# A New Computerized Boundary Element Algorithm for Cancer Modeling of Cardiac Anisotropy on the ECG Simulation


## ABSTRACT

**Aims:** The main aim of this paper is to propose a new boundary element method (BEM) algorithm for Cancer Modeling of Cardiac Anisotropy on the electrocardiogram (ECG) Simulation.
**Study design:** Original research paper.
**Place and Duration of Study:** Jamoum laboratory, June 2018 (Which Country ?)
**Methodology:** a new boundary element algorithm was proposed and implemented for solving the governing equations of new cancer mathematical modeling in conjunction with the governing equations of ECG simulation.
**Results:** The effect of cardiac anisotropy on the ECG. Also, the effect of anisotropy on the relation between healthy and infected tissues.
**Conclusion:** boundary element algorithm for cancer modeling of cardiac anisotropy on the ECG simulation. For a known set of conductivities, our results are in a very good agreement with the corresponding finite difference results.



*Keywords: Boundary Element Algorithm;* Cardiac Anisotropy*;* cancer mathematical modeling*;* electrocardiogram (ECG).

2010 mathematics subject classification: 65M38 - 65K05 - 74B05 - 74E05 - 74F05 - 74H05 - 74H15 - 74S20 - 90C31.


## 1. INTRODUCTION

our cancer mathematical modeling present the interaction between tumour and immune cells. Also, it investigates the importance of combining Immuno-Oncology (IO) with ionizing radiation (IR) [1-4]. An understanding of behaviour of electrocardiographic resulted in computer models of ECG which is an important role that has been filled the knowledge gaps [5, 6].

In the present paper, our cancer mathematical modeling investigates the relation between tumour and immune cells and the effects of radio and immuno therapies. Also, it investigates the importance of combining Immuno-Oncology (IO) with ionizing radiation (IR).

( The two paragraph are the same things)

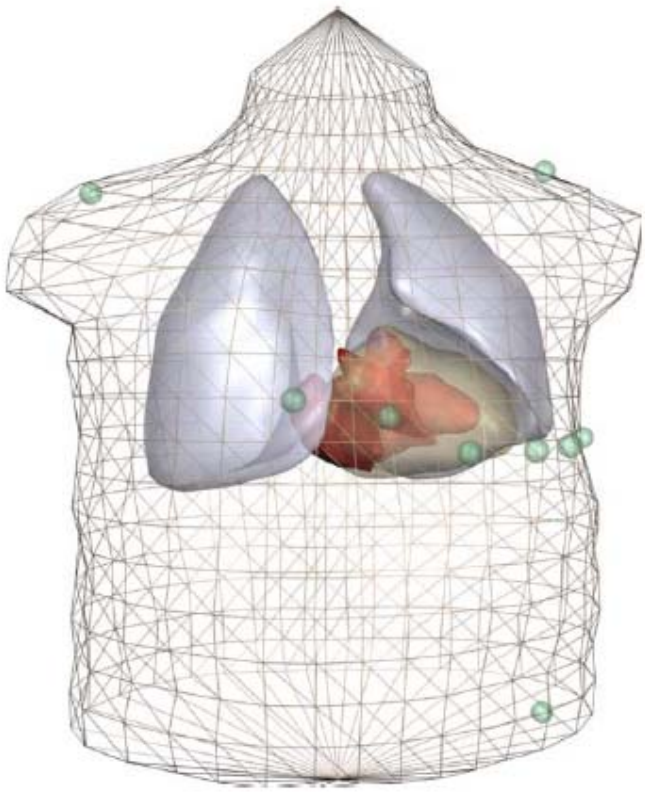

**Fig. 1. Boundary element anatomic model.**

## 2. BEM formulation and implementation

Recently, the BEM [7-55] has been used as a very important tool for ECG simulation to describe the torso, muscle layer, lungs and ventricular blood masses with thousand triangles shown in Fig. 1.
we consider the anisotropic bidomain model of cardiac tissue [1]

$$\nabla \cdot (G_i \nabla \Psi_i) = -\nabla \cdot (G_e \nabla \Psi_e) \quad (1)$$

where $\Psi_i$ and $\Psi_e$ are potential fields related to $J_i = G_i \nabla \Psi_i$ and $J_e = G_e \nabla \Psi_e$, respectively.
Making use of the following membrane potential $V_m = \Psi_i - \Psi_e$, we can write Eq. () as

$$\nabla \cdot ([G_i + G_e] \nabla \Psi_e) = -\nabla \cdot (G_i \nabla V_m) \quad (2)$$

In the current study of ECG, the boundary element model of membrane and finite difference (FD) model of human torso are simulated
where current density and conductivity tensor are given by

$$J_c = -G_c \nabla V_m, \qquad G_c = f_c G(R_c \sigma_{iT}, \sigma_{iT}) \quad (3)$$

The governing equation of reaction-diffusion model can be expressed as

$$C_m \frac{\partial V_m}{\partial t} = \beta^{-1} \nabla \cdot (G(\sigma_{mL}, \sigma_{mT}) \nabla V_m) - I_{ion} \quad (4)$$

where $\sigma_{mL}$ $and$ $\sigma_{mT}$ $\beta$, $C_m$ and $I_{ion}$ are membrane surface, capacitance and ionic currents summation, respectively and
The boundary integral equation corresponding to (4) can be written as

$$\Psi_{e\ell}(r) = \frac{1}{2\pi(\sigma_\ell^- - \sigma_\ell^+)} \cdot \left[ \int J_c(r') \cdot \frac{r - r'^3}{|r - r'|^3} dV' + \sum_k \int_{S_k} (\sigma_k^- - \sigma_k^+) \Psi_e(r'') dR_{rr''} \right] (5)$$

where $S_k, \sigma, \sigma_\ell^-, \sigma_\ell^+$ $and$ $J_c$ are set of surfaces $k$, continuous isotropic conductivity, conductivity inside surface $\ell$, conductivity outside surface and source current density field respectively

The mathematical cancer modeling of the considered problem (see Fig. 2) can be expressed as follows

$$\frac{ds}{dt} = g_1(S) - g_2(S,T,C) \quad (6)$$

$$\frac{dT}{dt} = g_3(S,T,I,C) - g_4(S,T,C) \quad (7)$$

$$\frac{dI}{dt} = -g_5(S,I) \quad (8)$$

$$\frac{dC}{dt} = -g_6(S,C) \quad (9)$$

Where $s$ is the tumour volume, $T-$cells is the tumour density which are only considered active against the cancer in our considered modeling, $I$ is the concentration of immune-agent, $C$ is the radioactivity administred, $g_1$, $g_2$, $g_3$, $g_4$, $g_5$, $g_6$ are tumour logistic growth, tumour death, T-cell activation, T-cell death, immunotherapy decrease and radiotherapy decrease, respectively.

## 3. Numerical algorithm, results and discussion

The numerical modeling considered in the current paper based on the following algorithm

1) Solving the governing equation of monodomain reaction-diffusion which is replaced by the boundary integral equation (5) following the boundary element technique of Fahmy [12-15]
2) Solving the mathematical cancer modeling system (6) - (9) using the technique of Fahmy [8-11] and Houbolt's algorithm
3) Find the solution that satisfy steps (1) and (2) simultaneously
4) Find the effect of anisotropy

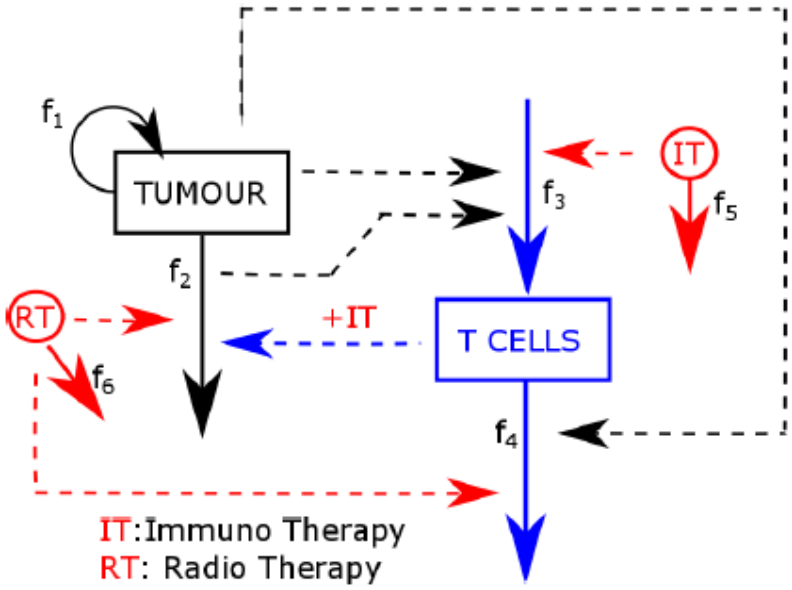


**Fig. 2 Model scheme representation.**

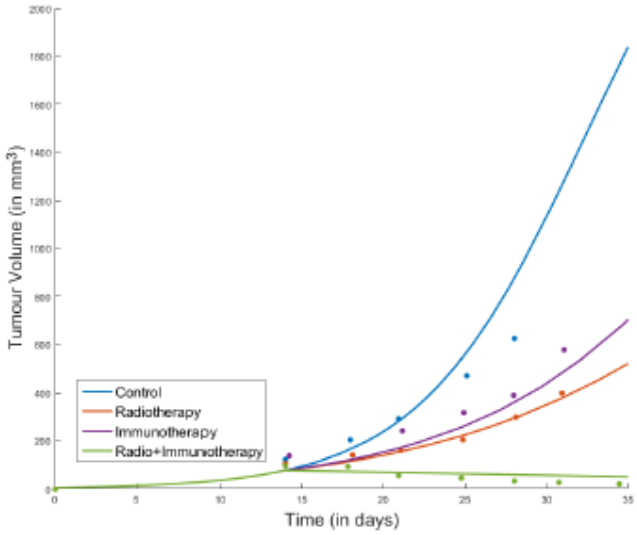


**Fig. 3. Variation of the tumour volume with time.**

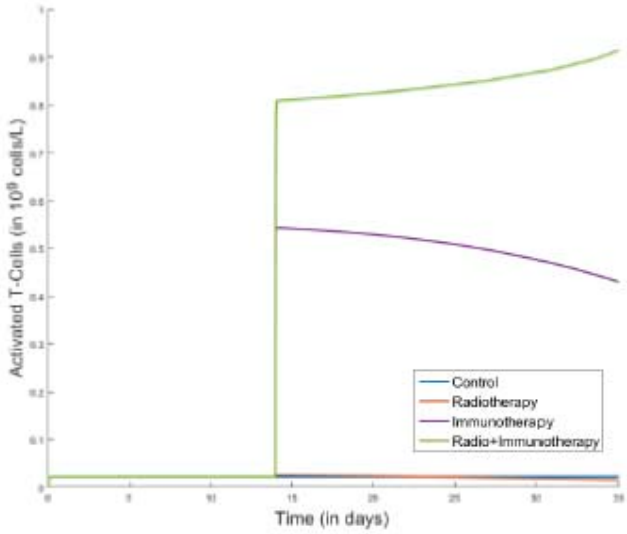


**Fig 4. Variation of the activated T-cells with time.**

It can be noticed from Fig. 3 that the IR and IO when used as a single agents can't reduce the tumour mass, but when they are used in a combination, the number of activated T-cells is higher than the single agents using of them as shown in Fig. 4

## 4. CONCLUSION

The boundary element algorithm for cancer modeling of cardiac anisotropy on the electrocardiogram (ECG) simulation. For a known set of conductivities, our results are in a very good agreement with the corresponding finite difference results. A lot of clinical applications neglect the effects of heart anisotropy, as an important result of our study, we concluded that the cardiac anisotropy has a strong effect on ECG simulation in comparison with considered isotropy effect. Also, if we considered the anisotropy effects, we can detect the heart cancer in people infected with it. The peoples eating plants, insects and animals. When they are eating plants such as vegetables and fruit with cancer can easily transmit it to humans when he takes it. When we make a sauce from rotten tomatoes, this sauce also can infect humans with heart cancer. Early detection of heart cancer can be difficult when we do not take into consideration cardiac anisotropy effect. It moves from infected tissues of organisms to healthy tissues of humans. For these reasons the anisotropy effect should be taken into consideration in clinical applications.



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