
Analytical solutions to generalized nonlinear Schrödinger equation by Adomian decomposition technique

**Original Research
Article**

Abstract

We study the higher-order nonlinear Schrödinger equation which takes care of the second as well as third order dispersion effects, cubic and quintic self phase modulations, self steepening and nonlinear dispersion effects. Taking advantage of the initial condition, we transform the previous equation into a nonlinear functional equation to which we apply a powerful analytical method called the Adomian decomposition method. We compute the Adomian's polynomials of corresponding infinite series solution. Assuming that the initial condition and all its derivatives converge to zero sufficiently rapidly as the time approaches to infinity, we established the convergence of the previous series. The last part of the paper describes applications resulting from nonlinear propagation phenomena in optical fibers. Numerical simulations are developed and it is further shown that comparison with other results yields a good qualitative agreement. These results demonstrate the robustness of the proposed method.

Keywords: higher-order nonlinear Schrödinger equation; Adomian decomposition technique; optical fibers

2010 Mathematics Subject Classification: 53C25; 83C05; 57N16

1 Introduction

For describing the dynamics of light pulses in optical fibers, one uses the mathematical model namely the nonlinear Schrödinger (NLS) equation [Agrawal (2007)]. In [Anderson (1983)], the NLS equation is formulated as a variational problem and comparison with scattering theory showed agreement. For picosecond light pulses, the NLS equation includes only the group velocity dispersion and the self-phase modulation, well known in fibers [Hong (2001)]. However, as one increases the intensity of the incident light power to produce shorter (femtosecond) pulses, additional nonlinear effects become important and the dynamics of pulses needs to be described in the frame of a generalized nonlinear Schrödinger equation (GNLSE) that includes higher-order nonlinear terms [Triki and Wazwaz (2016)], [Zayed and Amer (2017)]. Propagation of femtosecond pulse in fiber links can be described by the GNLSE which takes care of the second as well as third order dispersion effects, cubic and quintic self phase modulations, self steepening and nonlinear dispersion effects and can be written as [Shwetanshumala (2008)] and [Zhang G. and Zhang X. (2007)]:

$$\psi_z + a_1\psi + a_2\psi_{tt} + a_3|\psi|^2\psi + a_4\psi_{ttt} + a_5(|\psi|^2\psi)_t + a_6(|\psi|^4\psi)_t + a_7|\psi|^4\psi = 0, \quad (1.1)$$

where ψ is a complex valued function of the normalized spatial coordinate z (distance of transmission through the fiber) and the normalized time t (ψ the slowly varying envelope of the electric field), $a_j, j = 1, \dots, 7$ are complex parameters and the subscripts t and z denote the differentiation with respect to time and space, respectively; ψ_{tt} represents the normalized group velocity dispersion, with the next cubic term describing self phase modulations due to the Kerr effect. The term involving a_4 represents the third order dispersion, while the term involving a_5 is the self steepening effect. The sixth and seventh terms in the left hand side respectively describe the nonlinear dispersion and self phase modulation effects due to quintic nonlinearity. The original input optical pulse is defined as

$$\psi(z, t)|_{z=0} = g(t), \quad (1.2)$$

where g is a given complex valued function of the time t .

The GNLSE has become a standard tool for simulating the propagation of strong light pulses in optical fibers [Laegsaard (2007)], [Zayed and Amer (2014)], [Zheng (2011)]. For example, in order to describe the optical pulse propagation in a nonlinear dispersion medium, one derived this previous equation from Maxwell's equations under the slowly varying envelope approximation [Edah et al. (2015)]. For this equation, both the analytical and the numerical solutions have been subject to intensive investigations. It is a well known fact that a few exact solutions of the GNLSE are known even now. This has been largely due to the complexity of the partial differential equation. The construction of an exact solution for the nonlinear evolution equation is an important topic in the study of nonlinear physical phenomena because exact solutions can help one to well understand the mechanism of the complicated physical phenomena and dynamical processes modeled by nonlinear evolution equations. Various methods have been employed for the analytical study, the Lagrangian variational method being the most common of them. In this method, the Lagrangian density of the system is constructed and a suitable trial function for the pulse is assumed. In a series of remarkable papers [Shwetanshumala (2008)], [Artigas et al. (1997)], [Bogning (2006)], [Gulcin and Husnu (2004)], [Hasegawa and Tappert (1973)], [Kruglov et al. (2005)], [Mollenauer (1980)], [Rach and Adomian (1994)], [Scott and Sun (2007)], [Xiao et al. (2012)], [Wei et al. (2011)], [Ali et al. (2017)], [Li et al. (2017)] the authors have studied this equation using various approaches. These methods can be classified into two broad categories known as the finite-difference methods and the pseudo spectral methods [Edah et al. (2015)]. Nowadays the method that has been used extensively is the split-step Fourier method which has a faster calculation speed than the other finite-difference methods. So it is very important to develop efficient methods to solve this equation. One of the methods is the Adomian decomposition method for solving a wide range of physical problems [Adomian (1994)], [Bahman and Amaneh (2014)]. Several modifications were improved its ability in [Abbaoui (1994)], [Abdelrazec and Pelinosky (2008)], [Bahman and Amaneh (2014)],

[Cherruault (1989)], [Jin and Liu (2005)], [Ghosh and Roy (2007)], [Jafari and Daftardar-Gejji (2006)], [Pamuk (2005)], [Zhang G. and Zhang X. (2007)], [Kaya and Yokus (2002)]. An advantage of this method is that it can provide analytical approximation or an approximated solution to a wide class of nonlinear equations without linearization, perturbation, closure approximation or discretization methods [Weidman and Herst (1986)]. The paper is organized as follows. In section 2, we solve the generalized nonlinear Schrödinger equation and prove the convergence of the solutions. In section 3, numerical investigation of the obtained analytical solutions is given. A brief conclusion is incorporated in section 4.

2 Analytical solution

In this section, we provide the analytical solution to the problem (1.1) and (1.2). Taking account of the initial condition (1.2) and integrating the equation (1.1) from 0 to z , we have

$$\psi - N(\psi) = g(t) \tag{2.1}$$

where

$$N(\psi) = \int_0^z L[\psi(\xi, t), \psi^*(\xi, t), \psi_t(\xi, t), \psi_t^*(\xi, t), \psi_{tt}(\xi, t), \psi_{ttt}(\xi, t)] d\xi;$$

$$\begin{aligned} L(\psi, \psi^*, \psi_t, \psi_t^*, \psi_{tt}, \psi_{ttt}) &= -a_1\psi - a_2\psi_{tt} - a_3\psi^2\psi^* - a_4\psi_{ttt} - a_5(2\psi\psi_t\psi^* + \psi^2\psi_t^*) \\ &\quad - a_6(3\psi^2\psi_t\psi^{*2} + 2\psi^3\psi_t^*\psi^*) - a_7\psi^3\psi^{*2}. \end{aligned}$$

Here N is a nonlinear operator from a Hilbert space \mathcal{H} into \mathcal{H} .

In [Adomian (1994)]-[Bahman and Amaneh (2014)], Adomian decomposition method for solving linear and nonlinear equations was developed. In [Cherruault (1989)], [Abdelrazec and Pelinosky (2008)] it was shown that the Adomian decomposition method provides a fast convergent series. We assume that (2.1) has a unique solution. Let us write the solution of (2.1) as an infinite series as follows

$$\psi = \sum_{n \geq 0} \psi_n \tag{2.2}$$

using the following scheme:

$$\begin{aligned} \psi_0 &= g(t); \\ \psi_{n+1} &= A_n(\psi_0, \psi_2, \dots, \psi_n), \quad n = 0, 1, 2, 3, \dots, \end{aligned}$$

where $A_n(\psi_0, \psi_1, \dots, \psi_n)$ are polynomials of $\psi_0, \psi_1, \dots, \psi_n$ defined by

$$A_n = \frac{1}{n!} \left[\frac{d^n}{d\lambda^n} N \left(\sum_{i \geq 0} \lambda^i \psi_i \right) \right]_{\lambda=0}, \quad n = 0, 1, 2, 3, \dots$$

It comes that

$$A_n = \frac{1}{n!} \int_0^z \left[\frac{d^n}{d\lambda^n} L \left(\sum_{i \geq 0} \lambda^i \psi_i, \sum_{i \geq 0} \lambda^i \psi_i^*, \sum_{i \geq 0} \lambda^i \psi_{it}, \sum_{i \geq 0} \lambda^i \psi_{it}^*, \sum_{i \geq 0} \lambda^i \psi_{itt}, \sum_{i \geq 0} \lambda^i \psi_{ittt} \right) \right]_{\lambda=0} d\xi$$

Let us determine these polynomials. See Appendix for more details. We have

$$\psi_0 = g(t); \tag{2.3}$$

$$\psi_1 = zh_1(t); \tag{2.4}$$

$$\psi_2 = \frac{1}{1!} z^2 h_2(t); \tag{2.5}$$

$$\psi_3 = \frac{1}{2!} z^3 h_3(t) \tag{2.6}$$

and by induction

$$\psi_n = \frac{z^n h_n(t)}{(n-1)!}, \quad n \geq 1. \tag{2.7}$$

Therefore, the function ψ defined by

$$\psi(z, t) = g(t) + \sum_{n \geq 1} \frac{z^n h_n(t)}{(n-1)!} \tag{2.8}$$

is the exact solution of the previous equation satisfying the condition

$$\psi(z, t)|_{z=0} = g(t)$$

where h_i are functions depending on g, g^* and their derivatives, given by

$$h_1(t) = -a_1 g(t) - a_2 g''(t) - a_3 |g(t)|^2 g(t) - a_4 g'''(t) - a_5 [2|g(t)|^2 g'(t) + g^2(t)g^*(t)] - a_6 [3g'(t)|g(t)|^4 + 2g^2(t)|g(t)|^2 g^*(t)] - a_7 |g(t)|^4 g(t); \tag{2.9}$$

$$h_2(t) = \frac{1}{2} \left\{ h_1(t) \left[-a_1 - 2a_3 |g(t)|^2 - 2a_5 (g'(t)g^*(t) + g(t)g^{**}(t)) - 6a_6 (|g(t)|^2 g'(t)g^*(t) + g(t)|g(t)|^2 g^*(t)) - 3a_7 |g(t)|^4 \right] + h_1^*(t) \left[-a_3 g^2(t) - 2a_5 g(t)g'(t) - a_6 (6|g(t)|^2 g(t)g'(t) + 2g^3(t)g^*(t)) - 2a_7 g^2(t)|g(t)|^2 \right] + h_1'(t) \left[-2a_5 |g(t)|^2 - 3a_6 |g(t)|^4 \right] + h_1^{**}(t) \left[-a_5 g^2(t) - 2a_6 g^2(t)|g(t)|^2 \right] - a_2 h_1''(t) - a_4 h_1'''(t) \right\}; \tag{2.10}$$

$$h_3(t) = \frac{1}{3} \left\{ h_1^2(t) \left[-2a_3 g^*(t) - 2a_5 g^{**}(t) - 6a_6 (g'(t)g^{*2}(t) + 2|g(t)|^2 g^*(t)) - 6a_7 g(t)g^{*2}(t) \right] + h_1^{*2} \left[-6a_6 g^2(t)g'(t) - 2a_7 g^3(t) \right] + 2h_1(t)h_1^*(t) \left[-2a_3 g(t) - 2a_5 g'(t) - 6a_6 (2|g(t)|^2 g'(t) + g^2(t)g^*(t)) - 6a_7 g(t)|g(t)|^2 \right] + 2h_1(t)h_1'(t) \left(-2a_5 g^*(t) - 6a_6 |g(t)|^2 g^*(t) \right) + 2h_1(t)h_1^{**}(t) \left[-2a_5 g(t) - 6a_6 g(t)|g(t)|^2 \right] + 2h_1^*(t)h_1'(t) \left[-2a_5 g(t) - 6a_6 g(t)|g(t)|^2 \right] - 4a_6 g^3(t)h_1^*(t)h_1^{**}(t) + 2h_2(t) \left[-a_1 - 2a_3 |g(t)|^2 - 2a_5 (g'(t)g^*(t) + g(t)g^{**}(t)) - 6a_6 (|g(t)|^2 g'(t)g^*(t) + |g(t)|^2 g(t)g^*(t)) - 3a_7 |g(t)|^4 \right] + 2h_2^*(t) \left[-a_3 g^2(t) - 2a_5 g(t)g'(t) - a_6 (6|g(t)|^2 g(t)g'(t) + 2g^3(t)g^*(t)) - 2a_7 |g(t)|^2 g^2(t) \right] + 2h_2'(t) \left[-2a_5 |g(t)|^2 - 3a_6 |g(t)|^4 \right] + 2h_2^{**}(t) \left[-a_5 g^2(t) - 2a_6 |g(t)|^2 g^2(t) \right] - 2a_2 h_2''(t) - 2a_4 h_2'''(t) \right\}. \tag{2.11}$$

Finally we arrive at the following result Assume that the functions g, h_n satisfy the conditions

$$\begin{cases} g(t) \rightarrow 0 \text{ as } |t| \rightarrow +\infty \\ \sup_{t \in \mathbf{R}} |h_n(t)| \leq c_n, \quad \forall n \geq 1 \end{cases} \tag{2.12}$$

and the positive numerical series

$$\sum_{n \geq 1} c_n \tag{2.13}$$

converges.

Then the functional series

$$\sum_{n \geq 1} \frac{z^n h_n(t)}{(n-1)!} \tag{2.14}$$

converges uniformly on the set $[0, +\infty[\times \mathbf{R}$. Indeed,

$$\sup_{t \in \mathbf{R}} \left| \sum_{n \geq 1} \frac{z^n h_n(t)}{(n-1)!} \right| \leq \sum_{n \geq 1} \frac{z^n}{(n-1)!} \sup_{t \in \mathbf{R}} |h_n(t)|.$$

By virtue of the conditions (2.12) and (2.13),

$$\sup_{t \in \mathbf{R}} \left| \sum_{n \geq 1} \frac{z^n h_n(t)}{(n-1)!} \right| \leq \sum_{n \geq 1} \frac{c_n z^n}{(n-1)!}$$

By means of d'Alembert's test, the series

$$\sum_{n \geq 1} \frac{c_n z^n}{(n-1)!}$$

converges. Therefore,

$$\sum_{n \geq 1} \frac{z^n h_n(t)}{(n-1)!}$$

converges uniformly on the set $[0, +\infty[\times \mathbf{R}$. \square

3 Numerical approach and results

The purpose of this part is to provide numerical results for comparison with other results obtained using other methods of calculations both analytically and numerically. During the numerical calculation, different input sources with different duration, and different choices of parameters are used. As it is well known, the dispersion and nonlinear lengths characterize the pulse propagation in a given fiber. Physically, the dispersion length L_D is length at which a Gaussian pulse broadens by a factor of 2 due to group velocity dispersion. The dispersion length depends strongly on the half-width of the source slowly varying pulse envelope and the second order dispersion term and L_N the nonlinear length given by:

$$L_D = \frac{T_0^2}{\beta_2}, \quad L_N = \frac{1}{\gamma P_0}; \tag{3.1}$$

where P_0 is the peak power and γ is the nonlinear Kerr effect coefficient.

3.1 Case of picosecond pulses

We use the following parameters and compare our results with those in [Zhang G. and Zhang X. (2007)] where rapid numerical difference recurrent formula of nonlinear Schrödinger equation is used. i.e $T_0 = 20ps, \beta_2 = -4ps^2/km, \beta_3 = 0ps^3/km$, the fiber loss is equal to zero as well as the nonlinear Kerr effect parameter. The propagation distances are respectively: $z = 0km, z = 10km$, and $z = 16km$. The results presented in Fig. 1 and Fig. 2 provide the same Gaussian profiles as those presented in figure 1 in [Zhang G. and Zhang X. (2007)]. It is clearly observed that when the propagation distance increases the peak power decreases and the wave spreads while keeping its

constant energy. This result is expected and confirms the predictions. But unfortunately, what is not expected and constitute a limitation of our approach is that the distance propagation must remain small compared to the distance L_D . The validity of our method requires to restrict the propagation distance to the interval $[0, L_D]$. On the other side, the comparison is simply analytical and does not strictly take into account the numerical values of the output energy. In fact the initial idea was to compare numerical values of the energy but we arrived at results such that the propagation distances that are input parameters are not the same. We can conclude that the optical pulse propagation in optical medium can be obtained by Adomian decomposition method provided that the propagation distance is less than the dispersion length.

From Fig. 1 and Fig. 2 we can conclude that the shape of the pulse for a given propagation distance z depends on the input optical pulse. And as we have observed in Fig. 1, the peak power of the pulse at the input is high and as the pulse propagates in the fiber, the peak decreases and it spreads. We also restrict ourselves in the propagation interval to $[0, L_D]$.

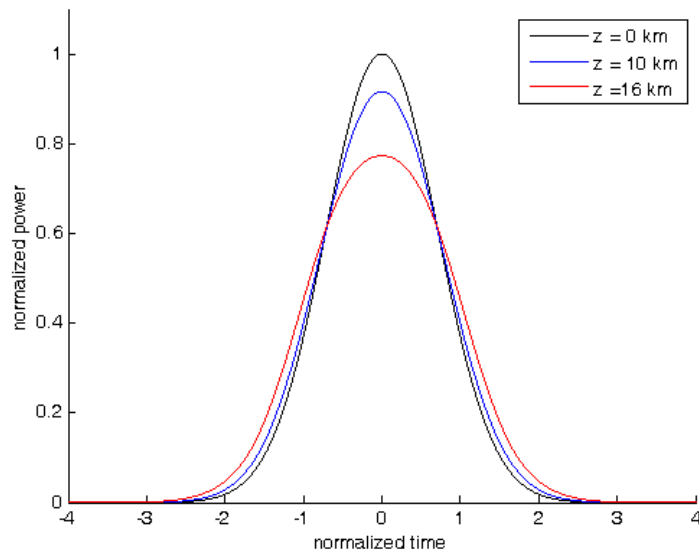


Figure 1: Adomian analytical solutions plots for distance propagation $z=0$ km, $z=10$ km and $z=16$ km respectively. The input optical pulse is Gaussian without chirp and loss.

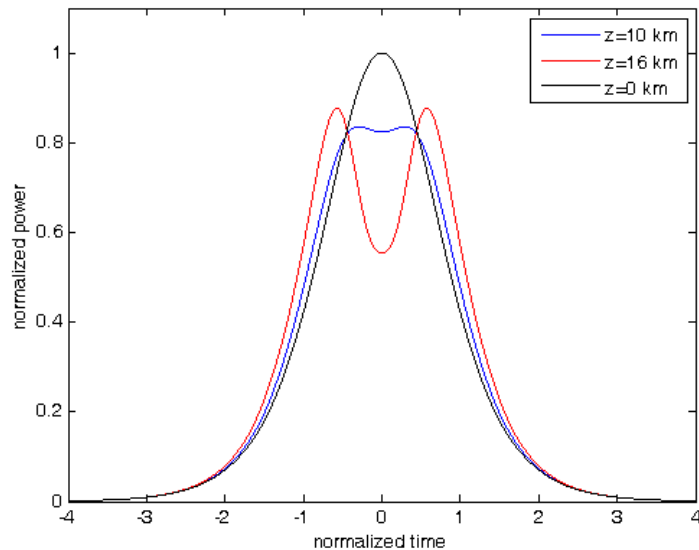


Figure 2: Adomian analytical solutions plots for distance propagation $z=0$ km, $z=10$ km and $z=16$ km respectively. The input optical pulse is hyperbolic secant without chirp and loss.

3.2 Case of femtosecond pulses

We consider that the input optical pulse is hyperbolic secant, the dispersive parameters used are the same as those in [Xiao et al. (2012)]: $\beta_2 = -1ps^2/km$, $\beta_3 = 0.0012ps^3/km$.

In Fig. 3 it can be observed that the pulse is shifted to the positive time axis during its propagation. In the numerical simulations, it is observed that the third order dispersion parameter and the self steepening parameter influence strongly the shape of the pulse during propagation. In Fig. 4 the influence of the fiber loss parameter a_1 is shown. The optical pulse energy dropped significantly. The arbitrary parameters used in Fig. 4 are: $a_2 = -i/2$, $a_3 = -i$, $a_4 = 0.02$, $a_5 = 0.8$, $a_6 = 0.1$, $a_7 = 0.5$.

4 Concluding remarks

In this paper, we have investigated the analytical solutions for the generalized nonlinear Schrödinger equation using the Adomian decomposition method. For the optical input pulse converging to zero sufficiently rapidly as the time approaches to infinity and for the bounded functions depending on the optical input pulse and its derivatives, we proved that the obtained solutions series of Adomian's polynomials converges uniformly on a given set. In order to validate the availability of our approach, we simulate numerically these obtained solutions by studying propagation of optical pulses inside a nonlinear dispersive medium. The picosecond and femtosecond optical input pulses with Gaussian and hyperbolic secant profiles are used. Thus the results showed an acceptable physical behavior of the solutions when the propagation distance is less than the dispersive length. We can conclude that Adomian decomposition method is an useful and important tool for scrutinizing the inside of an optical fiber provided that the propagation distance is less than the dispersive length.

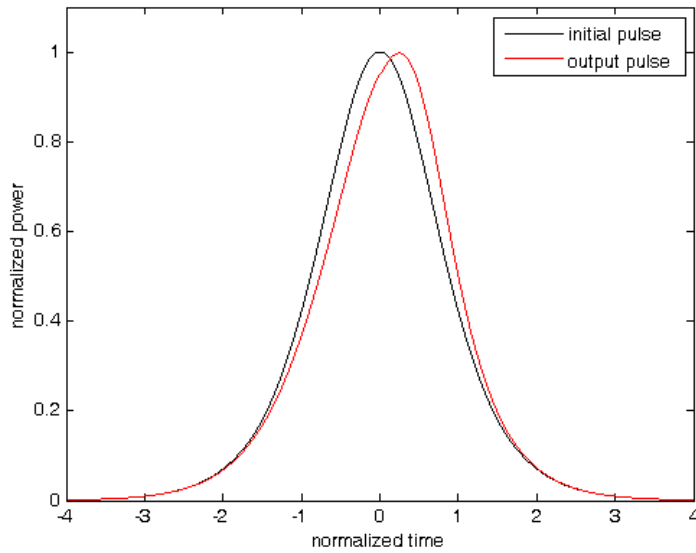


Figure 3: Adomian analytical solutions plots for distance propagation $z = 10^{-5} km$. The input optical pulse is hyperbolic secant.

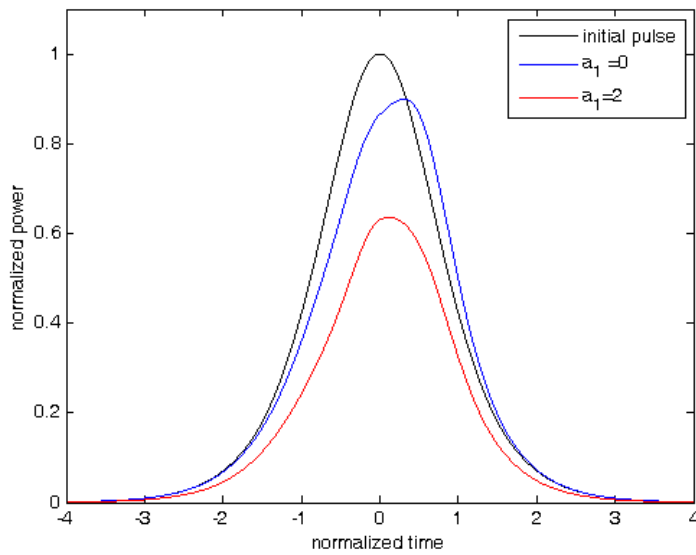


Figure 4: Adomian analytical solutions plots for distance propagation $z = 10^{-5} km$. The input optical pulse is hyperbolic secant; $\beta_2 = -1 ps^2/km$, $\beta_3 = 0.0012$; $a_1 = 0$; $a_1 = 2$

Appendix

In this appendix we list the first and second order derivatives of L with respect to $\psi, \psi^*, \psi_t, \psi_t^*, \psi_{tt}, \psi_{ttt}$ defining the expressions of $\frac{dL}{d\lambda}$ and $\frac{d^2L}{d\lambda^2}$:

$$\frac{dL}{d\lambda} = \frac{\partial L}{\partial \psi} \frac{d\psi}{d\lambda} + \frac{\partial L}{\partial \psi^*} \frac{d\psi^*}{d\lambda} + \frac{\partial L}{\partial \psi_t} \frac{d\psi_t}{d\lambda} + \frac{\partial L}{\partial \psi_t^*} \frac{d\psi_t^*}{d\lambda} + \frac{\partial L}{\partial \psi_{tt}} \frac{d\psi_{tt}}{d\lambda} + \frac{\partial L}{\partial \psi_{ttt}} \frac{d\psi_{ttt}}{d\lambda}$$

$$\begin{aligned} \frac{d^2L}{d\lambda^2} &= \frac{\partial^2 L}{\partial \psi^2} \left(\frac{d\psi}{d\lambda}\right)^2 + \frac{\partial^2 L}{\partial \psi^{*2}} \left(\frac{d\psi^*}{d\lambda}\right)^2 \\ &+ 2 \frac{\partial^2 L}{\partial \psi \partial \psi^*} \frac{d\psi}{d\lambda} \frac{d\psi^*}{d\lambda} + 2 \frac{\partial^2 L}{\partial \psi \partial \psi_t} \frac{d\psi}{d\lambda} \frac{d\psi_t}{d\lambda} + 2 \frac{\partial^2 L}{\partial \psi \partial \psi_t^*} \frac{d\psi}{d\lambda} \frac{d\psi_t^*}{d\lambda} \\ &+ 2 \frac{\partial^2 L}{\partial \psi^* \partial \psi_t} \frac{d\psi^*}{d\lambda} \frac{d\psi_t}{d\lambda} + 2 \frac{\partial^2 L}{\partial \psi^* \partial \psi_t^*} \frac{d\psi^*}{d\lambda} \frac{d\psi_t^*}{d\lambda} + \frac{\partial L}{\partial \psi} \frac{d^2\psi}{d\lambda^2} \\ &+ \frac{\partial L}{\partial \psi^*} \frac{d^2\psi^*}{d\lambda^2} + \frac{\partial L}{\partial \psi_t} \frac{d^2\psi_t}{d\lambda^2} + \frac{\partial L}{\partial \psi_t^*} \frac{d^2\psi_t^*}{d\lambda^2} + \frac{\partial L}{\partial \psi_{tt}} \frac{d^2\psi_{tt}}{d\lambda^2} + \frac{\partial L}{\partial \psi_{ttt}} \frac{d^2\psi_{ttt}}{d\lambda^2} \end{aligned}$$

with

$$\begin{aligned} \frac{\partial L}{\partial \psi} &= -a_1 - 2a_3\psi\psi^* - 2a_5(\psi_t\psi^* + \psi\psi_t^*) - 6a_6(\psi\psi_t\psi^{*2} + \psi^2\psi_t^*\psi^*) - 3a_7\psi^2\psi^{*2}, \\ \frac{\partial L}{\partial \psi^*} &= -a_3\psi^2 - 2a_5\psi\psi_t - a_6(6\psi^2\psi_t\psi^* + 2\psi^3\psi_t^*) - 2a_7\psi^3\psi^*, \\ \frac{\partial L}{\partial \psi_t} &= -2a_5\psi\psi^* - 3a_6\psi^2\psi^{*2}; \quad \frac{\partial L}{\partial \psi_t^*} = -a_5\psi^2 - 2a_6\psi^3\psi^*, \\ \frac{\partial L}{\partial \psi_{tt}} &= -a_2; \quad \frac{\partial L}{\partial \psi_{ttt}} = -a_4. \end{aligned}$$

$$\begin{aligned} \frac{\partial^2 L}{\partial \psi_t^2} &= 0; \quad \frac{\partial^2 L}{\partial \psi_t^{*2}} = 0; \quad \frac{\partial^2 L}{\partial \psi_{tt}^2} = 0; \quad \frac{\partial^2 L}{\partial \psi_{ttt}^2} = 0; \\ \frac{\partial^2 L}{\partial \psi \partial \psi_{tt}} &= 0; \quad \frac{\partial^2 L}{\partial \psi \partial \psi_{ttt}} = 0; \quad \frac{\partial^2 L}{\partial \psi^* \partial \psi_{tt}} = 0; \quad \frac{\partial^2 L}{\partial \psi^* \partial \psi_{ttt}} = 0; \quad \frac{\partial^2 L}{\partial \psi_t \partial \psi_t^*} = 0; \\ \frac{\partial^2 L}{\partial \psi_t \partial \psi_{tt}} &= 0; \quad \frac{\partial^2 L}{\partial \psi_t \partial \psi_{ttt}} = 0; \quad \frac{\partial^2 L}{\partial \psi_t^* \partial \psi_{tt}} = 0; \quad \frac{\partial^2 L}{\partial \psi_t^* \partial \psi_{ttt}} = 0; \quad \frac{\partial^2 L}{\partial \psi_{tt} \partial \psi_{ttt}} = 0. \end{aligned}$$

$$\begin{aligned} \frac{\partial^2 L}{\partial \psi^2} &= -2a_3\psi^* - 2a_5\psi_t^* - 6a_6(\psi_t\psi^{*2} + 2\psi\psi_t^*\psi^*) - 6a_7\psi\psi^{*2}; \\ \frac{\partial^2 L}{\partial \psi^{*2}} &= -6a_6\psi^2\psi_t - 2a_7\psi^3; \quad \frac{\partial^2 L}{\partial \psi \partial \psi_t} = -2a_5\psi^* - 6a_6\psi\psi^{*2}; \\ \frac{\partial^2 L}{\partial \psi \partial \psi^*} &= -2a_3\psi - 2a_5\psi_t - 6a_6(2\psi\psi_t\psi^* + \psi^2\psi_t^*) - 6a_7\psi^2\psi^*; \\ \frac{\partial^2 L}{\partial \psi \partial \psi_t^*} &= -2a_5\psi - 6a_6\psi^2\psi^*; \quad \frac{\partial^2 L}{\partial \psi^* \partial \psi_t} = -2a_5\psi - 6a_6\psi^2\psi^*; \quad \frac{\partial^2 L}{\partial \psi^* \partial \psi_t^*} = -2a_6\psi^3. \end{aligned}$$

References

Abbaoui, K. and Cherruault, Y. (1994). Convergence of Adomian's method applied to differential equations. *Comput. Math. Appl.* 28, 103-109.

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- Abdelrazec, A. and Pelinosky, D. (2011). Convergence of the Adomian decomposition method for initial value problems. *Numer. Methods Partial Differential Equations*, 27, 749-766.
- Adomian G. (1994) Solving frontier problems of physics: the decomposition method. Kluwer Academic, Dordrecht.
- Agrawal, G.P. (2007) *Nonlinear Fiber Optics*. 4th edn., Academic Press, New York.
- Anderson D. (1983). Variational approach to nonlinear pulse propagation in optical fibers. *Phys. Rev. A*, 27, 3135.
- Ali, A., Seadawy, A.R., Lu, D. (1973). Soliton solution of nonlinear Schrödinger solution with the dual power law nonlinearity and resonant nonlinear Schrödinger equation and their modulation instability analysis. *Optics International Journal of Light and Electron Optics*, 145, 79.
- Artigas, D., Torner, L., Torres, J.P., Akhmediev (1997). *N. Opt. Commun.* 143, 322.
- Bahman, G. and Amaneh, S. (2014). Adomian decomposition method for solving fractional Bratu-type equations. *Journal of Mathematics and Computer Science*, 8, 236-244.
- Bogning, J.R. (2006). *Chaos Solitons and Fractals*, 27, 148-153.
- Cherruault, Y. (1989). Convergence of Adomian's method, *Kybernetes*. 18, 31-38.
- Edah, G., Adanhounme, V., Kanfon, A., Guedje, F. and Hounkonnou, M.N (2015). Pulse Propagation in a Non-Linear Medium, 13, 151156.
- Ghosh, S. and Roy, D. (2007). An adaptation of Adomian decomposition for numeric-analytic integration of strongly nonlinear and chaotic oscillators. *Comput. Meth. Appl. Mech. Engrg.* 196, 1133-1153.
- Gulcin, M.M. and Husnu, A.E. (2004). Higher-Order Split-Step Schemes for the Generalized Nonlinear Schrödinger Equation. In: *Proceedings of ENUMATH 2003 the 5-th European Conference on Numerical Mathematics and Advanced Applications*, Prague, Springer Verlag., pp. 658-667.
- Hasegawa, A. and Kodama, Y. (1995). *Solitons in Optical Communications*, New York: Oxford University Press.
- Hasegawa, A. and Tappert, F.D. (1973). *Appl. Phys. Lett.* 23, 142.
- Hong, W. P. (2007). *Opt. Commun.* 194, 217.
- Jafari, H., Daftardar-Gejji, V. (2006). Revised Adomian decomposition method for solving a system of nonlinear equations. *Appl. Math. Comput.* 175, 17, 598-608.
- Jana, S. and Konar, S. (2007). *Phys. Lett. A*, 362, 435.
- Jin, C. and Liu, M. (2005). A new modification of Adomian decomposition method for solving a kind of evolution equation. *Applied Mathematics and Computation*, 169(2), 953-962.
- Kaya, D. and Yokus, A. (2002). A comparison of partial solutions in the decomposition method for linear and nonlinear partial differential equations. *Math. Comput. Simul.* 60, 507-512.
- Konar, S. and Jana, S. (2004). *Opt. Commun.* 236, 7.

-
- Konar, S., Sen, P. K., Kumar, J. J. (1999). *Nonlinear Opt. Phys. Mater.* 8, 492.
- Konar, S. and Sengupta (1994). *Opt. Soc. Am; B*, 11, 1644.
- Kumar, A., Kurrz, T., Lauterbon, W. (1997). *Phys. Lett.A*, 235, 367.
- Kruglov, V. I., Peacock, A. c., Harvey, J. D. (2005). *Phys. Rev. E-Stat. Nonlinear Soft Matter Phys.* 71, 056619-056630.
- Laegsgaard, J. (2007). Mode profile dispersion in the generalized nonlinear Schrödinger equation. *Optics Express*, 15, 16123.
- Li, S., Biondini, G., Schiebold, C. (2017). On the degenerate soliton solutions of the focusing nonlinear Schrödinger equation. *Journal of Mathematical Physics*, 58, 33507.
- Mollenauer, L. F., Stolen, L. F., Gordon, R.H. (1980). *Phys. Rev. Lett.* 45, 1095.
- Pamuk, S. (2005). Solution of the porous media equation by Adomian decomposition method. *Phys.Lett. A*, 344, 184-188.
- Rach, R. and Adomian, G. (1994). Modified decomposition solution of linear and nonlinear boundary-value problems. *Nonlinear Anal.*, 23, 615-619.
- Shwetanshumala, A. B., (2008). Femtosecond Pulse Propagation in Optical Fibers Under Higher Order Effects: A Collective Variables Approach. *Int. J. Theor. Phys.* 47, 1699-1709.
- Scott, K. and Sun, Y. P. (2007). Approximate analytical solutions for models of three-dimensional electrodes by Adomian's decomposition method, In: C.G. Vayenas, R.E. White, M.E. Gamboa-Adelco (Eds.). *Modern Aspects of Electrochemistry* 41, Springer, New York, 222-304.
- Triki, H. and Wazwaz, A. M. (2016). *Rom. Journ. Phys.* 61, 360-366.
- Wei, C., Yize, F., Jiacheng, C., Chunlin, F., Xiaoling D. (2011). *J. Mater Sci: Mater Electron* 22, 265.
- Weidman, J.A.C. and Herst, B. M. (1986). Split Step Methods for the Solutions of the Nonlinear Schrödinger Equation. *SIAM Journal on Numerical Analysis* 23, 485.
- Xiao, Y., Maywar, D. N., Agrawal, G. P. (2012). *J. Opt. Soc.: Am. B*, 20, 2958-2963.
- Zayed, E. M. E. and Amer, Y. A. (2017). Many exact solutions for a higher order nonlinear Schrödinger equation with non-Kerr terms describing the propagation of femtosecond optical pulses in nonlinear optical fibers. *Computational Mathematics and Modeling* 28, 118-139.
- Zayed, E. M. E. and Amer, Y. A. (2014). The first integral method and its application for finding the exact solutions of nonlinear fractional partial differential equations in mathematical physics, *Int. J. Phys. Sci.* 9, 174-183.
- Zhang, G. and Zhang, X. (2007). *Opt. Commun.* 270, 379-383.
- Zheng, B. (2011). Traveling wave solutions for some nonlinear evolution equations by the first integral method. *WSEAS Transactions on Mathematics*, 10, 249.
- Zhou, C., He, X.T., Chen, S. (1992). Basic dynamic properties of the high-order nonlinear Schrödinger equation. *Phys. Rev. A* 46, 2277.