Interactive learning and innovation: conceptual and mathematical models. A study of the Venetian district

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ABSTRACT

Aims: explaining how social, cognitive and personal proximities influence interactive learning and innovation in an industrial district.
Study design: drawing on a conceptual development, six proposals are presented and explored in an empirical study based in the Venetian district. On the basis of these proposals, a mathematical model for knowledge transfer and innovation is developed.
Results: a qualitative study of the Venetian glassmaking district shows how interactive learning in an industrial district occurs on both horizontal and vertical dimensions, along which proximities play different roles. Both horizontal and vertical learning takes place through social, cognitive and personal proximities. More precisely, it is demonstrated that knowledge of mathematical law, on both horizontal and vertical dimensions, are extensions of existing knowledge which can be found in the nevertheless scarce managerial literature on this subject.
Conclusion: this study contributes to the literature on proximity within industrial districts by highlighting the role of personal proximity, which has hitherto been largely unexplored. This paper also considers the coevolution between dimensions of proximity and provides empirical evidence of two mechanisms of coevolution: a compensation mechanism between social and cognitive proximity, and a substitution mechanism between personal proximity, and cognitive and social proximities. The elaboration of a mathematical model drawn from a qualitative conceptual model is rarely found in the existing literature. A theoretical expression of the equation of knowledge dynamics is also presented.

Keywords: network, equations of knowledge dynamics, interactive learning, innovation, proximity mathematical modeling, knowledge transfer mathematical modelling, glass district

1. INTRODUCTION

The present research is aligned with the current perspective of interdisciplinary research which simultaneously mobilizes management research and mathematical modeling regarding knowledge transfer laws [1], [2], [3], [4]. In this research, the laws of knowledge transfer are supposed to be a linear function of cognitive distance. Existing literature presents computational simulations in order to explore the renewal of knowledge within company networks. Nevertheless, no attempt is made to justify such knowledge transfer using empirical studies. Therefore, the present work focuses on this aspect.

This research is an extension of previous work exploring the influence of the proximity of workers in an industrial district on interactive learning and innovation**.** The district has consequently been conceptualized as a place which promotes interactive learning because of the local interactions and proximity of the actors therein [5], [6]. Thus, the interaction between different workers in the district occurs through interpersonal networks and interorganizational networks [7]. The dynamics of the network in industrial districts has led to the development of a theoretical stream which questions the role of proximity between workers in interactive learning and innovation within industrial districts. A growing stream of research proposes five types of proximity which can influence actors within industrial districts: geographical, cognitive, social, organizational and institutional [8], [9]. Recently, researchers underlined another aspect of proximity: personal proximity [10]. Considering the lack of theoretical sources, the roles of three dimensions of proximity are explored herein: social, cognitive and personal. The choice to focus on these three dimensions is driven by the critical role played by workers in an industrial district.

As De Clercq *et al*. argue, in an industrial district, individual behavior drives the development of organizational exchanges, and therefore tends to become the focus of facilitating conditions, particularly for learning and innovation. [11]. Furthermore, whereas social and cognitive proximities have been widely explored in existing theoretical contributions, personal proximity has often been equated with the dimension of social proximity.

The aim of the present research is thus to enrich the conceptual approaches of the different dimensions of proximity within industrial districts using mathematical modeling. The mathematical model is developed from empirical studies undertaken in the Murano industrial district. [12]. The assumptions of the mathematical model are simply the mathematical counterpart of the conceptual proposal.

The mathematical formulation shows that social and cognitive proximities are not independent: if one is known, a mathematical relation can provide the value of the other. The law of horizontal knowledge transfer is in fact standard, and the law of vertical knowledge transfer generalizes those laws anchored in managerial theoretical contributions, since it depends on personal proximity. Nevertheless, from simple mathematical argument, it is demonstrated how laws of knowledge transfer are a decreasing function of cognitive proximity. The law of vertical knowledge transfer is also presented as a necessary and increasing function of personal proximity.

In the literature review section here below, using a conceptual analysis of managerial theoretical contributions, six proposals about the effect of the three dimensions of proximity (social, cognitive and personal) and their interaction on interactive learning and innovation are presented. In the following section, an empirical study of the Murano industrial district is presented. Thus, a mathematical model for knowledge transfer applied to the Murano industrial district is proposed. Based on these findings, theoretical and managerial implications are discussed in the penultimate section, with a final section concluding the paper.

2. INTERACTIVE LEARNING AND INNOVATION IN INDUSTRIAL DISTRICT

Lundvall *et al.* defined interactive learning as ‘a process in which agents communicate and even cooperate in the creation and utilization of new economically useful knowledge’ [13](p. 226). Van de Ven and Polley suggest that, in a context marked by uncertainty, the innovative enterprise must develop its ability to adapt and learn while innovating. [14]. Thus, innovation refers to ‘the development and implementation of new ideas by people who over time engage in transaction with others within an institutional order’ [15]p. 591). The vertical dimension consists of co-located companies which are linked through input/output relations. In this dimension, knowledge is exchanged through market transactions between buyers and suppliers throughout the value chain, with little or no interactive learning taking place [16]. Through the horizontal dimension, the company population in an industrial district can be divided into homogenous groups, each composed of companies which share the same output combinations (a common product/market/technology).

3. PROXIMITY DIMENSIONS IN INDUSTRIAL DISTRICTS

The following three types of proximity are examined in relation to interactive learning and innovation in industrial districts: social, cognitive and personal.

3.1. Social proximity, interactive learning and innovation in industrial districts

Molina-Morales and Martinez-Fernandez highlight the role played by the dimensions of social capital, i.e., social interactions, trust, shared vision and involvement of local institutions, in the process and product innovation of companies inside an industrial district. [17]. According to this perspective, as argued by Caniëls [10](p. 234), social proximity is ‘a closeness between workers regarding their informal rules, hence referring to factors such as a common language and shared habits’. In his conceptualization of social proximity, Boschma [8] demonstrates the importance of the social community and of a shared history in building trust and reducing opportunism in social transactions. Thus, when knowledge is incorporated in a social context, it becomes even more specific, more difficult to imitate and thus more valuable for learning and innovation. The distinction between tacit and explicit knowledge is central to the debate on the role of proximity in learning and innovation. Cohendet and Meyer-Krahmer [18] highlight the fact that explicit knowledge is knowledge which is easy to identify, articulate, store, and transmit via formal means such as databases and other records. Tacit knowledge is implicit and more difficult to formalize, communicate, and store and thus tacit knowledge often emerges when engaging in direct experience. Indeed, internal individual processes, such as experience and talent, generate this tacit knowledge which is so difficult to code. It is generated through the implicit and non-codifiable accumulation of skills, which results from learning through the practical execution of tasks and requires face-to-face interaction. Tacit knowledge can play an important role in the initial stages of the innovation processes of companies [19]. Wineman *et al*. [20] affirms that social proximity encourages the development of structured communication, collaboration, access to knowledge, and knowledge transformation. The above theoretical developments lead to the formulation of the following proposal:

Proposal 1: Social proximity promotes interactive learning within industrial districts.

3.2. Cognitive proximity, interactive learning and innovation in industrial districts

Cognitive proximity refers to the degree to which the content of two workers’ knowledge bases and expertise overlap and is a means for reaching external knowledge sources [21], [22]. From this perspective, Hautala highlight how cognitive proximity (i.e. similar knowledge bases) is essential in creating knowledge when two workers have different professional and cultural backgrounds and is achieved through cooperation and suitable tasks. [23]. Thus, Huber, and Vom Stein and Sick, defines the following dimensions of cognitive proximity within the industrial district: a common technical language, a similar way of thinking about a technology or product, similar work-related technical details/facts, and similar work-related know-how (how to do things or to solve a problem). [24].

Petruzzelli *et al*. highlight how the effectiveness of external learning processes is positively influenced by cognitive proximity, consisting of a common knowledge base and expertise. [21], [22]. Thus, collaborators require similar but not necessarily identical knowledge bases to communicate and transfer new knowledge effectively. However, proximity may work differently in an area where the cognitive distance is relatively large (i.e. workers with very different skills) compared to a relatively low cognitive distance. Indeed, overly weak cognitive proximity increases the difference between workers' cognitive schemas, and thus diminishes their capacity to identify, interpret and exploit the knowledge possessed by other workers. [9]. However, overly high cognitive proximity is not conducive to innovation. In fact, the success of innovation depends not only on the generation of new ideas through access to diverse types and resources of knowledge, but also on the capacity to absorb external knowledge [9]. Starting with the Schumpeterian concept that innovation is the recombination of knowledge and ideas of entrepreneurs, this study proposes that industrial districts with diversified knowledge resources foster more innovation than do those with specialized knowledge. [25]. The above theoretical developments lead to the formulation of the following proposal:

Proposal 2: Low cognitive proximity between actors is more conducive to interactive learning and innovation within industrial districts.

3.3. Personal proximity, interactive learning and innovation in industrial districts

Research on the influence of personal proximity on interactive learning and innovation in industrial districts is still very limited. Personal proximity results from personal acquaintances and refers to a mutual feeling of acceptance, appreciation and interest in each other's ideas. [26]. Thus, the research of Werker *et al*. highlights how personal proximity facilitates collaborations and offers networking opportunities. [27]. Consequently, the low variability of personal characteristics of workers such as age, sex and seniority, as well as personality traits such as extraversion, openness, sympathy and awareness, increase the willingness of individuals to share knowledge and information. [28]. In these conditions, the performance of collaborations can be improved. For example, Werker *et al*. highlight how the similarity in age of individuals facilitates informal, non-technical communication within a network. [27]. Moreover, the similarity of personal characteristics makes it possible to collaborate under more pleasant conditions. Under thse circumstances, the willingness of collaborators to mutually share information facilitates interactive learning is the most important factor. Thus, Canïels *et al*. call this type of similarity 'homophilia', which stimulates learning and innovation for two main reasons. [10]. On the one hand, homophilia entails shared personal characteristics which facilitate communication between actors, and, on the other hand, workers who interact with others who are similar to them are also likely to find these interactions more enjoyable, which can promote professional cooperation.

In the same way, recent work on innovation networks emphasizes that personal proximity can create lasting relationships and improve communication by facilitating the interpretation of knowledge because of the existence of shared personal characteristics. [27]. In addition, emotional proximity promotes trust and allows individuals to better predict the behavior of their peers [29]. Hence the following proposal:

Proposal 3: Personal proximity promotes interactive learning and innovation within industrial districts.

3.4. The coevolution of proximity dimensions

According to Broekel, proximities can be interrelated in various ways. [29]. Firstly, they can be complementary; when two workers are close on one dimension, they are also likely to be close on another dimension of proximity. Secondly, these can also be substituted whereby the proximity on one dimension compensates for a lack of proximity on another dimension. Boschma emphasizes a mutual influence between social proximity and cognitive proximity by illustrating how social proximity decreases the heterogeneity of knowledge in an industrial district. [8]. Thus, the more the relations between the companies located within an industrial district are socially integrated, the more these companies will share common knowledge. In contrast, Giuliani and Bell report that collaborators in a local district who share a common language and a technical background will consult other collaborators in the same district and thereby develop networking practices which promote the spontaneous formation of a social community. [30]. Hence the following proposal:

Proposal 4: This research proposes the existence of a complementarity between social proximity and cognitive proximity within an industrial district.

In addition, Huber's research shows a substitution effect between cognitive proximity and personal proximity. [24]. The results of the present research argue that cognitively distant relationships require higher levels of personal proximity to actually work. Thus, emotional bonds reduce the tensions which arise due to differences in understanding and facilitate cooperation to integrate different sources of knowledge. In contrast, Maskell states that when companies share a common language for interpreting local knowledge, close personal contact is not necessary for learning: in these conditions, cognitive proximity compensates for the absence of personal proximity. [31]. Hence the following proposal:

Proposal 5: A substitution effect exists between personal proximity and cognitive proximity within an industrial district.

Cassi and Plunket argue that sharing a common personal relationship increases the possibility of forming a dyad between indirectly related collaborators. [32]. As a result, open triads tend to close over time as collaborators connect with their partners. This tendency to become acquainted with a colleague's friend increases social proximity as personal proximities are increasingly embedded in a growing network of mutual knowledge. [33]. Hence the following proposal:

Proposal 6: There exists a substitution effect between personal and social proximities.

4. EMPIRICAL STUDY: MURANO INDUSTRIAL DISTRICT

The new artistic and productive projects launched by artists/designers, coupled with practical knowledge of manufacturing processes and the know-how of master glassmakers and glassmakers constitute the fundamental elements of the added-value produced by the companies in the industrial district of Murano. Accordingly, this research distinguishes two types of interactions between workers within Murano as the main vectors of learning in this industrial district: horizontal interactions between glassmakers and master glassmakers, and vertical interactions between master glassmakers and artists/designers.

4.1. Interactive learning and innovation in Murano

At the level of horizontal relations, the learning process in Murano essentially equates to 'learning by doing', which has enabled glass masters to transmit their incremental knowledge to the next generations of glass makers. From the mid-twentieth century, the arrival of artists/designers in Murano provided the foundation for interactive learning on the vertical level of relationships which can be observed today between artists/designers and glass masters. The artists/designers of Murano provide the drawings for the new artistic project, and then follow the steps to complete the project provided to them by a master glassmaker, which is fundamental for the process of product-innovation in Murano.

4.2. The role of proximity in interactive learning and innovation in Murano

4.2.1. The influence of social proximity

The circulation of workers between the glassworks is central to the close ties between glassmakers and has thus promoted the development of a social community as well as the accumulation of strong social capital. This type of proximity is a means of transmitting the tacit knowledge needed for learning the craft. In addition, the study shows that the social proximity between artists/designers and glassmakers/master glassmakers (vertical level relationships) is low. Given these results, Proposal 1 is only supported for horizontal relationships.

4.2.2. The influence of cognitive proximity

On the horizontal dimension of relations, the cognitive proximity between glassmakers and master glassmakers is strong. Indeed, they use the same technology, the same know-how and the same technical language. This strong cognitive proximity promotes the transfer of knowledge between workers but represents a barrier for innovation.

Currently, the processes of creativity and innovation are generated within the links between glassmakers and artists/designers and in the low cognitive proximity which results from the different areas of expertise between these two categories of workers. It is precisely because of their low cognitive proximity that the creative communities of Murano artists and designers have been working to introduce new ways of thinking and processes which enable master glassmakers to adopt innovative practices. Thus, Proposal 2 is asserted only for vertical relations.

4.2.3. The influence of personal proximity

In terms of personal relationships, the results of the Murano district study highlight the influence of generational differences and emotional proximity in the process of learning and innovation. Indeed, the generational differences between glassmakers and master glassmakers can account for the lack of personal proximity which is reflected in their behavioral patterns. The interviews show that this low personal proximity does not prevent the transmission of glassmakers’ knowledge to younger glassmakers. This poor personal proximity does, however, represent a barrier to jointly developing innovation processes.

However, artists/designers have a strong influence on the way master glassmakers work, which leads to close personal proximity. In contrast to the above then, the personal proximity between glassmakers and artists/designers drives a significant amount of knowledge transfer and innovations.

Thus, Proposal 3 is affirmed for vertical relationships only.

4.3. Interaction of proximity dimensions

The results of this research show a reciprocity effect between social proximity and cognitive proximity on interactive learning and innovation processes within a regional industrial district. On the horizontal level of interactions, Murano district workers share a common base of knowledge and expertise and live within the same social community, which demonstrates that they have successfully communicated and understood the knowledge which is transferred between them. On the vertical level of relations, the study of the district of Murano shows that low social proximity co-occurs with low cognitive proximity. This situation stimulates cross-learning, which in turn leads to the generation of new ideas and thus initiates the process of innovation within vertical relationships. Proposal 4 is therefore affirmed for both vertical and horizontal relationships.

The results of this research also indicate a substitution effect between the personal and cognitive dimensions of proximity. Indeed, the weak cognitive proximity which exists between master glassmakers and artists/designers is offset by a strong personal proximity between these workers. The opposite phenomenon occurs in horizontal relationships between glassmakers and master glassmakers. In this situation, the low personal proximity between the glassmakers and the glassmakers is offset by their strong cognitive proximity.

Proposal 5 is thus validated for both vertical and horizontal relationships.

Finally, the study of the Murano cluster reveals a substitution effect between social and personal dimensions of proximity. Indeed, a strong social proximity between glassmakers and master glassmakers contrasts with a low personal proximity between these workers. In contrast, the low social proximity between master glassmakers and artists/designers is offset by a strong personal proximity between them.

Proposal 6 is consequently supported for both vertical and horizontal relationships.

5. MATHEMATICAL MODEL FOR KNOWLEDGE TRANSFER AND INNOVATION

5.1. Mathematical counterpart of conceptual Proposals 1 to 5.

Each conceptual variable is associated with a real positive variable.
From conceptual Proposals 1 to 3, mathematical assumptions 1 to 3 can be presented with confidence.
Assumption 1: Knowledge transfer (Kt) and innovation (I) are functions of social proximity (sp).
Assumption 2: Knowledge transfer and innovation are functions of cognitive proximity (cp).
Assumption 3: Knowledge transfer and innovation are functions of personal proximity (pp).
In fact, from the definition of the complementary of two proximities xp and yp, it can easily be seen that there exists a function F, such that F(xp, yp) = 0 and xp and yp increase simultaneously,
Moreover, if F is sufficiently regular, by applying the implicit function theorem it can be seen that xp (respectively yp) is an increasing function of yp (respectively xp).
So, from Proposal 4, it can be stated that: Assumption 4: Social proximity is an increasing function (f_1) of cognitive proximity (the reverse is obviously also true).
The mathematical treatment of Proposals 5 and 6 are different for horizontal and vertical relations, so arguments regarding these are directly presented in section 4.

5.2. Network structure

It is important to define the network of glassmakers, master glassmakers and designers. Let G = {1, …. , g} denote a finite set of glassmakers and master glassmakers, and D ={1, … , d} a finite set of artists/designers.
For any $(i, j) \in G \times G \cup G \times D$ define the binary variable $\chi(i, j)$ to take value $\chi(i, j)$= 1 if a connection exists between *i* and *j* and $\chi(i, j)$= 0 otherwise. The horizontal neighborhood of each glassmaker *i* is the set of glassmakers, such

that $\Gamma_{Gi} = \left\{ j \in G : \chi(i,j) = 1 \right\}$. The vertical neighborhood of each designer *i* is the set of master glassmakers, such that $\Gamma_{Di} = \left\{ j \in G : \chi(i,j) = 1 \right\}$.

5.3. Modeling knowledge transfer

Each agent (glassmakers, master glassmakers and designers) is characterized by a knowledge endowment which develops over time as the agent innovates and receives knowledge from other agents. For the sake of simplicity, let $k_{\alpha i}(t)$ ($k_{\alpha i}(t) \in [0,1]$, $\alpha = 1$ for $i \in G$ and $\alpha = 2$ for $i \in D$) denotes agent *i*'s knowledge endowment at time *t*, modeled as a discrete parameter: t = 0, 1, . . ., *i*,. . . [1], [3], [4]. This is a simplification because knowledge $k_{1i}(t)\,(i \in G)$ should be characterized by a knowledge vector, the dimension of this vector being the number of different aspects of knowledge needed by a glassmaker or master glassmaker for making glasses [1]. In the same way, knowledge $k_{2i}(t)\,(i \in D)$ should be characterized by a knowledge vector whose dimension represents the number of different aspects of knowledge the designer should transmit to the master glassmaker to enable them to make new types of glasses. Negative knowledge is not considered here, wherein validated statements are false [4]. From the conceptual model, the following constraints must be added:

(C1): $k_{2i}(0) = 0 \text{ for } i \in G$ (initially, master glassmakers do not control the designer's knowledge).

(C2): $k_{1i}(t) = 0 \text{ for } i \in D$ (the designer cannot at any time acquire the master glassmaker's knowledge).

5.4. Transfer of knowledge and innovation

For the Murano cluster, innovation can be defined as being the conversion of vertical knowledge transfer into the new design of glasses and the conversion of horizontal knowledge transfer into new manufacturing processes of glasses. As is implicitly assumed in the conceptual model, knowledge creation by agent i is not considered for any time interval [t, t + 1]. During each time interval [t, t + 1] the transfer of knowledge to agent i occurs only with one other agent belonging to his neighborhood [3], [4]. In this way, the reduction of efficiency and reliability associated with multi-tasking is avoided [34]. However, in the other direction, an agent may have more than one apprentice at the same time [1], [35]. At each time step [t, t + 1], agent i selects an agent j who belongs to his neighborhood. The selection rule may be either deterministic or random [36], with uniform [1] or non-uniform [3], [4] random selection probability.
For subsequent development, it is recalled that any distance *xd* is the mathematical inverse of the corresponding proximity *xp* i.e.

$$xd = \frac{1}{xp} \tag{1}$$

[37]p. 1021.

<u>5.4.1. Horizontal relations</u>

From the conceptual results of sections 3.1 and the mathematical analysis of section 4.1, only Assumptions 1, 4 and Proposals 5 and 6 are supported. From Assumption 1, it is easy to see that horizontal knowledge transfer from agent $i \in G$ to agent $j \in G$ (*i* ≠ *j*) during the time interval [t, t + 1] is given by:

$$Tk_{1ij}(t) = \left[\!\left[k_{1i}(t) - k_{1j}(t) > 0 \right]\!\right] f\left(sp_{ij}(t)\right). \tag{2}$$

in which $sp_{ij}(t)$ is social proximity between agents *i* and *j*, and where $[\![Q]\!]$ is the inversion bracket [38] converting Boolean values to numbers 0, 1: $[\![Q]\!]$=1 if *Q* is true, $[\![Q]\!]$= 0 if *Q* is false and finally *f* is a positive function.

The specific bracket $\left[\!\left[k_{1i}(t) - k_{1j}(t) > 0 \right]\!\right]$ implies that knowledge transfer may take place if and only if $k_{1i}(t) - k_{1j}(t) > 0$. Finally, for obvious reasons, the following must be true: $Tk_{1ij}(t) < k_{1i}(t) - k_{1j}(t)$.

From assumption 4, social proximity can be expressed in terms of cognitive proximity:

$$sp_{ij}(t) = f_1(cp_{ij}(t)). \tag{3}$$

in which f_1 is an increasing function, and $cp_{ij}(t)$ is cognitive proximity between agents *i* and *j.*

By considering formula (3), horizontal knowledge transfer (2) can be rewritten in the following form:

$$Tk_{1ij}(t) = \left[\!\left[k_{1i}(t) - k_{1j}(t) > 0 \right]\!\right] f\left(f_1(cp_{ij}(t))\right). \tag{4}$$

And the transfer of knowledge between agents *i* and *j* is a function of cognitive proximity between agent *i* and *j.*

At each time step $cp_{ij}(t)$ increases (because a transfer of knowledge occurs between agent i and j) so the transfer of knowledge between agent *i* and *j* must decrease with cognitive proximity and vanishes theoretically when $cp_{ij}(t)$ becomes infinite (i.e. $cd_{ij}(t)$=0). Thus, *f* must be a decreasing function.

For horizontal knowledge transfer, the substitution effect between both personal and cognitive proximities means that weak personal proximity is counteracted by strong cognitive proximity (Proposal 5). So therefore, personal proximity does not intervene in the rule of horizontal knowledge transfer (4) (strong cognitive proximity is sufficient to ensure horizontal knowledge transfer). For horizontal knowledge transfer, Proposal 6 shows that there is a substitution effect between both social and personal proximities. Strong social proximity between glassmakers and master glassmakers is compensated by weak personal proximity. Evidently, the rule of horizontal transfer of knowledge (2) only depends on social proximity, and Proposal 6 follows this.

The rule of knowledge transfer decreases with cognitive proximity and is thus conventional. Indeed, the usual rule of knowledge transfer commonly used in theoretical contributions [1], [2], [3], [4] is a linear function of cognitive distance and so decreases with cognitive proximity.

The knowledge of glassmaker j at t+1 is $k_{1j}(t+1) = k_{1j}(t) + Tk_{1ij}(t)$ and the cognitive distance between glassmaker *i* and glassmaker *j* at t+1 becomes $cd_{ij}(t+1) = k_{1i}(t) - k_{1j}(t+1)$ and finally relation (1) shows that the new cognitive proximity is $cp_{ij}(t+1) = \dfrac{1}{cd_{ij}(t+1)}$. Horizontal knowledge transfer does not involve innovation.

5.4.2. Vertical relations

From the conceptual results of sections 3.1 and the mathematical analysis of section 4.1, only Assumptions 2, 3, 4 and Proposals 5 and 6 are supported.
By considering assumptions 2 and 3, it can be seen that vertical knowledge transfer from designer $i \in D$ to master glassmaker $j \in G$ during the time interval [t, t + 1] is given by:

$$Tk_{2ij}(t) = \left[\!\left[k_{2i}(t) - k_{2j}(t) > 0 \right]\!\right] f_3\left(pp_{ij}(t), cp_{ij}(t)\right), \tag{5}$$

in which f_3 is a positive function defined on the set of couple (x, y) such that x and y are strictly positive and $pp_{ij}(t)$ is the personal proximity between agent *i* and *j*.
Since assumption 4 is valid, relation (3) is also valid for vertical knowledge transfer and may be, for example, chosen in the following form:

$$sp_{ij} = k\, cp_{ij}^{\gamma}$$

in which *k* and γ are positive constants.
Since assumption 4 is supported, the rule of vertical knowledge transfer (5) can be expressed in terms of both social and personal proximities.
It is equally important to consider the substitution effect between both cognitive and personal proximities (Proposal 5). For the rule of vertical knowledge transfer (5), the weakness of cognitive proximity between glassmakers/master glassmakers and artists/designers is compensated by strong personal proximity between these workers. For parsimony, formula (5) presents a decoupled effect between both personal and cognitive proximities in the following form:

$$f_3\left(pp_{ij}(t), cp_{ij}(t)\right) = f_4\left(pp_{ij}(t)\right) f_5\left(cp_{ij}(t)\right), \tag{6}$$

in which f_4 and f_5 are positive functions and, given the argument presented for horizontal knowledge transfer, f_5 must be a decreasing function.
Proposal 5 would mean that the strength of personal proximity enables vertical knowledge transfer. To obtain this function f_4 must be increasing and $f_4(0) = 0$, for example $f_4(pp_{ij}) = l\, pp_{ij}^{\beta}$ (*l > 0*, $\beta > 0$). Obviously formulae (5) and (6) show that strength of personal proximity enables vertical knowledge transfer (conversely if personal proximity diminishes so too does vertical knowledge transfer).
Proposal 6 states that there exists a substitution effect between both social and personal proximities. For the rule of vertical knowledge transfer (5), weak social proximity between master glassmakers and artists/designers is neutralized by their strong personal proximity. This can easily be seen by inserting the relationship in its inverse form ($cp_{ij}(t) = f_1^{-1}(sp_{ij}(t))$) in the rule of vertical knowledge transfer (5) ($f_3\left(pp_{ij}(t), cp_{ij}(t)\right) = f_4\left(pp_{ij}(t)\right) f_5\left(f_1^{-1}(sp_{ij}(t))\right)$, thus personal proximity is necessary for vertical knowledge transfer.
For the vertical knowledge transfer formula (5), a choice can be made (taking relation (1) into account) between cognitive proximity and cognitive distance:

$$f_5\left(cp_{ij}\right) = \frac{1}{f_5\left(\left(pp_{ij}\right)_{\max}\right) cp_{ij}} = \frac{cd_{ij}}{f_5\left(\left(pp_{ij}\right)_{\max}\right)}, \tag{7}$$

in which $\left(pp_{ij}\right)_{\max}$ is the maximum value of pp_{ij}.

By combining formulae (5), (6) and (7), the rule of vertical knowledge transfer can be presented in the following form:

$$Tk_{2ij}(t) = [\![k_{2i}(t) - k_{2j}(t) > 0]\!] \, w_{2ij}\left(pp_{ij}(t)\right) cd_{ij}(t), \qquad (8)$$

in which $w_{2ij}\left(pp_{ij}(t)\right) = \dfrac{f_5\left(pp_{ij}(t)\right)}{f_5\left(\left(pp_{ij}\right)_{\max}\right)}$.

Then version (8) of vertical knowledge transfer is the product of the knowledge distance between the agents and a weight $w_{2ij} \in [0, 1]$, this rule of knowledge transfer generalizes common knowledge transfer models [1], [2], [3], [4] since here w_{2ij} is not constant and depends on personal proximity.

The knowledge of master glassmaker j at t+1 is $k_{2j}(t+1) = k_{2j}(t) + Tk_{2ij}(t)$ and the cognitive distance between designer *i* and glassmaker *j* at t+1 becomes $cd_{ij}(t+1) = k_{2i}(t) - k_{2j}(t+1)$ and the new cognitive proximity is $cp_{ij}(t+1) = \dfrac{1}{cd_{ij}(t+1)}$.

Finally, vertical knowledge transfer involves innovation at time t+1

$$I_{ij}(t+1) = f_2\left(Tk_{2ij}(t)\right).$$

6. DISCUSSION

This research examined the effect of three dimensions of proximity, i.e. social, cognitive and personal, on interactive learning and innovation and their interactions within a specific Italian industrial district. A theoretical conceptual model was presented and tested in an empirical study of this Murano cluster, then a mathematical model for knowledge transfer was developed from this which shows that vertical transfer of knowledge is not standard and horizontal transfer of knowledge is standard.

6.1. The role of proximity in interactive learning and innovation: horizontal versus vertical relationships

This research shows that worker interactions are a powerful tool for knowledge transfer in industrial districts, both horizontally and vertically. On the one hand, on a horizontal level, effective transfer is linked to a strong social and cognitive proximity replacing personal proximity. On the other hand, on a vertical level, interactions are characterized by a weak social and cognitive proximity and a strong personal proximity. Vertical relationships are the source of innovation in the industrial district. This is an original result in that previous theoretical contributions do not distinguish the influence of proximity by the types of vertical and horizontal interactions.

Knowledge is created when actors within industrial districts become cognitively close through their knowledge base content, but workers develop a cognitive distance through a knowledge base structure. [23]. Thus, in the case of the Murano district, relations between master glassmakers and designers have stimulated the renewal of the glass industry. Furthermore, this study shows that personal proximity plays a pivotal role in the success of vertical learning, in that it acts as a substitute for social and cognitive proximity. The absence of personal proximity on a horizontal level hindered innovation.

The results of this research support the importance of the relationship between the different trades of workers within a regional industrial district for interactive learning and innovation.

The Murano glass industry district study supports several findings from previous research [39], [6] which demonstrate how too much social and cognitive proximity is a barrier to learning and innovation within districts.

6.2. Interactive learning and the interaction of dimensions of proximity

In line with the research which points to interaction between the different dimensions of proximity between workers within regional industrial districts [27], this study highlights a complementarity effect between the social and cognitive dimensions of proximity. Thus, strong social and cognitive proximities induce learning in horizontal interactions, whereas low proximity in these dimensions promotes learning in vertical interactions. This research also illustrates a substitution effect between personal proximity and the social and cognitive dimensions of proximity.

The results of this research corroborate the research of Huber who argues that low cognitive proximity is associated with strong personal proximity. [24]. Thus, if workers are emotionally distant, a high level of cognitive proximity will support the relationship. On the one hand, for vertical relations, where cognitive distance increases and social proximity is absent, actors may need to rely on other forms of proximity, such as friendships and knowledge sharing. As such, workers in close personal relationships tend to feel compelled to provide help and are concerned about the personal well-being of their relationships, regardless of the level of social and cognitive proximity. On the other hand, the opposite phenomenon occurs in horizontal relationships. When interpersonal connections are non-existent, interactive learning and innovation depend on a strong cognitive and social proximity. In keeping with the findings of Caniëls *et al*. these results provide an additional argument for separate consideration of personal and social proximity [10].

6.3. Conclusion

This research arguably makes several significant contributions. Firstly, the qualitative research empirically demonstrates the individual and joint influences of the different dimensions of proximity, interactive learning, and innovation within a specific regional industrial district. Existing empirical research on this topic is currently relatively limited. The proximity dimensions examined by this research include personal proximity, a dimension largely neglected in previous theoretical contributions. However, personal relationships, such as the friendship between workers of different companies, are of course present in regional industrial districts [40]. This study empirically confirms the important influence of personal proximity for learning and innovation in industrial districts.

The second contribution of this study is that it affirms the merits of a dynamic approach to the analysis of the influence of proximity in regional industrial districts. This study also shows that, while some dimensions of proximity appear to complement each other, other proximity dimensions act as substitutes for influencing interactive learning and innovation within an industrial district.

Thirdly, the unique contribution made by this research is the formulation of a mathematical model of knowledge transfer which is completely aligned with the conceptual model. The rule of horizontal knowledge transfer is demonstrated to be standard, and the rule of vertical knowledge transfer is an extension of rules previously proposed in theoretical contributions. The novelty provided in this research is its affirmation that the rule of knowledge transfer also depends on personal proximities, as well as that vertical knowledge transfer must be an increasing function of personal proximity.

The results of this qualitative research have major implications for regional industrial districts whose survival depends heavily on innovation [41]. These results emphasize the fact that companies should manage interactions between actors according to the objectives pursued. As a result, regional industrial districts pursuing development objectives should focus their attention on vertical interactions (between different trades). The results of this research call for interactions which primarily concern workers with similar personal characteristics, close emotional ties, different areas of expertise, and who are not part of the same social community. However, regional industrial districts which prioritize the transmission of their core business to future generations should foster the horizontal interactions of workers who practice very similar trades. These workers should also benefit from strong socialization and the very similar expertise required to understand and improve their businesses. The results of this research also emphasize how strong cognitive and social similarities can mitigate weak personal similarities between collaborators.

However, this research has several limitations. First, the research is focused on a single Italian industrial district. Future research combining the analysis of several industrial districts could increase the generalizability of the results of this study. Secondly, as the theoretical contributions on the development of industrial districts explain, the links between workers tend to develop over time, creating different effects on learning and innovation, including the opposite effects. This could not be illustrated by the data from this study, given that it was collected over a limited amount of time. In order to capture the development of the Murano industrial district, additional data collection over further years may shed light on this issue. Also, in order to numerically simulate the dynamic model of knowledge transfer and innovation stated in section 5, i.e. at each time step, the knowledge of each agent develops due to knowledge transfer, of which the general mathematical expressions of both the rules of vertical and horizontal transfer are rigorously deduced from the conceptual model (cf. section 2 and 3).

In conclusion, this research identifies proximity between collaborators as a fundamental factor of innovation. Thus, the industrial district of Murano glass making appears as a relatively static industrial space which merits stimulation to facilitate knowledge sharing and innovation. The results reveal a lack of personal closeness between Murano's professional glassmakers as being one of the main obstacles to innovation. However, for the transfer of knowledge from the designer to the master glassmaker (vertical knowledge transfer), personal proximity is essential for the transfer of knowledge and innovation. Indeed, Proposal 5 states that low cognitive proximity is neutralized by close personal proximity.

COMPETING INTERESTS

This research received no specific grant from any funding agency in the public, commercial, or not-for-profit sectors. None conflict of interest declared.

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