Earthquake Fastens Earth Rotation

Abstract

The relation between earthquake and planet motion is one of topics interesting to people. Based on conservation law of energy, this paper proves that the energy release by earthquake proportions to the square of velocity of Earth's rotation, while velocities of revolution of Earth and Moon remain unchanged. Further more, earthquake happening in pulse-mode is proved by Principle of minimum energy release. Testing examples of Japan 2011-3-11 earthquake, 1960 Great Chilean earthquake, and 2004 Indonesia Sumatra earthquake show that the shortening of a day caused by earthquake depends on the time of earthquake lasting, the shortening, the lasting.

Keywords: Earth rotation, conservation law of energy, earthquake energy release, Japan 2011-3-11 earthquake, 1960 Great Chilean earthquake, 2004 Indonesia Sumatra earthquake

1. Introduction

Relations between earthquakes and rotational variations of the Earth has been the subject of many journal publications since 1960. The methods used are statistic methods, e.g., a quote from the abstract of Chao, B.F. & R.S. Gross.(1995): "An extremely strong statistic is found for the earthquakes' tendency to increase the Earth's spin energy; the rate during

1977 to 1993 was +6.7 GW, about the same as the total seismic-wave energy release". Guo and Xu (1988) study great earthquake also by statistic method, using historical data, on the relation between earthquake and velocity of earth's rotation. Chen et al. (2.13) analyze the relationship between seismicity and the earth rotation by statistic method. However, statistic method does not give the reason of the relation between energy release of earthquake and velocity of earth's rotation, but just links the connection from phenomenon observation. Further more, earthquake is a temporal disturb, the results obtained by methods of using a long-term data to a temporal action seems to be lacking in logical reasonability. Gross, R. S. & B. F. Chao (2006, 2000) study the signature of the 2004-12-26 Sumatran earthquake, and the gravitational signature of earthquakes, in Gravity, Geoid, and Geodynamics. However, no relationship between the velocity of Earth's rotation and the earthquake scale has been established. Richard (2011) reported that according to the calculation of Richard Gross, NASA's Jet Propulsion Laboratory in Calf, the Japan 2011-03-11 earthquake shortening the length of day by 1.8 microseconds (μs). He also gives the shortening the length of day by 1.26 μs for 1960 Great Chilean earthquake and by 6.8μs for 2004 Indonesia Sumatra earthquake with the same model, reported by Baike 360 (2015). However, he does not give the details of model and calculation or equation linking to the **energy release of earthquake** (ERE) and shortening of day.

The aim of this paper is to establish an equation linking ERE and velocity of earth's rotation based on the **conservation law of energy** (CLE).

The CLE is one of the greatest discovers in 19^{th} century and has important use in wide fields. For examples, Yun and Li (1995) use CLE to prove the accuracy of calculation for an In-plane-hinge-joint rigid sloping piles' group, where no previous works can be use for comparing and no experiment can be done. Yun (2015) uses CLE to determine the seeking range of a missing plane MH370. Yun (2016) uses CLE to judge an investment will be success or failure. Here, CLE is again used to establish an equation linking up ERE and velocity of earth rotation.

2. Study range, coordinates system, and basic hypotheses

We study the isolated sun-earth-moon systems. An isolated system is defined the system in a stable equilibrium state that the total energy or work done by external forces and internal forces keeps unchanged . Thus, CLE holds for isolated system.

Cartesian coordinates and Cylindrical coordinates

Let (x, y, z) be the Cartesian coordinates with earth's center at $O_e(0, 0, 0)$, the z-axes, perpendicular to the equatorial plane xO_ey, be the earth's rotating axis with $z = 0$ at xO_ey.

Let (r, θ, z) be the cylindrical coordinates of the geometric center of the earth. The relation between (x, y) and (r, θ) is:

$$\begin{cases} x = r\cos\theta, \\ y = r\sin\theta, \end{cases} \quad (0 \le \theta \le 2\pi,\ 0 \le r < \infty, -\infty < z < \infty) \qquad (2\text{-}1)$$

Basic Hypotheses

A spherical earth with spherical-symmetry, continuously fully filled liquid mantle.

3. The method of analysis.

3.1 The CLE states that the change of energy of an isolated system is zero.

$$\Delta E_k + \Delta E_p + \Delta Q = C, \qquad (3\text{-}1)$$

Where $\Delta E_k = E_k(t_1) - E_k(t_0)$, $\Delta E_p = E_p(t_1) - E_p(t_0)$, $\Delta Q = Q(t_1) - Q(t_0)$, are the change of kinetic energy, potential energy and heat and electromagnetic energy respectively; C is a constant; t_0 is the time just before earthquake, t_1 is the time at earthquake over.

3.2 **The change of kinetic energies in sun-earth-moon system**

(1) The change of kinetic energy of earth's rotation

The change of kinetic energy for earth rotation is:

$$\Delta E_k = E_k(t_1) - E_k(t_0) = 2\pi \int_0^{R_e} dz \int_0^{r_z} [\int_{t_0}^{t_1} v^2(t)dt] r\rho(r,z,t)dr =$$

$$2\pi \int_0^{R_e} dz \int_0^{r_z} [v^2(t_1) - v^2(t_0)] r\rho(r,z,t)dr =$$

$$2\pi[\omega_c^2(t_1) - \omega_c^2(t_0)] \int_0^{R_e} dz \int_0^{r_z} r^3\rho(r,z,t)dr, \qquad (3\text{-}2)$$

Where $\rho = \rho(r,z,t)$ is mass density; $\omega_c = \omega = v/r$ is the rotation angular velocity of the crust. The appendix shows $\omega_c = \omega = v/r$ is a constant and can be moved out of the integral sign; $r_z = \sqrt{R_e^2 - z^2}$, a point on (r_z, z) of crust.

By continuity, using theorem of mean value of integration twice, we can move ρ out of the integral sign, then (3-2) becomes:

$$\Delta E_k = 2\pi[\omega_c^2(t_1) - \omega_c^2(t_0)]\rho_m \int_0^{R_e} dz \int_0^{r_z} r^3 dr$$

$$= \frac{\pi}{2}[\omega_c^2(t_1) - \omega_c^2(t_0)]\rho_m \left[R_e^5 - 2R_e^2 \frac{1}{3} R_e^3 + \frac{1}{5} R_e^5\right] = \frac{1}{5}[\omega_c^2(t_1) - \omega_c^2(t_0)]R_e^2 m_e, \quad (3\text{-}3)$$

$$m_e = \frac{4\pi}{3} R_e^3 \rho_m, \quad (3\text{-}4)$$

Where $R_e = 6{,}371{,}012\ \text{km}$ is the radius of the earth; $m_e = 5.976 \times 10^{21}\,(\text{kg})$ is the mass of the earth, ρ_m is the mean value of mass density, which can be calculated by (3-4).

(2) The change of kinetic energy of Earth revolving around Sun.

The kinetic energy of Earth revolving around Sun E_{krs} is:

$$E_{krs} = \frac{1}{2} m_e v_{rs}^2(t) = \frac{1}{2R_{se}^2} m_e \vartheta^2(t), \quad (3\text{-}5)$$

Where $R_{se} = R_{se}(t)$ is the distance between centers of earth and sun.

$v_{rs} = \vartheta R_{se}$ and ϑ are the velocity and angular velocity of earth revolved the sun respectively.

m_e is the mass of earth.

By the equilibrium equation of centrifugal force and force of universal gravitation,

$\sum F_{ri} = 0$, we have

$$m_e \frac{\vartheta^2}{R_{se}} = G\frac{m_e M_s}{R_{se}^2}, \quad (3\text{-}6)$$

Where G is gravitational constant; m_s is the mass of sun.

Substituting (3-6) into (3-5), we have

$$E_{krs} = \frac{1}{2}G\frac{m_e M_s}{R_{se}}, \quad (3\text{-}7)$$

$$\Delta E_{krs} = E_{krs}(t_1) - E_{krs}(t_0) = 0, \quad (3\text{-}8)$$

Since m_e, M_s, R_{se} are the same in $[t_0, t_1]$, then we have:

$$\vartheta(t_1) = \vartheta(t_0), \quad (3\text{-}9)$$

Eq. (3-9) shows that the velocity of earth revolved the sun keeps unchanged during earthquake.

(3) The change of kinetic energy of Moon revolving around Earth

The kinetic energy of Moon revolving around Earth E_{kmo} is:

$$E_{kmo} = \frac{1}{2}m_m v_{re}^2(t) = \frac{1}{2R_{em}^2}m_m\varphi^2(t), \quad (3\text{-}10)$$

Where $R_{em} = R_{em}(t)$ is the distance between centers of earth and moon. $v_{re} = \varphi R_{em}$ and φ are the velocity and angular velocity of moon revolved the earth respectively. m_m is the mass of moon.

By the equilibrium equation of centrifugal force and force of universal gravitation,

$\sum F_{ri} = 0$, we have

$$m_m \frac{\varphi^2}{R_{em}} = G\frac{m_e m_m}{R_{em}^2}, \quad (3\text{-}11)$$

Substituting (3-11) into (3-10), we have

$$E_{kmo} = \frac{1}{2}G\frac{m_e m_m}{R_{em}}, \quad (3\text{-}12)$$

$$\Delta E_{kmo} = E_{kmo}(t_1) - E_{kmo}(t_0) = 0, \quad (3\text{-}13)$$

Since m_e, m_m and R_{em} are the same at t_1 and t_0, then we have

$$\varphi(t_1) = \varphi(t_0), \quad (3\text{-}14)$$

Eq. (3-14) shows that the velocity of moon revolved the earth keeps unchanged during earthquake.

3.2 The change of potential energy E_p in sun-earth-moon system

Common type of potential energy includes: gravitational potential energy, deformation energy and electric potential energy, etc.

(1) The gravitational potential energy E_{pg}

Yun (2017) considered the potential energy in atmosphere involved gravity, buoyancy, centrifugal force and lateral buoyancy. However, the gravitational potential energy E_{pg} inside the earth is unknown yet, therefore its calculation is difficult.

(2) The deformation energy E_{deform}

One can calculate the deformation energy theoretically by definition, the displacement -strain relation, the constituting equation and the equilibrium equation. However, the theoretical calculation of the default deformation energy is nearly impracticable.

(3) The heat and electromagnetic energies E_Q

The calculation of E_Q is also nearly impossible. Since the individual calculation of E_{pg}, E_{deform}, and E_Q at a critical state of earthquake is nearly impossible, why not try to instead of these calculation by measuring the total value K of ERE ? That is, measuring K,

$$K = \Delta E_p = \Delta E_{pg} + \Delta E_{deform} + \Delta E_Q, \qquad (3\text{-}15)$$

If $K = \Delta E_p = 0$, it means no energy release, the system is in an equilibrium state.

To understand the sudden released of the stored deformation energy K, using a common tensile breaking test is best. A pair of tensile forces F apply to two ends of a testing specimen. The work done by forces F on elongation of the specimen is stored in deformation energy K. When the specimen broken, the stored energy is suddenly released and transferred by impacting to the testing frame with loud impacting sound, which likes an earthquake of suddenly breaking in some place of a plate. Similar laboratory test for seismic energy release can be found in (Boler & Spetzier, 1986).

Let the measured ERE between $[t_0, t_1]$ be K (j), then

$$\Delta E = \Delta E_k + K = C, \quad \text{(Joules)} \tag{3-16}$$

Substituting (3-3) into (3-16), we have

$$\omega_c^2(t_1) - \omega_c^2(t_0) = C - \frac{5K}{m_e R_e^2}, \tag{3-17}$$

If $K = \Delta E_p = 0$, then, $\omega_c^2(t_0) = \omega_c^2(t_1)$, by (3-16), C = 0.Then (3-17) becomes:

$$\omega_c^2(t_1) - \omega_c^2(t_0) = -\frac{5K}{m_e R_e^2}, \tag{3-18}$$

Where $K = -$, defines the system losing energy, or releasing energy; $K = +$, defines the system absorbing energy. Eq. (3-18) shows that $K = -$, $\omega_c^2(t_1) > \omega_c^2(t_0)$, the rotation velocity is increasing during earthquake.

Eq. (3-18) is one of main results of this paper. It states that the square of changing of earth rotation velocity is proportions to the release energy of the earthquake. It is a conservation law of energy that the release energy of earthquake is converted to the kinetic energy of earth rotation.

3.3 Principle of minimum energy release (PMER)

Here, we state the so-called Principle of minimum energy release as that if there are many possible paths to reach a goal, the actual carried out (or the best) path is that one which releases minimum energy.

Now, we use PMER to prove the mode of ERE is a pulse-mode in a short time interval.

Rewrite (3-18),

$$Y = \omega_c^2(t_1) - \omega_c^2(t_0) = -\frac{5K}{m_e R_e^2}, \qquad (3\text{-}19)$$

From the right hand side, Y represents releasing energy. Construct a path function $p(t) = \omega_t^2(t)$, such that

$$Y = \int_{t_0}^{t_1} p(t)dt = \int_{t_0}^{t_1} \omega_c^2(t)\, dt, \qquad (3\text{-}20)$$

There are many paths $p(t)$ or $\omega_c(t)$, satisfying (3-20). For example, a straight line or a curve line from t_0 to t_1. The actual one should be the minimum release of energy.

Proof:

This is an optimization problem, i.e.,

$$\min_{\omega_c} Y = \min_{\omega_c} \int_{t_0}^{t_1} \omega_c^2(t)dt, \qquad (3\text{-}21)$$

The necessary condition for this problem to be optimum is:

$$\frac{dY}{d\omega_c} = 0, \qquad (3\text{-}22)$$

Then, we have

$$2\int_{t_0}^{t_1}\omega_c(t)\frac{d\omega_c(t)}{dt}dt = 0, \quad (3\text{-}23)$$

Since t_0, t_1 can be arbitrary chosen, by Newton-Leibniz formula, the integrand of (3-23) must be zero. $\omega_c(t) \neq 0$, then, we have

$$\omega_c(t) = \text{const.} = \omega_c(t_0) \qquad (t_0 \leq t < t_1) \quad (3\text{-}24)$$

$$\omega_c(t) \to \omega_c(t_0), \qquad (t \to t_0) \quad (3\text{-}25)$$

Eqs. (3-24), (3-25) show that $\omega_c(t)$ is a broken line: for $(t_0 \leq t < t_1)$, $\omega_c(t) = \omega_c(t_0)$; for $(t \to t_0)$, $\omega_c(t) \to \omega_c(t_0)$. That means that the earthquake releases energy suddenly at $(t \to t_0)$ in a short time interval like few seconds or minutes, i.e., in a pulse-mode. □

4. Discussion

Question1**: Why the velocity of Earth rotation is increasing , while the velocities of Earth and Moon revolution remain unchanged during an earthquake?**

Answer: during earthquake, the system releases energy, according to CLE, the system must increase its kinetic energy, i.e., increasing the velocity of earth rotation to keep constant of the system's energy. Since the velocities of earth and moon revolution are determined by equilibrium equations (3-9) and (3-14) respectively, and they are independent to ERE, therefore they keep unchanged.

Question 2**: Why a larger earthquake usually happened to be at few seconds or minutes?**

Answer: According to PMER, if there are many possible paths to reach a goal, the actual one is that which releases minimum energy. The pulse-mode is the mode with minimum energy release.

5. Testing examples of calculation.

We use 4 examples of earthquakes to test the calculation.

5.1 The Japan 2011-03-11 earthquake, $M_w = 9.0$, $t_0 = 13{:}46$; $t_1 = 13{:}48$.

5.1.1 The formula of calculation of release energy of earthquake

There are various earthquake scaling laws (Robert A Meyers (Eds) 2009). Here, we use the formula of (Hanks & Kanamori, 1979) to calculate ERE, where assuming constant stress drop, Kanamori defines the moment magnitude M_w based on the empirical relation of (Gutengerg & Richttel, 1942, 1956) between surface wave and seismic energy change E_r.

$$\log E_r = 1.5M_w + 4.8, \qquad (5.1\text{-}1)$$

Where E_r is the change of the seismic energy in Joules.

Simplicity is the reason of why we choice Hanks &Kanamori's formula (5.1-1).

Substituting $M_w = 9.0$ into (5.1-1), we have

$$\log E_r = 18.3, \qquad (5.1\text{-}2)$$

then $E_r = 1.999 \times 10^{18}$, (Joules).

Where E_r is equal to ΔE_p of this paper, i.e.,

$$K = \Delta E_p = E_r, \qquad (5.1\text{-}3)$$

Substituting (5.1-3) into (3-18), we have

$$[\omega_c^2(t_1) - \omega_c^2(t_0)]m_e R_e^2 = 5 \times 1.999 \times 10^{18}(\text{Joules}), \tag{5.1-4}$$

Where $R_e = 6{,}371{,}012$ km; $m_e = 5.976 \times 10^{21}$(kg). 1(Joules) = 1(N.m), 1s = 10^6μs, $\omega_c^2(t_0) = \left(\frac{2\pi}{24\times60\times60}\right)^2 = 5.288 \times 10^{-21}(\mu s^{-2})$,

$$\omega_c(t_0) = 7.27320 \times 10^{-11}(\mu s^{-1}), \tag{5.1-5}$$

$$\omega_c(t_1) = 7.27321 \times 10^{-11}(\mu s^{-1}), \tag{5.1-6}$$

The above calculation shows that the influence calculated by formula (5.1-1) on the change of rotation velocity is two small. In order to compare with the results obtained by others, we use a modifying formula of (5.1-1).

5.1.2 . Modifying formula of Hanks &Kanamori's formula

$$\log E_r = BM_w^A + C, \tag{5.2-1}$$

Where parameters A, B, C are large, middle and small adjustors respectively.

Since the Hanks &Kanamori's formula is based on the empirical relation of (Gutengerg & Richttel , 1942, 1956) between surface wave and seismic energy change E_r, which is reflected only a small part of earthquake energy release, therefore herein, according to the comparing with results of Gross, we choose A=2, B=1.5, C=4.3 enough to emphasizes the release energy of earthquake.

$$\log E_e = 1.5M_w^2 + 4.3, \tag{5.2-2}$$

Repeating the process in section 5.1 with (5.2-2) instead of (5.1-1), we have

$\log E_r = 31.3,$ (5.2-3)

Then $E_r = 1.999 \times 10^{31}$(Joules). Substituting E_r into (3-18), we have

$\omega_c(t_0) = 7.27320 \times 10^{-11}(\mu s^{-1}),$ (5.2-4)

$\omega_c^2(t_1) = 5.700576 \times 10^{-21}(\mu s^{-2}),$ (5.2-5)

$\omega_c(t_1) = 7.549863 \times 10^{-11}(\mu s^{-1}),$ (5.2-6)

In order to compare with the result reported by Richard A. Lovett. (2011), let us change the above data to the time of one day. One cycle of revolution for initial velocity $\omega_c(t_0)$ needs time :

$T_0 = \frac{2\pi}{7.27320 \times 10^{-11}} = 8.6388188 \times 10^{10}(\mu s),$ (5.2-7)

Let us try a nearly a pulse-mode, i.e., in a time interval $[t_0, t_1]$, from t_0 to $t_2 = 0.9 \times (t_1 - t_0)$, the earth rotates with $\omega_c(t_0)$; from t_2 to t_1, the earth rotates with $\omega_c(t_1)$.

If $t_1 - t_0 = 120(s)$, then the earth only rotates 12 (s) with $\omega_c(t_1)$, the other time of a day rotates with $\omega_c(t_0)$.

In the time interval $t_2 - t_1 = 12(s)$, the Earth rotates an angle

$A_g = \omega_c(t_1) \times 12(\mu s^6) = 7.549863 \times 10^{-11}(\mu s^{-1}) \times 12 \times 10^6(\mu s)$
$= 1.0871862 \times 10^{-3},$ (5.2-8)

After t_1, the Earth rotates with initial velocity $\omega_c(t_0)$, the total time for one day is:

$T_{1day} = \frac{2\pi - A_g}{7.27320 \times 10^{-11}}(\mu s) + 12(s) = \frac{6.2820984738}{7.27320 \times 10^{-11}} + 12(s) = 8.6384239 \times 10^4(s),$

(5.2-9)

$T_{1day} - T_0 = -0.0003948 \times 10^4(s) = -3.948(s),$ (5.2-10)

If $t_1 - t_0 = 12(s)$, similar calculation, we get $A_g = \omega_c(t_1) \times 1.2(s) =$

1.087162×10^{-4},

$T_{1dat} = \frac{6.2830769414}{7.27320 \times 10^{-11}}(\mu s) + 1.2(s) = 0.8638789281 \times 10^{11}(\mu s)$,

$T_{1day} - T_0 = -0.29519 \times 10^{-5}(s)$, (5.2-11)

Eq. (5.2-10) shows that the shortening of time of one day reaches $3.948(s)$. Eq. (5.2-11) shows that the shortening of time of one day reduces to 0.29519 (μs). Which is near that of Gross (2011) 1.8 (μs).The result shows that the shortening of one day depends on the time of earthquake lasting $[t_0, t_1]$. The shorten the $[t_0, t_1]$, the shorten the day.

5.2 The 1960 Great Chilean earthquake, 1960-5-21, 15: 0, M_w = 9.5.

In order to compare different M_w of earthquakes, we use the same calculating formula, and the same earthquake lasting time. Substituting M_w into (5.2-2), we have $\log E_r = 32.8$, $E_r = 1.203 \times 10^{32}$ (J).

Substituting $E_r = K$ into (3.18), we have

$\omega_c(t_0) = 7.27320 \times 10^{-11}(\mu s^{-1})$, (5.3-4)

$\omega_c^2(t_1) = 77.6827904 \times 10^{-22}(\mu s^{-2})$, (5.3-5)

$\omega_c(t_1) = 8.81386 \times 10^{-11}(\mu s^{-1})$, (5.3-6)

$T_0 = \frac{2\pi}{7.27320 \times 10^{-11}} = 8.6388188 \times 10^{10}(\mu s)$, (5.3-7)

If $t_1 - t_0 = 120(s)$, then the earth only rotates 12 (s) with $\omega_c(t_1)$, the other time of a day rotates with $\omega_c(t_0)$.

In the time interval $t_2 - t_1 = 12(s)$, the Earth rotates an angle

$A_g = \omega_c(t_1) \times 12(\mu s^6) = 8.81386 \times 10^{-11}(\mu s^{-1}) \times 12 \times 10^6(\mu s)$

$= 1.0576632 \times 10^{-3}$, (5.3-8)

After t_1, the Earth rotates with initial velocity $\omega_c(t_0)$, the total time for one day is:

$$T_{1day} = \frac{2\pi - A_g}{7.27320\times 10^{-11}}(\mu s) + 12(s) = \frac{6.282179966}{7.27320\times 10^{-11}} + 12(s) = 8.63856457\times 10^4(s),$$

$$T_{1day} - T_0 = 8.63856457\times 10^4(s) - 8.6388188\times 10^4(s) = -2.5432(s), \qquad (5.3\text{-}9)$$

If $t_1 - t_0 = 12(s)$, similar calculation, we get $A_g = \omega_c(t_1)\times 1.2(s) =$

$$= 1.0576632\times 10^{-4}, \qquad (5.3\text{-}10)$$

$$T_{1day} = \frac{2\pi - A_g}{7.27320\times 10^{-11}}(\mu s) + 1.2(s) = \frac{6.283184603}{7.27320\times 10^{-11}} + 1.2(s) = 8.638817302\times 10^4(s), \qquad (5.3\text{-}11)$$

$$T_{1day} - T_0 = 8.638817302\times 10^4(s) - 8.6388188\times 10^4(s) = -1.498\times 10^{-4}(\mu s), \qquad (5.3\text{-}12)$$

5.3 The 2004 Indonesia Sumatra earth

$M_w = 9.0$. The data and calculating formula are the same as Section 5.1.

5.4 The 2008 Sichuan earthquake.

1960-5-21, 15: 0, $M_w = 8.0$. .For space reason, we do not discuss this case=

6. Conclusion

Using the modifying Hanks &Kanamori's formula to represent the release energy of earthquake, we can obtain the expect results of large M_w earthquakes. Which can be used to compare with others' results, e.g., the results of Gross of shortening a day by 1.8 (μs) for Japan 2011 earthquake, 6.9 (μs) for 2004 Indonesia Sumatra earthquake. All examples show that the shortening of a day depends on the time of earthquake lasting, the shortening, the lasting. Based on conservation law of energy, this paper proves that the energy release by earthquake proportions to the square of velocity of Earth's rotation. Furthermore, the pulse mode of earthquake is proved by PMER .The results show that during an earthquake, the Earth's rotation increases , while the velocities of revolution of Earth and Moon remain unchanged. □

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Appendix

Rotating angular velocity of a point of mantle due to Earth rotation .

Proposition: the rotating angular velocity of a point of mantle is equal to that of crust of Earth.

Proof: Suppose that the rotating angular velocity ω_N of a point N(r, 0, z) of mantle is different to that ω_C of a point C(r+dr, 0, z) of crust, say, $\omega_C > \omega_N$, then, a friction force $F_{friction}$ exists between C(r+dr, 0, z) and N(r, 0, z), such that $F_{friction}$ blocks ω_C meanwhile drags ω_N, until $\omega_C = \omega_N$. Similarly, the rotating angular velocity of a point of mantle is equal to that of its neighborhood. □

Vitae

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