## **Effect of Vadasz Number on Magnetoconvection in a Darcy Porous Layer With Concentration Based Internal Heating**

## **Abstract**

In this study, the effect of Vadasz number on magnetoconvection in a Darcy Porous Layer with concentration based internal heating is investigated using the linear stability analysis which is based upon the normal mode technique. The onset criterion for both stationary and oscillatory convection on the stability of the system is obtained. The results show that, the presence of Vadasz number destabilizes the system, whereas,  $Ha$ ,  $Rs$ ,  $Le$  stabilizes the system.

**Keywords:** Concentration based internal heat, normal mode analysis, Magnetoconvection, porous layer, double diffusive convection

 $\qquad \qquad \qquad \qquad \qquad \qquad$ 

## **1. Introduction**

Thermal instability with the presence of magnetic field in a Darcy porous layer with concentration based internal heating has attracted considerable interest especially due to its intrinsic properties in myriads of industrial problems, particularly in the chemical and nuclear industries, petroleumexploration, thermal insulation technology, nuclear reactors, solidification of binary alloys, cooling of electronic equipment, food processing industry, centrifugal casting of metals, reservoir modelling and building of thermal insulation.

Several researchers in the last decades have investigated thermal instability in porous medium by considering different fluids. The classical literatures [1, 2, 3, 4, 5, 6, 7, 8] investigated under various conditions of hydrodynamic and hydromagnetic, thermal instability of a Newtonian fluid. [9] have examined the stability of convective flow in a porous medium using Rayleigh's procedure. The Rayleigh instability of the thermal boundary layer in a flow through porous medium was considered by [10]. Exhaustive account of convection in porous medium can be found in [11, 12, 13,14, 15].

In recent years, considerable interest has been given to thermal instability in a porous medium which is heated and salted from below under varying conditions. [16] investigated effect of vertical magnetic field on the onset of double-diffusive convection in a horizontal porous layer with concentration based internal heat source. Soret and Magnetic field effects on Thermosolutal convection in a porous medium with concentration based internal heat source was considered by [17]. Double diffusive convection in a porous medium with reaction and cross-diffusion effects and concentration based internal heat source has been studied by [18, 19, 20]. In this study, we present the effect of Vadasz number on magnetoconvection in a Darcy porous layer with concentration based internal heating which hitherto has not been considered by other authors or researchers

## **2. Mathematical Formulation**

Consider an incompressible electrically conducting fluid induced by concentration based internal heating of the form  $Q_0(c^* - c_0)$  in the gap between two parallel horizontal plates located at  $z^* = 0$ and  $z^* = h$ . A cartesian coordinate system  $(x^*, y^*, z^*)$  is chosen such that the origin is at the bottom of the porous layer and the gravitational force  $\bar{g}$  acts vertically upwards, as shown in the Figure 1a. The flow occurs in the presence of a uniform externally applied vertical magnetic field  $\bar{B} = B_0 \bar{k}$ . The induced magnetic is neglected on the account that the magnetic Reynold number is small. Adverse temperature and concentration gradients are applied across the porous layer in such a way that the lower plate is maintained at temperature and concentration  $T_0 + \Delta T$  and  $c_0 + \Delta c$ ; while the upper plate at temperature and concentration  $T_0$  and  $c_0$ , where  $T_0$  and  $c_0$  are reference temperature and concentration respectively.



Figure 1a: Schematic diagram of the problem

Further we assume that the fluid is Newtonian with viscosity,  $\mu$ , thermal expansion coefficient,  $\beta_c$ , thermal conductivity  $\kappa_T$ , diffusion coefficient  $D_c$  and electric conductivity  $\sigma_c$ .

Following the usual Boussinesq approximation [12], the mixture density  $\rho$  depends linearly on both temperature and concentration defined by

$$
\rho = \rho_0 [1 - \beta_T (T^* - T_0) + \beta_c (c^* - c_0)] \tag{1}
$$

# where  $\rho_0$  is the reference density

By taking into account the Lorentz force and acceleration coefficient, while in the governing equations viscous heating effect

Where  $\vec{V} = (u^*, v^*, w^*)$  is the velocity,  $P^*$  is the pressure,  $\vec{E}$  is the electric field,  $\vec{J}$  is the current density,  $\kappa$  is the permeability of the porous medium,  $\phi$  is the porosity,  $Q_0 \frac{Q}{C_0 \epsilon_0}$  $(\rho c_p)_f$ is the thermal

conductivity,  $\vec{k}$  is the unit vector in the z -direction. The subscripts m and f denotes the medium and fluid respectively. Following the assumptions the governing equations becomes:

$$
\overline{\nabla^*} \cdot \overline{V^*} = 0 \tag{2}
$$

$$
\frac{\rho_0}{\phi} \frac{\partial \overline{V^*}}{\partial t^*} = -\nabla^* P^* + \rho_0 g [\beta_T (T^* - T_0) - \beta_c (c^* - c_0)]\overline{k} - \frac{\mu}{\kappa} \overline{V^*} + \mu_e \overline{V^{*2} V^*} + F_L
$$
\n(3)

$$
A\frac{\partial T^*}{\partial t^*} + (\overline{V^*}\cdot\overline{\nabla^*})T^* = \alpha_T \nabla^{*2} T^* + Q_0 (c^* - c_0)
$$
\n
$$
\tag{4}
$$

$$
\phi \frac{\partial c^*}{\partial t^*} + (\overline{V^*}. \overline{\nabla^*}) c^* = \kappa_c \overline{\nabla^*^2} c^*
$$
\n(5)

$$
\overline{J}^* = \sigma_c [E^* + \overline{V}^* \times \overline{B^*}], \qquad \overline{\nabla}^* . \overline{J}^* = 0 \qquad (6)
$$

The boundary conditions are  $-\rightarrow$ 

$$
V' = 0
$$
 on  $z' = 0, h$  (7a)

$$
T^* = T_0 + \Delta T, \quad c^* = c_0 + \Delta c \qquad \text{on} \qquad z^* = 0 \qquad (7b)
$$

$$
T^* = T_0, \ c^* = c_0 \qquad \text{on} \qquad z^* = h \qquad (7c)
$$

Here the last term in Equation (3),  $F_L$  represents the Lorentz force, which induces electromagnetic effect on the system. The appropriate form of Ohm's law given in Equation (6) for a moving medium employs the quasi-state approximation, whereby the electric field,  $\bar{E}^*$  can be written in terms of the electrostatic potential  $\varphi$ . That is,  $\bar{E} = -\nabla \varphi$ .

In this work, we consider the case of electrically insulating boundaries for which  $\bar{E} = 0$  on the account that the electrostatic potential is a constant, with this the current density given in equation  $(6)$  reduces to,

$$
\vec{J}^* = \sigma_c(\vec{V}^* \times \vec{B}^*)
$$
 (8)

And the Lorentz force  $F_L = \vec{J}^* \times \vec{B}$  becomes

$$
\vec{F}_L = \sigma_c (\vec{V}^* \times \vec{B}^*) \times \vec{B} \n= -\sigma_c B_0^2 (u^*, v^*, 0)
$$
\n(9)

Using Equation (9) and the scales  $h$ ,  $\frac{h^2 A}{\alpha_T}$ ,  $\frac{\alpha_T}{h}$ ,  $\frac{Kh}{(\alpha_T \mu)}$  for length, time, velocity and pressure respectively; together with  $T = \frac{(T^* - T_0)}{\Delta T}$  for temperature,  $c = \frac{(c^* - c_0)}{\Delta c}$  for concentrati  $\vec{\nabla} = h \vec{\nabla}^*$ , for the dimensionless equations governing the motion of the fluid are,

$$
\vec{\nabla}.\vec{V} = 0 \tag{10}
$$

$$
\left(\frac{1}{Va}\frac{\partial}{\partial t} + 1\right)\overrightarrow{V} + Ha^2(u, v, 0) = -\nabla P + RaT\vec{k} - Rsc\vec{k}
$$
\n(11)

$$
\frac{\partial T}{\partial t} + (\vec{V} \cdot \vec{\nabla})T = \nabla^2 T + R_i c \tag{12}
$$

$$
Le \varepsilon \frac{\partial c}{\partial t} + Le(\vec{V}.\vec{\nabla})c = \nabla^2 c \tag{13}
$$

The boundary conditions are:

$$
w = 0, T = 1, c = 1
$$
 on  $z = 0$  (14*a*)  
\n
$$
w = 0, T = 0, c = 0
$$
 on  $z = 1$  (14*b*)

The dimensionless parameters are:

 $Ra = \frac{\phi Pr}{Da}$  = Vadasz number,  $Pr = \frac{Av}{\alpha_T}$  = Prandtl number,  $Da = \frac{\kappa}{h^2}$  = Darcy number,  $Ra$  =  $\frac{\rho_0 g \beta_T \kappa h \Delta T}{\mu \alpha_T} = \text{Rayleigh number}, \quad Le = \frac{\alpha_T}{\alpha_c} = \text{Lewis number}, \quad Ha^2 = \frac{\alpha_c B_0^2 k}{\mu} = \text{Hartmann number},$  $Ri = \frac{h^2 Q_0 \Delta c}{\alpha_T \Delta T}$  = internal heat parameter,  $R_s = \frac{\rho_{0g\beta_c k h \Delta c}}{\mu \alpha_T}$  solutal Rayleigh number.

## 2.1 Basic State

The basic state of the system is assumed to be quiescent and is described by

$$
V_b = 0, T = T_b(z), c = c_b(z), p = p_b(z)
$$
\n(15)

Substituting Equation (15) into Equations (10) – (13) and the boundary condition (14) yield the equations governing the basic state as

$$
\frac{d^2T_b}{dz^2} + R_i c_b = 0 \tag{16}
$$

$$
\frac{d^2c_b}{dz^2} = 0\tag{17}
$$

$$
\frac{dP_b}{dz} = R a T_b - R s c_b \tag{18}
$$

Subject to

$$
T_b = c_b = 1 \qquad \qquad \text{on} \qquad \qquad z = 0 \qquad \qquad (19a)
$$

$$
T_b = c_b = 0 \qquad \qquad \text{on} \qquad z = 1 \qquad \qquad (19b)
$$

Under the boundary conditions  $(19)$ , the integration of Equations  $(16) - (18)$  yield

$$
T_b(z) = \frac{1}{6} \Big[ 6(1-z) + (2z - 3z^2 + z^3)R_i \Big]
$$
  
\n
$$
c_b(z) = 1 - z
$$
  
\n
$$
p_b(z) = \int (RaT_b(z) - Rsc_b(z))dz
$$
\n(20)

## **2.2 Linearization and Perturbation Solution**

To study the stability of the basic state, we now superimpose small perturbations in the form

$$
\vec{V} = \vec{V}_b + \vec{V}, \quad p = p_b(z) + p, \quad T = T_b(z) + \theta, \quad c = c_b(z) + \varphi
$$
\n<sup>(21)</sup>

where  $\vec{V}$ ,  $p$ ,  $\theta$  and  $\varphi$  are the perturbed quantities over their equilibrium counterparts and are assumed small. On substituting Equation (21) into Equations (10)-(13) and the boundary condition (14) and using Equation (21) we obtain the linearized equations as

$$
\vec{\nabla}.\vec{V} = 0 \tag{22}
$$

$$
\left(\frac{1}{Va}\frac{\partial}{\partial t} + 1\right)\overrightarrow{V} + Ha^2(u, v, 0) = -\nabla p + Ra\theta\overrightarrow{k} - Rs\varphi\overrightarrow{k}
$$
\n(23)

$$
\frac{\partial \theta}{\partial t} + (\vec{V} \cdot \nabla) \theta + f(z) w = \nabla^2 \theta + R_i \varphi
$$
\n(24)

$$
Le\varepsilon \frac{\partial \varphi}{\partial t} + Le(\vec{V}.\vec{\nabla})\varphi - Lew = \nabla^2 \varphi
$$
\n(25)

where  $f(z) = \frac{\partial T_b}{\partial z}$  $=\frac{\partial T_b}{\partial z}$  is the basic temperature gradient given by  $1 + \frac{R_i}{6} \left(2 - 6z + 3z^2\right)$  $\frac{T_b}{T_a} = -1 + \frac{R_i}{T_a} (2 - 6z + 3z)$ *z*  $\frac{\partial T_b}{\partial z} = -1 + \frac{R_i}{6} (2 - 6z + 3z^2)$ 

The impermeable boundary conditions are

$$
w = \theta = \varphi = 0 \qquad \text{on} \qquad z = 0, 1 \tag{26}
$$

Next, the pressure term is eliminated by taking the double curl of Equation (23) and keeping only the  $z$  – component. Then the system reduces to

$$
\left(\frac{1}{Va}\frac{\partial}{\partial t} + 1\right)\nabla^2 w + Ha^2 \frac{\partial^2 w}{\partial z^2} - Ra\nabla_h^2 \theta + Rs\nabla_h^2 \phi = 0
$$
\n
$$
f(z)w + \left(\frac{\partial}{\partial t} - \nabla^2\right)\theta - R_i\phi = 0
$$
\n
$$
Lew - \left(Le\varepsilon \frac{\partial}{\partial t} - \nabla^2\right)\phi = 0
$$
\n(27)

where  $\hat{c}_h^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial}{\partial y^2}$  is the Laplacian in the horizontal plane  $\nabla_h^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial}{\partial y^2}$  $\frac{\partial}{\partial x^2} + \frac{\partial}{\partial y^2}$  is the Laplacian in the horizontal plane

The boundary condition for the system  $(27)$  are

$$
w = \frac{\partial^2 w}{\partial z^2} = \theta = \varphi = 0
$$
 on  $z = 0, 1$  (28)

Further, assuming that, the conductive motion exhibit horizontal periodicity, then we seek a time dependent periodic disturbance of the form [21]

$$
\begin{pmatrix} w \\ \theta \\ \varphi \end{pmatrix} = \begin{pmatrix} W(z) \\ \Theta(z) \\ \Phi(z) \end{pmatrix} f(x, y) e^{\sigma t}
$$
 (29)

where  $\sigma = (\omega_r + i\omega_i)$  is the growth rate and is in general complex, with  $\omega_r$ ,  $\omega_i$  real and

 $f(x, y)$  is a horizontal plane tilting the  $xy$  -plane periodically.

The substitution of Equation  $(29)$  into  $(27)$  yield the eigenvalue problem.

$$
\left(\frac{\sigma}{Va} + 1\right)(D^2 - a^2)w + Ha^2w + a^2Ra\Theta - a^2Rs\Phi = 0
$$
  

$$
-f(z)w + (D^2 - a^2 - \sigma^2)\Theta + R_i\Phi = 0
$$
  

$$
Lew + (D^2 - a^2 - Le\varepsilon\sigma)\Phi = 0
$$
 (30)

where 
$$
D = \frac{\partial}{\partial z}
$$
 and  $\nabla_h^2 f + a^2 f = 0$ , *a* is the wave number  
\n $w = \Theta = \Phi = 0$  on  $z = 0, 1$  (31)

To study the boundary conditions  $(31)$ , we further assume that solution of Equation  $(30)$  in the form

$$
\begin{pmatrix} W(z) \\ \Theta(z) \\ \Phi(z) \end{pmatrix} = \begin{pmatrix} W_0 \\ \Theta_0 \\ \Phi_0 \end{pmatrix} \sin \pi z
$$
\n(32)

where  $W_0$ ,  $\Theta_0$ ,  $\Phi_0$  are constants. Substitution of Equation (32) into (30) yields

$$
H\overrightarrow{X} = 0 \tag{33}
$$

where

$$
H = \begin{pmatrix} J(1 + \frac{\sigma}{Va}) + \pi^2 Ha^2 & -a^2 Ra & a^2 Rs \\ 2F(z) & J + \sigma & -R_i \\ -Le & 0 & J + Le \epsilon \sigma \end{pmatrix}
$$

$$
\overline{X}
$$
 =  $(W_0, \Theta_0, \Phi_0)^T$ ,  $J = a^2 + \pi^2$  and  $F = \int_0^1 f(z) \sin^2 \pi z dz$ 

The solvability of the eigenvalue problem in Eq. (33) requires that  $|H| = 0$ . This requirement yields the thermal Rayleigh number as

$$
Ra = \frac{4\pi^2 J \left(1 + \frac{\sigma}{va}\right) (J + \sigma)(J + Le\epsilon\sigma)}{a^2 (b_2 + b_0 Le\epsilon\sigma)} + \frac{4\pi^4 Ha^2 (J + \sigma)(J + Le\epsilon\sigma)}{a^2 (b_2 + b_0 Le\epsilon\sigma)} + \frac{4\pi^2 LeRs (J + \sigma)}{b_2 + b_0 Le\epsilon\sigma} \tag{34}
$$

where

$$
b_0 = (4\pi^2 + R_i)
$$
  

$$
b_1 = 4\pi^2 LeR_i
$$

 $b_2 = b_0 J + b_1$ 

For neutral solutions, we set  $\sigma = i\omega$ , in Equation (34) and rearranging yields

$$
\Pr{Gr = Ra} = \frac{4\pi^2}{\Delta} \left[ \Delta_1 + i\omega_1 \Delta_2 \right]
$$
 (35)

where

$$
\Delta_{1} = \frac{1}{a^{2}} \left[ \int \left( 1 + \frac{f}{va} \right) + \text{Le}\varepsilon \left( f - \frac{\omega^{2}}{va} \right) \right] \text{Le}\varepsilon J\omega^{2} + Jb_{2} \left[ \int \left( f - \frac{\omega^{2}}{va} \right) - \text{Le}\varepsilon \left( 1 + \frac{f}{va} \right) \omega^{2} \right] + \text{Ha}^{2} \pi^{2} \left[ \text{Le}\varepsilon (1 + \text{Le}\varepsilon) b_{0} \omega^{2} + (J^{2} - \text{Le}\varepsilon \omega^{2}) b_{2} \right] \right] + \text{Le}\varepsilon Jb_{0} \omega^{2}
$$
\n
$$
\Delta_{2} = \frac{1}{a^{2}} \left[ \frac{J^{2} \text{Le}\varepsilon b_{0}}{Va} (\text{Le}\varepsilon + 1) \omega^{2} + \text{ILe}\varepsilon (\text{Le}\varepsilon \omega^{2} - J^{2}) \right] b_{0} + \left[ J^{2} \left( 1 + \frac{1}{Va} \right) + \text{ILe}\varepsilon \left( f - \frac{\omega^{2}}{Va} \right) \right] b_{2}
$$
\n
$$
+ \left[ (J + \text{Le}\varepsilon) b_{2} - (J^{2} - \text{Le}\varepsilon \omega^{2}) \text{Le}\varepsilon b_{0} \right] + \text{Le}\varepsilon S(b_{2} - \text{He}\varepsilon b_{0})
$$

$$
\Delta = b_2^2 + b_0^2 L e^2 \varepsilon^2 \omega^2
$$

Since  $R_a$  is a physical quantity, it must be real. Hence from Equation  $(35)$ , it follows that either  $\omega = 0$  for the onset of stationary convection or  $\Delta_2 = 0$ ,  $\omega \neq 0$  for the onset oscillatory convection.

## **3 Onset of Stationary Convection**

For the validity of principle of exchange of stabilities to hold for marginal stationary convection,  $\omega = 0$ . Setting  $\omega = 0$  in Equation (35) yields the Rayleigh number,  $Ra^{st}$  for the stationary convection as

$$
Ra^{st} = \frac{4\pi^2}{b_2} \left[ \frac{J^3}{a^2} + \frac{Ha^2\pi^2 J^2}{a^2} + LeRsJ \right]
$$
 (36)

In the absence of  $R_i = 0$ ,

$$
Ra^{st} = \frac{1}{a^2} [J^2 + Ha^2 \pi^2 J] + LeRs
$$

Further if  $Ha = 0$ 

$$
Ra^{st} = \frac{J^{2}}{a^{2}} + LeRs
$$
  
= 
$$
\frac{(a^{2} + \pi^{2})^{2}}{a^{2}} + LeRs
$$
 (37)

Which coincides with the earlier results of [22] and [16] Further, when  $Rs = 0$ , the stationary Rayleigh number given in Equation (37) reduces to classical result of  $[23]$  and  $[9]$ 

$$
Ra^{st} = \frac{(a^2 + \pi^2)^2}{a^2}
$$
 (38)

 $(37)$ In addition, Equation gives the critical Rayleigh number  $Ra_c^{st} = 4\pi^2$  with corresponding critical wave number,  $a_c^{st} = \pi$ 

## 3.1 Oscillatory Convection

For the onset of oscillatory convection  $\Delta_2 = 0$  and  $\omega \neq 0$ . Setting  $\Delta_2 = 0$  and  $Ra = Ra^{\omega s}$  in Equation (35) gives the expression for Rayleigh number as

$$
Ra^{OS} = \frac{4\pi^2}{a^2\Delta} \Biggl\{ \Biggl[ J\left(1 + \frac{J}{Va}\right) + Le\epsilon(J - \frac{\omega_i^2}{Va}) \Biggr] Le\epsilon J \omega_i^2 + Jb_2(J\left(J - \frac{\omega_i^2}{Va}\right) - Le\epsilon (1 + \frac{J}{Va})\omega_i^2 \Biggr] + Ha^2\pi^2 \Biggl[ JLe\epsilon (1 + Le\epsilon)b_0\omega_i^2 + (J^2 - Le\epsilon\omega_i^2)b_2 \Biggr] \Biggr\} + \frac{4\pi^2 LeRs}{\Delta} (Jb_2 + b_0\omega_i^2) \tag{39}
$$

where the frequency of oscillation is given by

 $X =$ 

$$
\frac{\omega_i^2 = \{[Ha^2\pi^2((J(J-1))(4\pi^2 + Ri) - 4\pi^2 L eRi) + JLe(a^2(4\pi^2 + Ri)Rs - 4\pi^2 JRi)\} - \left(4\pi^2 L eRi + J(4\pi^2 + Ri)\right)\left(\frac{J^3}{Va} + \frac{J^2 + Ha^2\pi^2 J + a^2 L eRs}{Va}\right)\}}{[Le\varepsilon]\frac{4\pi^2 J}{Va}(J-Ri) + (Ha^2\pi^2 + J)(4\pi^2 + Ri)}\tag{40}
$$

# **4 Discussion of Results**

The effect of Vadasz number on Magnetoconvection in Darcy porous layer with concentration based internal heating is studied analytically using the linear stability analysis technique. The expressions for both the stationary and oscillatory modes for different values of the governing parameters such as Magnetic field parameter, Ha, solute Rayleigh number, Rs, internal heat parameter,  $Ri$  and Lewis number,  $Le$ , are computed and the results are displayed in figures 1-8.



Fig. 1: Variation of thermal Rayleigh number for various values of internal heat parameter, *Ri* for  **stationary convection.** 

Figure 1. Influence of the internal heat parameter on the onset of instability for fixed values of Ha=2, Rs=5, Le=1 in stationary convection. It is observed that increase in in the internal heat decreases the thermal Rayleigh number, for stationary convection. This implies that internal heat hastens instability, which leads to a destabilization of the system.



## Fig. 2: **Variation of thermal Rayleigh number for various values of the magnetic parameter,** ࢇࡴ **for stationary convection**

Figure 2. Shows numerically the computed values for  $Rs = 2, Ri = 2$  and the influence of the Magnetic field parameter,  $Ha$  on the thermal Rayleigh umber. The result shows increase in Magnetic field increases the thermal Rayleigh number for the stationary mode. The result is an indication that Magnetic field stabilizes the system.



Fig. 3: Variation of thermal Rayleigh number for various values of the Lewis number, Le for  **stationary convection** 

Figure 3 depicts the influence of Lewis number, Le on the thermal Rayleigh number, Ra for fixed values of  $Ha = 2, Rs = 2, Ri = 1$  in stationary convection. It is observed that increase in Lewis number leads to a decrease in the thermal Rayleigh number,  $Ra$  which is an indication that Lewis number hastens the onset of instability in the system.



#### Fig. 4: Variation of thermal Rayleigh number for various values of solutal Rayleigh number, Rs in  **stationary convection**

Figure 4 depicts the influence of solutal Rayleigh number,  $\overline{Rs}$  on the thermal Rayleigh number for fixed  $Ha = 2, Ri = 1, Le = 1$ . The result shows increase in solutal Rayleigh number increases the thermal Rayleigh number. This implies that solutal Rayleigh number stabilizes the system for stationary convection.



Fig. 5: **Variation of thermal Rayleigh number with solutal Rayleigh number for various values of the internal heat parameter, Ri in stationary convection** 

Figure 5 shows the influence of the internal heat parameter,  $R_i$  on the thermal Rayleigh number for fixed  $Ha = 2$ ,  $Le = 1$ ,  $a_r = \pi$ . It is observed that increase in internal heat decreases the thermal Rayleigh number. This is an indication that, the system is destabilized in the presence of internal heat parameter.



## Fig. 6: Variation of thermal Rayleigh number for various values of magnetic parameters, *Ha* in  **oscillatory convection**

Figure 6 shows the linear relationship between the thermal Rayleigh number,  $Ra$  and the solutal Rayleigh number,  $Rs$  for variations in the Magnetic field parameter,  $Ha$ . Increase in Magnetic field increases the thermal Rayleigh number which is an indication that the system is stabilized in the presence of Magnetic field for stationary convection.



Fig. 7: Variation of thermal Rayleigh number for various values of the Vadasz number, Va in  **oscillatory convection**

Figure 7 depicts the influence of Vadasz number,  $Va$  on the thermal Rayleigh number,  $Ra$  for oscillatory convection with fixed values of  $Ha = 10$ ,  $Ri = 2$ . The result shows increase in Vadasz number increases the thermal Rayleigh number which indicates that Vadasz number delays the onset of instability in the system.



Fig. 8: **Variation of thermal Rayleigh number for various values of magnetic parameter,** ࢇࡴ **in oscillatory convection**

Figure 8 shows the influence of Magnetic field parameter,  $Ha$  on the thermal Rayleigh number,  $Ra$ for fixed values of  $Va = 50$ ,  $Ri = 2$  for oscillatory convection. It is evident that increase in Magnetic field increase the thermal Rayleigh number, which is an indication of the stabilization of the system.

## **5 Conclusion**

The effect of Vadasz number on magnetoconvection in a Darcy porous layer with concentration based internal heating has been studied analytically using the linear stability analysis. The porous layer is heated and salted from below. The roles of the governing parameters on the stability of the system was investigated. The result show that, the presence of the internal heat parameter,  $Ri$  and solutal Rayleigh number, Rs is to destabilize the system for both stationary and oscillatory modes. On the other hand, increase in the values of the magnetic parameter,  $Ha$  and vadasz number,  $Va$ stabilizes the system. Whereas, positive increase in the Lewis number stabilizes the system only for the stationary state and destabilize for the oscillatory state.

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