Original Research Article

Oblique Propagation of Nonlinear Solitary Waves in Magnetized Plasma with Nonextensive Electrons

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ABSTRACT

In this paper, authors have studied the properties of obliquely propagating nonlinear solitary waves in a plasma system consisting of warm ions and nonextensively distributed electrons. The nonlinear Korteweg-de-Vries (KdV) equation and its solution have been derived using the standard reductive perturbation method. The effect of ion temperature on the propagation of solitary waves has been investigated numerically. The critical value of nonextensivity at which solitary structures transit from negative to positive potential is found to shift to the lower value under the effect of finite temperature. The numerical results are interpreted graphically. The results may be useful for understanding the wave propagation in laboratory and space plasmas where magnetic field is present.

Keywords: Magnetized plasma, q-nonextensive distribution, reductive perturbation method, nonlinear waves and soliton

1. INTRODUCTION

The nonlinear wave structures have provided a fascinating field of research for the plasma physics community owing to their importance in explaining various laboratory, space and astrophysical atmospheres [1-3]. Nonlinear structures like solitons, shock waves, double layers etc. are observed in space and laboratory. Out of them, solitons have become the main source of interest for the researchers from across the globe owing to their rich physical insight underlying the various nonlinear phenomena. Solitons are stable nonlinear entities and arise due to a delicate balance of nonlinearity and dispersion. Nonlinear wave structures in various plasma models and compositions have been investigated theoretically and observationally for the last half century [4-8]. The existence of magnetic field in a plasma system has found a significant impact on the nonlinear wave propagation. Such a strong magnetic field is observed to exist on the surface of fast rotating neutron stars and in the pulsar magnetosphere [9-10]. Considering this, an immense interest has been developed in researchers to study nonlinear propagation of ion-acoustic waves in magnetized plasmas [11-15]. Dubouloz et al [16] reported that the electric field spectrum produced by an electron-acoustic solitary wave (EASW) is not significantly modified by the presence of a magnetic field. Mace and Hellberg [17] studied the influence of the magnetic field on the features of the weakly nonlinear electron-acoustic waves in magnetized plasma. They predicted the existence of negative potential structures in both magnetized and unmagnetized cases. Devanandhan et al [18] have investigated EASWs in two component magnetized plasma and predicted negative solitary potential structures. They further showed that with the increase in magnetic field, the soliton electric field amplitude increases while the soliton width and pulse duration decreases. The properties of small amplitude wave in magnetized plasma are investigated by Pakzad and Javidan [19]. They observed both rarefactive and compressive solitons whose profiles become narrower with the application of stronger magnetic fields.

The deviations of electron populations from their thermodynamic equilibrium have been reported by many space plasma observations. A nonextensive distribution is the most generalized distribution to study the linear and nonlinear properties of solitary waves in different plasma systems, where the non-equilibrium stationary states exist. The nonextensive statistical mechanics has gathered immense attention over the last two decades. This mechanics is based on the deviations of Boltzmann-Gibbs-Shannon (B-G-S) entropy measures first recognized by Renyi [20] subsequently proposed by Tsallis [21]. The Maxwellian distribution in Boltzmann–Gibbs statistics is valid universally for the macroscopic ergodic equilibrium systems. While for systems having long-range interactions, the complete description of the features becomes inadequate with Maxwellian distribution. The parameter q that underpins the generalized entropy of Tsallis, is associated to the underlying dynamics of the system and measures the amount of its nonextensivity. For q<1 (i.e. superextensivity), the generalized entropy of whole (i.e. S_{whole}) is greater than the entropies of subsequent parts (i.e. $S_{sub\text{-}parts}$). However q>1 i.e. subextensivity corresponds to $S_{whole} < S_{sub\text{-}parts}$. The nonextensive statistics has found applications in a large quantity of astrophysical and cosmological atmospheres such as steller polytropes [22], the solar neutrino problem [23], peculiar velocity distributions of galaxies [24] and systems with long range interactions and also fractal-like space-times. Different types of waves, viz. ion acoustic (IA) waves, electron-acoustic (EA) waves, or dust-acoustic (DA) waves in nonextensive plasmas are investigated by many researchers considering one or two components to be nonextensive [25-33]. Ferdousi et al [34] studied the properties of small amplitude ion-acoustic solitary waves (IASWs) in three component magnetized electron-positron-ion plasma. They considered Tsallis distributed electrons and cold ions for their analysis and discussed the effects of magnetic field and electron and positron nonextensivity on the propagation of solitary waves. However, in the present investigation, we aim at studying the effect of ion temperature on the obliquely propagating solitary waves in two component magnetized plasma system with nonextensive distributed electrons. The paper is organized as follows: in Sec. 2, the basic equations governing the plasma dynamics and the derivation of Korteweg-de Vries (KdV) equation is given. In Sec. 3, we present the numerical analysis and discussion of the results. Finally, we conclude the paper in Sec. 4.

2. BASIC EQUATIONS AND NONLINEAR ANALYSIS

Let us consider the homogeneous magnetized plasma containing q-nonextensive electrons and stationary warm ions. The external static magnetic field is assumed to point in the z-direction i.e. $B = B_0 \hat{z}$. The dynamics of the propagation of waves in such magnetized plasma is governed by the following set of normalized equations:

$$\frac{\partial n}{\partial t} + \nabla.(nu) = 0 \qquad (1)$$

$$\frac{\partial u}{\partial t} = (u.\nabla)u = -\nabla\phi - \omega_0 (u \times \hat{z}) - \frac{5}{3}\frac{\sigma}{n^{1/3}}\nabla n \qquad (2)$$

$$\nabla^2 \phi = n_e - n \qquad (3)$$

where *n* and *u* are the ion number density and ion fluid velocity normalized to equilibrium plasma density n_0 and ion acoustic speed $C_s = (T_e/m)^{1/2}$, T_e is the electron temperature and m is the mass of positively charged ions, respectively. ϕ is the electrostatic wave potential normalized to T_e/e, where e is the magnitude of electron charge and $\sigma = T_i/T_e$ with T_i being the ion temperature. In this plasma model, ion plasma period $\omega_p^{-1}=(m/4\pi n_o e^2)^{1/2}$, the Debye length $\lambda_D=(T_e/4\pi n_o e^2)^{1/2}$ and ion cyclotron frequency is given by $\omega_c=(eB_0/m\omega_p)$. The number density of electron fluid with nonextensive distribution is given by:

$$n_e = \left(1+\left(q-1\right)\phi\right)^{\frac{(q+1)}{2(q-1)}} \tag{4}$$

where q is the nonextensivity parameter. The electron distribution reduces to the well-known Maxwell Boltzmann distribution for the extensive limiting case q approaches to 1 [33]. In transformations given by Gardner and Morikawa [35] put $\alpha=1/2$ the stretched coordinates becomes $\xi=\varepsilon^{1/2}(l_x x+l_y y+l_z z-v_0 t)$, $\tau=\varepsilon^{3/2}t$. Here v_0 is the linear phase velocity and ε is a small parameter. l_x, l_y, l_z are the direction cosines of the wave vector with respect to the x, y and z axes respectively. The perturbed quantities are expanded in power series of ε as follows:

$$\begin{aligned}
n &= 1+\varepsilon n^{(1)}+\varepsilon^2 n^{(2)}+\varepsilon^3 n^{(3)}+\ldots\ldots\ldots\ldots \\
u_{x,y} &= 0+\varepsilon^{\frac{3}{2}} u_{x,y}^{(1)}+\varepsilon^2 u_{x,y}^{(2)}+\varepsilon^{\frac{5}{2}} u_{x,y}^{(3)}+\ldots\ldots\ldots \\
u_z &= 0+\varepsilon u_z^{(1)}+\varepsilon^2 u_z^{(2)}+\varepsilon^3 u_z^{(3)}+\ldots\ldots\ldots\ldots \\
\phi &= 0+\varepsilon \phi^{(1)}+\varepsilon^2 \phi^{(2)}+\varepsilon^3 \phi^{(3)}+\ldots\ldots\ldots\ldots
\end{aligned} \tag{5}$$

Now using the number density of electron fluid given by equation (4), stretching coordinates ξ and τ and the expansions (5) into (1)-(3). Comparing the coefficients of lowest order of ε i.e. $\varepsilon^{3/2}$, we get the linear dispersion relation which is given by the following expression.

$$v_0^2 = \frac{l_z^2}{c_1}\left[1+\frac{5}{3}\sigma c_1\right] \tag{6}$$

where $c_1=(q+1)/2$ and the phase velocity depends upon the ion to electron temperature ratio σ, the strength of nonextensivity q and obliqueness of propagation γ. It may be noted that in the limit $\sigma \to 0$, our expression of phase velocity becomes exactly similar to that derived by Ferdousi et al [34] for $\mu_p=0$. Mathematical relation (6) shows that phase velocity increases with ion to electron temperature ratio σ and decreases with non-extensive parameter (q) for all ranges of q. The q-dependence of phase velocity comes from the factor c_1 in the expression (6). Similar kind of behavior has been observed by Ferdousi et al [34], Akhtar et al [27] and Sahoo et al [36] in their respective researches. To investigate the effect of ion temperature, figure 1 shows the typical variation of the phase velocity v_0 with respect to angle of propagation γ for three different values of $\sigma=T_i/T_e$. It is observed that wave phase velocity decreases with angle between the direction of the wave propagation vector k and the external magnetic field B_0. The decrease of v_0 with γ also becomes clear from the expression (6) where $v_0 \propto \sqrt{\cos\gamma}$ and becomes zero for $\gamma = 90^0$. This decreasing trend of v_0 with γ is similar to that observed by Misra and Wang [37]. In order to investigate the electrostatic propagation, we consider small oblique angle. From the figure 1, it becomes clear that the phase velocity increases with increase in the temperature ratio σ. Hence ion temperature significantly effect the dynamics of given plasma system. Further, the wave phase velocity is found to be independent of the magnetic field strength and decreases with nonextensivity q (similar to the observations of Ferdousi et al [34]).

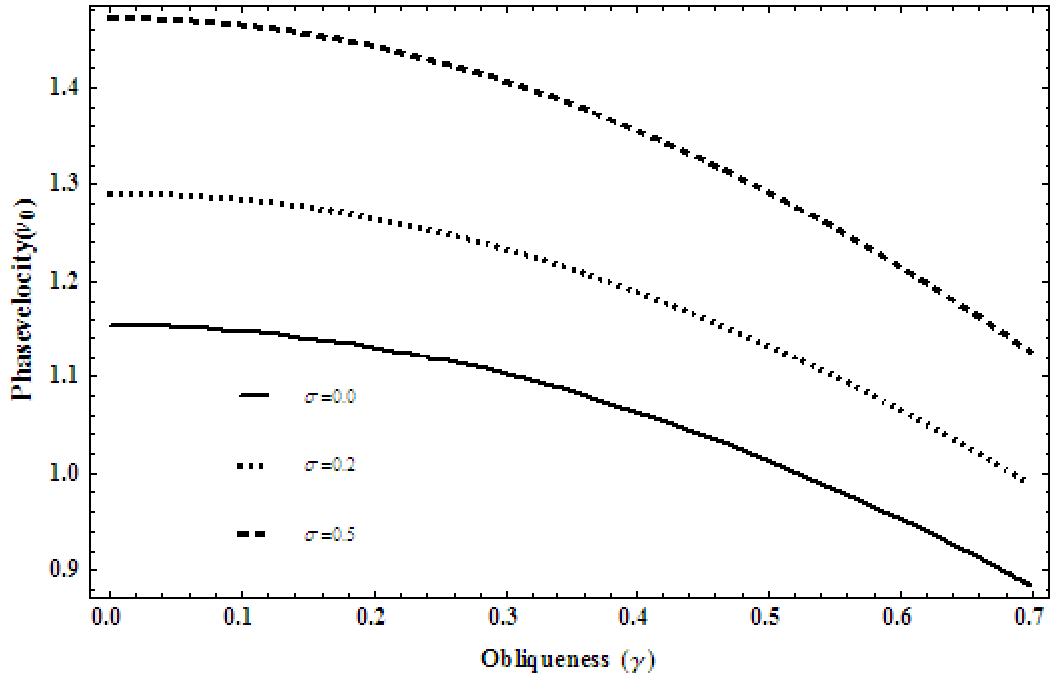

Fig.1. Variation of wave phase velocity (v_0) with angle of propagation (γ) for three values of ion temperature σ for q= 0.5.

Going to the next higher order of ε i.e. ε^2 and by doing algebraic manipulations, we get the following Korteweg-de Vries (KdV) equation (7) in which we have replaced $\phi^{(1)}$ with ϕ for simplicity.

$$\frac{\partial \phi}{\partial \tau} + A\phi \frac{\partial \phi}{\partial \xi} + B\frac{\partial^3 \phi}{\partial \xi^3} = 0 \qquad (7)$$

where A is non-linear and B is dispersion coefficients and are given as:

$$A = l_z\sqrt{c_1}\sqrt{1+\frac{5}{3}\sigma c_1}\left[\frac{3}{2} - \left[\frac{5\sigma}{18} + \frac{c_2}{c_1^3}\right]\frac{c_1}{\left[1+\frac{5}{3}\sigma c_1\right]}\right] \qquad (8)$$

$$B = \frac{1}{2}\frac{l_z}{c_1\sqrt{c_1}\sqrt{1+\frac{5}{3}\sigma c_1}}\left[1+\left[\frac{1-l_z^2}{\omega_0^2}\right]\left[1+\frac{5}{3}\sigma c_1\right]^2\right] \qquad (9)$$

where c_2=(q+1)(3-q)/8 and in the the limit $\sigma \rightarrow 0$, our expressions of nonlinear and dispersion coefficients A and B become exactly similar to that derived by Ferdousi et al [34] for μ_p=0 and μ_e=1. Now, the stationary solitary wave solution of Eq. (7) is directly given by

$$\phi = \phi_0\left[\sec h\left(\frac{\eta}{\delta}\right)\right]^2 \qquad (10)$$

where the amplitude ϕ_0 and width δ of the soliton are given by $\phi_0 = 3u_0/A$ and $\delta = (4B/u_0)^{1/2}$ and here $\eta = \xi - u_0\tau$. From the expressions of A and B (i.e. Eqns. (8) and (9)), it is found that the amplitude of the soliton depends on the ion and electron temperature ratio σ and independent of magnetic field. On the other hand, the width of the soliton depends on the strength of external magnetic field.

3. RESULTS AND DISCUSSION

In this paper, we have investigated the effects of ion temperature on the nonlinear wave propagation of small amplitude solitary waves in two component magnetized plasma. To describe the nonlinear propagation of the waves, we have derived a KdV equation (7) and obtained solitary wave solution (10). Depending upon the value of nonlinear coefficient A, the solitary wave might be associated with positive or negative potentials. Equation (8) indicates that A is dependent on parameters such as *q*, σ, $l_z = \cos(\gamma)$ which define the nature of solitary waves. We have concentrated our investigation to study the effect of ion temperature as much of other features are studied by Ferdousi et al [34].

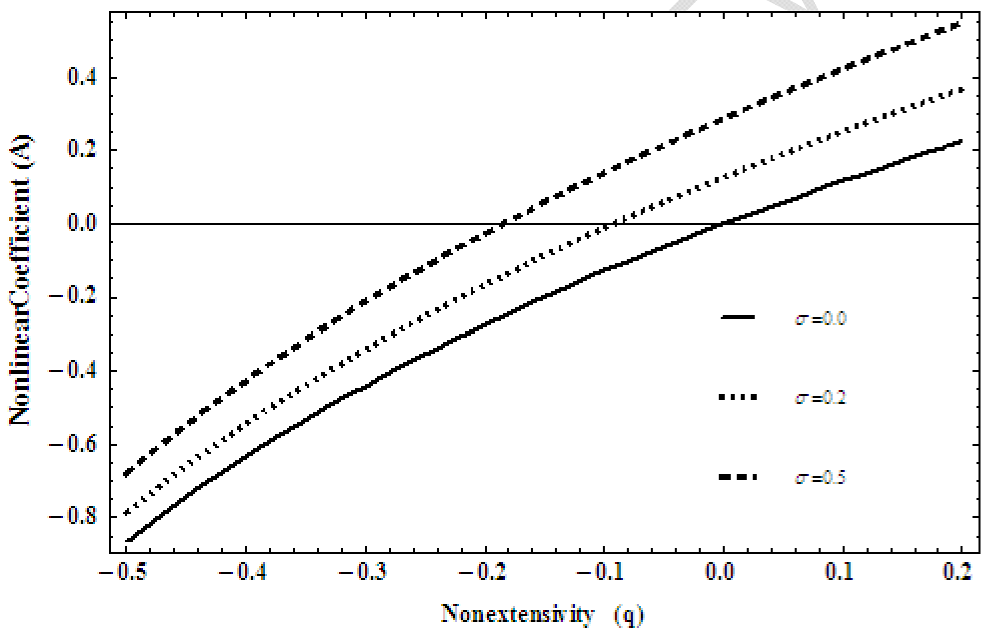

Fig.2. For $\gamma = 30^0$, variation of nonlinear coefficient (A) as a function of nonextensivity q at three different values of ion temperature σ. Here solid line stands for σ=0.0, dotted line for σ=0.2 and dashed line is for σ=0.5.

The nonlinear coefficient (A) as a function of nonextensivity (*q*) is displayed in Figure 2 for three different values of ion temperature σ. A transition from negative to positive potential structures results at a certain critical value of nonextensive parameter (q_c). We observe that at $q > q_c$, positive (hump shape or commonly known as compressive soliton) solitary waves exist, whereas at $q < q_c$, negative (dip shape or rarefactive solitons) solitary waves exist. Ferdousi et al [34] reported that the critical value q_c dependent on the parameters such as positron and electron density and electron-positron temperature ratio and independent of the obliqueness. In our case, the critical value of nonextensivity is also a function of ion temperature. It becomes obvious from figure 2, where a plot of nonlinear coefficient A as a function of nonextensivity is displayed for three values of ion temperature. The critical value of nonextensivity i.e. q_c decreases with increase in ion temperature.

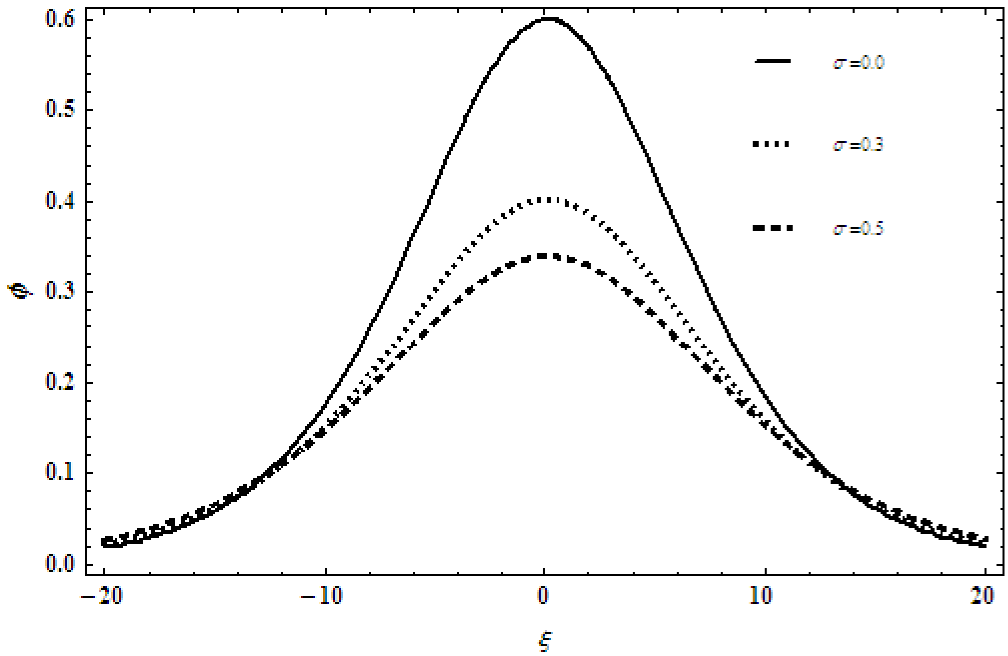

Fig. 3. For the range 0<q<1, variation of soliton solution ϕ as a function of parameter ξ at three different values of ion temperature i.e. σ=0.0 (Solid Line), σ=0.3 (Dotted Line) and σ=0.5 (Dashed Line) with other parameters as $\gamma = 30^0$, $\omega_0 = 0.40$ and $u_0 = 0.10$.

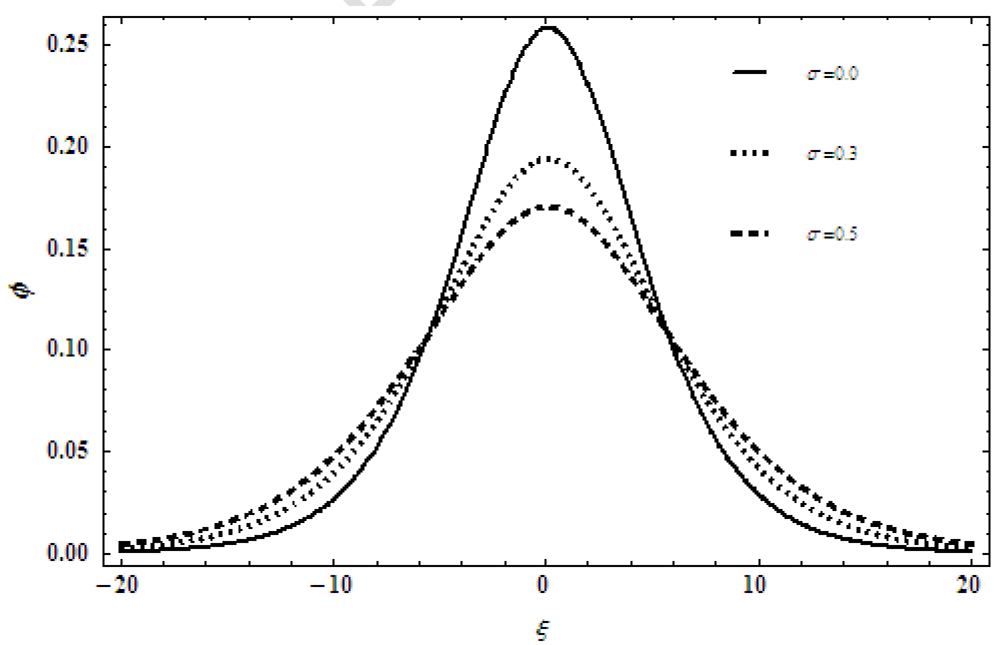

Fig. 4. For the range q>1, variation of soliton solution ϕ as a function of ξ for three different values of ion temperature i.e. σ=0.0 (Solid Line), σ=0.3 (Dotted Line) and σ=0.5 (Dashed Line) with other parameters as $\gamma = 30^0$, $\omega_0 = 0.40$ and $u_0 = 0.10$.

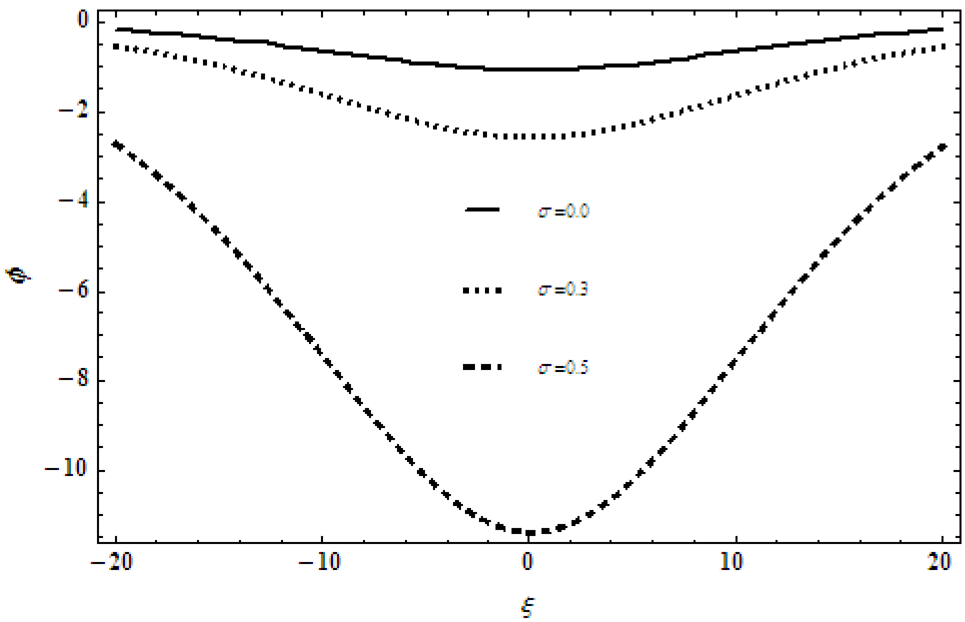

Fig. 5. For the range -1<q<0, variation of soliton solution ϕ as a function of ξ for three different values of ion temperature i.e. σ=0.0 (Solid Line), σ=0.3 (Dotted Line) and σ=0.5 (Dashed Line) with other parameters as $\gamma = 30^0$, $\omega_0 = 0.40$ and $u_0 = 0.10$.

In order to investigate the effect of ion temperature σ on the solitary wave profiles for three different ranges of nonextensivuty q viz. *0<q<1, q>1* and *-1<q<0,* graphs have been displayed in Figures 3, 4 and 5 respectively. The values of other parameters taken for the analysis are as follows: obliqueness $\gamma = 30^0$, $\omega_0 = 0.40$ and $u_0 = 0.10$. Here the solid line corresponds to σ=0.0 i.e. cold ions, dotted and dashed lines correspond to σ=0.2 and σ=0.5 respectively. For the ranges 0<q<1 and q>1, positive potential structures result as mentioned earlier. However, in the given parameter regimes, negative potential structures are observed for the range -1<q<0. It is observed that the peak amplitude of positive as well as negative potential structures decreases with increase in ion temperature. Hence the ion temperature plays a significant role here. Figure 6 presents a clearer picture of the dependence of ion temperature on the amplitude of solitary profiles. Here a plot of peak amplitude of solitary waves has been displayed as a function of nonextensivity q at three different values of ion temperature. Hence ion temperature is an important parameter in shaping the behavior of solitary structures.

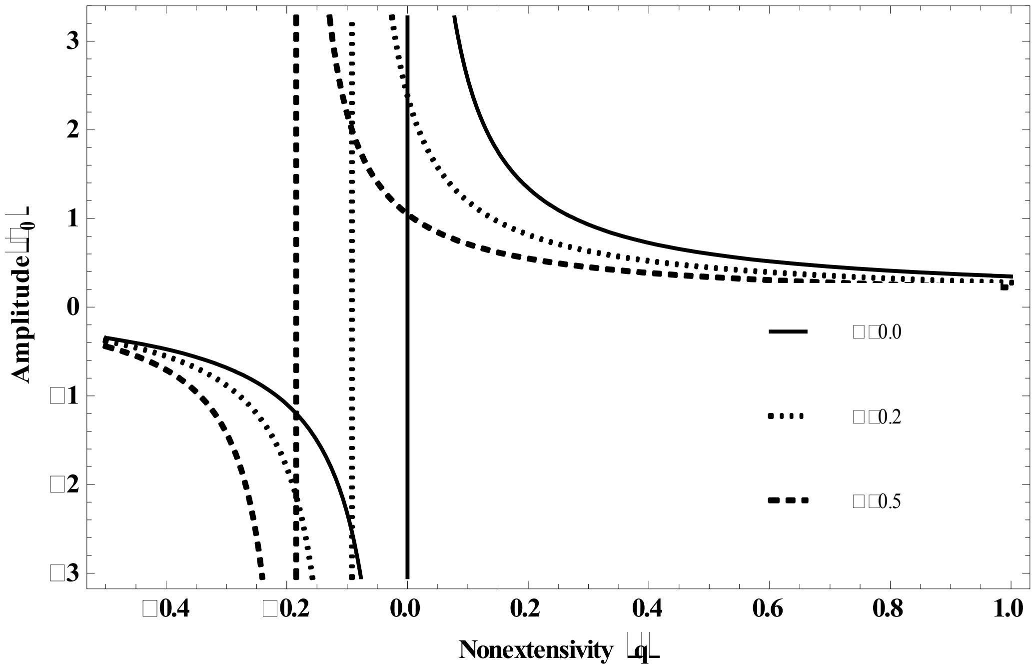

Fig. 6. Variation of soliton amplitude ϕ_0 as a function of q for three different values of σ = 0.0 (Solid Line), 0.2 (Dotted Line) and 0.5 (Dashed Line) with $\gamma = 30^0$, $\omega_0 = 0.40$ and $u_0 = 0.10$.

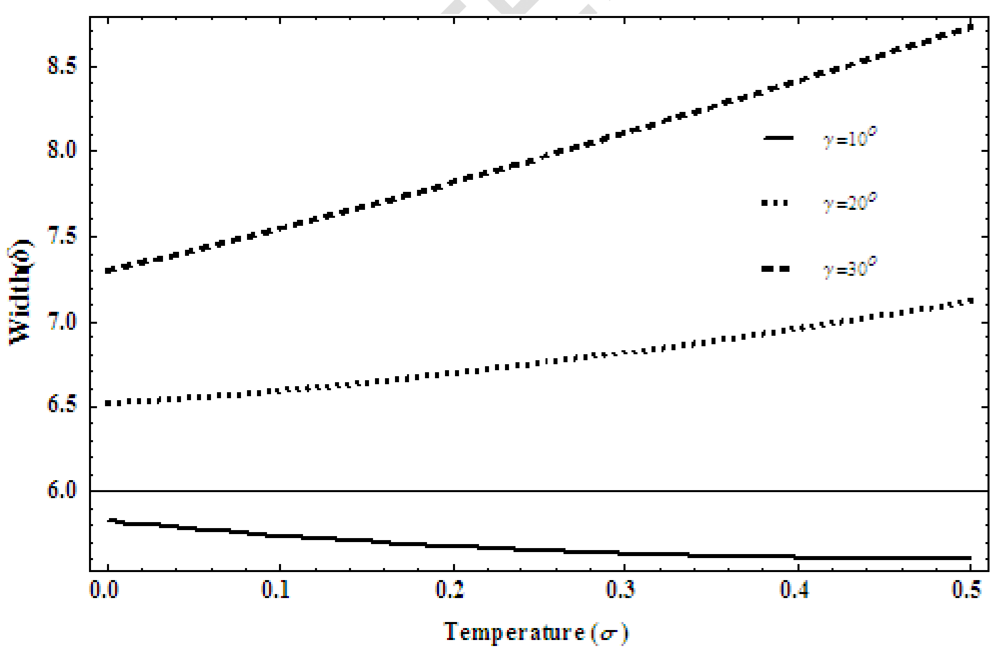

Fig.7. Variation of soliton width δ as a function of σ for different values of $\gamma = 10^0$ (Solid Line), 20^0 (Dotted Line) and 30^0 (Dashed Line) with q = 0.5, $\omega_0 = 0.5$ and $u_0 = 0.10$.

Figure 7 represents soliton width δ as a function of ion temperature (σ) at three different values of obliqueness with other parameters as q = 0.5, ω_0 = 0.5 and u_0 = 0.10. This figure clearly shows the impact of ion temperature on the width of the solitary structures. Here solid line corresponds to $\gamma=10^0$, dotted line for $\gamma=20^0$ and dashed line for $\gamma=30^0$. A peculiar behavior is observed at low and high value of obliqueness. It is found that for lower value of obliqueness i.e. $\gamma=10^0$, the width of solitary structure decreases with ion temperature. However. The trend becomes opposite on increasing the obliqueness as is clear from the dotted ($\gamma=20^0$) and dashed ($\gamma=30^0$) curves. Width starts increasing with ion temperature for higher values of obliqueness. Hence, introduction of finite ion temperature has significant effect here. It is evident that higher magnitudes of σ cause significant reduction in the amplitude of the solitary waves. But the soliton width increases with increase in ion temperature σ, a result which is in agreement with Akhtar et al [27].

4. CONCLUSION

The properties of wave propagation of solitary waves are greatly modified by the parameters like nonextensivity, strength of magnetic field, ion to electron temperature ratio and obliqueness. In the limit of σ□0, all our results become similar to that obtained by Ferdousi et. al [34. However our main findings with special reference to ion temperature are summarized below:

(i) Phase velocity of solitary wave increases with ion to electron temperature ratio.

(ii) The critical value of nonextensivity i.e. q_c decreases with increase in ion temperature.

(iii) The amplitudes of positive as well as negative potential structures decrease with ion temperature.

(iv) Width decreases with σ for lower value of obliqueness, while it shows an increase with σ at higher value of obliqueness.

The present investigation may be helpful in understanding the study of nonlinear electrostatic waves propagating in astrophysical and laboratory plasmas.

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