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# Effect of Vadasz Number on Magnetoconvection in a Darcy Porous Layer With Concentration Based Internal Heating

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## Authors' contributions

This work was carried out in collaboration between all authors. CIC and LE designed the study and performed the analysis. Author EA presented the plots/tables and interpreted the results. All authors read and approved the final manuscript.

## Abstract

In this research article, the effect of Vadasz number on magnetoconvection in a Darcy Porous Layer with concentration based internal heating is studied. The flow is governed by the Oberbeck-Boussineq model for Newtonian fluid. The stability analysis method based on the perturbation of infinitesimal amplitude is carried out using the normal mode analysis. The onset criterion for both the stationary and oscillatory convection on the stability of system is obtained. The analysis examines the effects of pertinent parameters on the stability of the system: magnetic field parameter, solutal Rayleigh number, Lewis number and Vadasz number. The result show that, internal heat parameter,  $Ri$  hastens the onset of instability in the system, whereas magnetic field,  $Ha$ , Lewis number,  $Le$ , Vadasz number,  $Va$  and solutal Rayleigh number,  $Rs$  delay the onset of instability.

**Keywords:** Thermal Rayleigh number, Double diffusive convection, normal mode analysis, Magnetoconvection, Concentration based internal heat.

## 1. Introduction

Studies on thermal instability with applied magnetic field in a Darcy porous layer with concentration based internal heating has received significant attention due to great number of areas of applications notably in industrial applications especially in nuclear reactors, cooling of electronic equipment, thermal insulation, food processing and reservoir modeling. Some of the researches in thermal instability in porous medium for different fluids and under various hydrodynamics and hydro magnetic conditions include [1,2,3,4,5,6,7,8]. Stability of a convective flow in a porous medium using Rayleigh's procedures was considered by [9]. Also, [10] considered Rayleigh instability of the thermal boundary layer in a flow through a porous medium. Comprehensive study of convection in a porous medium are contained in [11,12,13,14,15].

Thermal instability in a porous medium that is heated and salted from below under various conditions has equally received significant attention. The work of [16] is a classic example, where the effect of vertical magnetic field on the onset of double diffusion convection in a horizontal porous layer with convection based internal heat source is studied. Soret and magnetic field effect on thermosolutal convection in a porous medium with concentration based internal heat was considered by [17]. Cross-diffusion and reaction effect on double diffusive convection in a porous medium with concentration based internal heat source was studied by [18, 19,20].

In this study, we present the effect of Vadasz number on magnetoconvection in a Darcy porous layer with concentration based internal heating which hitherto has not been considered by other authors or researchers.

## 2. Mathematical Formulation

Consider an incompressible and electrically conducting fluid induced by concentration based internal heating of the form  $Q_0(c^* - c_0)$  at the center between two parallel horizontal plates located at  $z^* = 0$  and  $z^* = h$ . A Cartesian coordinate system  $(x^*, y^*, z^*)$  is chosen so that the origin is at the bottom of the porous layer and the gravitational force  $\vec{g}$  act downward, as shown in the Figure 1. The flow occur in the presence of a uniform externally applied vertical magnetic field  $\vec{B} = B_0 \vec{k}$ . The induced magnetic field is neglected because the magnetic Reynold number is small. Opposing temperature and concentration gradients are applied across the porous layer such that the lower plate is maintained at temperature and concentration  $T_0 + \Delta T$  and  $c_0 + \Delta c$ ; while the upper plate at temperature,  $T_0$  and concentration,  $c_0$ , where  $T_0$  and  $c_0$  are reference temperature and concentration respectively.

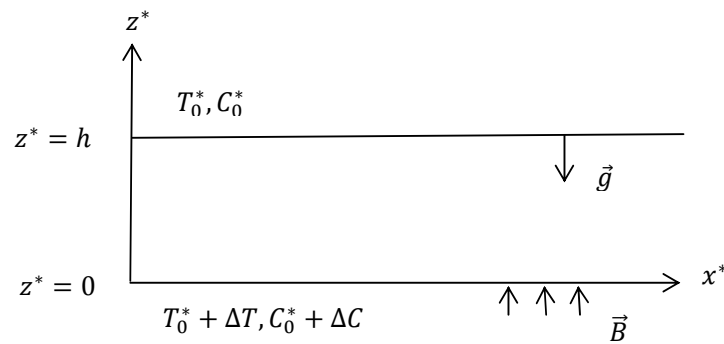


Figure 1: Schematic diagram of the problem

Also, we assume that the fluid is Newtonian with viscosity,  $\mu$ , thermal expansion coefficient,  $\beta_c$ , thermal conductivity,  $\kappa_T$ , diffusion coefficient,  $D_c$  and electric conductivity,  $\sigma_c$ .

We adopt the Boussinesq approximation [12]. The density of the mixture,  $\rho$ , is expressed linearly on both temperature and concentration in the form

$$\rho = \rho_0[1 - \beta_T(T^* - T_0) + \beta_c(c^* - c_0)] \quad (1)$$

where  $\rho_0$  is the reference density

Following [16], [21] and [22] and the assumptions made, the governing equations become

$$\vec{\nabla}^* \cdot \vec{V}^* \quad (2)$$

$$\frac{\rho_0}{\phi} \frac{\partial \vec{V}^*}{\partial t^*} = -\nabla^* P^* + \rho_0 g [\beta_T(T^* - T_0) - \beta_c(c^* - c_0)] \vec{k} - \frac{\mu}{\kappa} \vec{V}^* + \mu_e \vec{\nabla}^{*2} \vec{V}^* + \vec{F}_L \quad (3)$$

$$A \frac{\partial T^*}{\partial t^*} + (\vec{V}^* \cdot \vec{\nabla}^*) T^* = \alpha_T \vec{\nabla}^{*2} T^* + Q_0(c^* - c_0) \quad (4)$$

$$\phi \frac{\partial c^*}{\partial t^*} + ((\vec{V}^* \cdot \vec{\nabla}^*) c^* = \kappa_c \vec{\nabla}^{*2} c^* \quad (5)$$

$$\vec{j}^* = \sigma_c [E^* + \vec{V}^* \times \vec{B}^*], \quad \vec{\nabla}^* \cdot \vec{j}^* = 0 \quad (6)$$

With the following boundary conditions

$$\vec{V}^* = 0 \quad \text{on } z^* = 0, h \quad (7)$$

$$T^* = T_0 + \Delta T, c^* = c_0 + \Delta c \quad \text{on } z^* = 0 \quad (8)$$

$$T^* = T_0, c^* = c_0 \quad \text{on } z^* = h \quad (9)$$

where  $\vec{V} = (u^*, v^*, w^*)$  is the velocity,  $P^*$  is the pressure,  $\vec{E}$  is the electric field,  $\vec{j}$  is the current density,  $\kappa$  is the permeability of the porous medium,  $\phi$  is the porosity,  $Q_0 = \frac{Q}{(\rho c_p)_f}$  is the heat source parameter,  $\vec{k}$  is the unit vector in the  $z$ -direction. The subscripts  $m$  and  $f$  denotes the medium and fluid respectively.

The last term in Equation (3),  $\vec{F}_L$  represents the Lorentz force, which induces electromagnetic effect on the system. The appropriate form of Ohm's law given in Equation (6) for a moving medium employing the quasi-state approximation. The electric field,  $\vec{E}^*$  can be written in terms of the electrostatic potential,  $\varphi$  as

$$\vec{E} = -\nabla \varphi \quad (10)$$

In this article, we consider the case of electrically insulating boundaries for which  $\vec{E} = 0$  on the account that the electrostatic potential is a constant. With this the current density given in equation (6) reduces to,

$$\vec{j}^* = \sigma_c (\vec{V}^* \times \vec{B}^*) \quad (11)$$

Hence, the Lorentz force  $\vec{F}_L = \vec{j}^* \times \vec{B}$  becomes

$$\begin{aligned}\vec{F}_L &= \sigma_c(\vec{V}^* \times \vec{B}^*) \times \vec{B} \\ &= -\sigma_c B_0^2(U^*, V^*, 0)\end{aligned}\quad (12)$$

Using Equation (12) and the scales  $h$ ,  $\frac{h^2 A}{\alpha_T}$ ,  $\frac{\alpha_T}{h}$ ,  $\frac{Kh}{(\alpha_T \mu)}$  for length, time, velocity and pressure respectively; together with  $T = \frac{(T^* - T_0)}{\Delta T}$  for temperature,  $c = \frac{(c^* - c_0)}{\Delta c}$  for concentration,  $\varepsilon = \frac{\phi}{A}$  for porosity,  $\vec{V} = h\vec{V}^*$ , the dimensionless equations governing the motion of the fluid are:

$$\vec{\nabla} \cdot \vec{V} = 0 \quad (13)$$

$$\left(\frac{1}{\varepsilon} \frac{\partial}{\partial t} + 1\right) \vec{V} + Ha^2(U, V, 0) = -\nabla P + Ra T \vec{k} - Rs c \vec{k} \quad (14)$$

$$\frac{\partial T}{\partial t} + (\vec{V} \cdot \vec{\nabla})T = \nabla^2 T + Ri c \quad (15)$$

$$Le \varepsilon \frac{\partial c}{\partial t} + Le(\vec{V} \cdot \vec{\nabla})c = \nabla^2 c \quad (16)$$

The boundary conditions are:

$$w = 0, T = 1, c = 1 \text{ on } z = 0 \quad (17)$$

$$w = 0, T = 0, c = 0 \text{ on } z = 1 \quad (18)$$

The dimensionless parameters are:

$$\begin{aligned}Ra &= \frac{\phi Pr}{Da} = \text{Vadasz number}, \quad Pr = \frac{A\nu}{\alpha_T} = \text{Prandtl number}, \quad Da = \frac{\kappa}{h^2} = \text{Darcy number}, \quad Ra = \\ &\frac{\rho_0 g \beta_T \kappa h \Delta T}{\mu \alpha_T} = \text{Rayleigh number}, \quad Le = \frac{\alpha_T}{\alpha_c} = \text{Lewis number}, \quad Ha^2 = \frac{\alpha_c B_0^2 k}{\mu} = \text{Hartmann number}, \\ Ri &= \frac{h^2 Q_0 \Delta c}{\alpha_T \Delta T} = \text{internal heat parameter}, \quad Rs = \frac{\rho_0 g \beta_c \kappa h \Delta c}{\mu \alpha_T} = \text{solutal Rayleigh number}.\end{aligned}$$

## 2.1 Basic State

The basic state of the system is assumed to be quiescent and is described by

$$V_b = 0, T = T_b(z), c = c_b(z), p = p_b(z) \quad (19)$$

Substituting Equation (19) into Equations (13) – (16) and the boundary condition (17) and (18) yield the equations governing the basic state as

$$\frac{d^2 T_b}{dz^2} + Ri c_b = 0 \quad (20)$$

$$\frac{d^2 c_b}{dz^2} = 0 \quad (21)$$

$$\frac{dP_b}{dz} = Ra T_b - Rs c_b \quad (22)$$

Subject to

$$T_b = c_b = 1 \quad \text{on } z = 0 \quad (23)$$

$$T_b = c_b = 0 \quad \text{on } z = 1 \quad (24)$$

The integration of Equations (20) – (22), and application of boundary conditions (23) and (24), yield

$$T_b(z) = \frac{1}{6} [6(1 - z) + (2z - 3z^2 + z^3) Ri] \quad (25)$$

$$c_b(z) = 1 - z \quad (26)$$

$$P_b(z) = \int (RaT_b(z) - Rs c_b(z))dz \quad (27)$$

## 2.2 Linearization and Perturbation Solution

To study the stability of the basic state, we now superimpose small perturbations in the form

$$V = \vec{V}_b + \vec{v}, p = p_b(z) + p, T = T_b(z) + \theta, C = c_b(z) + \varphi \quad (28)$$

where  $\vec{v}, p, \theta$  and  $\varphi$  are the perturbed quantities over their equilibrium counterparts and are assumed small. On substituting Equation (28) into Equations (13)–(16) and the boundary condition (17) and (18), and then using Equation (28), we obtain the linearized equations as

$$\vec{\nabla} \cdot \vec{v} = 0 \quad (29)$$

$$\left(\frac{1}{\nu a} \frac{\partial}{\partial t} + 1\right) \vec{v} + Ha^2(u, v, 0) = -\nabla P + Ra \theta \vec{k} - Rs \varphi \vec{k} \quad (30)$$

$$\frac{\partial \theta}{\partial t} + (\vec{v} \cdot \vec{\nabla})\theta + f(z)w = \nabla^2 \theta + Ri \varphi \quad (31)$$

$$Le \varepsilon \frac{\partial \varphi}{\partial t} + Le(\vec{v} \cdot \vec{\nabla})\varphi - Lew = \nabla^2 \varphi \quad (32)$$

where  $f(z) = \frac{\partial T_b}{\partial z}$  is the basic temperature gradient given by

$$\frac{\partial T_b}{\partial z} = -1 + \frac{Ri}{6}(2 - 6z + 3z^2)$$

The impermeable boundary conditions are

$$w = \theta = \varphi = 0 \quad \text{on } z = 0, 1 \quad (33)$$

Next, the pressure term is eliminated by taking the double curl of Equation (30) and keeping only the  $z$  – component. Then the system reduces to

$$\left(\frac{1}{\nu a} \frac{\partial}{\partial t} + 1\right) \nabla^2 w + Ha^2 \frac{\partial^2 w}{\partial z^2} - Ra \nabla_h^2 \theta + Rs \nabla_h^2 \varphi = 0 \quad (34)$$

$$f(z)w + \left(\frac{\partial}{\partial t} - \nabla^2\right) \theta - Ri \varphi = 0 \quad (35)$$

$$Lew - \left(Le \varepsilon \frac{\partial}{\partial t} - \nabla^2\right) \varphi = 0 \quad (36)$$

where  $\nabla_h^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}$  is the Laplacian in the horizontal plane.

The boundary condition for the system (34) – (36) are

$$w = \frac{\partial^2 w}{\partial z^2} = \theta = \varphi = 0 \quad \text{on } z = 0, 1 \quad (37)$$

Assuming further that the conductive motion exhibit horizontal periodicity, then we seek a time dependent periodic disturbance of the form [23]

$$\begin{pmatrix} w \\ \theta \\ \varphi \end{pmatrix} = \begin{pmatrix} W(z) \\ \Theta(z) \\ \Phi(z) \end{pmatrix} f(x, y) e^{\sigma t} \quad (38)$$

where  $\sigma = (\omega_r + i\omega_i)$  is the growth rate and is in general complex, with  $\omega_r, \omega_i$  real and

$f(x, y)$  is a horizontal plane tilting the  $xy$  -plane periodically.  
The substitution of Equation (38) into Equations (34) - (36) yield the eigenvalue problem.

$$\left(\frac{\sigma}{Va} + 1\right)(D^2 - a^2)w + Ha^2w + a^2Ra \Theta - a^2Rs \Phi = 0 \quad (39)$$

$$-f(z)w + (D^2 - a^2 - \sigma^2)\Theta + Ri \Phi = 0 \quad (40)$$

$$Le w + (D^2 - a^2 - Le \varepsilon \sigma)\Phi = 0 \quad (41)$$

where  $D = \frac{\partial}{\partial z}$  and  $\nabla_h^2 + a^2 f = 0$ ,  $a$  is the wave number

$$w = \Theta = \Phi = 0 \quad \text{on } z = 0, 1 \quad (42)$$

We further assume the eigenvalue of Equations (39) – (41) of the form

$$\begin{pmatrix} W(z) \\ \Theta(z) \\ \Phi(z) \end{pmatrix} = \begin{pmatrix} W_0 \\ \Theta_0 \\ \Phi_0 \end{pmatrix} \sin \pi z \quad (43)$$

where  $W_0, \Theta_0, \Phi_0$  are constants. Substitution of Equation (43) into Equations (39) – (41) yield

$$H\bar{X} = 0 \quad (44)$$

where

$$H = \begin{pmatrix} J\left(1 + \frac{\sigma}{Va}\right) + \pi^2 Ha^2 & -a^2 Ra & a^2 Rs \\ 2F & J + \sigma & -Ri \\ -Le & 0 & J + Le \varepsilon \sigma \end{pmatrix}$$

$$\bar{X} = (W_0, \Theta_0, \Phi_0)^T, J = a^2 + \pi^2 \text{ and } F = \int_0^1 f(z) \sin^2 \pi z dz$$

The solvability of the eigenvalue problem in Equation (44) requires that  $|H| = 0$  This requirement yields the thermal Rayleigh number,  $Ra$  as

$$Ra = \frac{4\pi^2 J(1 + \frac{\sigma}{Va})(J + \sigma)(J + Le \varepsilon \sigma)}{a^2(b_2 + b_0 Le \varepsilon \sigma)} + \frac{4\pi^4 Ha^2(J + \sigma)(J + Le \varepsilon \sigma)}{a^2(b_2 + b_0 Le \varepsilon \sigma)} + \frac{4\pi^2 Le Rs(J + \sigma)}{b_2 + b_0 Le \varepsilon \sigma} \quad (45)$$

where

$$\begin{aligned} b_0 &= 4\pi^2 + Ri \\ b_1 &= 4\pi^2 Le Ri \\ b_2 &= b_0 J + b_1 \end{aligned}$$

For neutral solutions, we set  $\sigma = i\omega_i$  in Equation (45) and rearranging yields

$$Ra = \frac{4\pi^2}{\Delta} [\Delta_1 + i\omega_i \Delta_2] \quad (46)$$

where

$$\begin{aligned} \Delta_1 &= \frac{1}{a^2} \left[ J \left( 1 + \frac{J}{Va} \right) + Le \varepsilon \left( J - \frac{\omega^2}{Va} \right) \right] Le \varepsilon J \omega^2 + J b_2 \left[ J \left( J - \frac{\omega^2}{Va} \right) - Le \varepsilon \left( 1 + \frac{J}{Va} \right) \omega^2 \right] + Ha^2 \pi^2 [J Le \varepsilon (1 + \\ & Le \varepsilon) b_0 \omega^2 + (J^2 - Le \varepsilon \omega^2) b_2] + Le Rs (J b_2 + b_0 \omega^2) \end{aligned}$$

$$\Delta_2 = \frac{1}{a^2} \left[ \left[ \frac{J^2 Le \varepsilon b_0}{Va} (Le \varepsilon + 1) \omega^2 + J Le \varepsilon (Le \varepsilon \omega^2 - J^2) \right] b_0 + \left[ J^2 \left( 1 + \frac{1}{Va} \right) + J Le \varepsilon \left( J - \frac{\omega^2}{Va} \right) \right] b_2 \right. \\ \left. + [(J + Le \varepsilon) b_2 - (J^2 - Le \varepsilon \omega^2) Le \varepsilon b_0] \right] + Le Rs (b_2 - J Le \varepsilon b_0) \\ \Delta = b_2^2 + b_0^2 Le^2 \varepsilon^2 \omega^2$$

Since  $Ra$  is a physical quantity, it must be real. Hence from Equation (46), it follows that either  $\omega = 0$  for the onset of stationary convection or  $\Delta_2 = 0$ ,  $\omega \neq 0$  for the onset of oscillatory convection.

### 3 Onset of Stationary Convection

For the validity of principle of exchange of stabilities to hold for marginal stationary convection,  $\omega = 0$ . Setting  $\omega = 0$  in Equation (46) yields the Rayleigh number,  $Ra^{st}$  for the stationary convection as

$$Ra^{st} = \frac{4\pi^2}{b_2} \left[ \frac{J^3}{a^2} + \frac{Ha^2 \pi^2 J^2}{a^2} + Le Rs \right] \quad (47)$$

In the absence of  $Ri = 0$ ,

$$Ra^{st} = \frac{1}{a_2} [J^2 + Ha^2 \pi^2 J] + Le Rs$$

Further, if  $Ha = 0$ ,  $Ra^{st}$  simplifies to

$$Ra^{st} = \frac{(a^2 + \pi^2)^2}{a^2} + Le Rs \quad (48)$$

which coincides with the earlier results of [16] and [24]

Further, when  $Rs = 0$ , the stationary Rayleigh number given in Equation (48) reduces to the classical result of [9] and [25]

$$Ra^{st} = \frac{(a^2 + \pi^2)^2}{a^2} \quad (49)$$

In addition, Equation (48) gives the critical Rayleigh number  $Ra_c^{st} = 4\pi^2$  with corresponding critical wave number,  $a_c^{st} = \pi$

### 3.1 Oscillatory Convection

For the onset of oscillatory convection  $\Delta_2 = 0$  and  $\omega = 0$ . Setting  $\Delta_2 = 0$  and  $Ra = Ra^{(os)}$  in Equation (46) gives the expression for Rayleigh number as

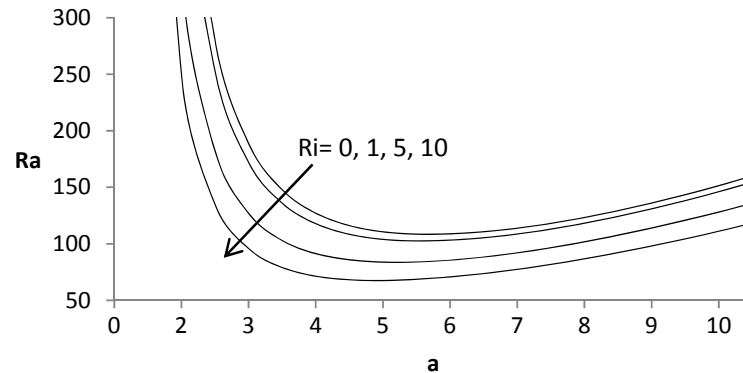
$$Ra^{os} = \frac{4\pi^2}{a^2 \Delta} \left\{ \left[ J \left( 1 + \frac{J}{Va} \right) + Le \varepsilon \left( J - \frac{\omega_i^2}{Va} \right) \right] Le \varepsilon J \omega_i^2 + J b_2 \left( J - \frac{\omega_i^2}{Va} \right) - Le \varepsilon \left( 1 + \frac{J}{Va} \right) \omega_i^2 \right\} + \\ Ha^2 \pi^2 [J Le \varepsilon (1 + Le \varepsilon) b_0 \omega_i^2 + (J^2 - Le \varepsilon \omega_i^2) b_2] + \frac{4\pi^2 Le Rs}{\Delta} (J b_2 + b_0 \omega_i^2) \quad (50)$$

where the frequency of oscillation is given by

$$\omega_i^2 = \frac{\{Ha^2 \pi^2 (J(J-1))(4\pi^2 + Ri) - 4\pi^2 Le Ri + J Le (a^2(4\pi^2 + Ri)Rs - 4\pi^2 J Ri) - (4\pi^2 Le Ri + J(4\pi^2 + Ri)) \left( \frac{J^3}{Va} + J^2 + Ha^2 \pi^2 J + a^2 Le Rs \right)\}}{\{Le \varepsilon \left[ \frac{4\pi^2 J}{Va} (J - Ri) + (Ha^2 \pi^2 + J)(4\pi^2 + Ri) \right]}}$$

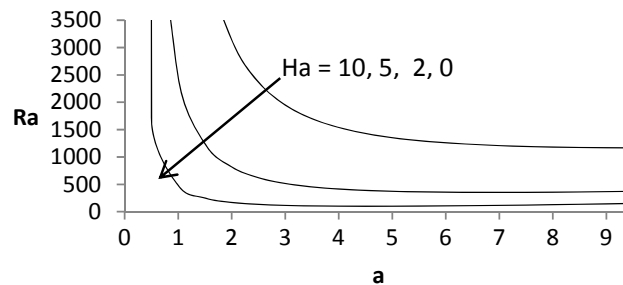
#### 4 Discussion of Results

The influence of Vadasz number on magnetoconvection in Darcy porous layer with concentration based internal heating is studied analytically using the linear stability analysis technique. The expressions for both the stationary and oscillatory modes for different values of the governing parameters such as magnetic field parameter,  $Ha$ , solute Rayleigh number,  $Rs$ , internal heat parameter,  $Ri$  and Lewis number,  $Le$ , are computed and the results are displayed in Figures 2-9.



**Figure 2:** Variation of thermal Rayleigh number for various values of internal heat parameter,  $Ri$  for stationary convection.

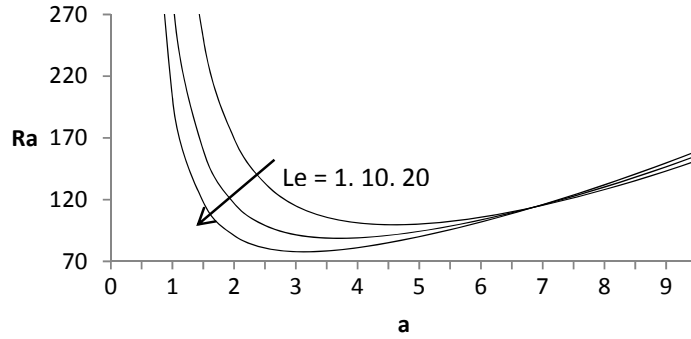
The Influence of the internal heat parameter on the onset of instability for fixed values of  $Ha = 2$ ,  $Rs = 5$ ,  $Le = 1$  in stationary convection is displayed in Figure 2. It is observed that increase in the internal heat decreases the thermal Rayleigh number, for stationary convection. This implies that internal heat hastens instability, which leads to a destabilization of the system.



**Figure 3:** Variation of thermal Rayleigh number for various values of the magnetic parameter,  $Ha$  for stationary convection

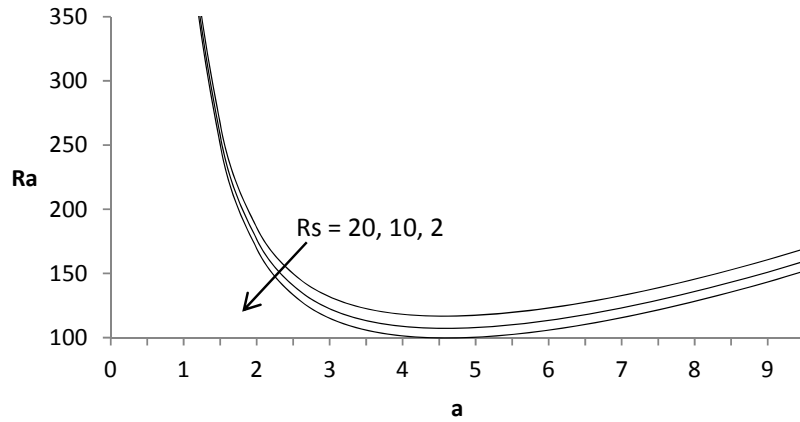
**Figure 3** show numerically the computed values for  $Rs = 2$ ,  $Ri = 2$  and the influence of the Magnetic field parameter,  $Ha$  on the thermal Rayleigh number. The result show increase in Magnetic field increases the thermal Rayleigh number for the stationary mode. The result is an indication that Magnetic field stabilizes the system.





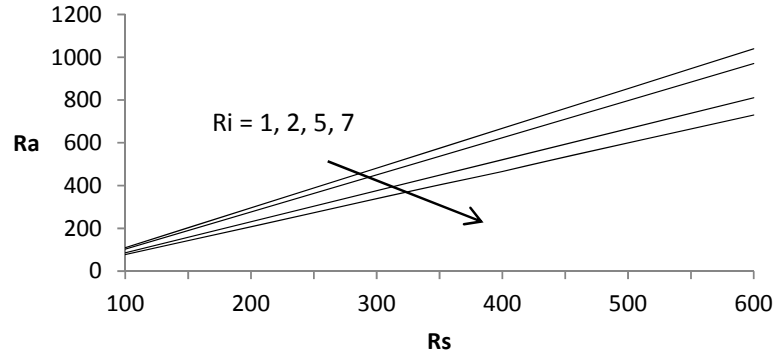
**Figure 4:** Variation of thermal Rayleigh number for various values of the Lewis number,  $Le$  for stationary convection

**Figure 4** depicts the influence of Lewis number,  $Le$ , on the thermal Rayleigh number,  $Ra$  for fixed values of  $Ha = 2, Rs = 2, Ri = 1$  in stationary convection. It is observed that increase in Lewis number leads to an increase in the thermal Rayleigh number,  $Ra$  which is an indication that Lewis number delays the onset of instability in the system.



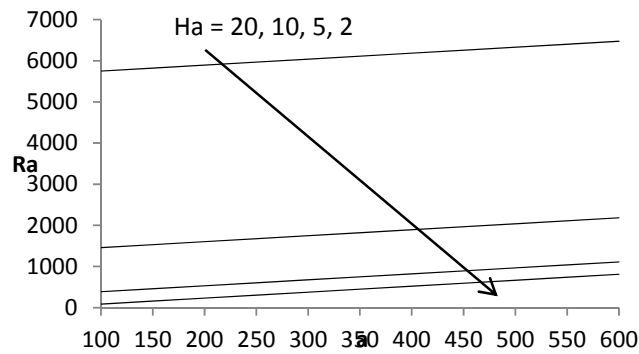
**Figure 5:** Variation of thermal Rayleigh number for various values of solutal Rayleigh number,  $Rs$  in stationary convection

The effect of solutal Rayleigh number,  $Rs$ , on the thermal Rayleigh number for fixed  $Ha = 2, Ri = 1, Le = 1$  is illustrated in **Figure 5**. The result shows increase in solutal Rayleigh number increases the thermal Rayleigh number. This implies that solutal Rayleigh number stabilizes the system for stationary convection.



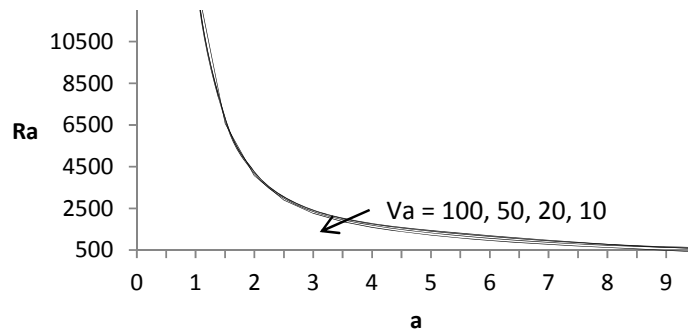
**Figure 6:** Variation of thermal Rayleigh number with solutal Rayleigh number for various values of the internal heat parameter,  $Ri$  in stationary convection

Figure 6 show the influence of the internal heat parameter,  $Ri$ , on the thermal Rayleigh number for fixed  $Ha = 2, Le = 1, a_c = \pi$ . It is observed that increase in internal heat decreases the thermal Rayleigh number. This is an indication that, the system is destabilized in the presence of internal heat parameter.



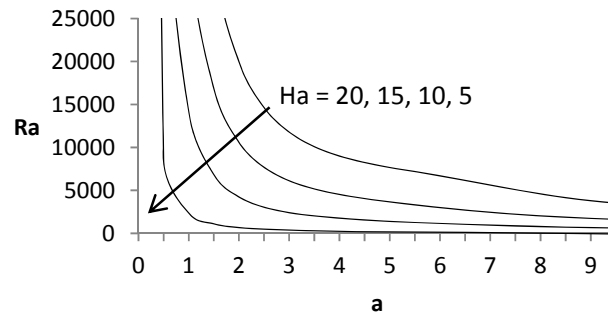
**Figure 7:** Variation of thermal Rayleigh number for various values of magnetic parameters,  $Ha$  in oscillatory convection

Figure 7 show a linear relationship between the thermal Rayleigh number,  $Ra$  and the solutal Rayleigh number,  $Rs$  for variations in the Magnetic field parameter,  $Ha$ . Increase in Magnetic field increases the thermal Rayleigh number which is an indication that the system is stabilized in the presence of Magnetic field for stationary convection.



**Figure 8:** Variation of thermal Rayleigh number for various values of the Vadasz number,  $Va$  in oscillatory convection

The influence of Vadasz number,  $Va$  on the thermal Rayleigh number,  $Ra$  for oscillatory convection with fixed values of  $Ha = 10, Ri = 2$  is depicted in Figure 8. The result show increase in Vadasz number increases the thermal Rayleigh number which indicates that Vadasz number delays the onset of instability in the system.



**Figure 9:** Variation of thermal Rayleigh number for various values of magnetic parameter,  $Ha$  in oscillatory convection

Figure 9 show the influence of Magnetic field parameter,  $Ha$  on the thermal Rayleigh number,  $Ra$  for fixed values of  $Va = 50, Ri = 2$  for oscillatory convection. It is evident that increase in Magnetic field increase the thermal Rayleigh number, which is an indication of the stabilization of the system.

## 5 Conclusion

The effect of Vadasz number on magnetoconvection in a Darcy porous layer with concentration based internal heating has been studied using the linear stability analysis. The porous layer is heated and salted from below. The roles of the governing parameters on the stability of the system were investigated. The result show that, the presence of the internal heat parameter,  $Ri$  destabilizes the system for both stationary and oscillatory modes. On the other hand, positive increase in the values of the magnetic parameter,  $Ha$ , Vadasz number,  $Va$ , Lewis number,  $Le$  and solutal Rayleigh number stabilizes the system.

## Competing Interests

Authors have declared that no competing interest exist.

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