Original Research Article

A Transmuted Lomax-Exponential Distribution: Properties and Applications

Abstract

In this article, the Quadratic rank transmutation map proposed and studied by [15] is used to construct and study a new distribution called the transmuted Lomax-Exponential distribution (*TLED*) as an extension of the Lomax-Exponential distribution recently proposed by [6]. Using the transmutation map, we defined the probability density function (*pdf*) and cumulative distribution function (*cdf*) of the transmuted Lomax-Exponential distribution. Some properties of the new distribution such as moments, moment generating function, characteristics function, quantile function, survival function, hazard function and order statistics are also studied. The estimation of the distributions' parameters has been done using the method of maximum likelihood estimation. The performance of the proposed probability distribution is being tested in comparison with some other generalizations of Exponential distribution using a real life dataset. The results obtained show that the *TLED* performs better than the other probability distributions.

Keywords: Exponential distribution, Quadratic rank transmutation map, Moments, Reliability analysis, Maximum likelihood estimation, Transmuted Lomax-Exponential distribution, parameters, Applications.

1. INTRODUCTION

An Exponential distribution which can be used in Poisson processes gives a description of the time between events. The distribution has been applied widely life testing experiments. The distribution exhibits memoryless property with a constant failure rate which makes the distribution unsuitable for real life problems and hence creating a vital problem in statistical modeling and applications. The cumulative distribution function (*cdf*) and probability density function (*pdf*) of an exponential random variable X are respectively given by;

$$G(x) = 1 - e^{-\theta x} \quad (1.1)$$

$$g(x) = \lambda e^{-\theta x} \quad (1.2)$$

where $\theta > 0$ is the exponential parameter ~~and $x > 0$ is the random variable~~.

There are several ways of adding one or more parameters to a distribution function which makes the resulting distribution richer and more flexible for modeling data. Some of the recent studies on the generalization of exponential distribution include the Lomax-exponential distribution by [6], the transmuted odd generalized exponential-exponential distribution by [1], the transmuted exponential distribution by [13], transmuted inverse exponential distribution by [11], the odd generalized exponential-exponential distribution by [9] and the Weibull-exponential distribution by [12]. Of interest to us in this article is the Lomax-exponential distribution (*LED*) which has been found to be useful in various fields to model variables whose chances of survival and failure decreases with time. It was also discovered that the *LED* is positively skewed and performed better than some existing distributions like Weibull-exponential and exponential distributions.

According to [3] the *cdf* and *pdf* of the Lomax-G family (Lomax-based generator) for any continuous probability distribution are given respectively as:

$$F(x)=1-\beta^{\alpha}\left(\beta-\log[1-G(x)]\right)^{-\alpha} \quad (1.3)$$

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$$f(x)=\alpha\beta^{\alpha}g(x)\left([1-G(x)]\{\beta-\log[1-G(x)]\}^{\alpha+1}\right)^{-1}, \quad (1.4)$$

Use the same notation

where g(*x*) and G(*x*) are the *pdf* and *cdf* of any continuous distribution to be generalized respectively and α>0 and β>0 are the two additional new parameters.

Recently, a new extension of the exponential distribution has been proposed in the literature by considering the Lomax-G family above where the random variable *X* is said to have follow the Exponential distribution with parameter *θ.* The distribution of *X* according to [6] is referred to as Lomax-Exponential distribution. The *pdf* of the Lomax-Exponential distribution is defined by

$$g(x)=\alpha\beta^{\alpha}\theta\left(\beta+\theta x\right)^{-(\alpha+1)}, x>0, \alpha, \beta, \theta>0 \quad (1.5)$$

Use f(x) and F(x) due to expression 1.3 and 1.4

The corresponding cumu xponential distribution is

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$$G(x)=1-\beta^{\alpha}\left(\beta+\theta x\right)^{-\alpha}, x>0, \alpha, \beta, \theta>0, \quad (1.6)$$

where, $x>0, \alpha>0, \beta>0, \theta>0$; α *and* β are the shape parameters and θ is a scale parameter.

The *cdf* and *pdf* of the transmuted Lomax-Exponential distribution are obtained using the steps proposed by [15]. A random variable *X* is said to have a transmuted distribution function if its *pdf* and *cdf* are respectively given by;

$$f(x)=g(x)[1+\lambda-2\lambda G(x)] \quad (1.7)$$

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and

$$F(x)=(1+\lambda)G(x)-\lambda[G(x)]^{2} \quad (1.8)$$

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where; $x>0$, and $-1\leq\lambda\leq 1$ is the transmuted parameter, *G(x)* is the *cdf* of any continuous distribution while *f(x)* and *g(x)* are the associated *pdf* of *F(x)* and *G(x)*, respectively.

The aim of this paper is to introduce a new continuous distribution called the Transmuted Lomax-Exponential distribution (*TLED)* from the proposed quadratic rank transmutation map by [15]. The remaining parts of this paper are presented in sections as follows: We defined the new distribution and give its plots in section 2. Section 3 derived some properties of the new distribution. Section 4 discusses reliability analysis of the *TLED.* The estimation of parameters using maximum likelihood estimation (MLE) is provided in section 5. In section 6, we carry out application of the proposed model with others using a real life dataset. Lastly, in section 7, we make some useful conclusions.

2. The Transmuted Lomax-Exponential Distribution *(TLED)*

Using equation (1.5) and (1.6) in (1.7) and (1.8) and simplifying, we obtain the *cdf* and *pdf* of the transmuted Lomax-Exponential distribution as follows:

$$F(x)=(1+\lambda)\left(1-\beta^{\alpha}(\beta+\theta x)^{-\alpha}\right)-\lambda\left(1-\beta^{\alpha}(\beta+\theta x)^{-\alpha}\right)^{2} \quad (2.1)$$

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and

$$f(x)=\alpha\beta^{\alpha}\theta(\beta+\theta x)^{-(\alpha+1)}\left[1+\lambda-2\lambda\left(1-\beta^{\alpha}(\beta+\theta x)^{-\alpha}\right)\right] \quad (2.2)$$

respectively. Where, $x>0,\alpha>0,\beta>0,$ ... scale parameter and λ is called the trans... The *pdf* and *cdf* of the *TLED* using som... follows.

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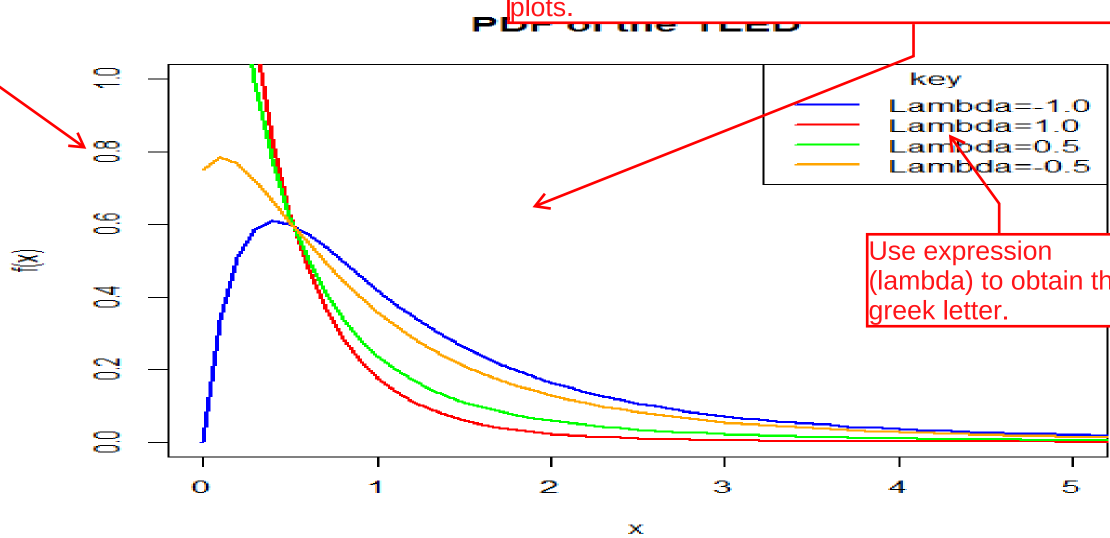

Use expression (lambda) to obtain the greek letter.

Figure 2.1: The graph of *pdf* of the *TLED* for $\alpha=3,\beta=2,\theta=1$ and different values of λ as displayed on the key in the plot above.

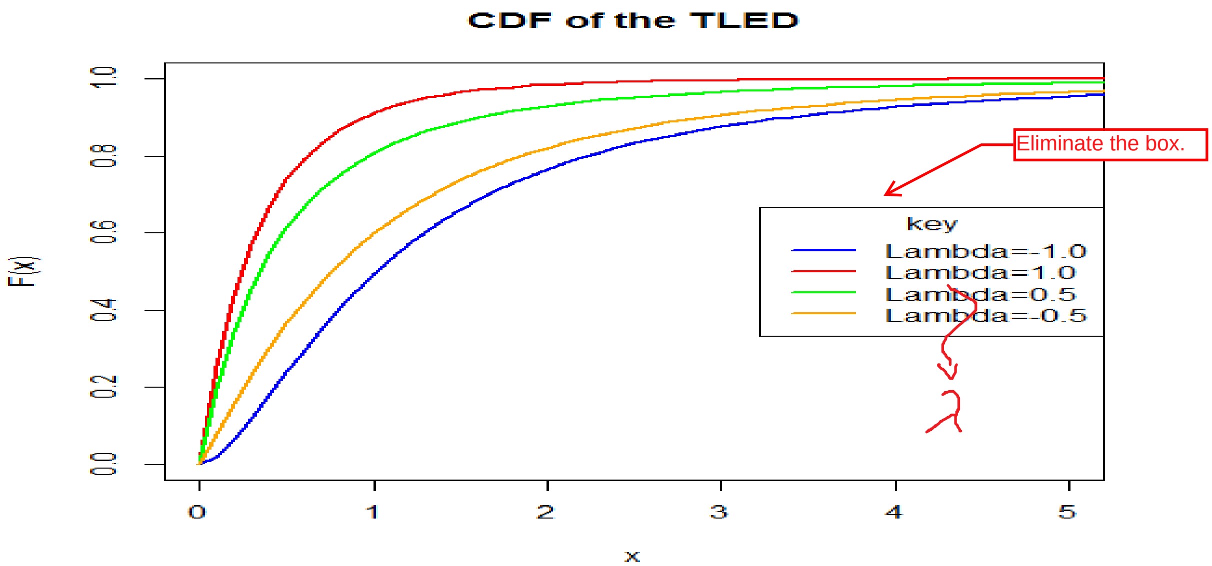

Figure 2.2: The graph of *cdf* of the *TLED* for $\alpha = 3, \beta = 2, \theta = 1$ and different values of λ as shown in the key on the figure above.

3. Statistical Properties of the *TLED*

3.1 The Quantile Function

This function is derived by inverting the *cdf* of any given continuous probability distribution. It is used for obtaining some moments like skewness and kurtosis as well as the median and for generation of random variables from the distribution in question. Hyndman and Fan [4] defined the quantile function for any distribution in the form $Q(u) = F^{-1}(u)$ where $Q(u)$ is the quantile function of $F(x)$ for $0 < u < 1$

Taking $F(x)$ to be the *cdf* of the *TLED* and inverting it as above will give us the quantile function as follows:

$$F(x) = (1+\lambda)\left(1-\beta^{\alpha}(\beta+\theta x)^{-\alpha}\right) - \lambda\left(1-\beta^{\alpha}\left(\beta+\theta x\right)^{-\alpha}\right)^{2} = u \qquad (3.1.1)$$

Simplifying equation (3.1.1) above, we obtain:

$$Q(u) = X_q = \frac{1}{\theta}\left\{\left(\frac{1}{\beta^{\alpha}}\left\{1-\left[\frac{(1+\lambda)-\sqrt{(1+\lambda)^2-4\lambda u}}{2\lambda}\right]\right\}\right)^{-\frac{1}{\alpha}} - \beta\right\} \qquad (3.1.2)$$

3.2 Skewness and Kurtosis

This paper presents the quantile based measures of skewness and kurtosis due to non-existence of the classical measures in some cases.

The Bowley's measure of skewness by [7] based on quartiles is given by:

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$$SK = \frac{Q\left(3/4\right) - 2Q\left(1/2\right) + Q\left(1/4\right)}{Q\left(3/4\right) - Q\left(1/4\right)} \qquad (3.2.1)$$

And the [10] kurtosis is on octiles and is given by;

$$KT = \frac{Q\left(7/8\right) - Q\left(5/8\right) - Q\left(3/8\right) + \left(1/8\right)}{Q\left(6/8\right) - Q\left(1/8\right)} \qquad (3.2.2)$$

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3.3 Moments

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Let X denote a continuous random variable, the n^{th} moment of X is given by;

$$\mu_n' = E\left[X^n\right] = \int_0^\infty x^n f(x)dx \qquad (3.3.1)$$

Taking *f(x)* to be the *pdf* of the *TLED* as given in equation (2.2) and simplifying the integral we have:

$$\mu_n' = \int_0^\infty x^n \left(\alpha\beta^\alpha\theta\left(\beta+\theta x\right)^{-(\alpha+1)}\left[1-\lambda+2\lambda\beta^\alpha\left(\beta+\theta x\right)^{-\alpha}\right]\right)dx$$

$$\mu_n' = \left(1-\lambda\right)\int_0^\infty \alpha\beta^\alpha\theta x^n\left(\beta+\theta x\right)^{-(\alpha+1)}dx + 2\lambda\int_0^\infty \alpha\beta^{2\alpha}\theta x^n\left(\beta+\theta x\right)^{-2\alpha-1}dx$$

$$\mu_n' = \alpha\beta^\alpha\theta\left(1-\lambda\right)\int_0^\infty x^n\left(\beta+\theta x\right)^{-(\alpha+1)}dx + 2\alpha\beta^{2\alpha}\theta\lambda\int_0^\infty x^n\left(\beta+\theta x\right)^{-2\alpha-1}dx$$

Using integration by substitution, let:

$$u = \beta+\theta x \Rightarrow x = -\tfrac{\beta}{\theta}\left(1-\tfrac{u}{\beta}\right)$$

$$\frac{du}{dx} = \theta \Rightarrow dx = \frac{du}{\theta}$$

Now, substituting for u, x and dx above, we have:

$$\mu_n' = \alpha\beta^\alpha\theta\left(1-\lambda\right)\int_0^\infty\left(-\tfrac{\beta}{\theta}\left(1-\tfrac{u}{\beta}\right)\right)^n\left(u\right)^{-(\alpha+1)}\frac{du}{\theta} + 2\alpha\beta^{2\alpha}\theta\lambda\int_0^\infty\left(-\tfrac{\beta}{\theta}\left(1-\tfrac{u}{\beta}\right)\right)^n\left(u\right)^{-2\alpha-1}\frac{du}{\theta}$$

$$\mu_n' = \tfrac{\alpha\beta^{\alpha+n}}{\theta^n}\left(-1\right)^n\left(1-\lambda\right)\int_0^\infty\left(u\right)^{-(\alpha+1)}\left(1-\tfrac{u}{\beta}\right)^n du + \tfrac{2\alpha\beta^{2\alpha+n}}{\theta^n}\left(-1\right)^n\lambda\int_0^\infty\left(u\right)^{-2\alpha-1}\left(1-\tfrac{u}{\beta}\right)^n du$$

$$\mu_n' = \tfrac{\alpha\beta^{n-1}}{\theta^n}\left(-1\right)^n\left(1-\lambda\right)\int_0^\infty\left(\tfrac{u}{\beta}\right)^{1-\alpha-1-1}\left(1-\tfrac{u}{\beta}\right)^{n+1-1}du + \tfrac{2\alpha\beta^{n-1}}{\theta^n}\left(-1\right)^n\lambda\int_0^\infty\left(\tfrac{u}{\beta}\right)^{1-2\alpha-1-1}\left(1-\tfrac{u}{\beta}\right)^{n+1-1}du$$

Recall that $B\left(x,y\right) = B\left(y,x\right) = \int_0^\infty t^{x-1}\left(1-t\right)^{y-1}dt$ and this implies that

$$\mu_n' = \left(\frac{\alpha}{\beta}\right)\left(-\frac{\beta}{\theta}\right)^n \left\{(1-\lambda)B\left(1-\alpha-1,n+1\right)+2\lambda B\left(1-2\alpha-1,n+1\right)\right\} \tag{3.3.2}$$

The mean, variance, skewness and kurtosis measures can also be calculated from the n^{th} ordinary moments as well as the moment generating function and characteristics function using some well-known relationships.

The Mean

The mean of the *TLED* can be obtained from the n^{th} moment of the distribution when $n=1$ as follows:

$$\mu_1' = \left(-\frac{\alpha}{\theta}\right)\left\{(1-\lambda)B\left(1-\alpha-1,2\right)+2\lambda B\left(1-2\alpha-1,2\right)\right\} \tag{3.3.3}$$

Also the second moment of the *TLED* is obtained from the n^{th} moment of the distribution when $n=2$ as

$$\mu_2' = \frac{\alpha\beta}{\theta^2}\left\{(1-\lambda)B\left(1-\alpha-1,3\right)+2\lambda B\left(1-2\alpha-1,3\right)\right\} \tag{3.3.4}$$

The Variance

The n^{th} central moment or moment about the mean of X, say μ_n, can be obtained as

$$\mu_n = E\left(X-\mu_1'\right)^n = \sum_{i=0}^{n}(-1)^i \binom{n}{i} \mu_1'^{\,i}\mu_{n-i}' \tag{3.3.5}$$

The variance ~~of X~~ for *TLED* is obtained from the central moment when $n=2$, that is,

$$Var(X) = E\left(X^2\right) - \left\{E\left(X\right)\right\}^2 \tag{3.3.6}$$

$$Var(X) = \mu_2' - \left\{\mu_1'\right\}^2 \tag{3.3.7}$$

Where μ_1' and μ_2' are the mean and second moment of the *TLED* all obtainable from equation ~~(3.3.2).~~ ← 3.3.3 and 3.3.4???

3.~~2~~4 Moment Generating Function

The moment generating is an important shape characteristic of a distribution and is always in one function that represents all the moments. In other words, the *mgf* produces all the moments of the random variable X by differentiation.

The *mgf* of a random variable X can be obtained by

$$M_x(t) = E\left(e^{tx}\right) = \int_0^\infty e^{tx} f(x)\,dx \tag{3.4.1}$$

$$M_x(t) = E\left(e^{tx}\right) = \sum_{n=0}^{\infty}\frac{t^n}{n!}\mu_n' \tag{3.4.2}$$

where

$$\mu_n' = \left(\frac{\alpha}{\beta}\right)\left(-\frac{\beta}{\theta}\right)^n \left\{(1-\lambda)B(1-\alpha-1,n+1)+2\lambda B(1-2\alpha-1,n+1)\right\}$$

is as defined in equation (10) previously.

3.3 Characteristics Function

is useful and has some properties which give it a genuine role in mathematical statistics. It is used for generating moments, characterization of distributions and in analysis of linear combination of independent random variables.

The characteristics function of a random variable *X* is given by;

$$\varphi_x(t) = E\left(e^{itx}\right) = E\left[\cos(tx) + i\sin(tx)\right] = E\left[\cos(tx)\right] + E\left[i\sin(tx)\right] \quad (3.3.1)$$

Simple algebra and power series expansion proves that

$$\phi_x(t) = \sum_{n=0}^{\infty} \frac{(-1)^n t^{2n}}{(2n)!} \mu_{2n}' + i\sum_{n=0}^{\infty} \frac{(-1)^n t^{2n+1}}{(2n+1)!} \mu_{2n+1}' \quad (3.3.2)$$

Where μ_{2n}' and μ_{2n+1}' are the moments of *X* for $n=2n$ and $n=2n+1$ respectively and can be obtained from μ_n as

$$\mu_{2n}' = \left(\frac{\alpha}{\beta}\right)\left(-\frac{\beta}{\theta}\right)^{2n} \left\{(1-\lambda)B(1-\alpha-1,2n+1)+2\lambda B(1-2\alpha-1,2n+1)\right\}$$

and

$$\mu_{2n+1}' = \left(\frac{\alpha}{\beta}\right)\left(-\frac{\beta}{\theta}\right)^{2n+1} \left\{(1-\lambda)B(1-\alpha-1,2n+2)+2\lambda B(1-2\alpha-1,2n+2)\right\}$$

respectively.

4. Some Reliability Functions

In this section, we present some reliability functions associated with *TLED* including the survival and hazard functions.

4.1 The Survival Function

The survival function describes the likelihood that a system or an individual will not fail after a given time. It tells us about the probability of success or survival of a given product or component. Mathematically, the survival function is given by:

$$S(x) = 1 - F(x) \quad (4.1.1)$$

Taking *F(x)* to be the *cdf* of the *TLED*, substituting and simplifying (4.1.1) above, we get the survival function of the *TLED* as:

$$S(x) = 1 - \left\{(1+\lambda)\left(1-\beta^{\alpha}(\beta+\theta x)^{-\alpha}\right) - \lambda\left(1-\beta^{\alpha}(\beta+\theta x)^{-\alpha}\right)^2\right\} \quad (4.1.2)$$

Below is a plot of the survival function at chosen parameter values in figure **4.1.1**

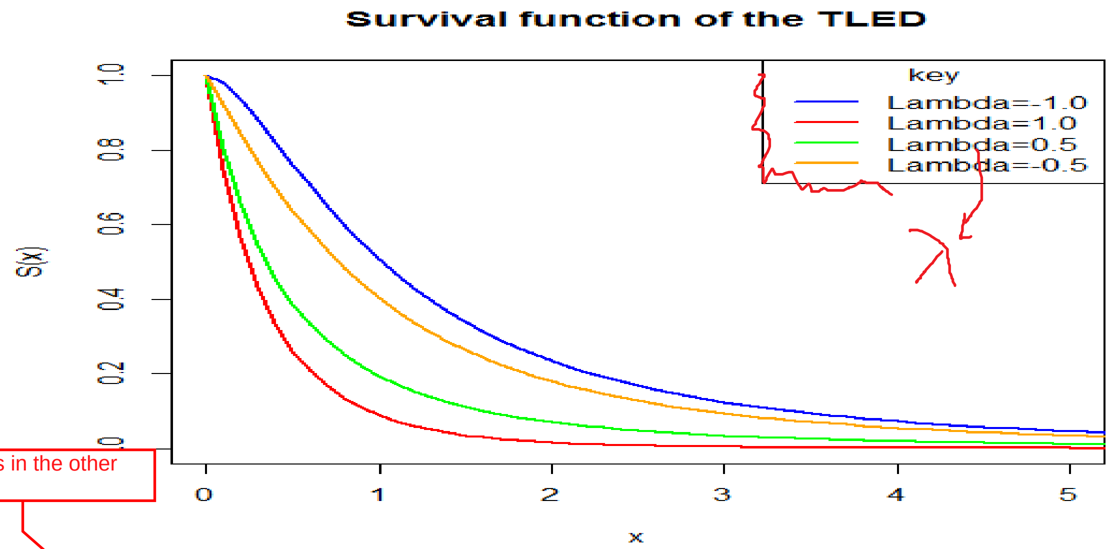

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Figure 4.1.1: Plot of the survival function of the *TLED* for $\alpha = 3, \beta = 2, \theta = 1$ and different values of λ as shown on the figure above.

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From the figure above, we observed that the probability of survival for any random variable following a *TLED* decreases with time, that is, as time or age grows the probability of life decreases. This implies that the *TLED* could be used to model random variables whose survival rate decreases as their age lasts.

4.2 The Hazard Function

Hazard function as the name implies is also called risk function, it gives us the probability that a component will fail or die for an interval of time. The hazard function is defined mathematically as;

$$h(x) = \frac{f(x)}{1 - F(x)} = \frac{f(x)}{S(x)} \qquad (4.2.1)$$

Taking *f(x)* and *F(x)* to be the *pdf* and *cdf* of the proposed Lomax-Exponential distribution given previously, we obtain the hazard function as:

$$h(x) = \frac{\alpha\beta^{\alpha}\theta(\beta+\theta x)^{-(\alpha+1)}\left[1+\lambda-2\lambda\left(1-\beta^{\alpha}(\beta+\theta x)^{-\alpha}\right)\right]}{(1+\lambda)\left(1-\beta^{\alpha}(\beta+\theta x)^{-\alpha}\right)-\lambda\left(1-\beta^{\alpha}(\beta+\theta x)^{-\alpha}\right)^{2}} \qquad (4.2.2)$$

The following is a plot of the hazard function at chosen parameter values in figure 4.2.1

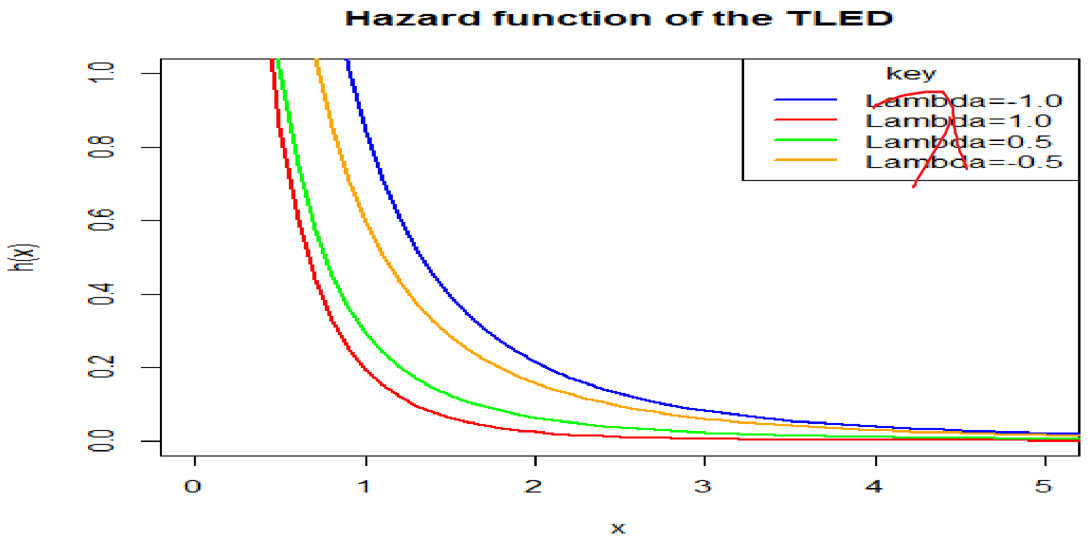

Figure 4.2.1: Plot of the hazard function of the *TLED* for $\alpha=3, \beta=2, \theta=1$ and different values of λ as shown on the plot above.

Interpretation: Figure 4.2.1 above shows the behavior of hazard function of the *TLED* and it means that the probability of failure for any *TLED* random variable is decreasing with respect to time that is, as the time increases, the probability of failure or death decreases.

5. Parameter Estimation via Maximum Likelihood

Let $X_1, ---, X_n$ be a sample of size "n" independently and identically distributed random variables from the *TLED* with unknown parameters α, β, θ, and λ defined previously. The *pdf* of the *TLED* is given as:

$$f(x)=\alpha\beta^{\alpha}\theta(\beta+\theta x)^{-(\alpha+1)}\left[1+\lambda-2\lambda\left(1-\beta^{\alpha}(\beta+\theta x)^{-\alpha}\right)\right]$$

The likelihood function is given by;

$$L(X/\alpha,\beta,\theta,\lambda)=\left(\alpha\beta^{\alpha}\theta\right)^{n}\prod_{i=1}^{n}(\beta+\theta x_i)^{-(\alpha+1)}\prod_{i=1}^{n}\left[1+\lambda-2\lambda\left(1-\beta^{\alpha}(\beta+\theta x_i)^{-\alpha}\right)\right] \quad (5.1)$$

Taking the natural logarithm of the likelihood function, i.e.,

Let, $l=\log L\left(x_1, x_2, ..x_n / \alpha,\beta,\theta,\lambda\right)$, such that

$$l=n\log\alpha+n\alpha\log\beta+n\log\theta-(\alpha+1)\sum_{i=1}^{n}\log(\beta+\theta x_i)+\sum_{i=1}^{n}\log\left[1+\lambda-2\lambda\left(1-\beta^{\alpha}(\beta+\theta x_i)^{-\alpha}\right)\right] \quad (5.2)$$

Differentiating l partially with respect to α, β, θ *and* λ respectively gives;

$$\frac{\partial l}{\partial \alpha} = \frac{n}{\alpha} + n\log\beta - \sum_{i=1}^{n}\log(\beta+\theta x_i) + \sum_{i=1}^{n}\left\{\frac{2\lambda\beta^{\alpha}(\beta+\theta x_i)^{-\alpha}\{\log\beta - \log(\beta+\theta x_i)\}}{1-\lambda+2\lambda\beta^{\alpha}(\beta+\theta x_i)^{-\alpha}}\right\} \quad (5.3)$$

$$\frac{\partial l}{\partial \beta} = \frac{n\alpha}{\beta} - (\alpha+1)\sum_{i=1}^{n}\left\{(\beta+\theta x_i)^{-1}\right\} - \sum_{i=1}^{n}\left\{\frac{2\lambda\beta^{\alpha}(\beta+\theta x_i)^{-\alpha}\{\alpha\beta^{-1}+(\beta+\theta x_i)^{-1}\}}{1-\lambda+2\lambda\beta^{\alpha}(\beta+\theta x_i)^{-\alpha}}\right\} \quad (5.4)$$

$$\frac{\partial l}{\partial \theta} = \frac{n}{\theta} - (\alpha+1)\sum_{i=1}^{n}\left\{x_i(\beta+\theta x_i)^{-1}\right\} + \sum_{i=1}^{n}\left\{\frac{2\alpha\lambda\beta^{\alpha}x_i(\beta+\theta x_i)^{-\alpha-1}}{1-\lambda+2\lambda\beta^{\alpha}(\beta+\theta x_i)^{-\alpha}}\right\} \quad (5.5)$$

$$\frac{\partial l}{\partial \lambda} = \sum_{i=1}^{n}\left\{\frac{2\beta^{\alpha}(\beta+\theta x_i)^{-\alpha}-1}{1-\lambda+2\lambda\beta^{\alpha}(\beta+\theta x_i)^{-\alpha}}\right\} \quad (5.6)$$

Equating equations (5.3), (5.4), (5.5) and (5.6) to zero and solving for the solution of the non-linear system of equations will give us the maximum likelihood estimates of parameters α, β & λ respectively. However, the solution cannot be obtained analytically except numerically with the aid of suitable statistical software like Python, R, SAS, etc., when data sets are given.

6 Applications

Here, we have applied and compared the performance of the Transmuted Lomax-exponential distribution *(TLED)* to that of Lomax-Exponential distribution *(LED),* transmuted odd generalized exponential-exponential distribution *(TOGEED),* odd generalized exponential-exponential distribution *(OGEED),* Weibull-Exponential distribution *(WED),* Transmuted Exponential distribution *(TED)* and the Exponential distribution (*ED)* using the following dataset.

Data set: This data represents the remission times (in months) of a random sample of 128 bladder cancer patients. It has previously been used by [5], [1], [14] and [8]. It's summarized as follows:

Table 6.1: Summary Statistics for the dataset

parameters	n	Minimum	Q_1	Median	Q_3	Mean	Maximum	Variance	Skewness	Kurtosis
Values	128	0.0800	3.348	6.395	11.840	9.366	79.05	110.425	3.3257	19.1537

From the descriptive statistics in table 6.1, we observed that the data set is positively skewed with a very high coefficient of kurtosis and therefore suitable for flexible and skewed distributions.

To compare the distributions listed above, we have used several measures of model fit such as *AIC* (Akaike Information Criterion), Cramèr-Von Mises *(W*),* Anderson-Darling *(A*)* Kolmogorov-smirnov (*K-S*) statistics.

Note that the model with the lowest values of these statistics shall be chosen as the best model to fit the data.

Table 6.2: The statistics *ll*, AIC, A^*, W^* and *K-S* for the fitted models to the dataset.

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Distributions	**Parameter estimates**	**ll=(log-likelihood value)**	***AIC***	A^*	W^*	***K-S***	***P-Value (K-S)***	**Ranks**
TLED	$\hat{\theta}$=0.4665 $\hat{\alpha}$=4.2157 $\hat{\beta}$ =9.7146 $\hat{\lambda}$ =-0.8445	409.6905	827.3809	0.1326	0.0210	0.0448	0.9593	1
LED	$\hat{\theta}$=0.1643 $\hat{\alpha}$=6.3108 $\hat{\beta}$ =9.9520	415.6839	837.3678	0.3392	0.0551	0.0988	0.1639	2
TOGEED	$\hat{\theta}$=0.0182 $\hat{\alpha}$=2.7822 $\hat{\lambda}$ =0.7591	416.5186	839.0372	1.0381	0.1747	0.1079	0.1014	3
TED	$\hat{\theta}$=0.1065 $\hat{\lambda}$=-0.2944	415.7532	835.5065	0.8349	0.1404	0.1322	0.0228	4
OGEED	$\hat{\theta}$=0.0346 $\hat{\alpha}$ =1.6066	439.5273	883.0546	3.2153	0.5463	0.2341	1.6e-06	5
WED	$\hat{\theta}$=0.0070 $\hat{\alpha}$=5.1855 $\hat{\beta}$=0.7814	465.8212	937.6424	0.5678	0.0924	0.2435	5.1[illegible]	[illegible]
	$\hat{\theta}$=0.1085	414.3576	830.7153	NaN	NaN	0.9465	2.2e-16	7

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It is shown from Table 6.2 above that the Transmuted Lomax-Exponential distribution *(TLED)* corresponds to the smallest values of *ll*, *AIC*, A^*, W^* and *K-S* compared to those of the Lomax-Exponential distribution *(TLED)*, Transmuted odd generalized exponential-exponential distribution *(TOGEED)*, odd generalized exponential-exponential distribution *(OGEED)*, Weibull-Exponential distribution *(WED)*, Transmuted Exponential distribution *(TED)* and the Exponential distribution (*ED)* and therefore we chose the *TLED* as the best model the fits the real life data. //

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7 Conclusion

In this article, we proposed a new distribution, *TLED*, derived and study some of its properties with graphical analysis and discussion on its usefulness and applications. Hence, having demonstrated earlier in the previous section, we have a conclusion based on our applications of the model to a real life data that the new distribution *(TLED)* has a better fit compared to the other six already existing models and hence a very competitive model for studying real life situations.

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