Adaptive Control of A Four-Dimensional Hyperchaotic System

Original Research Article

Abstract

This paper concerns the application of adaptive control method in a four-dimensional hyperchaotic system. Firstly, we carry out a systematic dynamic analysis including the properties of equilibrium point, stability, dissipation, Lyapunov exponent spectrum, and bifurcation. Both the existence of two positive Lyapunov exponents and the Lyapunov dimension value show the hyperchaotic property of the system. Based on Lyapunov stability theorem, we then construct an adaptive controller and the adaptive law to suppress hyperchaos to the origin, which is an unstable equilibrium point under a certain parameter set. The effectiveness of the adaptive control is verified by theoretical analysis and numerical simulation. We finally briefly demonstrate the control efficiency of self-linear feedback control and misaligned feedback control. For the four-dimensional hyperchaotic system, the adaptive control outperforms them from the view of control speed.

Keywords: Adaptive control; Control speed; Dynamic analysis; Hyperchaotic system; Misaligned feedback control; Self-linear feedback control

1 Introduction

Chaos is a complex phenomenon possessing characteristics of sensitivity on initial conditions, intrinsic randomness, and random order [1]. It may be undesirable in many applications of biological, physical, and engineering due to these sensitive features [2]. Theoretically, chaos is identified in the sense of Lyapunov when it's largest Lyapunov exponent is positive. A more complex phenomenon, called hyperchaos, occurs if there are more than one positive Lyapunov exponents in a system [3]. The concept of hyperchaos was first proposed by Rössler [4] who constructed a four-dimensional hyper Rössler system with computer numerical simulation. Compared with chaotic systems, a hyperchaotic system presents more complex dynamic behaviors and is yet technically more challenging [5].

In the past four decades, many hyperchaotic systems have been discovered in the fields of secure communications, image encryption, cryptography, optical, chemical, physical, biological systems, and neural networks [6, 7, 8]. Many hyperchaotic systems were constructed by modifying an already existed three-dimensional chaotic system. A continuous autonomous hyperchaotic system without equilibrium points was constructed by adding a feedback controller to a 3D autonomous chaotic Lorenz-type system [9]. Bonyah [10] proposed a new hyperchaotic system with four wings by changing the non-local and non-singular fractional derivatives. A novel 5D hyperchaotic system with infinitely many heteroclinic orbits was found through replicating and mutating the famous Shimizu-Morioka system [11]. Based on the method of system variable expansion, Sun et al. [12] constructed a new 4D hyperchaotic system.

Research on chaos and hyperchaos control has attracted considerable attention with the increase of hyperchaotic systems. A variety of chaos control methods have been proposed, such as impulsive control method, optimal control, adaptive control, linear feedback control, sliding mode control method, misaligned linear feedback control, and nonlinear hyperbolic function feedback control. In the Lorenz hyperchaotic system, Zhu [13] designed a linear feedback controller to control the hyperchaos of system to an unstable equilibrium point. The robust control with input nonlinear Rössler system was achieved via sliding mode control method [14]. Chen et al. [15] obtained some new and weak conservative conditions for projective and lag synchronization through an impulsive control method. According to optimal control method and adaptive control method, Effati et al. [16] achieved the chaos control of a 3D autonomous chaotic system and a 4D hyperchaotic system. Zhuang [17] confirmed the system's hyperchaotic attractors are controlled to unstable equilibrium point by using single state feedback control, misaligned linear feedback control, and nonlinear hyperbolic function feedback control, and the feedback coefficients in various control methods are compared.

Among the above mentioned control methods, adaptive control is prevalent in the literature owing to it's fast control speed and the capability of dealing with unknown parameters [18]. Wang et al. [19] designed an event-triggered adaptive backstepping controller in the attitude tracking of spacecraft. Their research results show that the controller could significantly reduce the communication burden and provided a stable and accurate response for attitude maneuvers. A new type of adaptive control that can well instruct the human arm model to imitate the arrived movements was found by Wang et al. [20]. According to an adaptive controller based on bio-inspired, Molina et al. [21] achieved highly efficient speed regulation of the DC motor under uncertain parameters. Teles and Lemos [22] designed a cancer therapy to eradicate metastatic renal cell carcinoma through adaptive control, the robustness in stability and performance is verified by numerical simulation. Palis [23] showed that the non-identifier-based adaptive control can control the nonlinear oscillations in the particle size distribution. Although the applications on adaptive control have been involved in many fields, the transient performance in adaptive control systems has remained a tricky issue. A large transient error could occur at the start of a control process and especially when there are a time-varying parameter [24].

Following the general stream of constructing hyperchaos, this paper proposes a 4D system by adding a state variable to the 3D Resource-Economy-Pollution system [25]. Let $x, y, z,$ and w represent the state variables. The dimensionless 4D system is as follows

$$\begin{cases} \dot{x} = a_1 x + a_2 y - a_3 yz - a_4 w, \\ \dot{y} = b_1 x(1 - x/M) - b_2 y - b_3 z + b_4 w, \\ \dot{z} = c_1 xy - c_2 z - c_3 w, \\ \dot{w} = dw(z - N), \end{cases} \tag{1.1}$$

where the dot '·' expresses the derivative with respect to time, parameter families a, b, and c together with d, M, N are positive. The number of equilibrium points depends on the values of parameters. However, the origin $O(0,0,0,0)$ is always an equilibrium point. This paper adopts adaptive control to suppress the hyperchaotic system to the origin which is an unstable equilibrium point. Numerical simulations will be used to show the effectiveness of the adaptive control. A comparison with other control methods will be applied to verify the advantage of adaptive control.

The outlines of this paper are organized as follows. The dynamic analysis of the hyperchaotic system is proposed in section 2. Section 3 provided the theoretical analysis of adaptive control. The numerical simulation and comparison analysis are given in section 4. In the last section, this paper draws the conclusions.

2 A hyperchaotic system and it's dynamic analysis

This section will show that the system (1.1) is hyperchaotic via analyzing the dynamic behaviors such as bifurcation diagram, Lyapunov exponents spectrum, dissipative analysis, equilibrium point analysis, max Lyapunov exponent, and Lyapunov dimension.

Given a set of parameters

$$\begin{array}{l} a_1 = 0.065, a_2 = 0.035, a_3 = 0.065, a_4 = 0.026, \\ b_1 = 0.6, b_2 = 0.088, b_3 = 0.07, b_4 = 0.066, \\ c_1 = 0.468, c_2 = 0.071, c_3 = 0.816, d = 0.035, \\ M = 6.6, N = 0.45. \end{array} \tag{2.1}$$

and initial value $[0.196, 0.36, 0.88, 0.29]$, the following dynamic analysis will be performed under the parameters and the initial value.

2.1 Bifurcation diagram and Lyapunov exponents spectrum

This subsection mainly proves the existence of hyperchaos by bifurcation diagram and Lyapunov exponents spectrum.

For one thing, the existence of chaos is tested by bifurcation diagram. The bifurcation diagram as shown in Fig. 1. There is an abrupt bifurcation occurs in the system about $c_2 = 0.122$. The chaos would appear at these parameters where it's value is smaller than c_2.

For another, the existence of hyperchaos is verified by Lyapunov exponents spectrum. Fig. 2 shows the Lyapunov exponents spectrum of the system. The system might be hyperchaos at specific parameters where the two of maximum Lyapunov exponents are positive.

2.2 Dissipative analysis

The dissipation of the system (1.1) will be discussed in the subsection. The system has a dissipative structure, which is a necessary condition for the generation of chaos.

The divergence of the system (1.1) is

$$\nabla V = \frac{1}{V}\frac{dV}{dt} = \frac{\partial \dot{x}}{\partial x} + \frac{\partial \dot{y}}{\partial y} + \frac{\partial \dot{z}}{\partial z} + \frac{\partial \dot{w}}{\partial w} = a_1 - b_2 - c_2 + d(z - N). \tag{2.2}$$

According to the parameters selected by Eq.(2.1), the divergence can be simplified to

$$\nabla V = 0.035z - 0.10375. \tag{2.3}$$

When $\nabla V < 0$, i.e., $0.035z < 0.10375$, the system (1.1) has been dissipated and convergence with the exponent $\frac{dV}{dt} = e^{0.035z - 0.10375}$. In pace with $t \to \infty$, each volume element containing the system trajectory converges to zero with $0.035z - 0.10375$. All system trajectories will be limited to a finite subset with zero volume, and it's asymptotic dynamic behaviors are fixed to an attractor, which further certificates the existence of attractors [26].

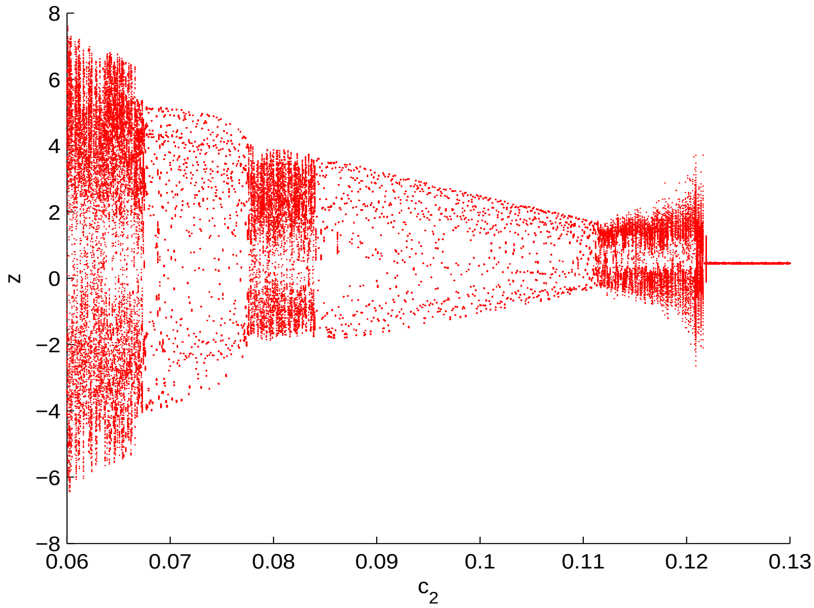

Figure 1: Bifurcation diagram of z with parameter c_2.

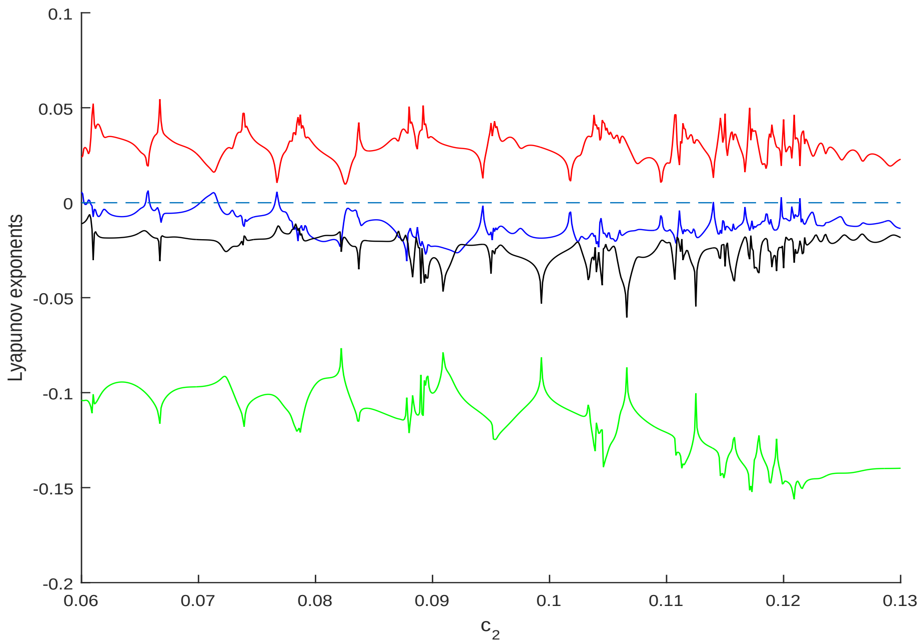

Figure 2: Lyapunov exponents spectrum.

2.3 Equilibrium point analysis

This subsection will analyze the equilibrium point of the system (1.1). The equilibrium point is an unstable saddle point is another necessary condition for the existence of chaos.

Let the four equations in system (1.1) are all equal to zero, we can get the equilibrium points of the system. Specifically, the origin $O(0,0,0,0)$ is always an equilibrium point. Below we only discuss the stability of the system at the equilibrium point $O(0,0,0,0)$.

The Jacobian matrix of system (1.1) at point $O(0,0,0,0)$ is

$$J=\begin{pmatrix} a_1 & a_2 & 0 & -a_4 \\ b_1 & -b_2 & -b_3 & b_4 \\ 0 & 0 & -c_2 & -c_3 \\ 0 & 0 & 0 & -dN \end{pmatrix}. \tag{2.4}$$

The characteristic equation is

$$(\lambda^2+(b_2-a_1)\lambda-a_1b_2-a_2b_1)(\lambda+c_2)(\lambda+dN)=0. \tag{2.5}$$

Under the parameters of Eq.(2.1), the four eigenvalues of the Eq.(2.5) are $\lambda_1=0.1524, \lambda_2=-0.1745, \lambda_3=-0.071, \lambda_4=-0.01575$, where λ_1 is a positive real number, $\lambda_2, \lambda_3,$ and λ_4 are a negative real number. By Routh-Hurwitz criterion, the equilibrium point $O(0,0,0,0)$ is an unstable saddle point, which might lead to chaos [27].

2.4 Hyperchaotic attractor

The hyperchaos will be proved by Lyapunov exponents and Lyapunov dimension under specific parameters in this subsection. The Lyapunov dimension between three and four is the sufficient condition to the existence of hyperchaotic attractor [28].

Under the set of parameters (2.1) and the initial value $[0.196, 0.36, 0.88, 0.29]$, system (1.1) has Lyapunov exponents $LE_1=0.0179, LE_2=0.0042, LE_3=-0.0197, LE_4=-0.0959$ by the Jacobian method [29]. There are two positive Lyapunov exponents and the sum of all Lyapunov exponents is negative.

The Lyapunov dimension is

$$D_L=3+\frac{LE_1+LE_2+LE_3}{|LE_4|}=3.025, \tag{2.6}$$

indicating that the system (1.1) is in hyperchaos.

The corresponding hyperchaotic attractor is shown in Fig. 3. The hyperchaotic attractor is a new attractor compared with the previous hyperchaotic attractors such as R$\ddot{o}$sser hyperchaotic attractor, Lorenz hyperchaotic attractor, and Chen hyperchaotic attractor. It has different hyperchaotic behaviors based on the above analysis.

3 Adaptive control

This section designs adaptive controller and adaptive law under unknown parameters. The theoretical analysis indicates that the hyperchaotic system can be controlled to the unstable equilibrium point by the adaptive control which based on the Lyapunov stability theory.

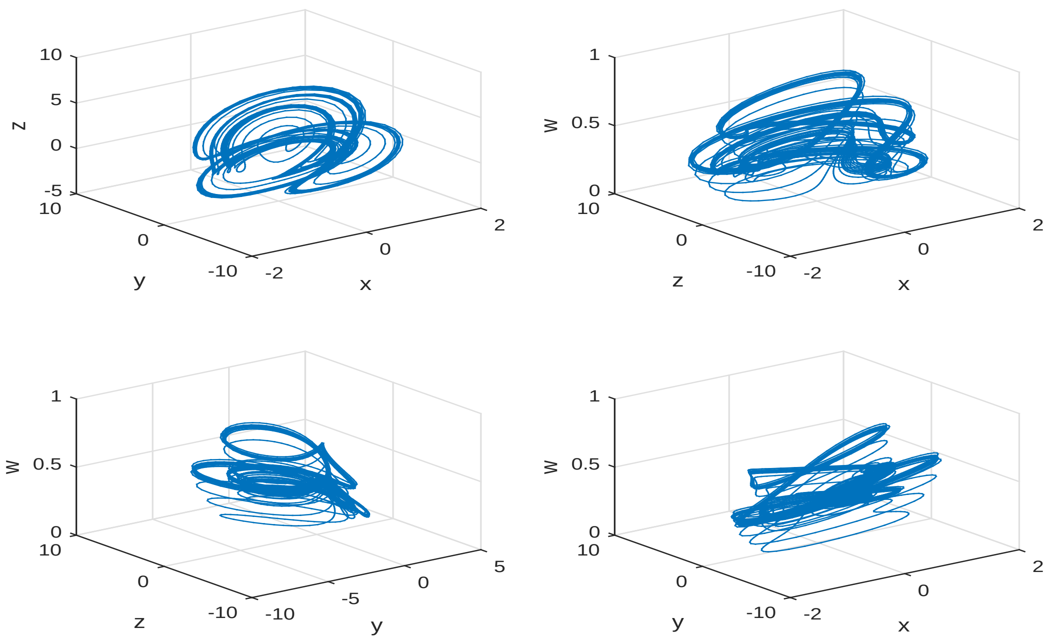

Figure 3: The 3-D view of the hyperchaotic attractor.

Traditionally, the controlled system can be obtained by introducing controller to the hyperchaotic system. The controlled system is

$$\begin{cases} \dot{x} = a_1x + a_2y - a_3yz - a_4w + u_1, \\ \dot{y} = b_1x(1 - x/M) - b_2y - b_3z + b_4w + u_2, \\ \dot{z} = c_1xy - c_2z - c_3w + u_3, \\ \dot{w} = dw(z - N) + u_4, \end{cases} \tag{3.1}$$

where u_1, u_2, u_3, u_4 are controllers.

The adaptive control aims to find the suitable adaptive controller and adaptive law to make hyperchaotic system (1.1) asymptotically converge to the unstable equilibrium point $O(0, 0, 0, 0)$. The adaptive controller is a reasonable control signal which can make a hyperchaotic system to a stable state. The adaptive law means the updated law of the estimate parameters [30].

Theorem 3.1. *The controlled system* (3.1) *is asymptotically converges to the unstable equilibrium point* $O(0, 0, 0, 0)$ *if the adaptive controller is set as*

$$\begin{cases} u_1 = -(\hat{A} + 1)x - 0.035y + 0.065yz + 0.026w, \\ u_2 = -0.6x(1 - x/6.6) + (\hat{B} - 1)y + 0.07z - 0.066w, \\ u_3 = -0.468xy + (\hat{C} - 1)z + 0.816w, \\ u_4 = (0.45\hat{D} - 1)w - \hat{D}wz, \end{cases} \tag{3.2}$$

and the adaptive law is set as

$$\begin{cases} \dot{\tilde{A}} = -x^2 - \tilde{A}, \\ \dot{\tilde{B}} = y^2 - \tilde{B}, \\ \dot{\tilde{C}} = z^2 - \tilde{C}, \\ \dot{\tilde{D}} = (0.45 - z)w^2 - \tilde{D}, \end{cases} \tag{3.3}$$

where $\tilde{A} = A - \hat{A}, \tilde{B} = B - \hat{B}, \tilde{C} = C - \hat{C}, \tilde{D} = D - \hat{D}$*;* $\dot{\tilde{A}}, \dot{\tilde{B}}, \dot{\tilde{C}}$*, and* $\dot{\tilde{D}}$ *are their derivatives;* $\hat{A}, \hat{B}, \hat{C}$*, and* $\hat{D}$ *are the estimated values of the unknown constants* A, B, C*, and* D *respectively.*

Proof. We first define the unknown parameter set. We pick up 10 parameters out of the 14 parameters. The fixed parameters are

$$\begin{array}{l} a_2 = 0.035, a_3 = 0.065, a_4 = 0.026, \\ b_1 = 0.6, b_3 = 0.07, b_4 = 0.066, \\ c_1 = 0.468, c_3 = 0.816, M = 6.6, N = 0.45. \end{array} \tag{3.4}$$

Assuming a_1, b_2, c_2, d are unknown. For clarify, we rename a_1, b_2, c_2, d to A, B, C, D.

Substituting the adaptive controller (3.2) and the fixed parameters (3.4) into the controlled system (3.1), we can get the following system

$$\begin{cases} \dot{x} = Ax - (\hat{A} + 1)x, \\ \dot{y} = -By + (\hat{B} - 1)y, \\ \dot{z} = -Cz + (\hat{C} - 1)z, \\ \dot{w} = Dw(z - 0.45) + (0.45\hat{D} - 1)w - \hat{D}wz. \end{cases} \tag{3.5}$$

We then define the Lyapunov function

$$V(x, y, z, w, \tilde{A}, \tilde{B}, \tilde{C}, \tilde{D}) = \frac{1}{2}x^2 + \frac{1}{2}y^2 + \frac{1}{2}z^2 + \frac{1}{2}w^2 + \frac{1}{2}\tilde{A}^2 + \frac{1}{2}\tilde{B}^2 + \frac{1}{2}\tilde{C}^2 + \frac{1}{2}\tilde{D}^2. \tag{3.6}$$

Taking the derivative of the Lyapunov function over time, we can get

$$\begin{aligned} \dot{V} &= x\dot{x} + y\dot{y} + z\dot{z} + w\dot{w} + \tilde{A}\dot{\tilde{A}} + \tilde{B}\dot{\tilde{B}} + \tilde{C}\dot{\tilde{C}} + \tilde{D}\dot{\tilde{D}} \\ &= x(Ax - (\hat{A} + 1)x) + y(-By + (\hat{B} - 1)y) + z(-Cz + (\hat{C} - 1)z) \\ &\quad + w(Dw(z - 0.45) + (0.45\hat{D} - 1)w - \hat{D}wz) + \tilde{A}\dot{\tilde{A}} + \tilde{B}\dot{\tilde{B}} + \tilde{C}\dot{\tilde{C}} + \tilde{D}\dot{\tilde{D}} \end{aligned} \tag{3.7}$$

Finally, substituting the adaptive law (3.3) into Eq.(3.8), the derivative of the Lyapunov function can be written as

$$\dot{V} = -x^2 - y^2 - z^2 - w^2 - \tilde{A}^2 - \tilde{B}^2 - \tilde{C}^2 - \tilde{D}^2 \tag{3.8}$$

We will show that the derivative of the Lyapunov function is negative when the Lyapunov function is positive. Therefore, the equilibrium solution of the controlled system (3.5) is asymptotically stable based on the Lyapunov stability theory, i.e., the controlled system (3.1) can asymptotically converge to the unstable equilibrium point $O(0, 0, 0, 0)$ with the adaptive controller (3.2) and the adaptive law (3.3). The proof is completed. □

In order to make a comparison, this paper also introduces self-linear feedback control and misaligned feedback control. The differences among the three proposed methods are the form of the controller and the condition of parameters. From the point of the form of the controller, adaptive controller may be nonlinear. Self-linear feedback controller is the function of the coefficient and the state variables. Misaligned feedback controller is the function of the coefficient and other state variable [4]. Adaptive control is under unknown parameters while the parameters of self-linear feedback control and misaligned feedback control are known.

We present two corollaries about the validation of the self-linear feedback control and the misaligned feedback control. Theoretical proofs are omitted for the length of the paper.

Corollary 3.2. *Under the hyperchaotic parameters* (2.1)*, the controlled system* (3.1) *is asymptotically stable to the unstable equilibrium point* $O(0,0,0,0)$ *by the self-linear feedback control with controllers* $u_1 = kx, u_2 = u_3 = u_4 = 0$*, where* $k > 0.3036$*.*

Corollary 3.3. *Under the hyperchaotic parameters* (2.1)*, the controlled system* (3.1) *is asymptotically stable to the unstable equilibrium point* $O(0,0,0,0)$ *by the misaligned feedback control with controllers* $u_1 = ky, u_2 = u_3 = u_4 = 0$*, where* $k > 0.0445$*.*

4 Numerical simulation

In this section, we give the numerical simulation results to verify the validity of the proposed control methods and make a comparison among their results. The simulation results are all obtained under the chaotic parameters (2.1) and the given initial value $[0.196, 0.36, 0.88, 0.29]$.

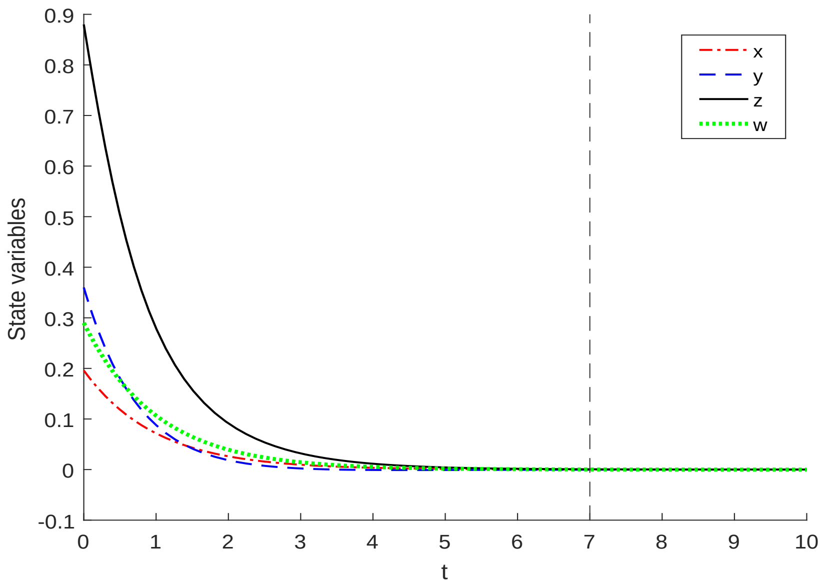

Figure 4: Evolution of state variables under adaptive control.

The trajectory of state variables is as shown in Fig. 4 when the unknown parameters are $a = 0.065, b = 0.088, c = 0.071, d = 0.035$, We find that $x, y, z,$ and w all monotonous decline until they tend to zero. And these state variables tend to the equilibrium point $O(0,0,0,0)$ about $t = 7$. It can be seen that the state variables asymptotically converge in a fast manner to the unstable equilibrium point $O(0,0,0,0)$ under the action of the adaptive controller.

Fig. 5 shows the estimating process of the unknown parameters under the adaptive control method, where the left vertical axis represents the estimated parameters and the right vertical indicates the convergent parameters. All the estimated parameters tend to their given values. But patterns of

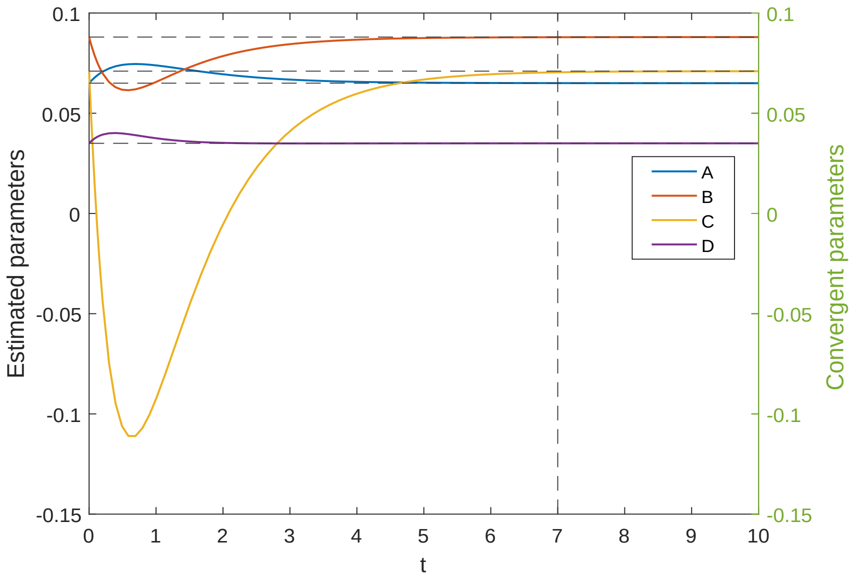

Figure 5: Estimating process of unknown parameters under adaptive control.

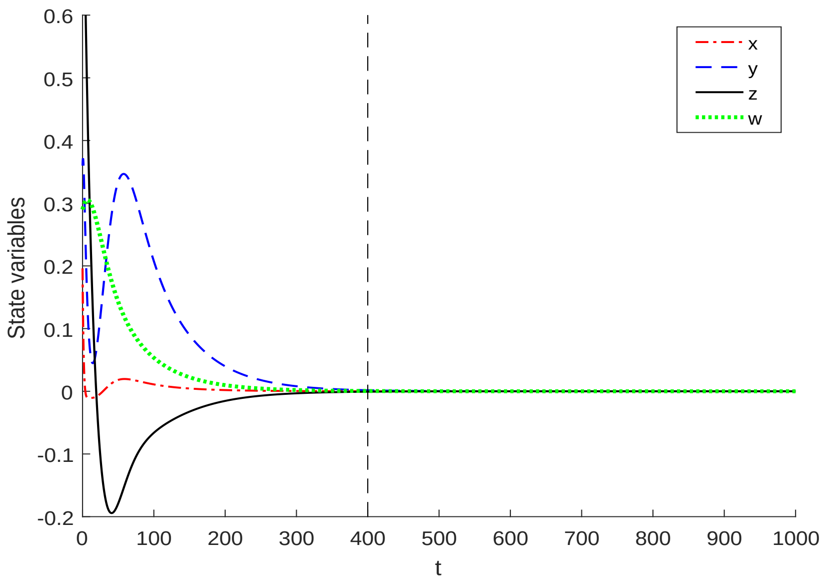

Figure 6: Evolution of state variables under self-linear feedback control.

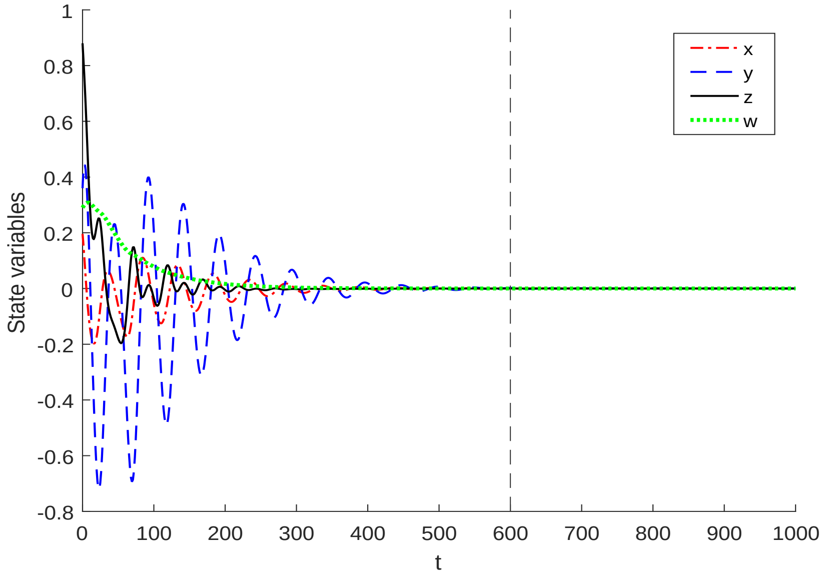

Figure 7: Evolution of state variables under misaligned feedback control.

the relative position between parameters are different. After one oscillation, A and D are always greater than their corresponding given value, while B and C are smaller. In addition, C converges with a significant large drop in the beginning. This indicates the effectiveness of adaptive control.

Fig. 6 indicates the evolution of state variables under self-linear feedback control with $k = 0.7$. In order to make fig. 6 more beautiful, we truncate the vertical axis to 0.6. We can see that, x, y, and z all drop while w goes upward before a low-frequent fluctuation. All the variables tend to zero approximately $t = 400$. This means the state variables converge to the equilibrium point $O(0, 0, 0, 0)$ under self-linear feedback control.

Fig. 7 denotes the evolution of the state variables under misaligned feedback control when $k = 0.07$. We find that at the beginning, both x and z fluctuate downward, while both y and w fluctuate upward. The state variables approach to the equilibrium point $O(0, 0, 0, 0)$ at $t = 600$ after dramatic fluctuations. This demonstrates the hyperchaotic system can be controlled to the equilibrium point $O(0, 0, 0, 0)$ under the misaligned feedback control.

We point out that the adaptive control has the fastest convergent speed among the three proposed methods. It takes only seven units of time for the state variables to converge under the adaptive control, which is merely 7/400, 7/600 of that under self-linear feedback control and misaligned feedback control. The reason lies in the convergence pattern. Under the adaptive control, the state variables convergence to the equilibrium point $O(0, 0, 0, 0)$ in a monotonous decreasing manner. The fluctuating convergence pattern in the other two control method take up much of time.

5 Conclusions

Adaptive control has been effectively applied to a four-dimensional hyperchaotic system in this study. Originated from a three-dimensional chaotic system, the hyperchaotic system we built has complex behaviors, such as hyperchaotic attractor, bifurcation diagram, and Lyapunov exponents spectrum.

This hyperchaotic attractor is controlled to the unstable equilibrium point $O(0, 0, 0, 0)$ through the adaptive control method. The validity of the adaptive control method is verified by theoretical analysis and numerical simulations, where Matlab simulations demonstrate the effectiveness of controller more intuitive. The theoretical analysis and numerical simulation in this study show the following conclusions:

1. The four-dimensional system is hyperchaotic because it has two positive Lyapunov exponents and the Lyapunov dimension is between three and four.
2. The origin is always an unstable equilibrium point.
3. The adaptive control is obviously superior to the other two methods in the control speed. However, the design of adaptive controller is also relatively complicated.
4. The self-linear feedback controller has a simple structure and a general control effect.
5. Although the misaligned feedback control can also achieve the control effect, it is the slowest in the control speed.
6. The choice of control method depends on the actual situation.

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