**Original Research Article**

# Entropy Generation Analysis of a Reactive MHD Third Grade Fluid in a Cylindrical Pipe with Radially Applied Magnetic Field and Hall Current

## ABSTRACT

The combined effects of chemical reaction, radially applied magnetic field and Hall effect on entropy generation of a steady third grade magnetohydrodynamics fluid flowing through a uniformly circular pipe was studied. The governing equations are presented and the resulting non-linear dimensionless equations are solved numerically using Galerkin Weighted Residual Method. The velocity, temperature and concentration profile were obtained and utilized in computing the entropy number. A parametric study of germane parameters involved are presented graphically and discussed. It was observed that irreversibility due to heat transfer dominates the flow compared to fluid friction and Hall parameter inhibits the Bejan number while Magnetic parameter enhances the Bejan number.

*Keywords: Magnetohydrodynamics, Hall crrent, Galerkin Weighted Residual Method, Bejan number.*

## 1. INTRODUCTION

Magnetohydrodynamics (MHD) flows in rectangular and cylindrical system continue to stimulate significant interest in the field of engineering science and applied mathematics. This interest is owned to the numerous important applications in biological and engineering industry such as reactive polymer flows, extraction of crude oil, synthetic fibres, paper production and also in absorption and filtration processes in chemical engineering [1]. The dynamics of reactive fluids through pipe at low Reynolds numbers has long been an important subject in the area of environmental engineering and science.

The steady flow of a reactive variable viscosity fluid in a cylindrical pipe with isothermal wall was studied by Makinde [2], reporting the dependence of the steady state thermal ignition criticality conditions on both Frank-Kamenetskii and viscous heating parameters. Makinde et al [3], numerical investigation for the entropy generation rates in an unsteady flow of a variable viscosity incompressible fluid through a porous pipe with uniform suction at the surface were examined. In Ajadi [4], closed-form solution using Homotopy Analysis method on the effect of variable viscosity and viscous dissipation on the thermal stability of a one-step exothermic reactive non-Newtonian flow in a cylindrical pipe assuming negligible reactant consumption were obtained. In 2013, Aiyesimi et al [5] considered a mathematical model for a dusty viscoelastic fluid flow in a circular channel was considered, observing that an increase in the value of magnetic field and viscoelastic parameter reduces the horizontal velocity of the fluid and particles, thereby reducing the boundary layer thickness, hence inducing an increase in the absolute value of the velocity gradient at the surface. And [6] examined the effect of radiation on unsteady MHD flow of a chemically reacting fluid past a hot vertical porous plate using finite difference approach. They reported the temperature and velocity of the fluid increases for increasing values of hest generation parameter, temperature and velocity decreases for increasing values of radiation parameter and temperature of the fluid increases for increasing values of Eckert number.

Hall current and chemical reaction effects on a hydromagnetic flow of a stretching vertical surface with internal heat generation/absorption was studied by Salem & Abd El-Aziz [7]. A finite element solution of heat and mass transfer flow with Hall current, heat source and viscous dissipation were presented in Sivaiah & Srinivasa [8]. A computational iterative approach known as Spectral Local Linearization Method (SLLM) blended with Chebyshev spectral method was used by Shateyi & Marewo [9], to study the effect of Hall current on MHD flow and heat transfer over an unsteady stretching permeable surface in the presence of thermal radiation and heat source/sink.

The thermodynamics second law analysis and its design-related concept of entropy generation minimization has been a cornerstone in the field transfer and thermal design. Several researchers were motivated to study fundamental and applied engineering problem based on second law analyses, due to the production of entropy resulting from combined effects of velocity and temperature gradient. Generating entropy is tied to thermodynamic irreversibility, which is common in all heat transfer process. Eegunjobi & Makinde [10] investigated the combined effects of buoyancy force and Navier slip on the entropy generation rate in a vertical porous channel with wall suction/injection. The combined effects of Navier slip, convective cooling, variable viscosity and suction/injection on the entropy generation rate in an unsteady flow of an incompressible viscous fluid flowing through a channel with permeable wall was studied by [11].

In this paper, the motivation comes from a desire to gain more understanding into the combined effect of radially applied magnetic field and Hall current on the flow of chemically reactive third grade fluid. The relevant governing equation have been solved numerically by Galerkin Weighted Residual Method [12, 13]. The effects of the various apposite parameters on the velocity, temperature and concentration are presented. In this work, entropy generation rate of a laminar MHD flow of a reactive third grade fluid is considered in a circular pipe, which is assumed electrically conducting and incompressible in the presence of an externally applied radially exponential magnetic field.

## 2. MATHEMATICAL FORMULATION

Considering a steady flow of electrically conducting, incompressible, third grade fluid in a non-conducting circular pipe in the absence of gravitational force. The z-axis is taken along the axis of flow. Radially exponential varying magnetic field $B_r = B_0 e^{r/2R}$ is applied [14] and no electric field is applied. The flow is induced due to constant applied pressure gradient in the z-direction and electron atom collision frequency is assumed to be relatively high compared to the collision frequency of ions. The equations which govern the MHD flow are the continuity, momentum and Maxwell equations. In fluid dynamics studies, it is assumed that the flows meet the Clausius-Duhem inequality and the specific Helmholtz free energy of fluid has a minimum at equilibrium [15]. Using the velocity field $V = (0,\ 0,\ \mathrm{w}(r)),$ the incompressibility condition is satisfied identically and momentum and Maxwell equations after the constitutive equations

$$
\begin{aligned}
T = & -pI + \mu A_1 + \alpha_1 A_2 + \alpha_2 A_1^2 + \beta_1 A_3 + \beta_2\left(A_2A_1 + A_1A_2\right) + \beta_3(trA_1^2)A_1 \\
& + \gamma_1 A_4 + \gamma_2(A_3A_1 + A_1A_3) + \gamma_3 A_2^2 + \gamma_4(A_2A_1^2 + A_1^2A_2) + \gamma_5(trA_2)A_2 \\
& + \gamma_6(trA_2)A_1^2 + \left(\gamma_7 trA_3 + \gamma_8 tr(A_2A_1)\right)A_1, \qquad (2.1)
\end{aligned}
$$

$$A_1 = gradV + (gradV)^T$$

$$A_n = \frac{dA_{n-1}}{dt} + A_{n-1}L + L^T A_{n-1}, \qquad (n > 1)$$

[16, 17] and the stated assumptions give

$$\frac{\partial w}{\partial t} = \frac{1}{r\rho}\left[\frac{\partial}{\partial r}\left(r\mu\frac{\partial w}{\partial r}\right) + \alpha_1\frac{\partial}{\partial r}\left(r\frac{\partial^2 w}{\partial r\partial t}\right) + 2\beta_3\frac{\partial}{\partial r}\left(r\left(\frac{\partial w}{\partial r}\right)^3\right)\right] - \frac{\partial \hat{p}}{\partial z} - \frac{\sigma B_r^2 w}{1+m^2} \qquad (2.2)$$

$$
\begin{aligned}
\frac{\partial T}{\partial t} = & \frac{\mu}{\rho c_p}\left(\frac{\partial w}{\partial r}\right)^2 + \frac{\alpha_1}{\rho c_p}\frac{\partial^2 w}{\partial r\partial t}\frac{\partial w}{\partial r} + \frac{2\beta_3}{\rho c_p}\left(\frac{\partial w}{\partial r}\right)^4 + \frac{k}{\rho c_p}\left[\frac{\partial^2 T}{\partial r^2} + \frac{1}{r}\frac{\partial T}{\partial r}\right] + \frac{Q_s}{\rho c_p}\left(T - T_0\right) \\
& - \frac{1}{\rho c_p}\frac{\partial q_r}{\partial r} + \frac{D_m \lambda_T}{\rho c_p c_s}\left(\frac{\partial^2 C}{\partial r^2} + \frac{1}{r}\frac{\partial C}{\partial r}\right) \qquad (2.3)
\end{aligned}
$$

$$\frac{\partial C}{\partial t} + w\frac{\partial C}{\partial r} = D_m\left(\frac{\partial^2 C}{\partial r^2} + \frac{1}{r}\frac{\partial C}{\partial r}\right) - k_c\left(C - C_0\right) \qquad (2.4)$$

where $w, T, B_0, \hat{p} = -p + \alpha\left(\frac{dw}{dr}\right)^2, \sigma, m, k, q, D_m, \lambda_T, c_p, k_c, T_0, T_w, C_0, C_w$ are fluid velocity, fluid temperature, applied magnetic field strength, modified pressure, electrical conductivity, Hall parameter, thermal conductivity, thermal radiation, molecular diffusivity, thermal diffusivity, specific heat capacity, chemical reaction rate constant, reference temperature, wall temperature, reference concentration and wall concentration.

Introducing the following non-dimensional quantities by Ellahi [18] into (2.2) to (2.5) and the boundary conditions

$$w = w_0 \bar{w},\quad t = \frac{\bar{t}R}{w_0},\quad r = R\eta,\quad T = (T_w - T_0)\theta + T_0,\quad C = (C_w - C_0)\chi + C_0$$

$$\Lambda = \frac{2\beta_3 w_0}{\rho R^3}, c = \frac{R}{\rho {w_0}^2}\left(\frac{\partial \bar{p}}{\partial z}\right), R_e = \frac{\rho w_0 R}{\mu_0}\ P_r = \frac{\mu_0 c_p}{k}, M = \frac{\sigma B_0^2 R^2}{\rho w_0}, Q_H = \frac{Q_s R}{\rho w_0 c_p}, \qquad (2.6)$$

$$E_c = \frac{w_0^2}{c_p (T_w - T_0)},\ K_R = \frac{K_c R^2}{D_m}, D_u = \frac{D_m \lambda_T (C_w - C_0)}{k c_p (T_w - T_0)}\ , R_p = \frac{16\sigma_* T_0^3}{3\delta_* \rho w_0 c_p R}, \mathrm{S}_c = \frac{w_0 R}{D_m}$$

and using Rosselands approximation

$$q_r = -\frac{4\sigma_*}{3\delta_*}\frac{\partial T^4}{\partial r} \qquad (2.7)$$

$\Lambda, M, c, P_r, E_c, Q_H, \beta_*, D_u, R_p, \mathrm{S}_c, K_R, \sigma_*, \delta_*$ denotes third grade parameter, magnetic parameter, pressure drop, Prandtl number, Eckert number, heat source/sink parameter, material constant parameter, Dufour number, radiation parameter, Schmidt number, chemical reaction parameter, Stefan-Boltzmann constant and mean absorption coefficient. For steady flow, the time dependent terms are set to zero and the following are equations were obtained respectively with the boundary conditions

$$\frac{1}{R_e}\frac{d^2 w}{d\eta^2} + \frac{1}{\eta R_e}\frac{dw}{d\eta} + \frac{\Lambda}{\eta}\left(\frac{dw}{d\eta}\right)^3 + 3\Lambda\left(\frac{dw}{d\eta}\right)^2 \frac{d^2 w}{d\eta^2} - c - \frac{Me^{\eta} w}{\left(1+m^2\right)} = 0 \qquad (2.8)$$

$$\frac{d^2\theta}{d\eta^2} + \frac{R_*}{\eta}\frac{d\theta}{d\eta} + P_r E_c R_* \left(\frac{dw}{d\eta}\right)^2 + \beta_* R_e P_r E_c R_* \left(\frac{dw}{d\eta}\right)^4 + Q_H R_e P_r R_* \theta$$
$$+ D_u P_r R_e R_* \frac{d^2\chi}{d\eta^2} + \frac{D_u P_r R_e R_*}{\eta}\frac{d\chi}{d\eta} = 0 \qquad (2.9)$$

$$\frac{d^2\chi}{d\eta^2} = S_c w \frac{d\chi}{d\eta} - \frac{1}{\eta}\frac{d\chi}{d\eta} - K_R \chi \qquad (2.10)$$

$$\frac{dw}{d\eta} = 0,\ \theta(\eta) = 0,\ \chi(\eta) = 0 \quad \text{at} \quad \eta = 0$$
$$w(\eta) = 0,\ \theta(\eta) = 1,\ \chi(\eta) = 1 \quad \text{at} \quad \eta = 1 \qquad (2.11)$$

Equations (2.8), (2.9), (2.10) and (2.11) comprise the boundary value problem to now be solved.

## 3. METHODS

### 3.1 Galerkin Weighted Residual Methods

Suppose an approximate solution is to be determined for the differential equation of the form

$$L(\phi) + f = 0 \qquad (3.1)$$

where $\phi(x)$ is an unknown dependent variable, $L$ is a differential operator and $f(x)$ is a known function. Let $\psi(x) = \sum_{i=1}^{N} c_i u_i(x)$ be an approximate solution to (2.8). On substituting $\psi(x)$ into (2.8), it is unlikely that (2.8) is satisfied i.e. $L(\psi) + f \neq 0$ therefore

$$L(\psi) + f = R \qquad (3.2)$$

where $R(x)$ is a measure of error called the Residual [13, 19]. Multiplying (3.2) by an arbitrary weight function $u(x)$ and integrating over the domain to obtain

$$\int_D u(x)\left[L(\psi)+f\right]dD=\int_D u(x)R(x)dD\neq 0 \qquad (3.3)$$

Galerkin Weighted Residual method ensures equation (3.3) vanishes over the solution domain and the weight function is choosing from the basis functions $u(x)=u_i(x) \quad (i=0,...,N)$ hence

$$\left\langle u,R\right\rangle=\int_D u(x)R(x)dD=\int_D u_i(x)\left[L(u_0(x)+\sum_{i=1}^{N}c_iu_i(x))+f\right]dD=0 \qquad (3.4)$$

These are a set of n-order linear equations to be solved to obtain all the $c_i$ coefficients. The trial functions can be polynomials, trigonometric functions etc. The trial functions are usually chosen in such that the assumed function $\psi(x)$ satisfies the global boundary conditions for $\phi(x)$ though this is not strictly necessary and certainly not always possible [12].

To apply the method to (2.8)-(2.10), we select an approximate solutions of the form $\psi_w(\eta)=a_0+a_1\eta+a_2\eta^2$, $\psi_\theta(\eta)=b_0+b_1\eta+b_2\eta^2$, $\psi_\chi(\eta)=c_0+c_1\eta+c_2\eta^2$ for the velocity, temperature and concentration respectively, which satisfies the boundary conditions (2.11). Applying the boundary conditions on the approximate solution we obtain the following:

$w(\eta)=a_0(1-\eta^2)$, $\theta(\eta)=\eta^2+b_1(\eta-\eta^2)$, $\chi(\eta)=\eta^2+c_1(\eta-\eta^2)$ and

$u_1=(1-\eta^2)$, $u_2=(\eta-\eta^2)$, $u_3=(\eta-\eta^2)$ are the weighting functions $u_i$, where $a_0,b_1,c_1$ are the coefficients to be determined.

The residue $R$ for (2.7)-(2.9) respectively are given by

$$R_a=1-\frac{4a_0}{R_e}-32\Lambda a_0^3\eta^2+\frac{Me^n a_0\eta^2}{1+m^2}-\frac{Me^n a_0}{1+m^2} \qquad (3.5)$$

$$\begin{aligned}R_b&=2(1-R_*)+R_*b_1(\frac{1}{\eta}-2)-2b_1+4P_rE_cP_*\eta^2a_0^2+16\Lambda P_rE_cR_eP_*\eta^4a_0^4-Q_HP_rR_eP_*\eta^2b_1\\&+Q_HP_rR_eP_*\eta^2+Q_HP_rR_eP_*\eta b_1-4D_uP_rR_eP_*c_1+4D_uP_rR_eP_*+\frac{D_uP_rR_eP_*c_1}{\eta}\end{aligned} \qquad (3.6)$$

$$\begin{aligned}R_c&=4(1-c_1)+\frac{c_1}{\eta}-2S_ca_0c_1\eta^3+2S_ca_0\eta^3+2S_ca_0c_1\eta^2+2S_ca_0c_1\eta-2S_ca_0\eta\\&-S_ca_0c_1-K_Rc_1\eta^2+K_R\eta^2+K_Rc_1\eta\end{aligned} \qquad (3.7)$$

Taking into account of orthogonality of the residues above, we have

$$\left\langle u_1,R_a\right\rangle=\int_0^1\left[(1-\eta^2)(1-\frac{4a_0}{R_e}-32\Lambda a_0^3\eta^2+\frac{Me^n a_0\eta^2}{1+m^2}-\frac{Me^n a_0}{1+m^2})\right]d\eta=0$$

$$\left\langle u_2,R_b\right\rangle=\int_0^1\left[\begin{aligned}&(\eta-\eta^2)(2(1-R_*)+R_*b_1(\frac{1}{\eta}-2)-2b_1+4P_rE_cP_*\eta^2a_0^2\\&+16\Lambda P_rE_cR_eP_*\eta^4a_0^4-Q_HP_rR_eP_*\eta^2b_1+Q_HP_rR_eP_*\eta^2\\&+Q_HP_rR_eP_*\eta b_1-4D_uP_rR_eP_*c_1+4D_uP_rR_eP_*+\frac{D_uP_rR_eP_*c_1}{\eta})\end{aligned}\right]d\eta=0$$

$$\left\langle u_3,R_c\right\rangle=\int_0^1\left[\begin{aligned}&(\eta-\eta^2)(4(1-c_1)+\frac{c_1}{\eta}-2S_ca_0c_1\eta^3+2S_ca_0\eta^3+2S_ca_0c_1\eta^2\\&+2S_ca_0c_1\eta-2S_ca_0\eta-S_ca_0c_1-K_Rc_1\eta^2+K_R\eta^2+K_Rc_1\eta)\end{aligned}\right]d\eta=0$$

The symbolic calculation software MAPLE 2016 is used to compute the values of $a_0,b_1,c_1$ and the approximate solutions.

### 3.2 Entropy Generation

Inherent irreversibility in a pipe flow occurs owing to exchange of energy and momentum within the fluid and the solid boundaries. The entropy generation is owed to heat transfer and the effects of fluid friction. The equation for rate of entropy generation per unit volume [3, 11] is given

$$S^{m} = \frac{k}{T_w^2}\left(\frac{dT}{dr}\right)^2 + \frac{\mu}{T_w}\left(\frac{dw}{dr}\right)^2 + \frac{2\beta_3}{T_w}\left(\frac{dw}{dr}\right)^4 \qquad (4.1)$$

where the first term in (4.1) is the irreversibility due to heat transfer, the second and third term are entropy generation due to viscous dissipation. Introducing the dimensionless quantities in (2.6) to (4.1), we have

$$N_s = \frac{r^2 S^{m}}{k} = \frac{\eta^2}{\Omega^2}\left(\frac{d\theta}{d\eta}\right)^2 + \frac{B_R\eta^2}{\Omega}\left(\frac{dw}{d\eta}\right)^2 + \frac{\beta_*\eta^2}{\Omega}\left(\frac{dw}{d\eta}\right)^4 \qquad (4.2)$$

where $\Omega = \frac{T_w}{T_w - T_0}, B_R = \frac{\mu w_0^2}{k(T_w - T_0)}, \beta_* = \frac{\beta_3 w_0^4}{kR^2(T_w - T_0)}$ are temperature difference parameter, Brickman number and third grade parameter and

$$N_1 = \frac{\eta^2}{\Omega^2}\left(\frac{d\theta}{d\eta}\right)^2, N_2 = \frac{B_R\eta^2}{\Omega}\left(\frac{dw}{d\eta}\right)^2 + \frac{\beta_*\eta^2}{\Omega}\left(\frac{dw}{d\eta}\right)^4 \qquad (4.3)$$

where $N_1$ is irreversibility due to heat transfer and $N_2$ gives entropy generation due to viscous dissipation. The Bejan number is defined as

$$B_e = {N_1}/{N_s} \qquad (4.4)$$

such that $0 \le B_e \le 1$ denoting $B_e = 1$ is the limit at which heat transfer irreversibility dominates, $B_e = 0$ is the limit at which fluid friction irreversibility dominates, and $B_e = {1}/{2}$ connotes equal contribution [20].

## 3. RESULTS AND DISCUSSION

In this section, results are presented and discussed. Fig. 1 depicts the influence of magnetic parameter, increasing the magnetic parameter decreases the flow profile of the system owning to the Lorentz force acting in contradiction of the flow. Fig. 2 shows the Hall parameter enhancing the flow profile with increasing Hall values. Increasing the Reynolds number enhances the velocity profile as shown in Fig. 3. In Fig. 4, the thickening effect of the fluid in regard to increasing thirdgrade parameter inhibits the flow field.

Fig. 5-7 portrays the effect of Eckert, Prandtl and Reynolds number on the temperature profile. Considerable increase in the Eckert number slightly increases the temperature profile then increasing the Prandtl number and Reynolds number decreases the temperature field of the system. The temperature field in Fig. 8 is enhanced with increasing the radiation parameter.

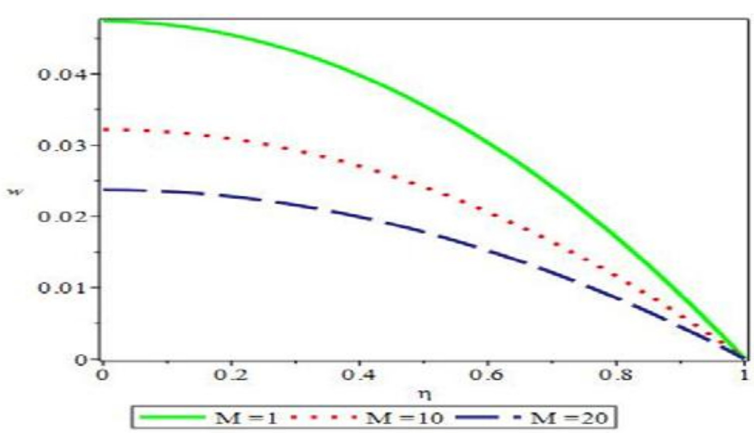


**Fig. 1. Effect of varying magnetic parameter (M=1, M=10, M=20) on velocity profile.**

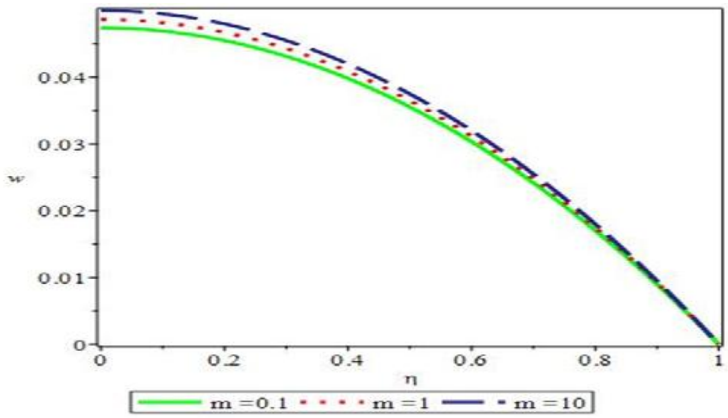


**Fig. 2. Effect of varying Hall parameter (m=0.1, m=1, m=10) on velocity profile.**

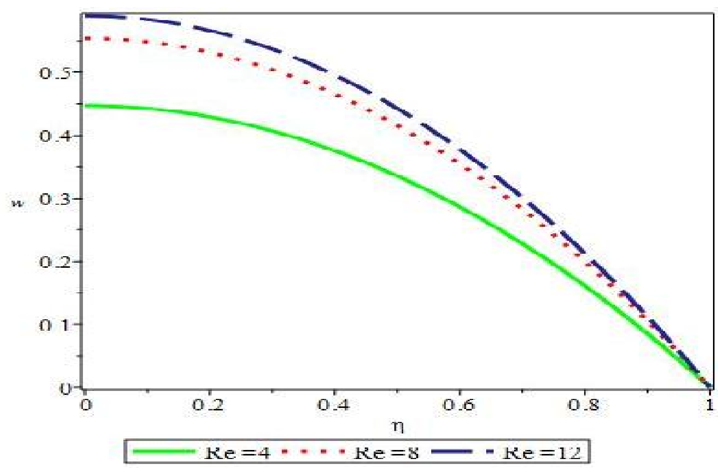


**Fig. 3. Effect of varying Reynolds number (Re=4, Re=8, Re=12) on velocity profile.**

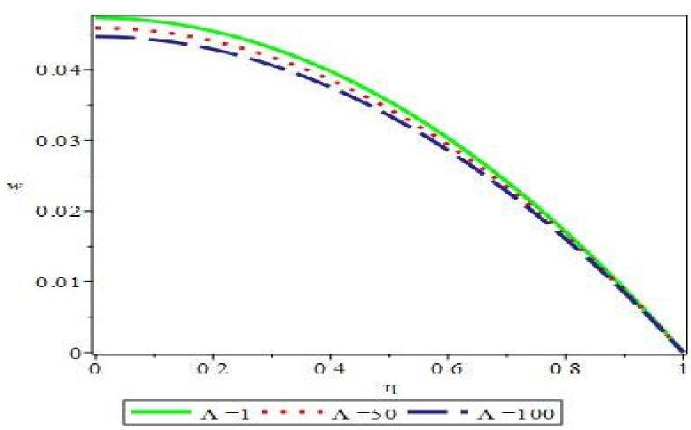


**Fig. 4. Effect of varying Thirdgrade parameter ( $\Lambda$ =1, $\Lambda$ =50, $\Lambda$ =100) on velocity profile.**

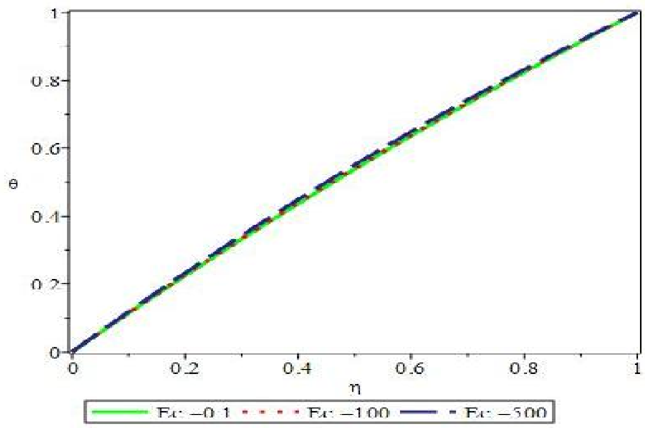


**Fig. 5. Effect of varying Eckert number (Ec=0.1, Ec=100, Ec=500) on velocity profile.**

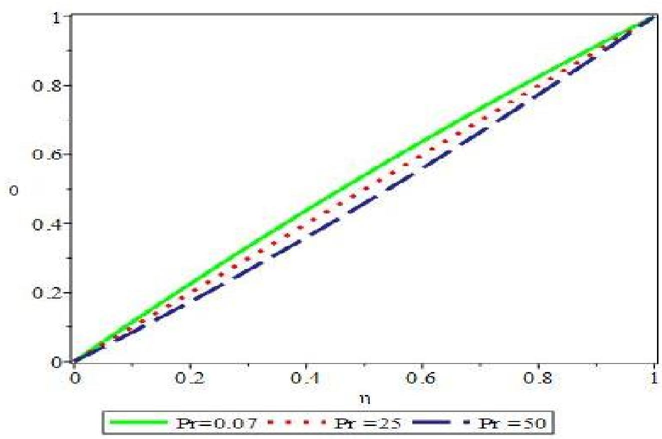


**Fig. 6. Effect of varying Prandtl number (Pr=0.07, Pr=25, Pr=50) on temperature profile.**

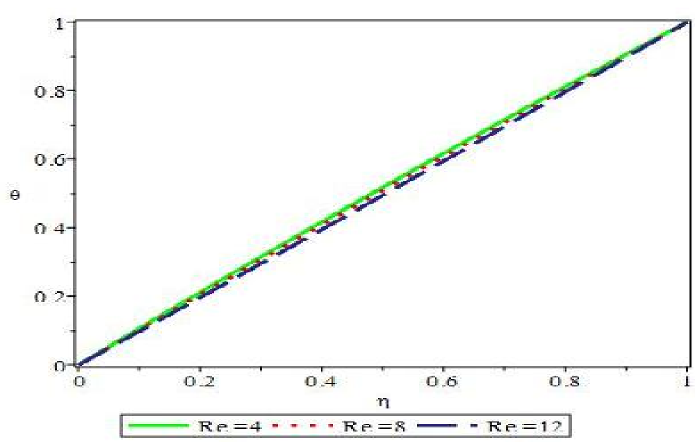


**Fig. 7. Effect of varying Reynolds number (Re=4, Re=8, Re=12) on temperature profile.**

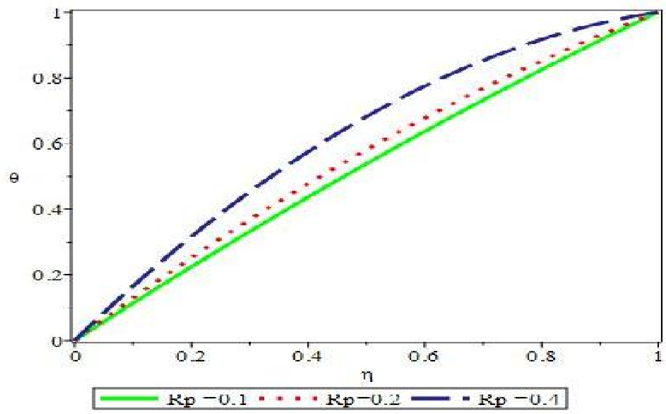


**Fig. 8. Effect of varying Radiation parameter (Rp=0.1, Rp=0.2, Rp=0.4) on temperature profile.**

Figures 9-10 depicts the influence of Dufour and Schmidt numbers on the concentration profile. Increasing the Dufour number increases the concentration field while the concentration profile decreases with increasing values of Schmidt number. The entropy generation profile is portrayed in Fig. 11-14 with influences of Reynolds, Prandtl, Eckert numbers and radiation parameter. Increasing the Reynolds number enhances the entropy generation while increasing Eckert number inhibits entropy generation. Increasing the Prandtl number decreases the entropy generation firstly around the pipe centreline then it enhances entropy rapidly towards the pipe wall while increasing the radiation parameter enhances the entropy generation around the centreline firstly then it inhibits it rapidly towards the pipe wall.

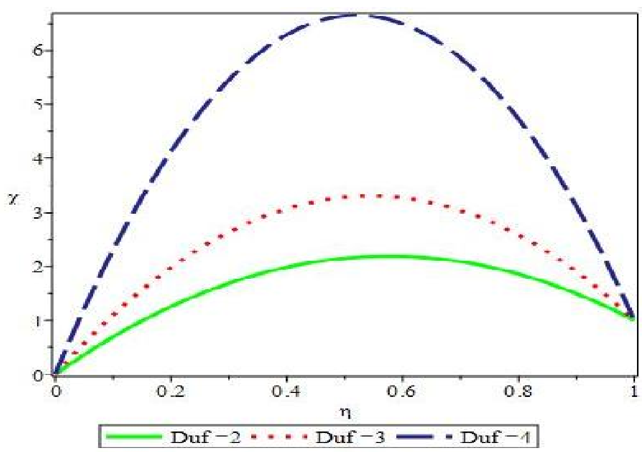


**Fig. 9. Effect of varying Dufour number (Duf=2, Duf=3, Duf=4) on concentration profile.**

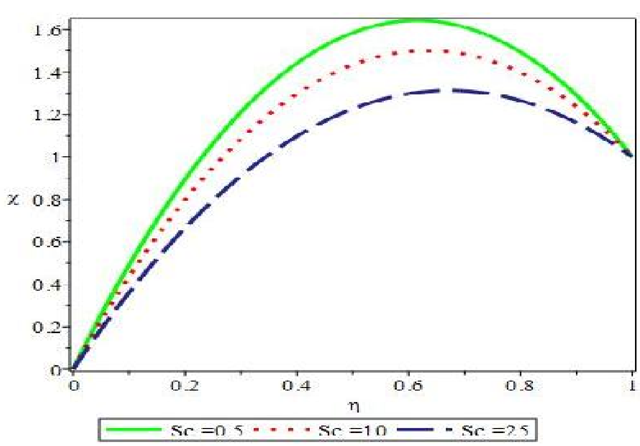


**Fig. 10. Effect of varying Schmidt number (Sc=0.5, Sc=10, Sc=25) on concentration profile.**

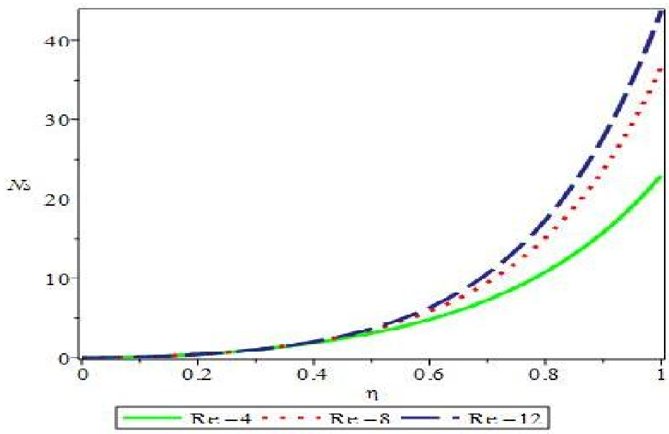


**Fig. 11. Effect of varying Reynolds number (Re=4, Re=8, Re=12) on entropy generation profile.**

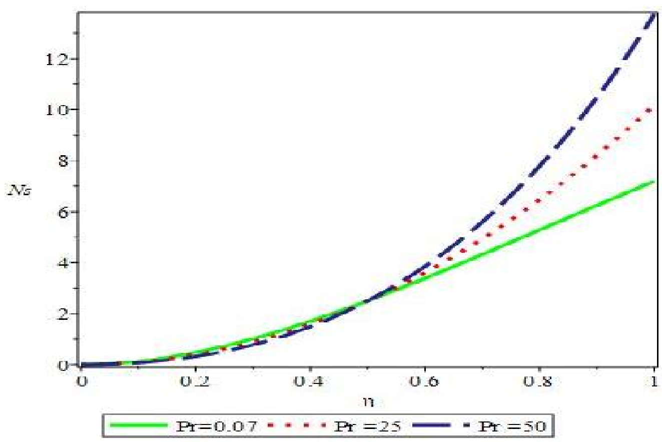


**Fig. 12. Effect of varying Prandtl number (Pr=0.07, Pr=25, Pr=50) on entropy generation profile.**

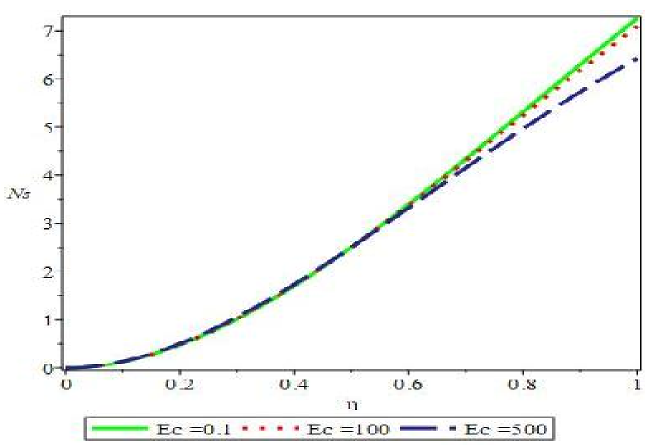


**Fig. 13. Effect of varying Eckert number (Ec=0.1, Ec=100, Ec=500) on entropy generation profile.**

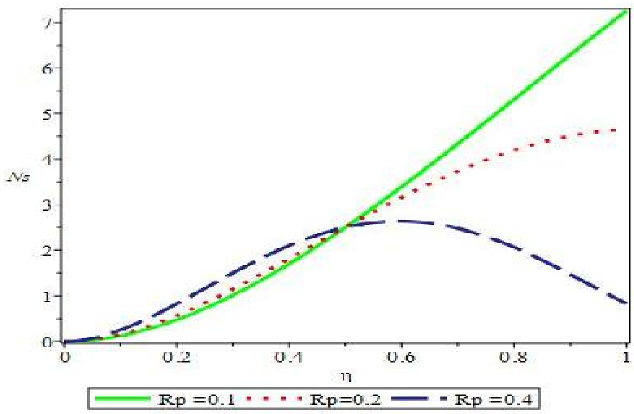


**Fig. 14. Effect of varying Radiation parameter (Rp=0.1, Rp=0.2, Rp=0.4) on entropy generation profile.**

Figures 15-22 presents the influence of Hall parameter, magnetic parameter, Prandtl number, Eckert number Reynolds number, thirdgrade parameter, Dufour number and reaction parameter on Bejan number. Increasing the Hall parameter, Eckert number and Reynolds number inhibits the Bejan number and the irreversibility due to heat transfer dominates over fluid friction irreversibility from the pipe centreline to pipe wall except for Reynolds number where irreversibility due to fluid friction dominates gradually towards the pipe wall. On increasing the magnetic parameter, thirdgrade parameter, Dufour number and reaction parameter enhances the Bejan number and the irreversibility due to heat transfer dominates over fluid friction irreversibility. Increasing the Prandtl number firstly inhibits the Bejan number around the pipe centreline then it enhances Bejan number towards the wall of the pipe and the flow is dominated by heat transfer irreversibility.

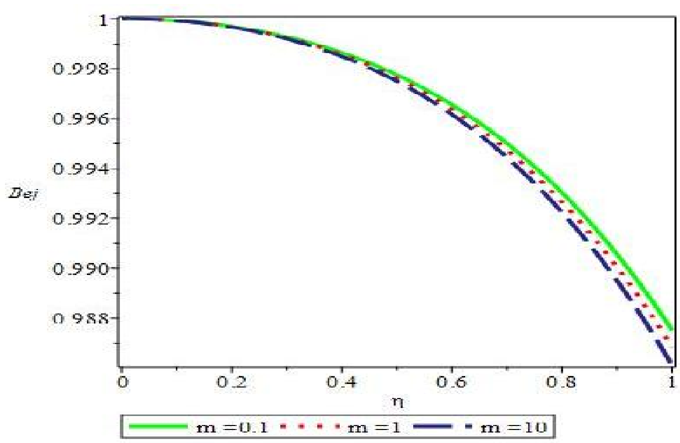


**Fig. 15. Effect of varying Hall parameter (m=0.1, m=1, m=10) on Bejan number.**

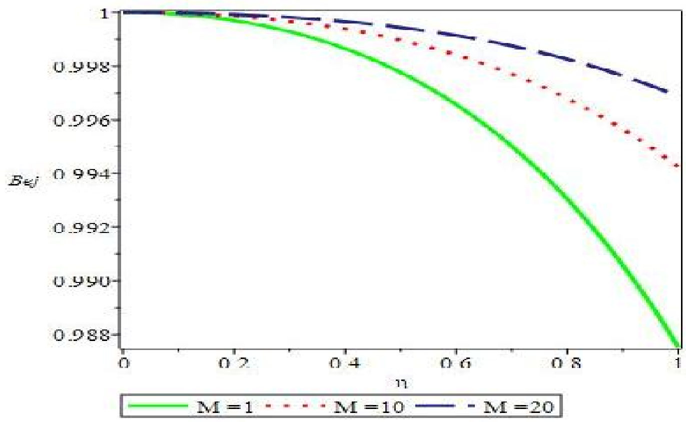


**Fig. 16. Effect of varying Magnetic parameter (M=1, M=10, M=20) on Bejan number.**

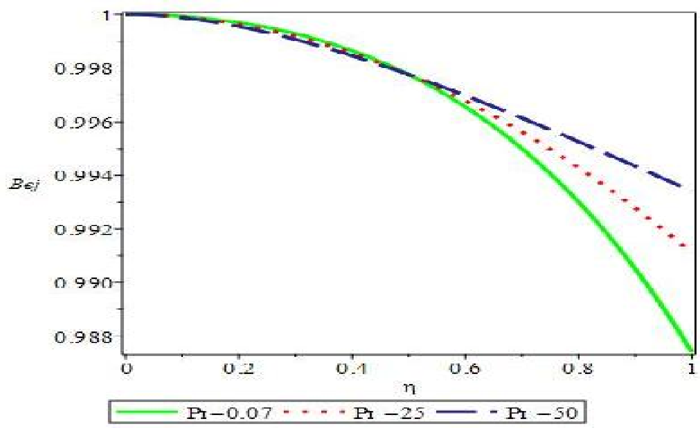


**Fig. 17. Effect of varying Prandtl number (Pr=0.07, Pr=25, Pr=50) on Bejan number.**

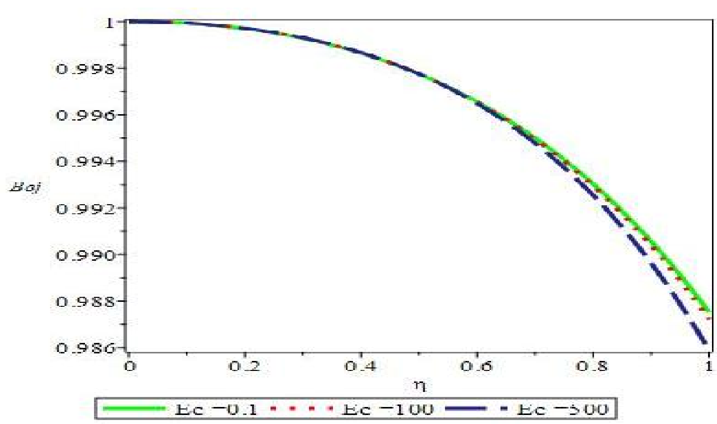


**Fig. 18. Effect of varying Eckert number (Ec=0.1, Ec=100, Ec=500) on Bejan number.**

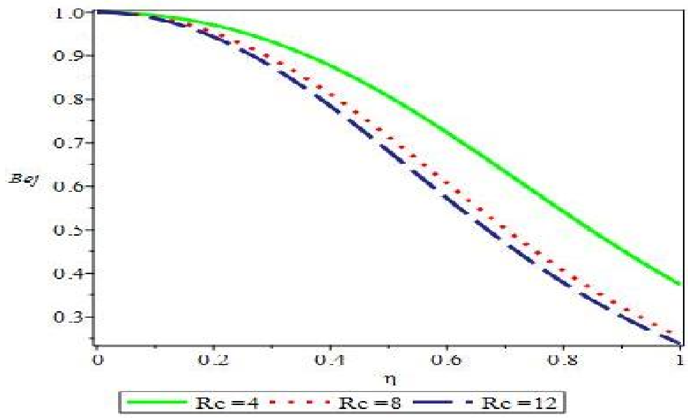


**Fig. 19. Effect of varying Reynolds number (Re=4, Re=8, Re=12) on Bejan number.**

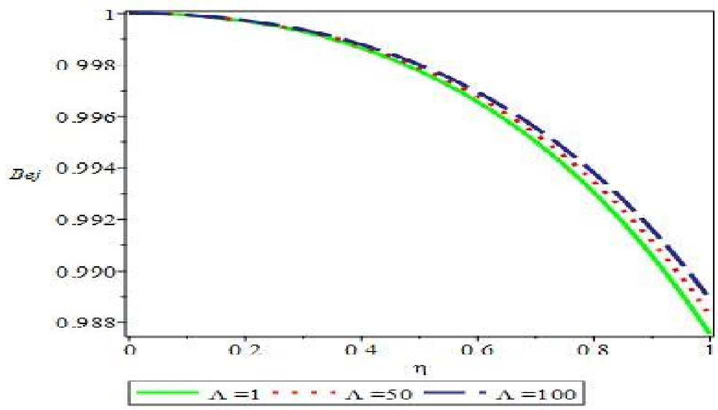


**Fig. 20. Effect of varying Thirdgrade parameter ( $\Lambda$ =1, $\Lambda$ =50, $\Lambda$ =12) on Bejan number.**

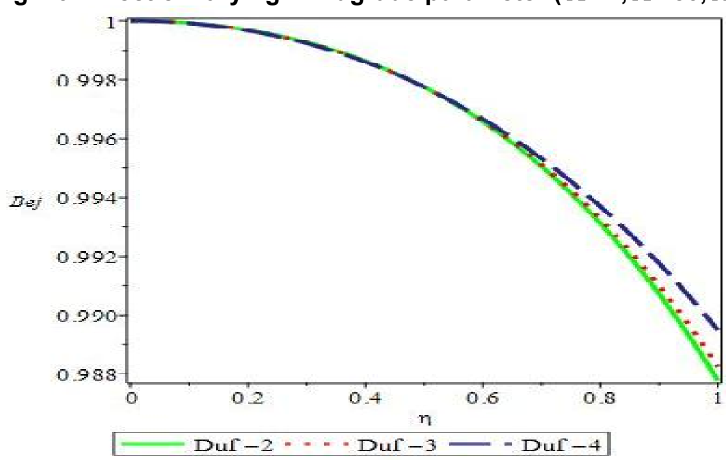


**Fig. 21. Effect of varying Dufour number (Duf=2, Duf=3, Duf=4) on Bejan number.**

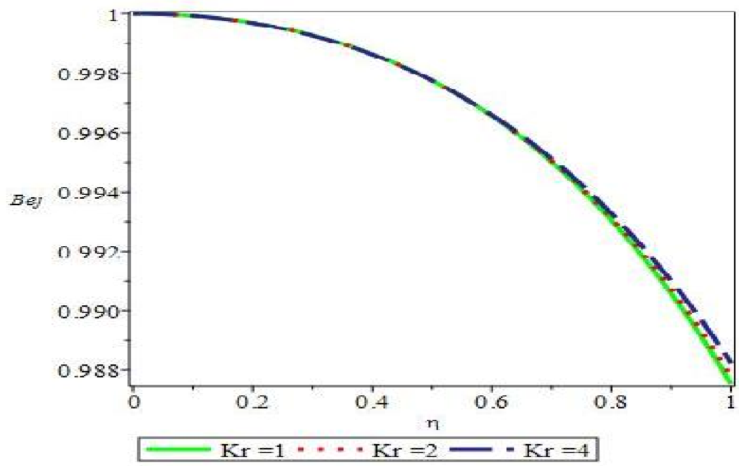


**Fig. 22. Effect of varying reaction parameter (Kr=1, Kr=2, Kr=4) on Bejan number.**

## 4. CONCLUSION

In this numerical investigation, the entropy generation rate of steady reactive magnetohydrodynamic third grade fluid flow in a circular pipe is presented using the Galerkin method. Numerical expression for the velocity, temperature and concentration was obtained which were used to compute the entropy generation number. Special emphasis has been focused here to the variations of pertinent parameter of physical significance on the entropy generation rate and Bejan. The main findings of the present analysis are:

- The velocity is enhanced for increasing values of $m, \mathrm{Re}$ and inhibited for $M, \Lambda$
- The temperature is enhanced for values of $Ec, Rp$ and inhibited for $\mathrm{Pr}, \mathrm{Re}$ and $Du$
- The concentration is enhanced values of $Du, K_R$ and inhibited for $Sc$ and $\mathrm{Re}$
- $\mathrm{Re}, K_R$ and $Du$ have enhancing effects on the entropy generation rate.
- $M, Du, K_R$ and $\Lambda$ enhances the entropy generation rate while it is inhibited for $\mathrm{Re}$ and $Ec$ .

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## DEFINITIONS, ACRONYMS, ABBREVIATIONS

Here is the Definitions section. This is an optional section.

**Term**: Definition for the term

## APPENDIX