A theoretical investigation on sound transmission loss through multi-walled plates with air space

In this study, a theoretical model is proposed for the diffuse incidence, to investigate the sound transmission loss through multi-walled plates consisting air layers and decompression spaces. Using the present approach, the influences of various parameters, such as the wall thickness, the decompressed air and the thickness of air space, on the sound transmission loss through are simulated and discussed in detail. The result shows that the wave frequency of mass-air-mass resonance which occurs in double walled glass plates improve the sound transmission loss at low frequency range. The sound transmission loss increases with the decrease of air pressure since the sound is not transmitted in vacuum space. The advantage of the simulation procedure is easily used for designing the layer structures with different parameter to improve the sound insulation effect.

Introduction

Sound insulating characteristics of wall, window, ceilings and floors of general buildings and structures are very important aspects of their design [1-3]. Many advanced materials and composites, such as lightweight sandwich structures and multi-layered panels, have been widely used in application fields including environment protection and energy conservation [4]. Important design parameters of walls or panels, such as material properties, the thickness and the layer structures, affects largely on their sound insulation [5-8]. In the basic design stage of sound insulation products, the investigation of design parameters is very important before the products are developed and manufactured. Theoretical studies of the sound insulation are based on the physical consideration of sound propagation. The mass law of sound transmission is a classical method which is generally used for predicting the transmission loss through panels or walls at low frequency noise [9, 10]. Using governing equations of motion, many simulations on the sound insulation of plate structures at high frequency vibration were carried out, and there is good agreement between the theoretical and experimental results. Zhang et al. developed an effective medium model to calculate the sound transmission loss of a metamaterial thin plate, consisting of periodic subwavelength arrays of shunted piezoelectric patches bonded to the two surfaces of an unbounded homogeneous thin plate [11]. The sound absorption performance of the metamaterial plates could be well understood by explicit formulations derived based on the effective medium method. For porous materials and perforated plates, the sound transmission loss of micro-perforated panel system was investigated theoretically by lumped and distributed models, and using transfer matrix method [12-15]. The measurement of sound absorption coefficient in the micro-perforated panel was carried out in the impedance tube and the reverberation room to verify the theoretically predicted results [12]. Based on the classical thin-plate theory, Takahashi and Tanaka developed an analytical model by introducing flow continuity at the panel surface in a spatially mean sense and air-solid interaction within the panel material [16]. Moreover, some fundamental acoustic problems were analyzed and discussed in relation to the interactive effect of flexural vibration and plate permeability. Mahjoob et al. investigated and discussed the acoustic insulation of multi-layered panels containing Newtonian fluids [17]. The progressive impedance model was used to predict the sound insulation provided by the panel in a normal incidence field. The sound transmission loss through sandwich panels with damping layers were analyzed for the roles of various structural and material properties [18-20]. Their results reported that the sandwich panel had better sound transmission characteristics than a homogeneous elastic panel of equivalent weight [18]. The sound insulation was largely affected by the damping materials and their properties for high frequency range [19]. The investigation of sound transmission loss revealed that the skin damping alone did not reduce wave coincident peaks significantly. The damping materials in the core layer were more important than those in the skin layers, resulting in an increase of transmission loss with increasing damping property. Natsuki [21] reported that the sound transmission loss through multi-layered panels can be improved by separated air space thickness or decreasing air pressure due to a change in wave resonance frequencies of mass-air-mass.

In this study, a mathematical model and solution were derived for the diffuse field incidence of acoustic load through multi-layer panels. Based on the developed theoretical equations of the proposed model [21], the effects of various parameters on the sound transmission loss through multi-walled panels were investigated in detail. The analytical model can be used to explain the effects of various system parameters on the performance characteristics of sound insulation.

2. Theoretical analytical model

2.1 Governing equation

The progressive impedance model with scattering incidence is used to derive the sound transmission loss of a multi-walled plates separated air space. To analyze the acoustic insulation property, we present an analytical model of three-walled plates separated by air spaces or decompressed air layers as depicted in Fig. 1. The size of structure in the *x*-direction is taken as infinite, and the *z*-coordinate is taken along the thickness direction of plate. p_i, p_r and p_t are the incident, the reflected and the transmitted acoustic pressures, respectively. w_j denotes the transverse displacement of different plates with $j = 1, 2, 3$. ρ_0 and c_0 are the air density and the speed of sound at atmosphere, respectively. ρ' and c' are the decompressed density and the sound under the decompressed air, respectively.

The sound transmission is caused by the bending wave propagation in walls. See Fig. 1, the governing equations of motion for the transverse deflection are given as

$$D_1 w_1^{(4)} + \rho_1 h_1 \ddot{w}_1 = \left[p_{1i} + p_{1r} - p_{1t} - p_{tr}\right]_{z=0} \tag{1}$$

$$D_2 w_2^{(4)} + \rho_2 h_2 \ddot{w}_2 = \left[p_{2i} + p_{2r} - p_{2t} - p'_{tr}\right]_{z=0} \tag{2}$$

$$D_3 w_3^{(4)} + \rho_3 h_3 \ddot{w}_3 = \left[p_{3i} + p_{3r} - p_{3t}\right]_{z=0} \tag{3}$$

where ρ and h are the mass density and the thickness of plates, respectively. D is the flexural rigidity, given by

$$D_j = \frac{E_j h_j^3}{12\left(1-\nu_j^2\right)}, \qquad j = 1, 2, 3 \tag{4}$$

where E is the elastic modulus of plates, and ν is the Poisson's ratio.

2.2 Analytical solution of sound transmission loss

Considering harmonic wave vibration, the acoustic pressures of the incident, reflected and transmitted sound waves in Eqs. (1)-(3) can be expressed as

$$p_i = P_i e^{i\omega t - ik\left(x\sin\theta + z\cos\theta\right)} \tag{5}$$

$$p_r = P_r e^{i\omega t - ik\left(x\sin\theta - z\cos\theta\right)} \tag{6}$$

$$p_t = P_t e^{i\omega t - ik\left(x\sin\theta + z\cos\theta\right)} \tag{7}$$

and, the displacement of plates can be written as

$$w_j = W_j e^{i\left(\omega t - kx\sin\theta\right)}, \quad j = 1, 2, 3 \tag{8}$$

where θ is the angle of incidence, ω is the angular frequency and $k = \omega/c$ is the wave number of the sound velocity in air. W, P_i, P_r and P_t are the amplitudes of the vertical displacement of plates, the incident, reflected and transmitted acoustic pressures, respectively.

Substituting Eqs.(5)-(8) into Eqs.(1)-(3), we obtain

$$D_1 k^4 \sin^4\theta W_1 - \rho_1 h_1 \omega^2 W_1 = P_{1i} + P_{1r} - P_{1t} - P_{tr} \tag{9}$$

$$D_2 k^4 \sin^4\theta W_2 - \rho_2 h_2 \omega^2 W_2 = P_{2i} + P_{2r} - P_{2t} - P'_{tr} \tag{10}$$

$$D_3 k^4 \sin^4\theta W_3 - \rho_3 h_3 \omega^2 W_3 = P_{3i} + P_{3r} - P_{3t} \tag{11}$$

According to boundary conditions for plates in the coupled analysis, the wave velocities at the left and right sides of each panel are continuous, resulting the following equations:

$$P_{1t} - P_{tr} = \frac{iZ'\omega W_1}{\cos\theta} \qquad P_{2i} - P_{2r} = \frac{iZ'\omega W_2}{\cos\theta} \tag{12}$$

$$P_{2t} - P'_{tr} = \frac{iZ'\omega W_2}{\cos\theta} \qquad P_{3i} - P_{3r} = \frac{iZ'\omega W_3}{\cos\theta} \tag{13}$$

$$P_{1i} - P_{1r} = \frac{iZ\omega W_1}{\cos\theta} \qquad P_{3t} = \frac{iZ\omega W_3}{\cos\theta} \tag{14}$$

where Z' is the impedance of decompression air space, and Z is the impedance of air at atmospheric pressure. Next, the relationship of sound pressure between the plates with air space *d*, is given as

$$P_{1t} = e^{ikd\cos\theta} P_{2i} \qquad P_{tr} = e^{-ikd\cos\theta} P_{2r} \tag{15}$$

$$P_{2t} = e^{ikd\cos\theta} P_{3i} \qquad P'_{tr} = e^{-ikd\cos\theta} P_{3r} \tag{16}$$

Substituting Eq. (15) into Eq. (12), and Eq. (16) into Eq. (13), we obtain

$$P_{2i} = \frac{iZ'\omega}{\cos\theta} \frac{W_2 e^{-ikd\cos\theta} - W_1}{e^{-ikd\cos\theta} - e^{ikd\cos\theta}} \qquad P_{2r} = \frac{iZ'\omega}{\cos\theta} \frac{W_2 e^{ikd\cos\theta} - W_1}{e^{-ikd\cos\theta} - e^{ikd\cos\theta}} \tag{17}$$

$$P_{3i} = \frac{iZ'\omega}{\cos\theta} \frac{W_3 e^{-ikd\cos\theta} - W_2}{e^{-ikd\cos\theta} - e^{ikd\cos\theta}} \qquad P_{3r} = \frac{iZ'\omega}{\cos\theta} \frac{W_3 e^{ikd\cos\theta} - W_2}{e^{-ikd\cos\theta} - e^{ikd\cos\theta}} \tag{18}$$

Furthermore, substituting Eqs. (17)-(18) into Eqs. (9)-(11), the motion equations of plates can expressed as matrix form

$$\begin{bmatrix} s_{11} & s_{12} & 0 \\ s_{21} & s_{22} & s_{23} \\ 0 & s_{32} & s_{33} \end{bmatrix} \begin{Bmatrix} W_1 \\ W_2 \\ W_3 \end{Bmatrix} = \begin{Bmatrix} 2P_{1i} \\ 0 \\ 0 \end{Bmatrix} \tag{19}$$

where

$$s_{11} = D_1 (k \sin\theta)^4 - \rho_1 h_1 \omega^2 + \frac{iZ\omega}{\cos\theta} - \frac{iZ'\omega}{\cos\theta} \cdot \frac{1+e^{i2kd\cos\theta}}{1-e^{i2kd\cos\theta}} \qquad s_{12} = \frac{2iZ'\omega}{\cos\theta} \cdot \frac{e^{ikd\cos\theta}}{1-e^{i2kd\cos\theta}} \qquad s_{13} = 0 \tag{20}$$

$$s_{21} = s_{12} \qquad s_{22} = D_2 (k \sin\theta)^4 - \rho_2 h_2 \omega^2 - \frac{i2Z'\omega}{\cos\theta} \cdot \frac{1+e^{i2kd\cos\theta}}{1-e^{i2kd\cos\theta}} \qquad s_{23} = \frac{i2Z'\omega}{\cos\theta} \cdot \frac{e^{ikd\cos\theta}}{1-e^{i2kd\cos\theta}} \tag{21}$$

$$s_{31} = 0 \qquad s_{32} = s_{23} \qquad s_{33} = D_3 (k \sin\theta)^4 - \rho_3 h_3 \omega^2 + \frac{iZ\omega}{\cos\theta} - \frac{iZ'\omega}{\cos\theta} \cdot \frac{1+e^{i2kd\cos\theta}}{1-e^{i2kd\cos\theta}} \tag{22}$$

According to Eq. (14) and Eq. (19), the transmission coefficient is defined as the ratio of the transmitted sound power to the incident sound power, given as

$$\tau(f,\theta) = \left| \frac{P_{3t}}{P_{1i}} \right|^2 = \left| \frac{i2Z\omega s_{21} s_{32} / \cos\theta}{s_{11}s_{22}s_{33} - s_{11}s_{32}s_{23} - s_{33}s_{12}s_{21}} \right|^2 \tag{23}$$

The transmission coefficient is a function of the incident angle θ. For a diffuse sound field, the averaged form of the transmission coefficient $\tau(f,\theta)$ can be defined as integrating over a semi-circular surface over all the angles. The averaged transmission coefficient over all angles of incidence θ is given by

$$\bar{\tau} = \frac{\int_0^{\theta_0} \tau(f,\theta) \sin\theta \cos\theta \, d\theta}{\int_0^{\theta_0} \sin\theta \cos\theta \, d\theta} \tag{24}$$

where the limiting θ_0 is usually taken to be 78°, above which it is assumed that no sound is received. Thus, the sound transmission loss (STL) with random incidence is obtained as

$$STL = -10 \log_{10} |\tau| \tag{25}$$

3. Analytical results and discussion

In the simulation, the multi-walled plates are considered to be two- or three-layered glass panels with an air layer therebetween. The Young's modulus of a glass plate is 72 GPa, the density is 2200 kg/m^3, and the Poisson's ratio is 0.3. The influence of decompressed air on the sound transmission loss of panel was investigated based on the present theoretical model. The density (ρ) and the sound velocity (c) of air layers were calculated by air pressure, given by

$$\rho = \frac{p}{RT}, \qquad c = \sqrt{\frac{\kappa p}{\rho}} \tag{25}$$

where $R = 287\, J/kg \cdot K$ is the specific gas constant for air, and $\kappa = 1.403$ is the heat capacity ratio for air. T is the absolute temperature, and p is the atmospheric pressure in air.

Figure 2 shows the sound transmission loss through two-layer glass plate subjected to the incident angles of 0 and 45 degrees, and the scattering diffuse angle. It is found that the sound transmission loss depends largely on the incidence angles. Comparing the result of scattering angle with oblique incidences, the sound transmission loss is the maximum for the case of normal incidence. The dips in the Fig. 2 indicate the mass-air-mass resonance frequencies (coincidence critical frequency) of glass panels separated by air space. The coincidence critical frequency obviously exists in an identical incidence, but the dip become weak for scattering incidence. According to the present prediction, the coincidence frequency is about 900 Hz for the glass panels subjected to a normal incidence, shifting to higher frequency as the incidence angle increases. The peaks of the critical frequency are found to be lower for scattering incidence compared with identical incidence, and the sound insulation decreases.

In the following simulation, we considered general forms of sound propagation and analyzed the sound transmission losses through panels based on scattering incidence of sound. The effects of layer numbers on the sound transmission loss are shown in Fig. 3 as a function of frequency. The multi-walled panels are considered to have the same total thickness of 6 mm and the air space of 2 mm thick. It is found that the number of coincidence critical frequency increases with increasing air layers due to the interference of vibration between adjacent plates. The number of the coincidence frequency increase with increasing layer number. At low frequency less than 400 Hz (coincidence frequency), the multi-walled glass panels can provide more higher sound transmission loss compared with single panel due to middle air space. The single glass panel exhibits larger sound insulation than the multi-walled glass panels when vibration frequency is over 500 Hz. Figure 4 shows the influences of air space on the sound insulation of double-walled panels when the air interval between the two plates are 2 mm, 5mm, and 10 mm, the thickness of each glass plate is 2 mm. It can be seen that the coincidence critical frequency of the glass pales decreases with the increase of air space. The influence of the air space on the sound transmission loss is very small for high except for range of coincidence frequencies.

Figure 5 shows the sound transmission loss of the double-walled panels with separated by air space of 2.0 mm. The total panels are 6.0 mm thick, in which the thickness ratio $(t_1 : t_2)$ are $(1,1)$, $(1,1.2)$ and $(1,2)$, respectively. It is observed that the thickness ratio of the double-walled panel affects the sound transmission loss, especially for the coincidence frequency larger than 1000 Hz. The sound insulation of double-walled panels can be largely improved only when little variation exists in the plate thickness. This can be explained that the difference in plate thickness cause more energy loss of sound pressure due to the mismatch in the resonance frequency between two plates.

Figures 6 and 7 shows the influence of the decompressed air pressure on the sound transmission loss of double-walled panels separated by an air space of 2.0 mm. The pressure values of internal air layers are taken as the standard atmospheric pressure (1.0 atm), 1/5 atm and 1/10 atm. pressure with decompressed air. The simulations were carried out for identical wall thickness of 3.0 mm (Fig. 6), and different wall thickness ratio (Fig. 7). It is seen that the air pressure influences significantly the resonance frequency of mass-air-mass and the sound transmission loss. The coincidence critical frequencies decrease with decreasing air pressure, resulting an improvement of the sound insulation ability in high frequency range. For the frequency range larger than 2000 Hz, the sound transmission loss can be increased by about 5 dB when the air pressure decreases by 10% of atmosphere under condition of unchanged other parameters. Comparing the results of Fig. 6 with Fig. 7, the effect of decompressed air pressure on the sound insulation will increase for the double-walled panel with different wall thickness ratio.

In summary, the coincidence critical frequency can be moved out of interest frequency by adjusting such as the decompressed air, the air space and the different thickness of plates. Therefore, the sound transmission loss of multi-walled panels will be improved above the coincidence frequency.

4. Conclusions

A theoretical model and an exact solution procedure are developed to investigate the sound transmission characteristics of multi-walled structures with air space. The governing equations of motion for the multi-walled panels is presented based on the bending vibration mode, and the solution of the sound transmission loss is derived by using acoustic pressure method. In the simulation, the influences of various parameters on the sound insulation characteristics of multi-walled panels are simulated and discussed in detail based on the proposed theoretical model and analysis. The result shows that the sound transmission loss through panels with air space is can be improved by resonance frequencies of mass-air-mass. Therefore, the sound insulation characteristics of the multi-walled panels is largely affected by air-space interval, plate thickness ratio and decompressed air pressure.

References

1. J.H. Rindel, Sound insulation of buildings. *INTER-NOISE*, Istanbul, Turkey 28-31 August (2007)
2. C. Hoeller, B. Zeitler, J. Mahn, Direct sound transmission loss of heavy gauge steel stud walls, EuroNoise, Maastricht, Holland 31 May - 3 June (2015) 1121-1126.
3. J. L. Davy, Predicting the sound insulation of walls. *J. Build. Acoust.* **16** (2009) 1-20
4. G. Gupta, A. Kumar, R. Tyagi, S. Kumar, Application and future of composite materials: A Review, *International Journal of Innovative Research in Science, Engineering and Technology* **5** (2016) 6901-6911
5. A. Dijckmansa, G. Vermeir, Sound transmission through finite lightweight multilayered structures with thin air layers. *J. Acoust. Soc. Am.* **128** (2010)3513-3524
6. J. Wanga, T. J. Lub, J. Woodhouseb, R. S. Langleyb, J. Evans, Sound transmission through lightweight double-leaf partitions: theoretical modelling. *J. Sound Vib.* **286** (2005) 817-847
7. J. Zhou, A. Bhaskar, X. Zhang, Optimization for sound transmission through a double-wall panel. *Appl. Acoust.* **74** (2013) 1422-1428
8. C. Shen, F. X. Xin, T. J. Lu, Theoretical model for sound transmission through finite sandwich structures with corrugated core. *Inter. J. Non-Linear Mechan.* **47** (2012) 1066-1072
9. A. Osipov, P. Meesb, G. Vermeif, Low-frequency airborne sound transmission through

single partitions in buildings. *Appl. Acoust.* **52** (1997) 213-288

10. A. J. B. Tadeu, D. M. R. Mateus, Sound transmission through single, double and triple glazing. Experimental evaluation. *Appl. Acoust.* **62** (2001) 307-325
11. H. Zhang, J. Wen, Y. Xiao, G. Wang, X. Wen, Sound transmission loss of metamaterial thin plates with periodic subwavelength arrays of shunted piezoelectric patches. *J. Sound Vib.* **343** (2015) 104-120
12. S. S. Jung, Y. T. Kim, D. H. Lee, Sound absorption of micro-perforated panel. J. Korean Phys. Soc. **50** (2007) 1044−105
13. F. Asdrubali, G. Pispola, Properties of transparent sound-absorbing panels for use in noise barriers, *J. Acous. Soc. America* **214** (2007) 121-221
14. J. Kang, H. V. Fuchs, Predicting the absorption of open weave textiles and micro-perforated membranes backed by an air space, *J. Sound Vib.* **220** (1999) 905-920
15. D. H. Leea, Y. P. Kwon, Estimation of the absorption performance of multiple layer perforated panel systems by transfer matrixmethod. *J. Sound Vib.* **278** (2004) 847-860
16. D. Takahashi, M. Tanaka, Flexural vibration of perforated plates and porous elastic materials under acoustic loading. *J. Acoust Soc. Am.* **112** (2002) 1456-1464
17. M. J. Mahjoob, N. Mohammadi, S. Malakooti, An investigation into the acoustic insulation of triple-layered panels containing Newtonian fluids: Theory and experiment. *Applied Acoustics,* **70** (2009) 165-171
18. S. Narayanan, R. L. Shanbhag, Sound transmission through a damped sandwich panel. *J. Sound Vib.* **80** (1982) 315-327
19. C. L. Dym, D. C.Lang, Transmission loss of damped asymmetric sandwich panels with orthotropic cores. *J. Sound Vib.* **88** (1983) 299-319
20. N. Mohammadi, Acoustic modeling of a 3-Layered panel incorporating electro-rheological/magneto-rheological (EMR) fluids. *Open Journal of Acoustics.* **12**

(2014) 1-12

21. T. Natsuki, J. Natsuki, Theoretical Analysis of Sound Insulation Performance through Multi-Layered Structures. *J. Sci. Eng. Res.* **5 (2018)** 446-451

Figure captions

Figure 1 Schematic illustration showing a three-layered construction

Figure 2 Effect of the incidence angle on the sound transmission loss through two layers of panels with 2 mm thick and separated by an air space of 2 mm.

Figure 3 Sound transmission loss through different panels, having the same total thickness but different layer numbers

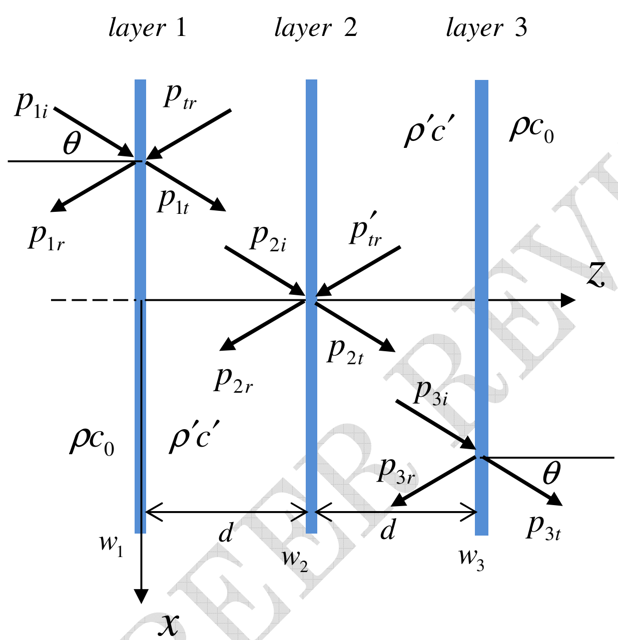
Figure 4 Influences of different air space on the sound insulation of panels with 2 mm thick

Figure 5 Sound transmission loss of double-walled panels with a different thickness ratio and an air space of 2 mm

Figure 6 Influence of the decompressed air layer on sound transmission loss of double-walled panels with each 3 mm thick and separated by 2 mm air layer

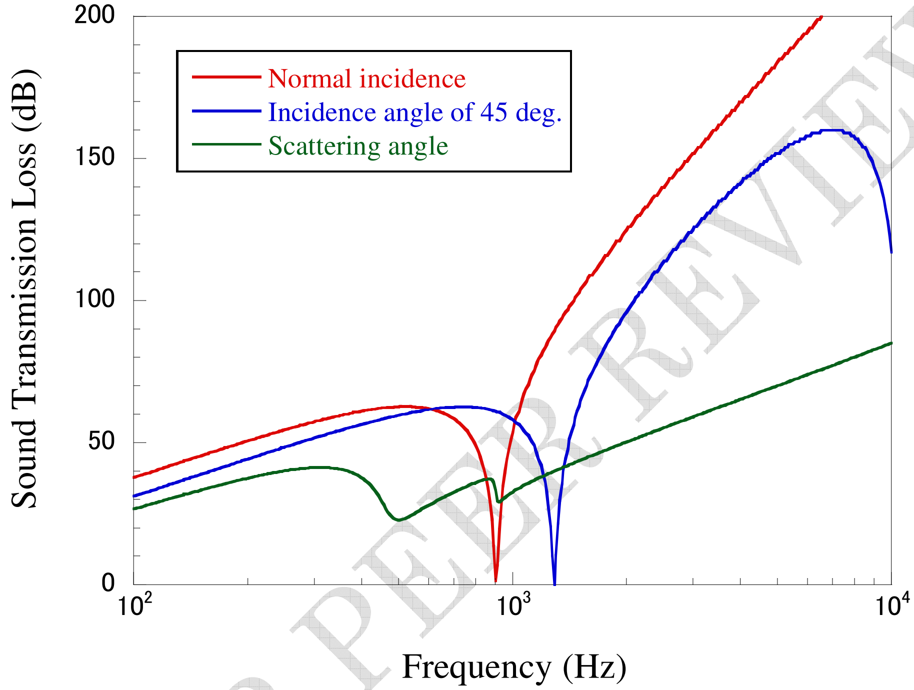
Figure 7 Influence of the decompressed air layer on sound transmission loss of double-walled panels with total thickness of 3 mm, but the thickness ratios $(t_1 : t_2)$ are $(1, 1)$, $(1, 1.2)$ and $(1, 2)$, and separated by 2 mm air space

Figure 1



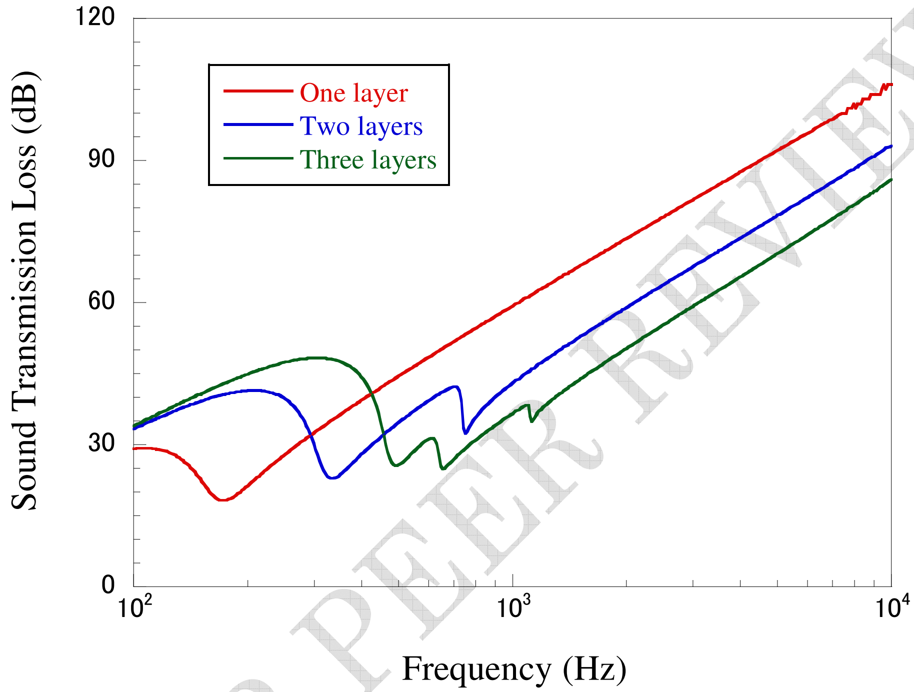
$layer\ 1$
$layer\ 2$
$layer\ 3$
p_{1i}
p_{tr}
θ
p_{1t}
p_{1r}
$\rho' c'$
ρc_0
p_{2i}
p'_{tr}
z
p_{2t}
p_{2r}
p_{3i}
ρc_0
$\rho' c'$
p_{3r}
θ
p_{3t}
d
d
w_1
w_2
w_3
x

Figure 2



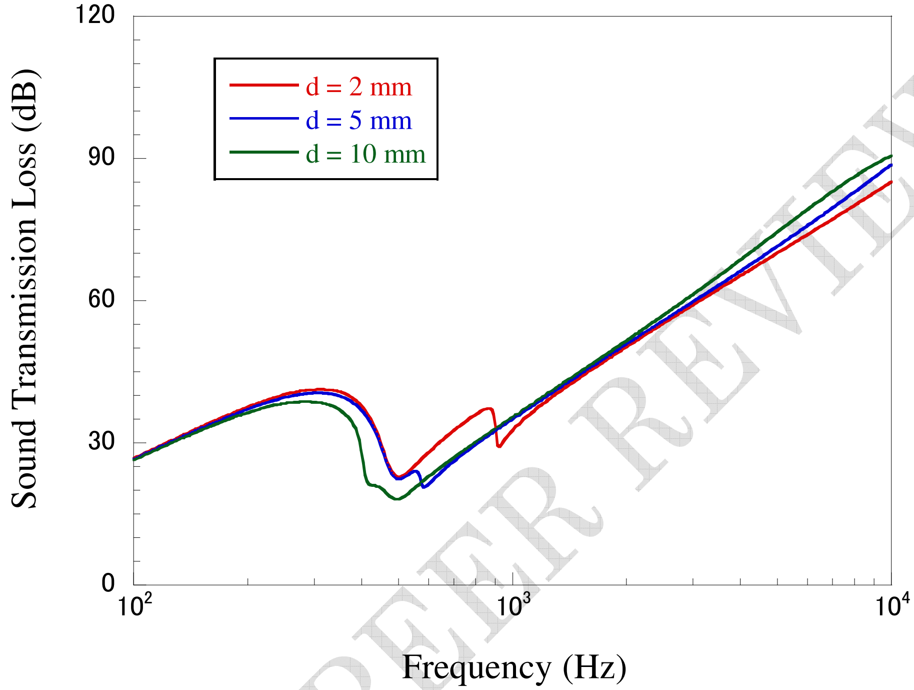
Sound Transmission Loss (dB)
200
150
100
50
0
Normal incidence
Incidence angle of 45 deg.
Scattering angle
10^2
10^3
10^4
Frequency (Hz)

Figure 3



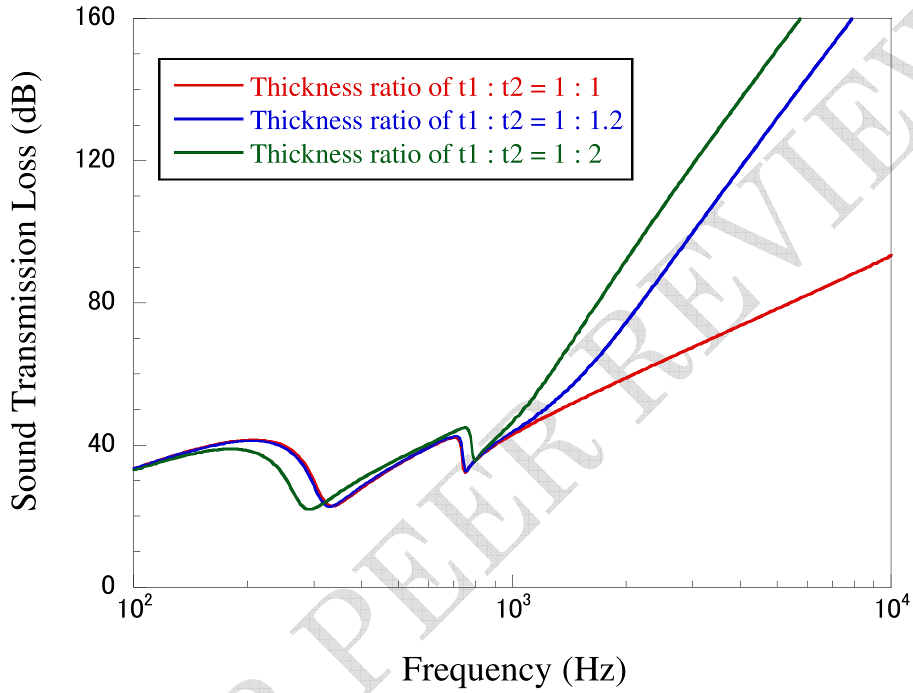
120
90
60
30
0
10^2
10^3
10^4
Sound Transmission Loss (dB)
Frequency (Hz)
One layer
Two layers
Three layers

Figure 4



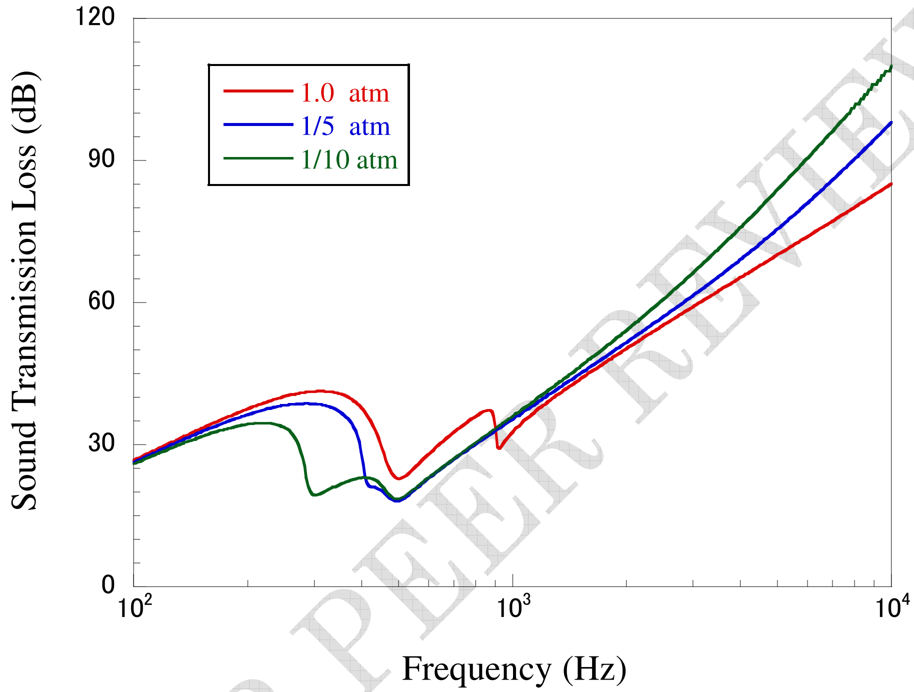
120
90
60
30
0
Sound Transmission Loss (dB)
d = 2 mm
d = 5 mm
d = 10 mm
10^2
10^3
10^4
Frequency (Hz)

Figure 5



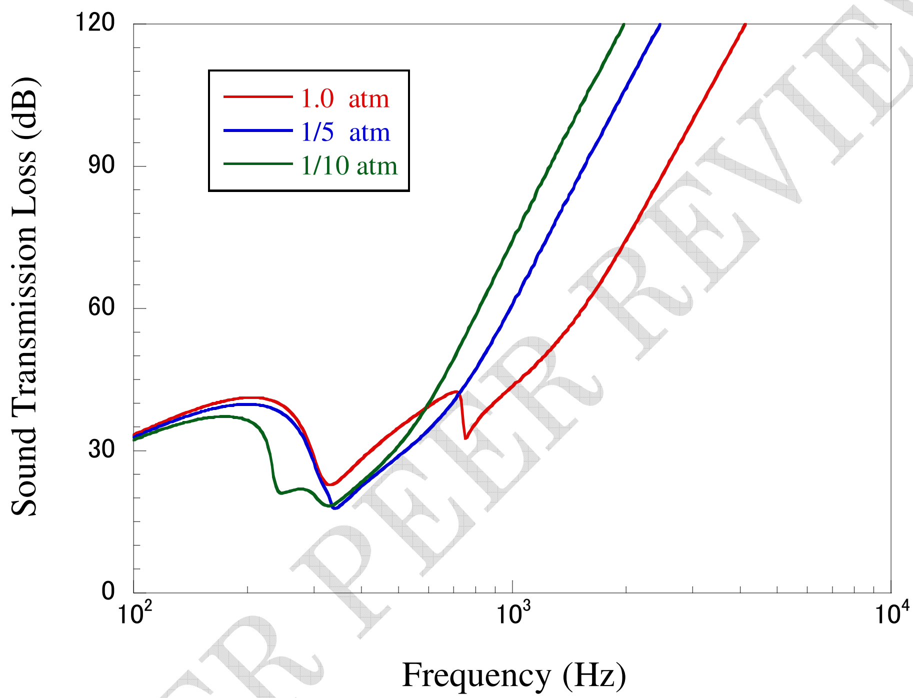
Thickness ratio of t1 : t2 = 1 : 1
Thickness ratio of t1 : t2 = 1 : 1.2
Thickness ratio of t1 : t2 = 1 : 2
Sound Transmission Loss (dB)
160
120
80
40
0
10^2
10^3
10^4
Frequency (Hz)

Figure 6



120
90
60
30
0
Sound Transmission Loss (dB)
1.0 atm
1/5 atm
1/10 atm
10^2
10^3
10^4
Frequency (Hz)

Figure 7



120
90
60
30
0
10^2
10^3
10^4
Sound Transmission Loss (dB)
Frequency (Hz)
1.0 atm
1/5 atm
1/10 atm