{t} = \frac{N_3(0)}{D_2'(0)}$$

Where

$N_3(0) =$
$(p_{01} + p_{02})[(1-p_{16})(1-p_{27}) -$
$p_{28}p_{14}] + [p_{01}(1-p_{27}) + p_{02}$

$p_{28}](1-p_{10}) + [p_{02}(1-p_{16}) + p_{01}$
$p_{14}](1-p_{20})$

Now the expected visits by the regular repairman in (0,t]

$\mu_{vr}(t) = \int_0^t V_0(u)du$

X1, EXPECTED NUMBER OF VISITS BY EXPERT REPAIRMAN

Using the definition of $V_i^E(t)$, the recursive relations among $V_i^E(t)$ can be easily developed.

Number of visits in steady state by repairman, is given by

$$V_0(0) = \lim_{t\to\infty} \frac{V_0(t)}{t} = \frac{N_5(0)}{D_2'(0)}$$

Where,

$N_6(0) =$
$p_{03}[(1-p_{16})(1-p_{27}) - p_{28}p_{14}] +$
$p_{01}[(1-p_{27}) + p_{02}p_{28}]$

$(1-p_{10}) + [p_{02}(1-p_{16}) + p_{01}p_{14}]$
$(1-p_{20})$

$D_2'(0)$ is same as availability analysis

Now the expected number of visits by the expert repairman in (0,t]

$\mu_{ve}(t) = \int_0^t V_0^E(u)du$

XII. PROFIT ANALYSIS

Therefore, profit analysis of the system can be given as:

$$P_1 = K_0A_0 - K_1B_0 - K_2B_0^E - K_3V_0 - K_4V_0^E$$

Where, $K_0, K_1, K_2, K_3, K_4 =$ Cost associated per unit time for which regular and expert repairman is available, busy and visits of repairman in system respectively.

XIII. STUDY OF SYSTEM BEHAVIOUR

The behavior of availability and profit analysis of the system is studied

TABLE-1: Effect of α_1 and fixed parameters $\alpha_2,\ \lambda,\ \gamma_1,\ n_1$ *and* n_2 on Availability.

α_1	Availability		
	$\alpha_2 = 0.50, \lambda = 0.34.,$ $\gamma_1 = 0.08, n_1 = 0.57,$ $n_2 = 0.63,$	$\alpha_2 = 0.55, \lambda = 0.37,$ $\gamma_1 = 0.07, n_1 = 0.52,$ $n_2 = 0.65,$	$\alpha_2 = 0.60, \lambda = 0.41,$ $\gamma_1 = 0.09, n_1 = 0.53,$ $n_2 = 0.67,$
0.1	0.732143	0.718196	0.693035
0.2	0.729272	0.715739	0.690012
0.3	0.726475	0.713335	0.687058
0.4	0.723747	0.710984	0.684187
0.5	0.721086	0.708683	0.681395
0.6	0.718491	0.706431	0.67868
0.7	0.715958	0.704227	0.676037
0.8	0.713486	0.702068	0.673466
0.9	0.711071	0.699954	0.670962
1.0	0.708713	0.697882	0.668522

TABLE 2: Effect of γ_1 and fixed parameters $\alpha_1,\ \alpha_2, \lambda, n_1, n_2$ on Availability

γ_1	Availability		
	$\alpha_1 = 0.95, \alpha_2 = 0.40,$ $\lambda = 0.38, n_1 = 0.30,$ $n_2 = 0.42,$	$\alpha_1 = 0.90, \alpha_2 = 0.43,$ $\lambda = 0.32, n_1 = 0.38,$ $n_2 = 0.58,$	$\alpha_1 = 0.99, \alpha_2 = 0.52,$ $\lambda = 0.34, n_1 = 0.41,$ $n_2 = 0.53,$
0.1	0.724638	0.757576	0.746269
0.2	0.72606	0.758407	0.747547
0.3	0.727356	0.759165	0.748686
0.4	0.728454	0.759815	0.749642
0.5	0.729333	0.760343	0.750407
0.6	0.730004	0.760754	0.750998
0.7	0.730493	0.761062	0.751442
0.8	0.730831	0.761283	0.751765
0.9	0.731052	0.761436	0.751995
1.0	0.731181	0.761536	0.752152

TABLE-3: Effect of α_1 and fixed parameters $\alpha_2, \lambda, \gamma_1, n_1, n_2, k_0, k_1, k_2, k_3, k_4$ on Profit.

α_1	Profit		
	$\alpha_2 = 0.31, \lambda = 0.20,$ $\gamma_1 = 0.09, n_1 = 0.37,$ $n_2 = 0.02, k_0 = 1000$ $k_1 = 500, k_2 = 470$	$\alpha_2 = 0.30, \lambda = 0.18,$ $\gamma_1 = 0.08, n_1 = 0.32,$ $n_2 = 0.03, k_0 = 990$ $k_1 = 400, k_2 = 350$	$\alpha_2 = 0.28, \lambda = 0.14,$ $\gamma_1 = 0.06, n_1 = 0.22,$ $n_2 = 0.13, k_0 = 950$ $k_1 = 420, k_2 = 380$

TABLE-4: Effect of γ_1 and fixed parameters $\alpha_1, \alpha_2, \lambda, n_1, n_2, k_0, k_1, k_2, k_3, k_4$ on Profit

γ_1	Profit		
	$\alpha_1 = 0.48, \alpha_2 = 0.54,$ $\lambda = 0.11, n_1 = 0.24,$ $n_2 = 0.13, k_0 = 1000$ $k_1 = 420, k_2 = 200,$ $k_3 = 300, k_4 = 320$	$\alpha_1 = 0.46, \alpha_2 = 0.52,$ $\lambda = 0.12, n_1 = 0.25,$ $n_2 = 0.14, k_0 = 980$ $k_1 = 400, k_2 = 250,$ $k_3 = 290, k_4 = 310$	$\alpha_1 = 0.44, \alpha_2 = 0.50,$ $\lambda = 0.16, n_1 = 0.39,$ $n_2 = 0.18, k_0 = 970$ $k_1 = 390, k_2 = 280,$ $k_3 = 295, k_4 = 330$
0.1	3.96394	36.25	6.55174
0.2	172.844	180.677	112.459
0.3	264.886	260.275	177.319
0.4	316.422	304.823	217.228
0.5	345.256	329.426	241.494
0.6	361.255	342.648	255.975
0.7	370.024	349.445	264.423
0.8	374.74	352.672	269.229
0.9	377.204	353.964	271.891
1.0	378.431	354.246	273.333

CONCLUSION:

In Table 1, Availability w.r.t. α_1 and fixed values of a parameter $\alpha_2, \lambda, \gamma_1, n_1, n_2$ is studied. Also in Table 3, Profit w.r.t. α_1 and fixed values of parameters $\alpha_2, \lambda, \gamma_1, n_1, n_2, k_0, k_1, k_2, k_3, k_4$ is calculated and in both cases, it is analyzed that Availability and Profit analysis of the system decreases w.r.t. α_1 (failure rate) keeping other parameters fixed. In Table 2, Availability γ_1 w.r.t. and fixed values of parameter $\alpha_1, \alpha_2, \lambda, n_1, n_2$ is computed. Also, In Table 4, Profit w.r.t. γ_1 and fixed values of a parameter $\alpha_1, \alpha_2, \lambda, n_1, n_2, k_0, k_1, k_2, k_3, k_4$ is studied and It is seen that Availability and Profit analysis of the system increases w.r.t. γ_1 (Repair rate) keeping other parameters fixed. Therefore, it can be concluded that expected life of the system increases with decreasing the failure rate of a unit in failure mode (α_1) and increases with an increasing repair rate of a unit in repair mode (γ_1) which in turn increases system reliability.

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