

**BAYESIAN ESTIMATION OF THE SCALE PARAMETER OF THE WEIMAL
DISTRIBUTION**

Abstract

This article aims at estimating the scale parameter of the Weimal distribution using Bayesian method and comparing the estimators obtained to the estimator of the scale parameter obtained from the method of maximum likelihood. Under Bayesian approach, the estimators are obtained by using uniform prior and Jeffrey's prior with the adoption of the precautionary, quadratic and square error loss functions. A derivation and discussion under maximum likelihood estimation is also done. The above methods of estimation employed in this paper are compared based on their mean square errors (MSEs) through a simulation study carried out in R statistical software with different sample sizes. The results indicate that the most appropriate method for the scale parameter is precautionary loss function under either uniform or Jeffrey's prior irrespective of the sample sizes allocated and the values taken by the other parameters.

Keywords: Weimal distribution; Bayesian Methods; Prior distributions; Loss functions; Maximum likelihood Estimation; Mean Square Error; Sample size.

1. Introduction

Ieren and Yahaya [1] developed a new distribution named Weimal distribution as an extension of the Normal distribution with two additional parameters for the scale and shape of the new distribution. The maximum likelihood estimates of parameters were obtained by the method of maximum likelihood in [2]. The fitness of Weimal distribution was tested by using two lifetime datasets and it was discovered that the new distribution provides a better fit for the skewed datasets when compared to other existing generalizations of the normal distribution including Kumaraswamy-Normal and Beta-Normal as well as the normal distribution.

In statistics, we have two basic methods of parameter estimation and these are the classical and the non classical methods. In the classical theory of estimation, the parameters are taken to be

30 fixed but unknown whereas we consider the parameters to be unknown and random just like
31 variables. The most popular and unique method under classical theory is the method of
32 maximum likelihood estimation while the Bayesian estimation method is considered under non
33 classical theory. But, in common real life problems described by life time distributions, the
34 parameters cannot be treated as fixed in all the life testing period according to [3] as well as [4]
35 and [5]. Based on this fact, it becomes obvious the frequentist or classical approach can no
36 longer handle adequately problems of parameter estimation in life time models and therefore the
37 need for non classical or Bayesian estimation in life time models.

38 In order to achieve the gap above, many researchers have used Bayesian estimation method for
39 parameters of different probability distributions and a list of some of these studies is as follows:
40 Bayesian estimation for the extreme value distribution using progressive censored data and
41 asymmetric loss by [6], Bayesian estimators of the shape and scale parameters of modified
42 Weibull distribution using Lindley's approximation under the squared error loss function,
43 LINEX loss function and generalized entropy loss function by [7], comparison of Bayesian
44 estimates of the shape parameter of Generalized Exponential Distribution based on a class of
45 non-informative prior under the assumption of quadratic loss function, squared log-error loss
46 function and general entropy loss function (*GELF*) and maximum likelihood estimates by [8],
47 Bayesian Survival Estimator for Weibull distribution with censored data by [9] as well as [10],
48 [11]. Similarly, [12] studied the shape parameter of generalized Rayleigh distribution under non-
49 informative priors with a comparison to the method of maximum likelihood. Besides, a good
50 number of loss functions have been shown to be performing during estimation under Bayesian
51 method in so many studies including [13], [14], [15], [16], [17], [18] and [19] etc.

52
53 Since the approach of estimating a parameter differs from one parameter of a distribution to
54 another, this study aims at estimating the scale parameter of the Weibull distribution using
55 Bayesian approach and making a comparison between the Bayesian approach and the method of
56 maximum likelihood estimation approach. The rest of this paper organized in sections as follows:
57 section 1 presents the introduction, section 2 gives the materials and methods used in the article
58 beginning with the distribution and likelihood function in subsection 2.1, estimation under
59 uniform prior in 2.2, estimation under Jeffrey's prior in 2.3 and estimation using method of

60 maximum likelihood in subsection 2.4. In section 3 we presents the results and discussions and
 61 finally the conclusion in section 4.

62 2. Materials and Methods

63 2.1 PDF and Likelihood function

64 The *pdf* of the Weimal distribution with unknown parameter vector $\theta = (\alpha, \beta, \mu, \sigma)^T$ is given as:

$$65 \quad f(x; \theta) = \frac{\alpha\beta}{\sigma} \phi\left(\frac{x-\mu}{\sigma}\right) \frac{\left[\Phi\left(\frac{x-\mu}{\sigma}\right)\right]^{\beta-1}}{\left[1-\Phi\left(\frac{x-\mu}{\sigma}\right)\right]^{\beta+1}} \exp\left\{-\alpha \left[\frac{\Phi\left(\frac{x-\mu}{\sigma}\right)}{1-\Phi\left(\frac{x-\mu}{\sigma}\right)}\right]^{\beta}\right\} \quad (2.1.1)$$

66 respectively, where $-\infty < X < \infty$ represent any continuous random variable, $\sigma > 0$ is the
 67 dispersion parameter, $-\infty < \mu < \infty$ is the location parameter, $\alpha > 0$ is the scale parameter and
 68 $\beta > 0$ is the shape parameter of the distribution.

69 The total log-likelihood function for θ is obtained from $f(x)$ as follows:

$$70 \quad L(X_1, X_2, \dots, X_n | \alpha, \beta, \mu, \sigma) = \left(\frac{\alpha\beta}{\sigma}\right)^n \sum_{i=1}^n \phi\left(\frac{x_i-\mu}{\sigma}\right) \frac{\sum_{i=1}^n \left[\Phi\left(\frac{x_i-\mu}{\sigma}\right)\right]^{\beta-1}}{\sum_{i=1}^n \left[1-\Phi\left(\frac{x_i-\mu}{\sigma}\right)\right]^{\beta+1}} \exp\left\{-\alpha \sum_{i=1}^n \left[\frac{\Phi\left(\frac{x_i-\mu}{\sigma}\right)}{1-\Phi\left(\frac{x_i-\mu}{\sigma}\right)}\right]^{\beta}\right\} \quad (2.1.2)$$

71 The likelihood function for the scale parameter, α , is given by;

$$72 \quad L(\underline{X} | \alpha) \propto (\alpha)^n \exp\left\{-\alpha \sum_{i=1}^n \left[\frac{\Phi\left(\frac{x_i-\mu}{\sigma}\right)}{1-\Phi\left(\frac{x_i-\mu}{\sigma}\right)}\right]^{\beta}\right\}$$

73 Hence, for simplicity and ease of derivation and computation, we let $\sum_{i=1}^n \left[\frac{\Phi\left(\frac{x_i-\mu}{\sigma}\right)}{1-\Phi\left(\frac{x_i-\mu}{\sigma}\right)}\right]^{\beta} = \omega$ such

74 that the above likelihood function becomes

$$75 \quad L(\underline{X} | \alpha) \propto (\alpha)^n \exp\{-\alpha\omega\} \quad (2.1.3)$$

76 Where
$$\omega = \sum_{i=1}^n \left[\frac{\Phi\left(\frac{x_i - \mu}{\sigma}\right)}{1 - \Phi\left(\frac{x_i - \mu}{\sigma}\right)} \right]^\beta$$

77 **2.2 Bayesian Analysis under the Assumption of Uniform Prior Using Three Loss**
 78 **Functions**

79 One crucial aspect when dealing with Bayesian approach is the selection of a prior distribution
 80 for the parameter of interest. Most at times priors are chosen according to one's subjective
 81 knowledge and beliefs. Another important aspect of it is the choice of a loss function.

82 To derive the posterior distribution of a parameter given some sample observations, we apply
 83 Bayes' Theorem which is stated as follows:

84
$$p(\alpha | \underline{X}) = \frac{p(\alpha)L(\underline{X} | \alpha)}{\int_0^\infty p(\alpha)L(\underline{X} | \alpha)d\alpha} \quad (2.2.1)$$

85 where $p(\alpha)$ and $L(\underline{X} | \alpha)$ are the prior distribution and the Likelihood function respectively.

86 The uniform prior is defined as:

87
$$p(\alpha) \propto 1; 0 < \alpha < \infty$$

88 The posterior distribution of the scale parameter α under uniform prior is obtained from
 89 equation (2.2.1) using integration by substitution method as

90
$$p(\alpha | \underline{X}) = \frac{\alpha^n e^{-\alpha\omega}}{(\omega)^{-(n+1)} \Gamma(n+1)} \quad (2.2.2)$$

91
 92 The Bayes estimators and posterior risks under uniform prior using *SELF*, *QLF* and *PLF* are
 93 given respectively as follows:

94
$$\alpha_{SELF} = \frac{n+1}{\omega} \quad (2.2.3)$$

95
$$P(\alpha_{SELF}) = \frac{(n+2)(n+1) - ((n+1))^2}{(\omega)^2} \quad (2.2.4)$$

96
 97
$$\alpha_{QLF} = \frac{n-1}{(\omega)} \quad (2.2.5)$$

98
$$P(\alpha_{QLF}) = \frac{1}{n} \quad (2.2.6)$$

99

100

101
$$\alpha_{PLF} = \frac{[(n+2)(n+1)]^{\frac{1}{2}}}{\omega} \quad (2.2.7)$$

102
$$P(\alpha_{PLF}) = 2 \left\{ \frac{\{(n+2)(n+1)\}^{\frac{1}{2}} - (n+1)}{\omega} \right\} \quad (2.2.8)$$

103

104 **2.3 Bayesian Analysis under the Assumption of Jeffrey's Prior Using Three Loss**
 105 **Functions**

106 Also, the Jeffrey's prior is defined as:

107
$$p(\alpha) \propto \frac{1}{\alpha}; 0 < \alpha < \infty \quad (2.3.1)$$

108 The posterior distribution of the scale parameter α for a given data under Jeffrey prior is
 109 obtained from equation (2.2.1) using integration by substitution method as

110
$$p(\alpha | \underline{X}) = \frac{\alpha^{n-1} (\omega)^n e^{-\alpha\omega}}{\Gamma(n)} \quad (2.3.2)$$

111 The Bayes estimators and posterior risks under uniform prior using *SELF*, *QLF* and *PLF* are
 112 given respectively as follows:

113
$$\alpha_{SELF} = \frac{n}{\omega} \quad (2.3.3)$$

114

115
$$P(\alpha_{SELF}) = \frac{n}{(\omega)^2} \quad (2.3.4)$$

116

117

118
$$\alpha_{QLF} = \frac{n-2}{\omega} \quad (2.3.5)$$

119

$$P(\alpha_{QLF}) = \frac{1}{n-1} \quad (2.3.6)$$

121

$$\alpha_{PLF} = \frac{[n(n+1)]^{\frac{1}{2}}}{\omega} \quad (2.3.7)$$

122

$$P(\alpha_{PLF}) = 2 \left\{ \frac{\{n(n+1)\}^{\frac{1}{2}} - n}{\omega} \right\} \quad (2.3.8)$$

123

124 2.4 Maximum Likelihood Estimation

125 This part of the article estimates the scale parameter of the Weimal distribution using the method
 126 of maximum likelihood estimation. Let X_1, X_2, \dots, X_n be a random sample from the Weimal
 127 distribution with unknown parameter vector $\theta = (\alpha, \beta, \mu, \sigma)^T$. The overall log-likelihood
 128 function for θ is obtained from $f(x)$ as follows:

$$L(X_1, X_2, \dots, X_n / \theta) = \left(\frac{\alpha\beta}{\sigma} \right)^n \prod_{i=1}^n \phi \left(\frac{x_i - \mu}{\sigma} \right) \frac{\sum_{i=1}^n \left[\Phi \left(\frac{x_i - \mu}{\sigma} \right) \right]^{\beta-1}}{\sum_{i=1}^n \left[1 - \Phi \left(\frac{x_i - \mu}{\sigma} \right) \right]^{\beta+1}} \exp \left\{ -\alpha \sum_{i=1}^n \left[\frac{\Phi \left(\frac{x_i - \mu}{\sigma} \right)}{1 - \Phi \left(\frac{x_i - \mu}{\sigma} \right)} \right]^{\beta} \right\} \quad (2.4.1)$$

130 The likelihood function for the scale parameter, α , is given by;

$$L(\underline{X} | \alpha) \propto (\alpha)^n \exp\{-\alpha\omega\} \quad (2.4.2)$$

132

133 Let the log-likelihood function, $l = \log L(\underline{X} | \alpha)$, therefore

$$l = n \log \alpha - \alpha\omega \quad (2.4.3)$$

134

135
 136 Differentiating l partially with respect to α , the scale parameter and solving for $\hat{\alpha}$ gives;

$$\frac{\partial l(\theta)}{\partial \alpha} = \frac{n}{\alpha} - \omega = 0$$

137

$$\hat{\alpha} = \frac{n}{\omega} \quad 2.4.4$$

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()

3. Results and Discussions

3.1 Simulation and Comparison

In this section, a package in R software named *newdistr* developed by [20] has been used to generate random samples of sizes $n = (5, 10, 15, 20, 25, 30, 35, 55, 75, 100, 150)$ from Weimal distribution by using different values for the distribution parameters as stated in the headings of the tables below. These tables present the results of our simulation study by providing the Mean Square Errors (*MSEs*) for the estimators of the scale parameter of the Weimal distribution under the some of the concern estimation methods or loss functions such as Maximum Likelihood Estimation (*MLE*), Squared Error Loss Function (*SELF*), Quadratic Loss Function (*QLF*), and Precautionary Loss Function (*PLF*) under both Uniform and Jeffrey prior.

Table 3.1: Mean Square Errors (*MSEs*) for estimate of the scale parameter based on different sample sizes for $\alpha = 0.5$, $\beta = 3.5$, $\mu = 1.0$ and $\sigma = 1.0$.

Sample sizes	<i>MLE</i>	Uniform Prior			Jeffrey's Prior		
		<i>SELF</i>	<i>QLF</i>	<i>PLF</i>	<i>SELF</i>	<i>QLF</i>	<i>PLF</i>
5	0.4504	0.6854	0.2803	0.8216	0.4504	0.1752	0.5544
10	0.1297	0.1501	0.1152	0.1622	0.1297	0.1066	0.1389
15	0.0899	0.0924	0.0890	0.0943	0.0899	0.0897	0.0909
20	0.0819	0.0811	0.0835	0.0809	0.0819	0.0859	0.0814
25	0.0814	0.0796	0.0836	0.0789	0.0814	0.0862	0.0805
30	0.0817	0.0796	0.0840	0.0786	0.0817	0.0866	0.0806
35	0.0835	0.0814	0.0857	0.0805	0.0835	0.0880	0.0824
55	0.0913	0.0897	0.0930	0.0889	0.0913	0.0948	0.0905
75	0.0978	0.0965	0.0991	0.0959	0.0978	0.1004	0.0972
100	0.1037	0.1027	0.1047	0.1022	0.1037	0.1057	0.1032
150	0.1116	0.1109	0.1122	0.1106	0.1116	0.1129	0.1112

From table 3.1, it is observed that *MSEs* of the estimates increases as we increase the sample sizes and we also found that for all the samples the *PLF* has a minimum bias under both priors

153 irrespective of the variation in the samples indicating that the *PLF* under both priors is the best
 154 method for the scale parameter of the Weimal distribution.

155 **Table 3.2:** Mean Square Errors (*MSEs*) for estimate of the scale parameter based on different
 156 sample sizes for $\alpha = 1.0$, $\beta = 0.5$, $\mu = 1.5$ and $\sigma = 2.5$.

Sample sizes	<i>MLE</i>	Uniform Prior			Jeffrey's Prior		
		<i>SELF</i>	<i>QLF</i>	<i>PLF</i>	<i>SELF</i>	<i>QLF</i>	<i>PLF</i>
5	0.5882	0.7009	0.5406	0.7782	0.5882	0.5579	0.6339
10	0.4647	0.4436	0.4917	0.4354	0.4647	0.5246	0.4537
15	0.4938	0.4732	0.5159	0.4637	0.4938	0.5398	0.4835
20	0.5206	0.5041	0.5377	0.4963	0.5206	0.5556	0.5124
25	0.5441	0.5308	0.5577	0.5243	0.5441	0.5718	0.5374
30	0.5616	0.5505	0.5730	0.5451	0.5616	0.5845	0.5561
35	0.5746	0.5651	0.5842	0.5605	0.5746	0.5939	0.5699
55	0.6155	0.6098	0.6213	0.6069	0.6155	0.6271	0.6126
75	0.6401	0.6360	0.6442	0.6340	0.6401	0.6483	0.6380
100	0.6596	0.6567	0.6625	0.6552	0.6596	0.6655	0.6581
150	0.6841	0.6823	0.6860	0.6813	0.6841	0.6878	0.6832

157 In the table above, it is also clear that *MSEs* for all the estimators gets larger as sample size is
 158 increased. Again the *PLF* has the minimum *MSEs* independent of the sample size and prior
 159 distribution which still indicates that it is a perfect estimator for the scale parameter of the
 160 Weimal distribution irrespective of the value of the shape, location and dispersion parameter.

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168 **Table 3.3:** Mean Square Errors (*MSEs*) for estimate of the scale parameter based on different
 169 sample sizes for $\alpha = 1.5$, $\beta = 0.5$, $\mu = 2.5$ and $\sigma = 1.5$.

Sample sizes	<i>MLE</i>	Uniform Prior			Jeffrey's Prior		
		<i>SELF</i>	<i>QLF</i>	<i>PLF</i>	<i>SELF</i>	<i>QLF</i>	<i>PLF</i>
5	1.2261	1.2163	1.3009	1.2347	1.2261	1.4407	1.2133
10	1.2998	1.2372	1.3683	1.2087	1.2998	1.4427	1.2685
15	1.3976	1.3540	1.4429	1.3332	1.3976	1.4898	1.3760
20	1.4592	1.4272	1.4919	1.4116	1.4592	1.5254	1.4433
25	1.5067	1.4819	1.5319	1.470	1.5067	1.5574	1.4944
30	1.5415	1.5214	1.5619	1.5115	1.5415	1.5824	1.5315
35	1.5656	1.5488	1.5826	1.5405	1.5656	1.5998	1.5573
55	1.6397	1.6298	1.6496	1.6250	1.6397	1.6595	1.6348
75	1.6824	1.6756	1.6892	1.6722	1.6824	1.6961	1.6790
100	1.7155	1.7107	1.7204	1.7082	1.7155	1.7253	1.7131
150	1.7566	1.7536	1.7597	1.7521	1.7566	1.7627	1.7551

170 Again from table 3.3, it is obvious that *PLF* (under uniform and Jeffrey priors) method yielded
 171 the best estimate for the scale parameter despite the changes in the sample sizes. Besides, the
 172 *MSEs* still increase as sample sizes becomes larger and there is no change even with the different
 173 parameter values.

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183 **Table 3.4:** Mean Square Errors (*MSEs*) for estimate of the scale parameter based on different
 184 sample sizes for $\alpha = 2.0$, $\beta = 0.5$, $\mu = 0.5$ and $\sigma = 0.5$.

Sample sizes	<i>MLE</i>	Uniform Prior			Jeffrey's Prior		
		<i>SELF</i>	<i>QLF</i>	<i>PLF</i>	<i>SELF</i>	<i>QLF</i>	<i>PLF</i>
5	2.3640	2.2318	2.5612	2.1913	2.3640	2.8234	2.2928
10	2.6348	2.5307	2.7448	2.4819	2.6348	2.8607	2.5832
15	2.8015	2.7348	2.8698	2.7026	2.8015	2.9398	2.7685
20	2.8978	2.8503	2.9461	2.8270	2.8978	2.9952	2.8743
25	2.9693	2.9330	3.0060	2.9152	2.9693	3.0430	2.9513
30	3.0214	2.9923	3.0508	2.9780	3.0214	3.0803	3.0070
35	3.0567	3.0325	3.0811	3.0205	3.0567	3.1057	3.0447
55	3.1639	3.1499	3.1779	3.1430	3.1639	3.1919	3.1569
75	3.2246	3.2150	3.2342	3.2103	3.2246	3.2439	3.2199
100	3.2714	3.2646	3.2783	3.2612	3.2714	3.2851	3.2680
150	3.3292	3.3250	3.3334	3.3229	3.3292	3.3376	3.3271

185 More so the result from table 3.4 corresponds with the previous results showing that uniform and
 186 Jeffrey's priors with *PLF* have the smallest *MSEs* which by comparison produces the best
 187 estimates for the scale parameter, and looking at all the results presented in the tables, we can
 188 conclude that Bayes estimates under precautionary loss function (*PLF*) using uniform prior and
 189 Jeffrey's prior are associated with minimum *MSEs* when compared to those obtained using *MLE*,
 190 *SELF*, and *QLF* under both uniform and Jeffrey's priors irrespective of the assumed parametric
 191 values and allocated sample sizes of $n=5, 10, 15, 20, 25, 30, 55, 75, 100$ and 150 .

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193 4. Summary and Conclusion

194 In summary, we obtained Bayesian estimators of the scale parameter of the Weimal distribution
 195 under Posterior distributions assuming Uniform and Jeffrey's priors. Bayes estimators and their
 196 posterior risks have been derived and presented using three loss functions, namely: Squared
 197 Error Loss Function (*SELF*), Quadratic Loss Function (*QLF*) and Precautionary Loss Function
 198 (*PLF*). The performance of these estimators is assessed on the basis of the Mean Square Errors
 199 (*MSEs*) of the estimates. A simulation study is carried out in R statistical software to compare the
 200 performance of the estimators from the two methods considered in this paper and it is discovered

201 that the *PLF* (under uniform and Jeffrey priors) produces estimates with minimum *MSEs*
202 consistently irrespective of the parameter values and differences in sample size. Therefore, we
203 conclude that Bayesian Method under both uniform and Jeffrey's priors using precautionary loss
204 function (*PLF*) is better compared to Maximum Likelihood Estimation and should be considered
205 when estimating the scale parameter of the Weimal distribution irrespective of the differences in
206 sample sizes and the parameter values.

207 REFERENCES

- 208 [1] Ieren TG, Yahaya A. The Weimal Distribution: its properties and applications. *J. of the*
209 *Nigeria Ass. of Math. Physics*, 2017; 39: 135-148.
- 210 [2] Yahaya A, Ieren TG. Estimation of Parameters of the Weibull-Normal Distribution using
211 Maximum Likelihood Approach, *Abacus: The J. of Math. Ass. of Nigeria*, 2017; 44(1):
212 265-269.
- 213 [3] Martz HF, Waller RA. *Bayesian reliability analysis*. New York, NY: John Wiley, 1982.
- 214 [4] Ibrahim JG, Chen MH, Sinha D. *Bayesian survival analy..* New York, NY: Springer-Verlag,
215 2001.
- 216 [5] Singpurwalla ND. *Reliability and risk: A Bayesian perspective*. Chichester: John Wiley,
217 2006..
- 218 [6] Al-Aboud FM. Bayesian estimations for the extreme value distribution using progressive
219 censored data and asymmetric loss, *Int. Math. Forum*, 2009; 4(33), 1603–1622.
- 220
- 221 [7] Preda V, Eugenia P, Alina C. Bayes Estimators of Modified-Weibull Distribution parameters
222 using Lindley's approximation. *WSEAS TRANSACTIONS on MATHEMATICS*, 2010; 9
223 (7), 539-549.
- 224 [8] Dey S, Maiti SS. Bayesian estimation of the parameter of Maxwell distribution under
225 different loss functions. *Journal of Statistical Theory and Practice*, 2010; 4(2), 279-287.
- 226 [9] Ahmed AOM, Ibrahim NA, Arasan J, Adam MB. Extension of Jeffreys' prior estimate for
227 weibull censored data using Lindley's approximation, *Austr. J. of B. and Appl. Sci.*, 2011;
228 5(12), 884–889.

- 229 [10] Pandey BN, Dwividi N, Pulastya B. Comparison between Bayesian and maximum
230 likelihood estimation of the scale parameter in Weibull distribution with known shape
231 under linex loss function, *J. of Sci. Resr.*, 2011; 55: 163–172.
- 232 [11] Al-Athari FM. Parameter estimation for the double-pareto distribution, *J. of Math. and Stat.*,
233 2011; 7(4), 289–294.
- 234 [12] Aliyu Y, Yahaya A. Bayesian estimation of the shape parameter of generalized Rayleigh
235 distribution under non-informative prior. *Int. J. of Adv. Stat. and Prob.*, 2016; 4 (1):1-10
- 236 [13] Ahmad SP, Ahmad K. Bayesian analysis of weibull distribution using R software, *Austr. J.*
237 *of B. and Appl. Sc.*, 2013; 7(9), 156–164.
- 238 [14] Ahmad K, Ahmad SP, Ahmed A. On parameter estimation of erlang distribution using
239 bayesian method under different loss functions, *Proc. of Int. Conf. on Adv. in Computers,*
240 *Com., and Electronic Eng.*, 2015; 200–206, University of Kashmir.
- 241 [15] Ahmad K, Ahmad SP, Ahmed A. Classical and Bayesian Approach in Estimation of Scale
242 Parameter of Nakagami Distribution. *J. of Prob. and Stat.*, 2016; Article ID 7581918, 8
243 pages <http://dx.doi.org/10.1155/2016/7581918>
- 244 [16] Gupta PK, Singh AK. Classical and Bayesian estimation of Weibull distribution in presence
245 of outliers. *Cogent Math.*, 2017; 4: 1300975.
- 246 [17] Gupta I. Estimation of Parameter And Reliability Function of Exponentiated Inverted
247 Weibull Distribution using Classical and Bayesian Approach. *Int. J. of Recent Sci. Res.*,
248 2017; 8(7): 18117-18819.
- 249 [18] Ieren TG, Oguntunde PE. A Comparison between Maximum Likelihood and Bayesian
250 Estimation Methods for a Shape Parameter of the Weibull-Exponential Distribution.
251 *Asian J. of Prob. and Stat.*, 2018; 1(1): 1-12.
- 252 [19] Ieren TG, Chukwu AU. Bayesian Estimation of a Shape Parameter of the Weibull-Frechet
253 Distribution. *Asian J. of Probab. and Stat.*, 2018; 2(1):1-19. DOI:
254 10.9734/AJPAS/2018/44184
- 255 [20] R Core Team (2018). R: A language and environment for statistical computing. R
256 Foundation for Statistical Computing, Vienna, Austria. URL <https://www.R-project.org/>.