

# MODELLING THE NIGERIA CRUDE OIL PRICES USING ARIMA, PRE-INTERVENTION AND POST-INTERVENTION MODEL

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## Abstract

*The study applied Autoregressive Integrated Moving Average Intervention in modelling crude oil prices in Nigeria between January 1986 to June 2017. The time plot of the series showed an abrupt increase in series and this call for intervention model. The data was divided into three set (actual series, pre-intervention and post-intervention series). The Augmented Dickey Fuller (ADF) was used to test for unit root on each of the variables at level and the series were found to be non-stationary. The first differences showed the presence of unit root in all the three series (actual, pre and post- intervention series). Eighteen models were estimated and the best model was the pre-intervention model that minimise the Akaike information criterion (AIC) (ARIMA (111) with AIC of (4.4.578). The plot of the residual correlogram showed adequacy of the model. The model was adequate since there was no spike that cut the level of the correlogram and the histogram of the residual was normally distributed with probability values (0.0000)*

**KEY WORDS:** ARIMA, Pre- Intervention, Post-intervention, Crude oil Prices, Nigeria.

## 1.0 INTRODUCTION

The major source of income to Nigeria government is crude oil production and sales. According to Nigerian National Petroleum Corporation (NNPC 2013) Oil was first discovered in 1956 at Oloibiri in Bayelsa State, The discovery of oil was made by Shell D'Arcy now Shell Petroleum company after numerous exploration efforts which began in 1938. The abundance of oil in Nigeria gave her a place of reputation in the world market as it joined the oil producing country in 1958 when its first oil field came on the stream producing 5,100 barrels per day. Today, Nigeria is the largest oil producer in Africa, with about 33 -35 percent of Africa's oil and gas capitals, the fifth largest exporting country in the Organization of Petroleum Exporting countries (OPEC) and the fifth largest oil-exporting country to the U.S.A. [17].

The production and sales of the commodity has been on the decrease for some period and sundered increase, this call for intervention models. The most widely used method for modelling time-series data is the Box Jenkins' Autoregressive integrated moving average (ARIMA) model. However when the patterns of the series under study are affected by some external event such as increase, decrease, and regular movement, the ARIMA model may be affected. Then, one of the most appropriate techniques is the ARIMA-Intervention model. There are three types of intervention, step, pulse/ point and ramp. Step Intervention occurs at particular period of time and exists in the subsequent time-periods. The effect of step intervention may decrease, increase or remain constant over time. In the foreign market such type of intervention occurs due to new economic policy [8]. The intervention model is used to study the increase and decrease that occur in the event of interest [3]. The purpose of

applying autoregressive integrated moving average (ARIMA) intervention model to any series is to find out the dynamic effect on the means level of the variable and other event that affect the series [16].

The aim of the study is to apply Box-Tiao method of ARIMA intervention in modelling crude oil price in Nigeria from (January 1986 to June 2017).

## 2.0 LITERATURE REVIEW

Intervention modelling was introduced by [5] to examine the impact of air pollution controls on smog-producing oxidant level in the Los Angeles area and of economic controls on the consumer price index in the United States [16]. Used ARIMA intervention model to analyse the Chinese stock price. [10] has fitted a SARIMA (011)\*(011)<sub>12</sub> model for Nigeria inflation rate. [11] Studied monthly Nigeria Treasury bill rates using Box-Jenkins techniques. The acceptable model for Treasury bill rates is SARIMA (011)\*(011)<sub>12</sub> model. [1] Studied rainfall pattern in Ghana as a seasonal ARIMA process using the Box- Jenkins method. Four model were estimated for the series but the best model was choose base on the lest BIC, which is estimated as SARIMA (0,0,0)\*(2,1,0)<sub>12</sub>. [19]. Examined time series intervention model and forecasting cotton yield in Gujarat and Maharashtra in India. Intervention was found to be superior to the conventional ARIMA model. [12] fitted an intervention model of daily Moroccan Dirhan to Nigeria Naira exchange rate. The pre-intervention model was used to describe the increase in the amount of Nigeria Naira per Moroccan Dirhan. [17] carried out an investigation on the production performance of cumin in india, he adopted the ARIMA and GACH and forested the production of cumin up to 2020.

## 3.0 METHODOLOGY

The conditional means and the error terms of a series is model using autoregressive integrated moving average (ARIMA) modelled. The movement of the crude oil price in Nigeria has the component of the ARIMA model and an intervention term [21].

### 3.1 ARIMA Model

Many series are non-stationary, [7] proposed that differences up to an appropriate order make it stationary. Suppose (d) is the minimum order of differencing necessary for stationary to be attained, then (c<sub>t</sub>) is said to follow an autoregressive integrated moving average of order p, d and q denoted as (ARIMA(p d q)). If the series is seasonal in nature, the ARIMA model will incorporate both the non-seasonal and the seasonal component in a multiplicative model. The notation for the model is

$$\text{SARIMA}(p\ d\ q) \times (P\ D\ Q)_S \quad (3.1)$$

Where  $p$  = non-seasonal AR,  $d$  = non-seasonal differencing,  $q$  = non-seasonal MA,  $P$  = seasonal AR,  $D$  = seasonal differencing,  $Q$  = seasonal MA, and  $S$  = time span of repeating seasonal pattern. Without differencing operations, the model could be written more formally as. [1]

$$\phi(B^S)\psi(B)(C_t) - \mu = \theta(B^S)\Theta(B)\varepsilon_t \quad (3.2)$$

The non-seasonal components are:

$$\text{AR: } \psi(B) = 1 - \varphi_1 B_1 - \dots - \varphi_p B_p \quad (3.3)$$

$$\text{MA: } \Theta(B) = 1 + \Theta_1 B_1 + \dots + \Theta_q B_q \quad (3.4)$$

The seasonal components are: Seasonal

$$\text{AR: } \phi(B^S) = 1 - \phi_1 B^S - \dots - \phi_p B^{Sp} \quad (3.5)$$

$$\text{Seasonal MA: } \theta(B^S) = 1 + \theta_1 B^S + \dots + \theta_Q B^{SQ} \quad (3.6)$$

Seasonal differencing is defined as a difference between a value and a value with lag that is a multiple of  $S$ . Where  $S = 12$ , which may occur with monthly data, a seasonal difference is

$$B^{12} c_t = c_t - c_{t-12} \quad (3.7)$$

The differences (from the previous year) may be about the same for each month of the year giving us a stationary series. Seasonal differencing removes seasonal trend and can also get rid of a seasonal random walk type of non-stationary. Non-seasonal differencing: If trend is present in the series after seasonal differencing, we may also need non-seasonal differencing. Often a first non-seasonal difference will “de-trend” the data [21].

#### 4.0 INTERVENTION

The simple ways to study intervention analysis (event study) is to consider some simple dynamic models which involved two stages, pulse function and step function.

##### 4.1 Pulse Function

A pulse function is a function which shows that intervention only occur in a single time index ( $t_1$ ). Mathematically the pulse function is

$$P_t = \begin{cases} 0 & \text{if } T \neq t_1 \\ 1 & \text{if } T = t_1 \end{cases} \quad (4.1)$$

##### 4.2 Step Function

A step function is function which define the intervention process as a continual stage starting with time index ( $t_1$ ). Mathematically the step function is

$$S_t = \begin{cases} 0 & \text{if } t < t_1 \\ 1 & \text{if } t \geq t_1 \end{cases} \quad (4.2)$$

##### 4.3 Arima Intervention Model

The intervention model discussed by [3] is of the form

$$Y_t = \Gamma(B)I_t + K_t \quad (4.3)$$

Where,  $Y_t$  = dependent variable,  $I_t$  = is the time of intervention (indicator variable),  $I_t = P_t = S_t$

$$\Gamma(B) = \frac{\xi(B)}{\eta(B)} B^r, K_t = \frac{\theta(B)}{\varphi(B)} \varepsilon_t, B^r = \text{backshift operator}$$

$$Y_t = \frac{\xi(B)}{\eta(B)} B^r I_t + \frac{\theta(B)}{\varphi(B)} \varepsilon_t \quad (4.4)$$

##### 4.2.0 Step Involving Arima Intervention Model

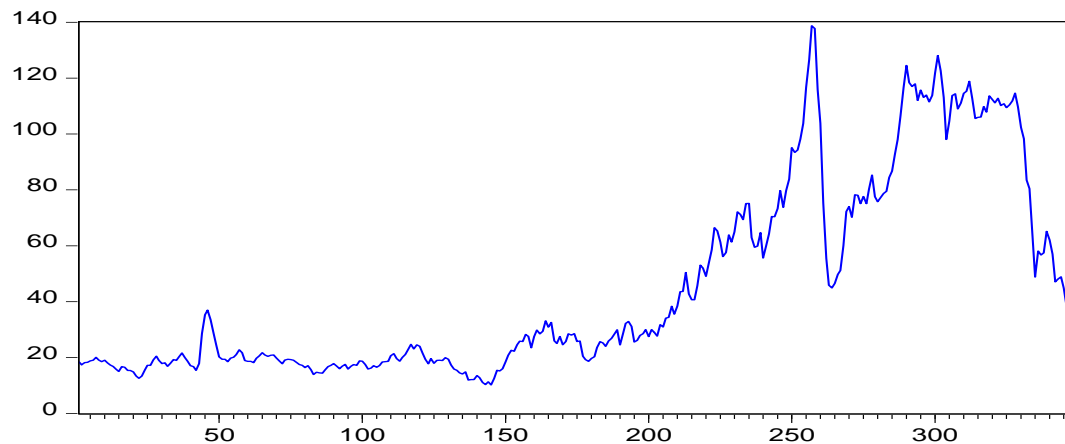
The general step for estimation of ARIMA intervention model consists of the following three stages:

- 1 Time plot of the original series.
- 2 Test for stationarity of the data using Augmented Dickey-Fuller (ADF) test where the null hypothesis of unit root exist.
- 3 Model Estimation and selection.
- 4 Diagnosis check of the parameters of the model
- 5 Residual diagnostic check using correlogram of Q-statistics, ACF and PACF of the residual is used for residual diagnostic check.

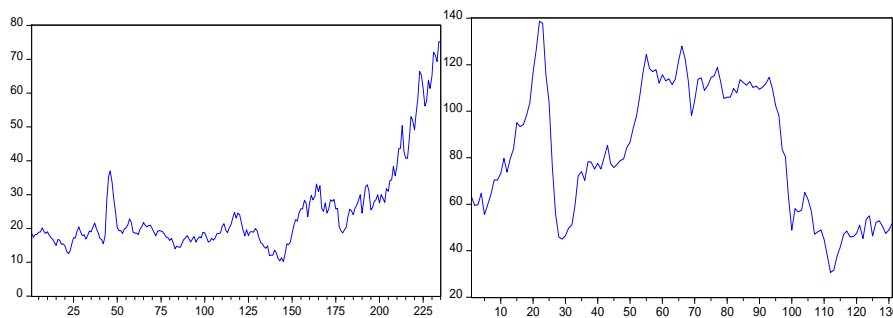
## 5.0 RESULTS AND DISCUSSION

The series is a monthly crude oil prices in US dollars per barrel from January 1986 to June 2017 from Central Bank of Nigeria website. [www.cenbank.org](http://www.cenbank.org). The researcher used eviews 10 (student version) in the statistical analysis.

**Figure 1.0. Time Plot of Crude Oil Price**



Examining Figure 1.0, we noticed an irregular movement, with sudden jump in the year 2010 the effect beginning at time  $t_1 = 251$  suggesting a new set of governmental intervene in crude oil production and sale. Therefore, there is one intervention point, meaning production of crude oil and the sale was on increase. Thus, we identify the intervention point; it can be defined as follows; pre-intervention period (0 to 235) and post-intervention period (235 to 366). The data was divided into two part, see Figure 1.1 and figure 1.2 for pre-intervention and post-intervention time plot.



*Figure 1.1. Pre-Intervention*

*Figure 1.2 Post-Intervention*

The pre-intervention series shows irregular and linear trend, with gradual increase to December 235 (meaning irregular increase in the price of crude oil). From figure 1.2 the post-intervention graph show irregular movement upward and downward trend (meaning increase and decrease in the sales of crude oil price)

## 5.1 Stationarity Test

**Table (1.0) Unit Roots Test**

| Variable          | Levels              | 1 <sup>st</sup> difference | Test critical value |         |
|-------------------|---------------------|----------------------------|---------------------|---------|
| Crude oil price   | -1.9555<br>(0.3067) | -13.036 (0.000)            | 1% level            | -3.4816 |
| Pre-intervention  | 1.266(0.9976)       | -13.225 (0.000)            | 5% level            | -2.8829 |
| Post-intervention | -2.153(0.332)       | -7.2933 (0.000)            | 10% level           | -2.5787 |

The series are required to be stationary in order to carry out joint significant test on the lags of the variable and the method used to test for stationary is the Augmented Dickey Fuller (ADF) test. The test is used to check for unit root on each of the variables [6]. Table (1.0) represent result of The Augmented Dickey Fuller (ADF) test at level and first differences and probability values in brackets. The probability values (p-values) at level is greater than 0.05 (p-values >0.05), the result showed the present of unit root. The p-value of all the variables at first difference were tested for stationarity and the series was stationary. The probability values is (0.000) for all the variables and null hypothesis was rejected since the series are stationary [13].

## 5.2 Model Estimation and AIC and SIC Values

Six models each were estimated for ARIMA, Pre-Intervention and Post-Intervention with Akaike information criterion (AIC) and Schwarz Information Criterion (SIC) are shown in appendix (1). The best model for Nigeria Crude Oil Price is ARIMA-Intervention (pre-intervention series in appendix (4)) the model (ARIMA (111)) minimised the information criterion AIC (4.579) and SIC (4.638) [2].

The pre-intervention (ARIMA (111)) model for crude oil price in Nigeria is given below

$$C_t = \frac{\xi(B)}{\eta(B)} B^r I_t + \frac{\theta(B)}{\varphi(B)} \varepsilon_t \quad (5.1)$$

Where

$C_t$  = crude oil price at time t,  $\xi(B)$  = Impact parameter,  $\eta(B)$  = slop parameter,  $\varphi(B)$  = AR(1) parameter,  $\theta(B)$  = MA(1) parameter

$I_t$  = indicator variable coded according to the type of intervention involved in the study. The type of intervention in this is call a pulse function (intervention only occur in a single time index  $t_1 = 235$ ) [8].

The pre-intervention model obtained is given as follow

$$C_t = \frac{-7.88}{1-0.0439B} B I_t + \frac{1-0.764B}{1+0.61B} \varepsilon_t \quad (5.2)$$

The model can also be represented lineally as

$$C_t = 1.045C_{t-1} + 0.268C_{t-2} - 7.88I_{t-1} - 4.81I_{t-2} - 1.2\varepsilon_{t-1} + 0.374\varepsilon_{t-2} + \varepsilon_t \quad (5.3)$$

### 5.3 Diagnostic Test

The plot of the residual correlogram showed adequacy of the model since there is no spike that cut off level in appendix (2). The histogram of the residual is normally distributed with probability values (0.000) in appendix (3). The test statistics of the residual showed that the data was normally distributed with constant mean and variance (White noise Process) [7].

### 5.4 Conclusion

It has been found that in all the three models (ARIMA, pre- intervention and post intervention) of modelled of crude oil price in Nigeria, the pre- intervention models was found to be more superior to the conventional ARIMA and post intervention models. It maybe conclude that pre-intervention model (ARIMA (111)) is appropriate in modelling movement of Nigeria crude oil price with the minimise Akaike information criterion (AIC) of (4.4.578), [15].

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## Appendix (1)

### Parameter Estimation Of Arima And Arima+Interventions Models

| MODELS                                 | PARAMETER CO-EFFICIENT |                   | AIC          | SIC          |
|--|------------------------|-------------------|--------------|--------------|
|  | AR(p)                  | MA(q)             |              |              |
| ARIMA MODEL OF CRUDE OIL PRICES        |                        |                   |              |              |
| ARIMA(111)                             | 0.3823                 | -0.0260           | 5.785        | 5.800        |
| ARIMA(011)                             |                        | 0.3386            | 5.766        | 5.798        |
| ARIMA(110)                             | 0.3598                 |                   | 5.7528       | 5.784        |
| ARIMA(210)                             | 0.356<br>0.00923       |                   | 5.758        | 5.80         |
| ARIMA(012)                             |                        | 0.344<br>0.0972   | 5.764        | 5.80         |
| ARIMA(212)                             | -0.2806<br>0.334       | 0.6552<br>-0.1337 | 5.75         | 5.823        |
| ARIMA-INTERVENTION (PRE-INTERVENTION)  |                        |                   |              |              |
| <b>ARIMA (111)</b>                     | <b>-0.6095</b>         | <b>0.7638</b>     | <b>4.578</b> | <b>4.637</b> |
| ARIMA (110)                            | 0.1383                 |                   | 4.587        | 4.892        |
| ARIMA (011)                            |                        | 0.16456           | 4.584        | 4.627        |
| ARIMA (210)                            | 0.1526<br>-0.1014      |                   | 4.586        | 4.644        |
| ARIMA (012)                            |                        | 0.1492<br>-0.1111 | 4.58         | 4.644        |
| ARIMA (212)                            | 0.158<br>0.372         | -0.0274<br>-0.542 | 4.59         | 4.67         |
| ARIMA-INTERVENTION (POST-INTERVENTION) |                        |                   |              |              |
| ARIMA(111)                             | 0.477                  | -0.0834           | 6.55         | 6.66         |
| ARIMA(110)                             | 0.40766                |                   | 6.5366       | 6.58         |
| ARIMA(011)                             |                        | 0.37067           | 6.564        | 6.617        |
| ARIMA(210)                             | 0.922<br>0.0373        |                   | 6.55         | 6.617        |
| ARIMA(012)                             |                        | 0.366<br>0.136    | 6.564        | 6.63         |
| ARIMA(212)                             | -0.2056<br>0.3811      | 0.6249<br>-0.1583 | 6.58         | 6.717        |



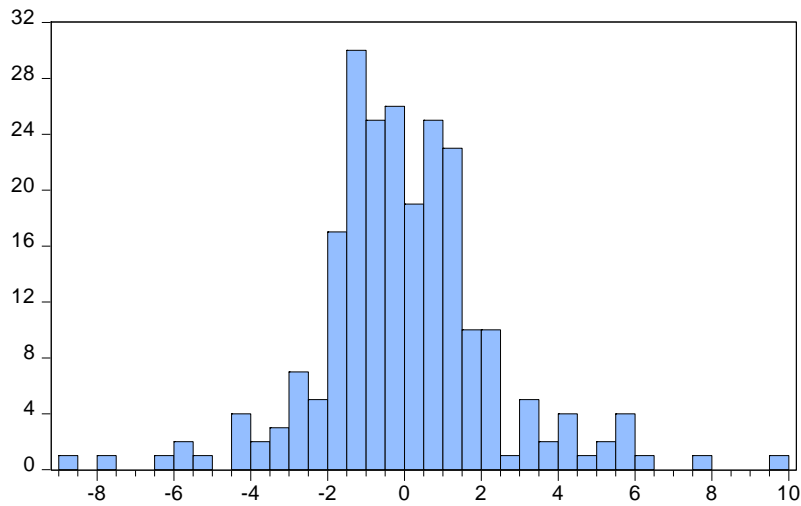
## Appendix (2).

### Corrrogram of Nigeria Crude Oil Price.

Number of observations: 257

| Autocorrelation | Partial Correlation | AC | PAC    | Q-Stat | Prob   |       |
|-----------------|---------------------|----|--------|--------|--------|-------|
|                 |                     | 1  | 0.139  | 0.139  | 4.5621 | 0.033 |
|                 |                     | 2  | -0.078 | -0.099 | 6.0195 | 0.049 |
|                 |                     | 3  | -0.080 | -0.056 | 7.5481 | 0.056 |
|                 |                     | 4  | -0.156 | -0.149 | 13.414 | 0.009 |
|                 |                     | 5  | 0.029  | 0.064  | 13.622 | 0.018 |
|                 |                     | 6  | -0.028 | -0.077 | 13.818 | 0.032 |
|                 |                     | 7  | -0.010 | -0.004 | 13.841 | 0.054 |
|                 |                     | 8  | -0.041 | -0.071 | 14.242 | 0.076 |
|                 |                     | 9  | 0.039  | 0.068  | 14.616 | 0.102 |
|                 |                     | 10 | 0.217  | 0.184  | 26.234 | 0.003 |
|                 |                     | 11 | 0.117  | 0.074  | 29.623 | 0.002 |
|                 |                     | 12 | 0.093  | 0.102  | 31.778 | 0.001 |
|                 |                     | 13 | -0.035 | -0.004 | 32.080 | 0.002 |
|                 |                     | 14 | -0.096 | -0.008 | 34.399 | 0.002 |
|                 |                     | 15 | -0.099 | -0.078 | 36.855 | 0.001 |
|                 |                     | 16 | -0.105 | -0.073 | 39.673 | 0.001 |
|                 |                     | 17 | 0.076  | 0.076  | 41.150 | 0.001 |
|                 |                     | 18 | 0.020  | -0.019 | 41.255 | 0.001 |
|                 |                     | 19 | 0.107  | 0.099  | 44.174 | 0.001 |
|                 |                     | 20 | 0.117  | 0.045  | 47.697 | 0.000 |
|                 |                     | 21 | 0.009  | -0.001 | 47.720 | 0.001 |
|                 |                     | 22 | -0.029 | -0.076 | 47.944 | 0.001 |
|                 |                     | 23 | -0.103 | -0.077 | 50.720 | 0.001 |
|                 |                     | 24 | 0.009  | 0.049  | 50.743 | 0.001 |
|                 |                     | 25 | 0.024  | 0.051  | 50.895 | 0.002 |
|                 |                     | 26 | -0.026 | 0.013  | 51.079 | 0.002 |
|                 |                     | 27 | -0.049 | -0.060 | 51.722 | 0.003 |
|                 |                     | 28 | -0.037 | -0.009 | 52.094 | 0.004 |
|                 |                     | 29 | 0.001  | -0.081 | 52.095 | 0.005 |
|                 |                     | 30 | 0.139  | 0.085  | 57.314 | 0.002 |
|                 |                     | 31 | 0.021  | -0.075 | 57.429 | 0.003 |
|                 |                     | 32 | 0.001  | 0.058  | 57.430 | 0.004 |
|                 |                     | 33 | -0.089 | -0.045 | 59.622 | 0.003 |
|                 |                     | 34 | -0.074 | -0.003 | 61.152 | 0.003 |

### Appendix (3) Histogram of residuals of Nigeria crude oil price



|                   |           |
|-------------------|-----------|
| Series: Residuals |           |
| Sample 2 235      |           |
| Observations 234  |           |
| Mean              | 0.000813  |
| Median            | -0.230061 |
| Maximum           | 9.909967  |
| Minimum           | -8.702704 |
| Std. Dev.         | 2.351997  |
| Skewness          | 0.308722  |
| Kurtosis          | 5.618035  |
| Jarque-Bera       | 70.54461  |
| Probability       | 0.000000  |

### Appendix (4) ARIMA(1,1,1)

Dependent Variable: DE

Method: ARMA Maximum Likelihood (OPG - BHHH)

Date: 10/30/18 Time: 15:57

Sample: 2 235

Included observations: 234

Convergence achieved after 27 iterations

Coefficient covariance computed using outer product of gradients

| Variable           | Coefficient | Std. Error            | t-Statistic | Prob.    |
|--------------------|-------------|-----------------------|-------------|----------|
| C                  | 0.239790    | 0.177461              | 1.351225    | 0.1780   |
| AR(1)              | -0.609500   | 0.119080              | -5.118389   | 0.0000   |
| MA(1)              | 0.763892    | 0.099669              | 7.664325    | 0.0000   |
| SIGMASQ            | 5.508249    | 0.340418              | 16.18086    | 0.0000   |
| R-squared          | 0.036335    | Mean dependent var    |             | 0.242521 |
| Adjusted R-squared | 0.023765    | S.D. dependent var    |             | 2.395928 |
| S.E. of regression | 2.367286    | Akaike info criterion |             | 4.578684 |
| Sum squared resid  | 1288.930    | Schwarz criterion     |             | 4.637750 |
| Log likelihood     | -531.7061   | Hannan-Quinn criter.  |             | 4.602500 |
| F-statistic        | 2.890700    | Durbin-Watson stat    |             | 1.977482 |
| Prob(F-statistic)  | 0.036222    |                       |             |          |
| Inverted AR Roots  | -.61        |                       |             |          |
| Inverted MA Roots  | -.76        |                       |             |          |

## Appendix (5)

Dependent Variable: Z

Method: Least Squares (Gauss-Newton / Marquardt steps)

Date: 10/30/18 Time: 16:58

Sample (adjusted): 2 131

Included observations: 130 after adjustments

Convergence achieved after 33 iterations

Coefficient covariance computed using outer product of gradients

$Z = C(1) * (1 - C(2))^{(T - 235) / (1 - C(2))}$

|                    | Coefficient | Std. Error            | t-Statistic | Prob.  |
|--------------------|-------------|-----------------------|-------------|--------|
| C(1)               | -7.875241   | 351.5401              | -0.022402   | 0.9822 |
| C(2)               | -0.043943   | 0.398930              | -0.110152   | 0.9125 |
| R-squared          | 0.000277    | Mean dependent var    | -0.008154   |        |
| Adjusted R-squared | -0.007533   | S.D. dependent var    | 1.747367    |        |
| S.E. of regression | 1.753936    | Akaike info criterion | 3.976867    |        |
| Sum squared resid  | 393.7653    | Schwarz criterion     | 4.020983    |        |
| Log likelihood     | -256.4963   | Hannan-Quinn criter.  | 3.994793    |        |
| Durbin-Watson stat | 1.063710    |                       |             |        |