Original Research Article

Perceptual judgments of logical propositions

Aims: This study is part of a series aiming to evaluate the computational complexity used by the human brain while ~~parceptually~~ perceptually judging a logic proposition.

Methods: In the present article we report a psychophysical study in which the hypothesis that efficiency and efficacy of these perceptual decisions depend on the proposition but not on the validating sensory stimuli was tested. Subjects had to judge whether a color stimulus verifies a proposition under a go/no-go protocol. Different protocols were used for the evaluation of the relative weight of proposition connectors on the latency, accuracy and precision of the responses**.**

Results and Discussion: Errors and latencies increase with the minimum description length of the proposition, but the relative weight of absences was double than the weight of presences (even when brief and single color stimuli ruled out visual search). However, values predicted by this rule are smaller than those found for conditionals and larger than those found for biconditionals and exclusive disjuntions. We postulate that the brain uses a “one and only one is valid” operator (which is equivalent to exclusive disjunction in dyadic statements) to deal with these propositions.

Conclusions: Decision difficulty (including within this term time and accuracy) depends on proposition structure. We provide a heuristic rule that predicts evaluation time better than previously proposed hypotheses.

1. INTRODUCTION

The rules of language-based logical decisions have been studied in formal logic from the classical Aristotle School to present. However, less attention has been paid to perceptual decisions in which the subject has to evaluate whether a proximal stimulus (e.g. the stimulus on the sensory system) verifies a distal stimulus (e.g. the object that causes sensory stimulation) abstractly defined by a logical proposition. These decisions can be achieved by a form of non-verbal thinking which supports of human perceptual creativity since it implies the construction of engrams from elemental percepts, allowing a subject to judge whether sensory signals match a still not experienced percepts. The focus of this arti address what are the possible rules matching the accuracy and rapidness of this mental ability.

Comment [c1]: Rephrase, not clear.

A first source of knowledge relevant to this problem is that formal logic language owes many ways to express the same proposition. Statements in logic language are called propositional formulas. These are built up from atoms (or elementary propositions which may have the value true or false) and connectors.

The logical operators negation (¬), conjunction (AND), and disjunction (OR) are often combined to represent and also to construct new human concepts from elemental ones [1] . This conceptual framework is called Boolean algebra. Even though this framework is the more often used by psychologists and computer scientists, several equivalent forms can be used to represent and construct concepts in an equivalent way. In fact, all possible truth tables can be expressed by combining conjunction (AND) and negation ("a OR b" can be expressed as "¬ [¬ a AND ¬b]"). Strikingly, even a single operator can be used to enunciate all binary propositions. Two of these singleton sets of connectors are Peirce's (Peirce's "ampheck" equivalent to "¬ [a OR b]", and Sheffer's connector (equivalent to "¬ [a AND b]" [2]).

Despite these multiple grammars are formally equivalent to the "machine language" used by the human brain to construct logic engrams (and also to judge whether sensory inputs validate them) it is yet unknown which (if any) human-created logic languages resemble the mechanisms implemented in the human brain. Nonetheless, as the efficiency and efficacy for making a logic decision depend on the performance of the neural circuitry subservient brain logic operations a first step to advance in this knowledge in to reduce the possible brain's logic grammars to those matching brain performances.

Two other areas of knowledge give background and are relevant for focusing our study: perceptual decision making and concept learning.

Perceptual evaluation of logically defined categories was a very active field in the middle of last century. Most studies focused on timing and accuracy of the brain to evaluate Boolean operators. It was shown that judging whether at least one out of two (a green figure OR a triangle) or out of three features (a green figure OR a triangle OR the letter "A") takes, respectively, two or three times longer than the time added to reaction time by the process of recognizing a single distinctive attribute [3]. Also, it was shown that the time taken to decide on the truth value of binary conjunctions, inclusive disjunctions and other binary propositions (exclusive disjunctions XOR, conditionals and biconditionals follows the order: inclusive disjunction = conjunction < conditional < biconditional; [4]). This is consistent with the classification in three levels of complexity previously proposed by Neisser and Weene [5]: at the simplest level are the concepts defined in terms of the value taken by a single variable (e.g., the presence or absence of an attribute); at the middle level are conjunctions, disjunctions, and conditionals; and at the most complex level are biconditionals and their negations, exclusive disjunctions.

More recently, visual search studies indicate that a) decision time increase with the number of items to be evaluated (so called "set size") and b) the slope of such increment is larger if the valence of the item is negative [6,7]. This difference between affirming and denying the target presence has been attributed to the time taken while the subject performs a visual scanning of the image until it rules out the presence of the distinctive feature [7]. However, experiments made with stimulus shorter than the reaction time for a saccade (e.g, the decision was taken the absence of ocular movements) showed that identifying the absence of a target also requires an extra time depending on the number of image locations in which the target could had been placed [6] or the number of items necessary to be "mentally" scanned to make the decision. However, some reports have shown that perceptual evaluation of a single feature takes longer for target absence than for its presence [8,9].

Another source of knowledge comes from the study of concept learning. A study suggests that learning Boolean concepts depend on its minimum propositional formula [10,11]. Under this approach the complexity of a concept was defined as the minimum number of values taken by the whole set of variables involved in a concept. Further studies have challenged this simple view. Information theory based studies stressed the role of regularities (as for example parity and invariance [11-15]. Moving forward this idea Vigo [15,16] introduces a Boolean differential operator allows to measure the degrees of invariance of the category with respect to its dimensions, and proposes that the structural complexity of a Boolean category (increasing with the quotient cardinality over invariance), determines the degree of learning difficulty.

Despite it is plausible to believe that concept learning and perceptual decision making share similar mechanisms, literature report important differences [17]: a) the difficulty introduced by the operation OR is larger than that introduced by AND during concept learning [18,19] but this is not the case in perceptual decision making [4]; b) the role of negated concepts on learning difficulty of dyadic propositions appears to be negligible [10,16] which contradicts experimental findings when using simple negation to increase the difficulty of a recognition test [9].

On a different approach, the theory of mental models highlights that given a set of primitive operators that includes XOR, minimal descriptions no longer predict difficulty in concept learning [20, 21]. Under the framework of mental models theory only positive models (i.e. those that match a concrete feature that distinguish a concept) can be identified [22]. Nonetheless, when this theory faces the decision making process instead of concept comprehension, it is difficult to explain the order of decision times obtained in early psychophysical studies [4,5], since it predicts a single model to be required validating propositions of the form [a AND b], two models for [a XOR b] and three models for [a OR b].

Despite the mentioned studies shed light on how the brain implements perceptual evaluation of logically defined engrams, taken together they ~~do not allowed~~ did not allow us to conceive a common, and integrative rule that explain brain performance.

In the search for such a proposition-dependent rule, here we explore efficacy and efficiency of perceptual decisions when an individual identifies a stimulus as belonging to a category defined by monadic or dyadic propositions. Our data is well fitted by a heuristic model, in which the response time depends on the structure of the proposition as a whole, and in which decision times increase 75 ms with every positive value and 150 ms with every negative value that must be evaluated. Additionally our data indicates that an "one and only one of these is valid" evolved to evaluate evenly-paired propositions can be used also for evaluating other propositions as conditionals.

2. MATERIAL AND METHODS

Experiments were performed by 36 healthy, right-handed, voluntary subjects (age range 16-55). According to the Helsinski declaration, participants were previously informed of the procedure and signed a written consent statement approved by the ethics committee of the Instituto de Investigaciones Biológicas Clemente Estable (Note Nº5, 2013).

2.1. Experimental conditions.

In all experiments, subjects had to use color information to assess the truth value of a proposition under a go (true) /no-go (false) random presentation protocol. Subjects were seated in a dim-lightened cabin looking at a computer screen (48 x 26 cm, 1680 x 1050 pixels, 60 Hz, maximum screen illumination) where a colored circle was displayed on a black background (Fig. 1A). In any experiment the number of target colors exceeded three on the previously reported basis that when the number of variables is equal or less than four, the size working memory is not a critical factor [23]. Three types of stimulus images covering either 7º, 3.5º, or 1º of the visual field were used. In two cases stimulus images consisted of three non-contiguous identical and homogeneously colored (either b=blue, y=yellow, g=green, v=violet, c=cyan and r=red) 60º circle sectors centered at clock dial positions 12, 4 and 8 separated by a randomly pixelated color pattern in which every mentioned color was equally present (neutral image, Figure 1B and C). In the other case a small circle homogeneously colored was briefly displayed at the center of the neutral image (Figure 1D).

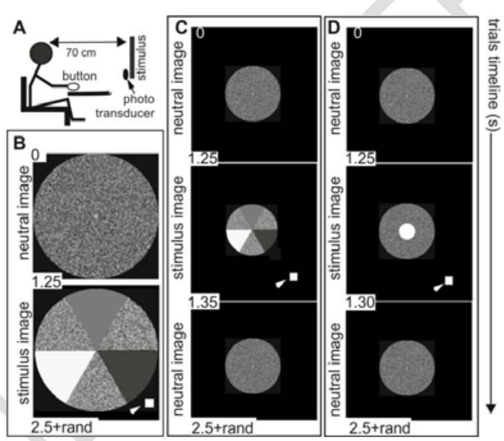

Figure 1. Experiment al design and stimulus types. A. Schematics representing the setup. Stimulus time were recorded by a photo-transducer located on a corner of the screen where a white spot was lightened during the stimulus image presentation (indicated by the arrowhead in B, C and D) Response time was recorded by directly connecting a mouse switch to the parallel port input. B. Image series during a trial in experiments 1 and 2. C. Image series during a trial in experiment 3. D. Image series during a trial in experiment 4.

Subjects had to fix their sight on a central spot, to respond as soon as possible by pressing a mouse button when the colors in homogeneous sectors of the stimulus image truly verified the proposition (e.g. "a blue sector is present", "a blue sector is not present") and should not respond in the opposite case. The experiments started after the subject performed several trials of practice until the consistence of the results fully convinced the experimenter that the subject had fully understood each task. Digital signals recorded from the stimulus screen using a photo-transducer and directly from a switch were fed into the parallel port of the computer where both of them were sampled using computer's clock at 30 kHz. Each experimental session consisted of at least 96 trials where target (e.g. verifying the proposition) and non-target (e.g. falsifying the proposition) stimulus images were presented with equal probability in a pseudorandom sequence. Trials started with a neutral image during 1.25 + 0.2 followed by a stimulus image during either 50 (small circle), 100 (brief sectors stimuli) or 1250 (long-lasting sectors stimuli) ms, depending on the experimental series, and were repeated at regular but not fixed intervals (2.5 + 0.2 ms). An in-house program was used to evaluate the number and latency of the true positive (hits) responses as well as the false-positive (false alarms) and also the number of false-negative (misses) and true-negative responses. With these data we calculated the accuracy and precision of the responses. Hits and false alarm latencies were defined as the median values of their trial latency distribution while each subject evaluated each proposition.

We performed 3 series of experiments in which the studied propositions and experimental protocols were varied in order to answer different questions. In the 11 subjects having 288 trials for the most difficult proposition (y XOR ¬b) we checked that the median latency values from the first 5 and last 5 hits were not significantly different (sign rank test P=0.63 N=12).

2.2. Experimental series 1.

The objectives of this experimental series (6 subjects, 4 females) were to rule out the effect of visual search on the extra time taken by negation. To avoid either overt or covert visual scanning, a single target color was presented in a small (1º) circle at the center of the neutral image during 50 ms. While the use of a single spot rules out mental scanning, the stimulus brevity rules out the possibility of ocular movements searching the image. Six experiments were run on each subject; each had a different target color (r, g, b, ¬r, ¬g, and ¬b). Two way Friedman ANOVA test was used to depict systematic differences in the latency of the responses depending either on the target color used or on the affirmed or negated version of the target color. Bonferroni-corrected signranks test was used for post-hoc analysis.

2.2. Experimental series 2.

The objectives of this series (3 male and 13 female subjects) was to test the relative weight of negation and number of variable values on the decision time and errors and whether the evaluation of different valid stimuli in each proposition yielded different decision times. The stimulus used was large (7º) and long-lasting (1.25 ms) as in Figure 1B. We explored the propositions “a”; “¬ a”; “y AND b”; “¬[a AND b]”; “y OR b”; “¬[y OR b]”; “y XOR b”; “y XOR (¬ b)” in the mentioned order. Median response time values from 96 trials per proposition for hits and false alarms and the percentage of errors for false alarms and misses were computed for each subject. Friedman tests were used to depict systematic differences in the latency of the response and percentage of errors (either misses or false alarms) among the group of propositions “b”, “b AND y”, “b OR y”, and the group of their negations. Friedman-test was also applied to the latencies to test the effect of the number of variable values. Bonferroni-corrected signranks test was used for post-hoc analysis. This test was also used to contrast “b XOR y” with “b XOR (¬ y)” (bi-conditional). Least mean squares curve fitting procedures followed by Pearson correlation was performed to test the hypothesis of linear dependence of the false alarms vs. hits latencies.

In 11 of the 16 subjects (3 males, 8 females) propositions “a”; “¬ a”; “y OR b”; “¬(y OR b)”; “y XOR b”; “y XOR (¬ b)” were explored in 3 sesions (288 trials per proposition) to check whether the evaluation time was dependent on the proposition as a whole or differentially on the stimuli validating the proposition. The rationale behind this experiment was that while serial computer machines yield different processing times for the same proposition depending on the validating stimuli, it is not known whether the brain operates using a stimulus dependent decision tree or a holistic evaluation of the proposition. Then, in this subset of experiments we separately measured median response times for each valid stimuli and proposition. A three-way ANOVA was performed to evaluate whether a) the explored propositions, b) the different stimulus combination, c) the subjects or d) their interaction had a significant effect on the latency of the responses. In addition, for each stimulus having 3 valid stimuli combination Kruskal-Wallis test followed by post-hoc Bonferroni’s-corrected signrank test were used to compare the subset of responses coming from the evaluation of different propositions.

2.3. Experimental series 3.

The objectives of this experimental series were a) to explore all possible binary propositions in a single population, and b) to rule out the effect of foveation in searching the image (overt search). To avoid the possibility that ocular movements participate in the visual search the stimulus was of small size (3.5 º of the visual field) and brief duration (100 ms). We explored all possible binary propositions using blue and yellow as target colors. Median response time values from 96 trials per proposition and the percentage of errors were computed for each of the 12 subjects (8 females) that participated in this series. Friedman test was used to depict systematic differences in the latency of the responses. Bonferroni-corrected signranks test was used for post-hoc analysis. This test was also used to contrast “b XOR y” with “b XOR (¬ y)” (bi-conditional) and also to contrast the “b AND (¬ y)” with “b XOR y” and also to contrast “b AND (¬ y)” with “b XOR y” and conditional (“b OR (¬ y)”) with bi-conditional (“b XOR (¬ y)”) propositions. Least mean squares curve fitting procedures followed by Pearson correlation was performed to test the hypothesis of linear dependence of the false alarms vs. hits latencies.

3. RESULTS AND DISCUSSION

Here we report the influence of various factors on the latency, accuracy and precision of humans visual decisions on the truth value of a given proposition involving colors. This report focuses only in monadic and dyadic propositions. For monadic propositions we compared recognition of a color with the recognition of the absence of such a color (Experiment 1, testing “a” is true vs “¬ a” is true). For dyadic propositions we explored how the time taken and the accuracy depends on the minimum number of variable values (including the result of “¬a” as one of the two values that the variable a can take) and the connectors. We found that i) decision times and accuracy obtained while evaluating the same stimuli were consistently different for different propositions. ii) number and sign (e.g. presence and absence) of variable values were both determinant of both decision time and accuracy and iii) the two alternative forced choice operation (“a XOR b”) might also be implemented independently of the operators OR, AND and NOT.

3.1. Judging presence is easier than judging absence

In the first experimental series we used a single colored small spot presented very briefly (50 ms) to rule out the influence of covert visual search on the difference between response times and accuracy. We found that all 6 subjects show a larger decision time for deciding the absence than for deciding the presence of the target color (Friedman ANOVA test for presence vs absence of the target: χ^2= 4.52, P=0.033, df =1). No differences were observed between colors (Friedman ANOVA test for effects of color χ^2= 0.99, P=0.61, df =2). Post hoc analysis showed significant paired differences (mean + SE presence vs absence in each case, N= 6; red: 399+30 vs. 453 + 39, P= 0.031; green: 418+38 vs. 475+46, P= 0.031; blue: 389+40 vs. 446+39, P= 0.031; Fig 2).

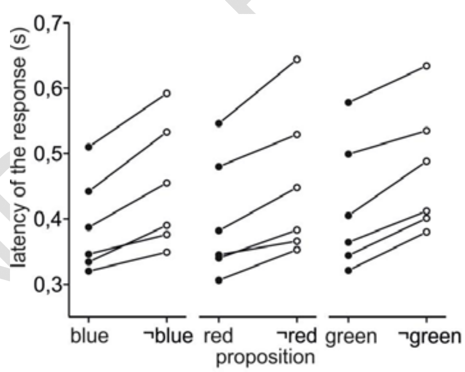

Figure 2. Denying takes longer than affirming. The use of a single colored spot presented briefly (50 ms) rules out that this result is due to either overt or covert visual search. A. Line series plots correspond to the latencies during the evaluation of blue vs not blue, red vs not red and green vs not green by the same six subjects. Two way Friedman ANOVA test showed a significant effect for presence vs absence of the target (χ2= 4.52, p=0.033, df =1) but no differences were observed between colors (χ2= 0.99, p=0.61, df =2).

In the other two series, similar experiments using different stimuli showed the same results. For large stimuli, affirming the presence of a blue sector took significantly shorter than denying it (462 + 91 ms, vs. 544 + 112 ms, mean and standard error respectively, sign rank test, P= 2.3*10-3, N=16). For small stimuli, affirming the presence of a blue sector took significantly shorter than denying it (462 + 91 ms, vs. 544 + 112 ms, mean and standard error respectively, sign rank test, P= 2.3*10-3, N=16, Fig 3, 4 and 5). Accuracy (hits plus correct rejections over total trials) and precision (hits over total positive responses) were also better for affirming than for denying in all experiments (sign rank test, P< 0.01 after Bonferroni's correction, Fig. 4 and 5).

Detecting a feature amongst a complex array of stimuli typically involves either ocular movements to scan the image (overt search), or an attentional scan (covert search) of the sensory image stored in a short-term memory without using eye movements [6,7, 23, 24, 25]. However our stimulus design (having in any case convergent sectors at the center of the circle where the sight is fixed) and the experimental series 1 and 3 performed using brief stimuli (shorter than the start timing of ocular saccades [6]) preclude overt scanning of the stimulus. Taking together the three experimental series in particular the first one using a single colored spot of 1º and 50 ms stimulus duration clearly shows that the extra time added while denying the presence of an object attribute does not depend on either overt or covert visual search.

3.2. Each proposition is evaluated in a holistic way.

The truth values of dyadic propositions depend on the identification of the stimuli belonging to the four possible that make them valid. As we used only two target variables ("blue" and "yellow"), the target stimulus images across different experiments had 4 possible color combinations "blue and not yellow sectors" and "yellow and not blue sectors", "blue and yellow sectors", and "neither blue nor yellow sectors". The 16 possible propositions depending of the independent presence of two colors in the stimulus can be reduced to 10 (excluding tautology, contradiction, and those in which one variable exchanged with the other results in an equivalent proposition). Among these 10 propositions 3 are validated by three stimuli ("b OR y", "b OR ¬y", "¬ [b AND y]"), 4 by two ("b", "¬b", "b XOR y", "b XOR¬y") and 3 ("b AND y", "b AND ¬y", "¬ [b OR y]") by only one. Thus, some propositions are validated by various types of stimuli each of which in turn is the only stimulus validating another proposition. For example, the biconditional proposition [b AND y] OR ¬[b OR y] is validated by stimuli showing blue and yellow sectors and by stimuli lacking blue and yellow sectors which considered separately validate "b AND y" and "¬[b OR y]", respectively. Therefore, we asked whether the latency of the response is either have the same or different for different sensory stimulus when validating the same proposition. To answer such question, we tested whether the timing and accuracy of the response was dependent on the proposition or on the color combination of the stimulus image, in 11 subjects (out of the 16 performing series 2) who evaluated six propositions ("b", "¬b", "b OR y", "¬[b OR y]","b XOR y" ,b XOR [¬ y]") in 3 sesions of 96 trials each (288 in total).

Our results ruled out a serial decision tree as used in serial computer programming. A 3-way ANOVA analysis for testing the effects of the five evaluated propositions (we excluded "¬[b OR y]" from the analysis because it has a single validating stimulus), the target set of validating stimulus and the subjects, indicates that the response time has a strong dependence on the proposition (F=84,7, $p<3{:}10^{-26}$; DF: 4) and subject (F=34.5, $p<3{:}10^{-23}$; DF: 10) but fails to show dependence on stimuli (F=2.0, P=0.14; DF: 3).

For each color stimulus combination we performed a separated analysis (we excluded "yellow but not blue sectors" because it was equivalent to "blue but not yellow sectors"). In the case of the three propositions validated by stimuli showing "blue but not yellow sectors" (e,g, "b"; "b OR y" and "b XOR y") the latency was different (Kruskal-Wallis test χ2=14.33, DF = 2, P=0.0008). Post hoc analyses showed that the latency order from shortest to longest is as follows: "b"< "b OR y" < "b XOR y" (signrank test after Bonferroni correction in every pairwise comparison, P= 0.003, N=11, Fig 3A).

In the case of the three propositions validated by stimuli showing "blue and yellow sectors" ("b", "b OR y", and "b XOR ¬y") the latency was different (Kruskal-Wallis test χ2=17.16, DF =2, P=0.0002). Post hoc analyses) showed that the latency order from shortest to longest is as follows: "b" < "b OR y" < "b XOR ¬y" (signrank test after Bonferroni correction in every pairwise comparison, P=0.0117, N=11, Fig 3B).

In the case of the three propositions validate by stimuli "lacking blue and lacking yellow sectors" ("¬b", "¬[b OR y]", and "b XOR ¬y"), the latency was different (Kruskal-Wallis test χ2=8.51, DF=2, P=0.0142). Post hoc analyses showed that the latency order from shortest to longest is as follows: "¬b"<"¬[b OR y]" <"b XOR ¬y" (signrank test after Bonferroni correction in every pairwise comparison, P=0.003, N=11, Fig 3 C) .

Comparisons between the time taken for decisions after different valid target stimuli when evaluating the same proposition did not show significant differences for simple recognition "b" is true (blue and not yellow stimuli vs. blue and yellow stimuli signrank test P= 0.1 N=11); “b OR y” (Kruskall Wallis among the effects of the three valid stimuli, $\chi 2$=2.11, P= 0.34 DF=2); and bi-conditional (blue and yellow vs. not blue and not yellow, signrank test P= 0.83 N=11).

Last, plotting the latencies of the decisions evoked by each validating stimuli vs. the global latency obtained for each proposition showed a tight fitting to the identity (r = 0.98, P= 8*10-9, N=12, Fig 3D).

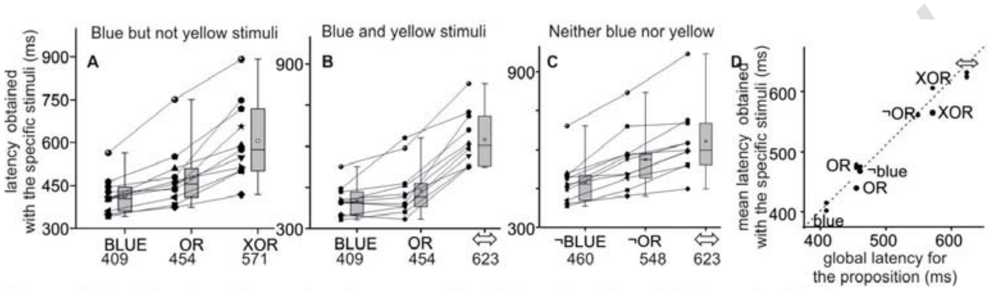

Figure 3. Decision time depend on the proposition but not on the validating stimulus images.
A) Comparison between decision times corresponding to the three propositions in which the blue but not yellow stimuli was a valid target. Note that for all subjects the latency of the response to the same stimuli follows the same order the same order as the mean latency for each proposition across the population (Kruskal-Wallis test χ^2=14.33, DF = 2, p=0.0008, post-hoc analyses: signrank test after Bonferroni correction in every pairwise comparison, P= 0.0029, N=10. B) Comparison between decision times corresponding to the three propositions in which the blue and yellow stimuli was a valid target. Note that in most subjects the latency of the response to the same stimuli follows the same order as the mean latency across the populations (Kruskal-Wallis test χ^2=17.16, DF =2, P=0.0002) post-hoc analyses: signrank test after Bonferroni correction in every pairwise comparison, P=0.0117, N=10). C) Comparison between decision times corresponding to the three propositions in which the neither blue not yellow stimuli was a valid target. Note that in most subjects the latency of the response to the same stimuli follows the same order as the mean latency across the populations (Kruskal-Wallis test χ^2=8.51, DF=2, P=0.0142, post hoc analyses signrank test after Bonferroni correction in every pairwise comparison, P=0.0029, N=10,). D) Plot of the latency observed with a specific valid stimuli vs. the global latency observed for the proposition (r=0.98 P=8*10^{-9} N=12).

The difficulty expressed by the latency, accuracy and precision do not depend on the number of target color combinations potentially validating each proposition (which is 3 in “y OR b”, “y OR ¬b”, “¬y OR b”, and “¬y OR ¬b”; 2 in “y XOR¬b” and “y XOR b”, and one in the rest) since the latency and accuracy order is not predicted by this variable. The same argument is true for the validating images. Furthermore, the timing of the response to the same validating color combination in the stimulus varies with the proposition, and all valid stimuli for a proposition take the same time to evoke the response. Therefore, differences in efficiency and efficacy of perceptual judgments must depend on the logic complexity of the evaluated proposition and not just correspond to image recall and servant matching.

3.3. Role of number of items, valence and connectors.

In experimental series 2 we compared 4 propositions having exclusively positive values (“b”, “b OR y”, “b AND y”, and “b XOR y”) with their negations including the same number of items but also negative valences (“¬b”, “¬[b OR y]”, “¬[b AND y], “b XOR y” vs.“b XOR [¬ y]”. (Fig. 4). A two way non-parametric ANOVA analysis (4x2 with repetitions according to the number of subjects) indicated a main effect of the proposition type (Friedman test for rows: $\chi 2$=26.2, P=8.5*10^{-6}, df =3) and also a main effect of the positive vs. negative forms (Friedman test for columns: $\chi 2$=8.53, P=0.0035, df=1).

For dyadic operations, (i.e. AND, OR) the latency was significantly shorter than for their negated forms (signrank test, P values calculated after Bonferroni’s correction are shown in Fig 4A, N=16). The extra time added by negation was very similar in both cases (“b AND y”: 560 + 36 ms; “b OR y”: 557 + 37 ms, “[¬b] OR [¬y]”=“¬[b AND y]”: 682 + 45 ms;

"[¬b] AND [¬y]" = "¬[b OR y]": 663 + 48 ms). As the processing time added by negation in all three cases (simple affirmation, AND, and OR) was non-significantly different (Kruskall-Wallis, $\chi2$ =2.68, P=0.26), this suggests that either negating one ("b" vs. "¬blue" and "blue AND yellow" vs. "¬blue AND ¬ yellow" or three mental models ("blue OR yellow" vs. "¬blue OR ¬yellow]" OR "¬blue AND ¬yellow") adds a similar extra time regardless of the number of target images required to verify it.

When comparing the forced single choice (XOR) and bi-conditional propositions we found significant differences in the latency of the response ("b XOR y": 705 + 118 ms, vs. "b XOR [¬y]": 752 + 198 ms; signrank test P= 0.032, N=16, p calculated after Bonferroni's correction) and a larger number of errors when evaluating bi-conditional.

Regarding response correctness, we tested the effect of connector (none, OR, AND, XOR) and valence (present, absent) on two parameters: accuracy (e.g. number of correct responses over the total of trials), and precision (hits over total positive responses). While the last evaluate the decay in correctness by false alarms, the first includes false alarms and misses as sources of errors. Friedman test showed significant main effects of the proposition connector (χ^2=29.9, P=1.4*10^{-6}, df =3) and valence (χ^2=, P=8.9*10^{-7}, df=1) on the accuracy of the responses. A similar analysis for precision yield only a significant main effect of proposition connector (proposition: χ^2=13.05, P=0.003, df =3; valence: χ^2=3.5, P=0.06, df =1, Fig 4 B).

When false alarms latencies were plotted as a function of hits latencies, they were well fitted by a line of slope near one (r=0.78, N=80, Fig. 4C). This confirms previous suggestion that there is a top-down routing of cognitive processing that pre-determines the timing of the evaluation once the task is loaded in memory.

To rule out the potential effect of overt visual search and to complete the exploration of all possible propositions having two target variables we repeated the experiment using a half diameter and 100 ms duration (less than the reaction time for a saccade [6] stimuli in 12 (6 females) subjects non-overlapping with those of experimental series 2.

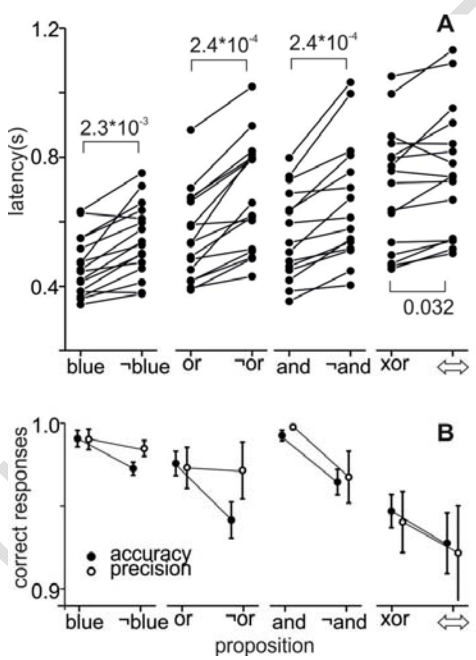

Figure 4. Role of number of items, valence and connectors on proposition evaluation difficulty (second experimental series with large and long lasting stimuli). A) Line series plot showing the consistent increase in the latency of the response to negated propositions for every subject. Signrank tests for paired propositions yielded Bonferroni's corrected p values indicated on the plots, N=16 in each case). B) There is consistent reduction of accuracy (number of correct responses over the total of trials) and precision (number of hits over positive responses) of the response to negated propositions. .

Results were qualitatively the same as those found for large and long-lasting stimuli including the effect of negation (see details in Fig. 5). Friedman test showed a significant main effect of negated values on latency ($\chi2$=24.5, P=7*10^{-7}, Fig. 5A; post-hoc pairwise comparisons signrank tests p<0.01 in all cases) and accuracy (χ^2=16.71, P=4.3*10^{-5}, df =1, Fig 5B). When false alarms latencies were plotted as a function of hits latencies, they were also well fitted by a line (r=0.78, N=80, Fig. 5C).

In this series we added two additional odd paired propositions ([blue and ¬yellow] and [yellow OR ¬ blue]) to complete the 10 possible dyadic cases. In these two cases, in which one of the variables was affirmed and the other negated, the decision time was equivalent to those corresponding even-parity propositions. “b AND [¬ y]” had a decision time not significantly different from “b XOR y” (which is equivalent to “[b AND ¬ y] OR [¬ b AND y]”; signrank test P=0.28; N=13), and “y OR ¬ b” (conditional) had a decision time not significantly different from “[b AND y] OR [¬b AND ¬y]” (biconditional; signrank test P=0.90; N=13).

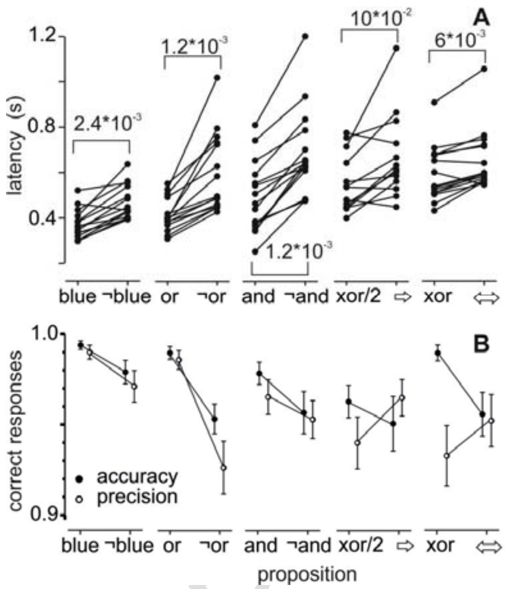

Figure 5. Role of number of items, valence and connectors on proposition evaluation difficulty (third experimental series with small and brief stimuli). A) Line series plot showing the consistent increase in the latency of the response to negated propositions for every subject. Signrank tests for paired propositions yielded Bonferroni's corrected P values indicated on the plots, N=12 in each case). B) There is consistent reduction of accuracy (number of correct responses over the total of trials) and precision (number of hits over positive responses) of the response to negated propositions.

Confirming previous reports [9] and experimental series 1 we found that evaluating absence takes longer than evaluating presence. It should be stressed that our data were obtained after the subject had fully comprehended the logic proposition and therefore our design excludes that the time taken for decision making includes a language comprehension component in each trial. This means that when the task is understood (prior to the experimental series of trials), brain circuits appear to be specifically tuned in order to set up a central expectation engram and, when the task is performed, such engram is conceptually compared with sensory data. Consistently with this hypothesis, there was a good correlation between the latency of the false alarms and hits suggests that th ere is a preparatory top-down process that sets the processing route for the visual image depending on the complexity of the proposition.

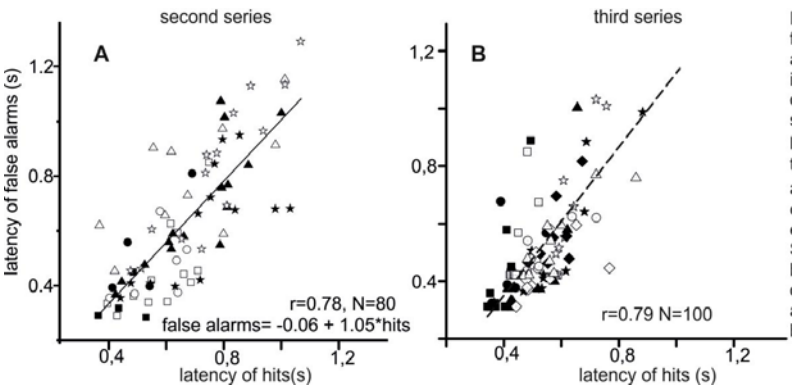

Figure 6. Latency of false alarms as a function of the latency of hits, False alarms were not frequent but informative since their latencies also depend on the task. A. Data from the second experimental series with large long-lasting stimuli. B. Data from the third experimental series with short and brief stimuli. each symbol correspond to the latency of an experiment in a single subject. Symbol types: blue (black square), ¬ blue (white square), or (black circle), ¬ or (white circle), and (black triangle), ¬ and (white triangle), XOR (black star), bi-conditional (white star).

Although this is similar to the findings in language based reasoning [26, 27], this extra time cannot be explained by difficulty in sentence comprehension because our experimental design involves the full understanding of the task prior to the experiment. This indicates that the extra time is introduced during the perceptual evaluation process.

These differences in latency and accuracy while processing positive and negative valences in monadic propositions suggest the existence of two different processes. While both require a central expectation about the same perceptual attribute, in the fastest process the task is to detect the matching between sensory data and the internal engram, in the other the task is to detect a mismatch.

In addition, negation adds extra time to the evaluation of binary propositions when it is combined with other operators. Interestingly, negating a whole proposition (¬ [a AND b], or ¬ [a OR b]) increases the processing time as much as when negating an elemental feature (¬ [a]).

Despite the differences between visual search experiments and ours, one might hypothesize that visual search and the assessing of whether a visual stimulus matches a logically constructed engram might share common mechanisms. It is well known that the time taken in visual search experiments is proportional to the number of possible values expressed in the image ("set size"), but having a different slope for target (20-30 ms per item) and non-target (40-60 ms per item) values ([7, 23, 24]). In our case the set size was always 3 and therefore equally affecting all experiments by a factor of 3. Decision times between 60 and 90 ms for present and between 120 and 180 ms as derived from these studies clearly match our means of 75 ms and 150 ms respectively, suggesting that common mechanisms might be employed by the brain in covert visual search experiments and perceptual evaluation of logic propositions.

Consistently with this view, a most parsimoneous model states that while the decision time increase by steps of 75 ms positive values it doubles this time for negative values. This heuristic rule was valid for the monadic ("a" and "¬a") and some dyadic propositions combining conjunctions and negations "y AND b" "y AND ¬b", "¬(yAND b)", and "¬y AND ¬b)".

This rule was valid for disjunctions only when both variable values were positive ("y OR b") or negative which is the de Morgan's equivalent to "¬(y AND b)" but fails by defect to fit brain performance with the simple conditional ("y OR ¬b") and by excess to fit "y XOR b" and "y XOR ¬b".

Then we postulate that an additional operator has evolved in humans to deal more advantgeosly with these complex propositions. One alternative is that this operator is related to the exclusive disjunction (XOR). The "mental models" theory suggest that this operation can be implemented aside from Boolean operators by the human brain [21]. In fact, decision time for "b XOR y" is larger than for "b OR y" or "b AND y" but shorter than the expected value using the Boolean rule "(b AND ¬y) OR (¬b AND y) ". The longer time for XOR compared to the other two operators is consistent with theoretical studies indicating that simple networks are only able to perform basic Boolean operations but only complex networks with recurrent loops are able to evaluate XOR and biconditional (Mizraji and Lin, 2003, 2011).

The shorter time for XOR compared to the Boolean rule is an evidence that the brain additionally evolved a powerful simplifying operator that can be considered a dyadic form of NOT [28,29]. Importantly, XOR deals intrinsically with evenly-paired variable values expressed as “mirror images” in the alternative instance. Consistently, [y XOR b] operation takes almost the same time as the conjunction ([y AND ¬ b]) suggesting that the difficulty for deciding on the truth of evenly paired propositions is not incremented by the attentional evaluation of the mirror image ([b AND ¬ y]).

On the other hand, the longest decision times and largest number of errors result from simple conditional (y OR [¬ b]) and bi-conditional propositions (y XOR [¬ b])) in which two instances (one affirming and other negated) are operated by a disjunction. One possible hypothesis to explain the largest decision time and number of errors that characterize simple and bi-conditionals is the use of XOR in both cases. The extra time taken for bi conditionals (either in the form “y XOR [¬ b]”, or “b XOR [¬ y]”) compared to (y XOR b) could be explained by the presence of a negated value. More significantly, our data implies that the brain evaluates ([¬b] XOR (a AND b)) instead of [a OR ¬b] and the use of the XOR brain mechanism for evaluating conditionals may be the origin of errors and cognitive illusions frequently observed conditional inferences [30,31,32].

Taken our data together, we constructed a simple heuristic model that linearly fit decision time (DT) in obtained in experimental series 2 (r = 0,976, p= 3.2*10-16, N=24) and 3 (r = 0,976, p= 3.2*10-16, N=24. Fig 7).

Consistently with the minimum description length hypothesis [10], difficulty increases with the number of variable values. However, we found some discrepancies with such hypothesis.

First, the increment in time taken caused by a positive value (presence of a feature) was about half of that caused by a negative one (absence of a feature). This might involve a dual neural system, involving matching and mismatching detector circuits (i.e. NOT operator) for evaluating positive and negative values.

Secondly, the exclusive disjunction (XOR), which involves the implicit negation of the alternative, appears to have evolved for dealing with propositions where negative and positive variable values are simultaneously evaluated. This might be consistent with the importance that this operation has in the “mental models” theory [21]. Last, but not least our experiments unveil an interesting problem that popped out when exploring our data under the light of Mizraji and Lin study [28,29]: Does brain truly solves the truth value of the expression “y XOR b” defined in Boolean terms as “(b AND ¬y) OR (¬b AND y)” or the mental operation that solves such judgment works not only with even but also odd number of concepts? A strictly logic XOR operator would have a false outcome when evaluating “we are in the downtown theatre, XOR at the beach XOR deep in the forest” [29]: but a biological operator that choose one between three (or more) options would give the same result as XOR in a dyadic operations but could also constitute a more powerful mental tool for processing triadic or higher order propositions.

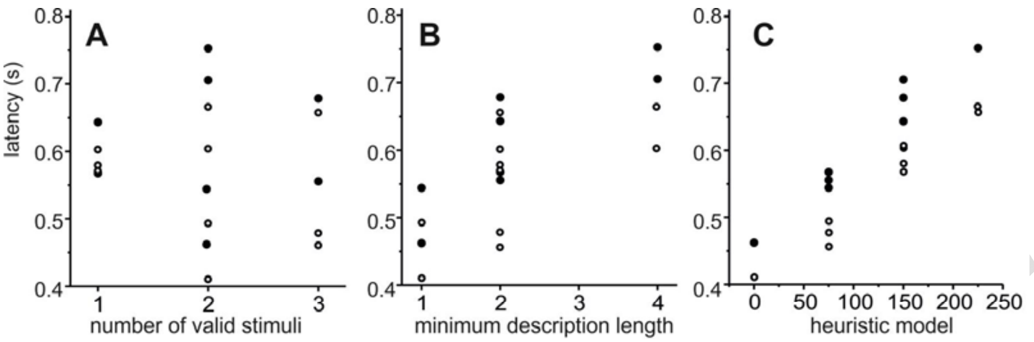

Figure 7. Models. Experimental data from series 2 (filled symbols, obtained with large and long-lasting stimuli, 8 propositions, 16 subjects) and 3 (open symbols, small and brief stimuli, 10 propositions, 12 subjects) are used to contrast three models. Each point corresponds to the average data for each operation. A) Decision time as a function of number of validating stimuli (r = 0.04 F=0.012, p=0.91 for series 2 and r = -0.25, F=0.58, p=0.47 for series 3). B) Decision time depends on the minimum description length (r= 0.86; F=17.27; p=0.006 for series 2 and r=0.65 F= 5.88; p=0.042 for series 3). However, this criterion does not take into account the propositions with negative valences (squares) which appear to be shifted up. C) Data are optimally fitted by the model "Decision time= (number of positive values + 2* number of negative values) *K + (number of XORs)*C -basal time" (r= 0.98; F= 129.0 p=2.79*10^{-5} for series 2 filled symbols and r= 0.98; F= 164.9 p=1.27*10^{-6} for series 3 open symbols). Model parameters K=75 ms, C=150.

4. CONCLUSIONS

Our data suggests that: a) Humans are able to evaluate the presence of an object precisely described by a logic proposition with a decision time and errors that depend on the proposition structure but not on the validating stimuli. This suggests a top down tuning of the decision process, b) This top-down routing of cognitive processing that pre-determines the timing of the evaluation once the task is loaded in a working memory; c) Decision timing, accuracy and precision depend on proposition connectors including NOT; d) The analysis of the responses to XOR, simple-conditionals and bi-conditionals, suggests that a fourth operator (e.g. "only one of the variable values is valid") is implemented by the brain. This hypothesis postulate a process separated from that supporting the classical Boolean operations (NOT, AND and OR).

To sum up, our findings reduce the spectrum of brain implemented connectors to a redundant set (NOT, AND, OR and XOR).which influence on decision time is represented in a heuristic model that fits the data better than previous ones either based on minimum description length or number of validating stimuli. Thus, if one accept the hypothesis that these logic connectors express independent brain operations which are "recruited" selectively depending on the proposition structure one must ask next what mechanisms are involved in routing sensory information through the decision process.

Comment [c2]: Rephrase.

CONSENT (WHERE EVER APPLICABLE)

All authors declare that 'written informed consent was obtained from the patient (or other approved parties) for publication of this case report and accompanying images. A copy of the written consent is available for review by the Editorial office/Chief Editor/Editorial Board members of this journal.

ETHICAL APPROVAL (WHERE EVER APPLICABLE)

All authors hereby declare that all experiments have been examined and approved by the appropriate ethics committee and have therefore been performed in accordance with the ethical standards laid down in the 1964 Declaration of Helsinki.

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DEFINITIONS, ACRONYMS, ABBREVIATIONS

¬:Logical operator not
AND: Logical operator and
OR: Logical operator inclusive or

¬AND: Logical operator not AND
¬OR: Logical operator not OR
XOR: Logical operator exclusive or.
¬XOR: Logical operator not exclusive or; equivalent to biconditional also labeled with a double arrow in the figures
XOR/2: Logical operator for A and not B
Single arrow: simple conditional judgment