A theoretical investigation on sound transmission loss throughmulti-walled plates with air space

In this study, ananalyticalmodel is proposed to investigate the sound transmission loss through multi-walledplates with airlayers ordecompression air layers, under thediffuse incidencefield.Using the present approach, the influences of various parameters, such as the wallthickness, the decompressed air and the thickness of air space, on the sound transmission lossthroughare simulated and discussedin detail. It is seen that, due to the wave frequency of mass-air-mass resonance between double-walled glass plates, the sound transmission loss of the plates can be improved at low frequency range. The sound transmission loss tends to increase with decreasing air pressure because thesound is not transmitted through vacuum space.The advantage of the simulation procedure is easily used for designing the layer structures with different parameter to improve the sound insulation effect.

Introduction

Improving the sound insulation performance of wall, window, ceilings and floors of general buildings, and building structuresare veryimportant aspectsin living environment maintenance.[1-3].Many advanced composites, such as lightweight sandwich materials and multi-layered panels, have been widely applied in the fields including environment protection and energy conservation[4].Important design parameters of walls or panels, such as material properties, the structures and the dimensions,affectlargely on theirsound insulation[5-8]. In the basic design stage of sound insulation products,the prediction of design parametersis very importantbeforethe products are developed and manufactured. Theoretical studies on the sound absorption and insulation are mainly based on the physical consideration of sound propagation. The mass law of transmission losswas a classical method,andgenerallyadopted to predict thetransmission loss through panels or walls, and used for frequencies between $2f_{resonancy}$ and $0.5f_{critical}$ [9, 10].For the range at high frequency vibration, a number of simulation approaches on the sound insulation of plate structureswere carried out using governing equations of motion, and there wasgood agreement between the theoretical and experimental results.Zhang et al. reported an effective medium model to predict the sound transmission loss ina thin plate. They suggested a model based on the two surfaces bonded by shunted piezoelectric patches[11]. The sound absorptionproperties of the thin platescould beinvestigated by explicit formulations derived from the effective medium method. For porous materials and perforated plates, the sound transmission loss of micro- perforated panel system waspredictedtheoreticallyusing transfer matrix methodbased onlumped and distributed models[12-15].In order to verify the theoretically predicted results,the experimental measurement of the sound absorptioncoefficient in the micro-perforated panel was performed using theimpedance tube method and the reverberation room [12].Takahashi and Tanaka[16]presented a method for analyzing acoustic coupling due to flexural vibration of perforated plates and plates of poroelastic materials.Moreover, some fundamental acoustic problems,such as the interactive effect of flexural vibration and plate permeability,werealso investigated.Mahjoob et al. [17]investigated and discussed the acoustic insulation of multi-layered panels containing Newtonian fluids. A progressive impedance model wasproposed to predictthe sound insulationof the plateat normal incidence condition.The sound transmission loss through sandwich panels with damping layers were analyzed according tovarious structural and material properties[18-20]. Their results reported that the sandwich panel had better sound transmission characteristics than a homogeneous elastic panel of equivalent weight [18].The sound insulation was largely affected bythe damping materials and theirpropertiesfor high frequency range [19]. The result shows that the skin damping alone did not decrease wave coincident peaks significantly. However, the damping in the core layer were more important than those in the skin layers, resulting in an increase of sound transmission loss with increasing damping.Natsuki [21] reported recently that the sound transmission loss through multi-layered panels can be improved based on the separated air space thickness or decreasing air pressure due to the influence of mass-air-mass resonance.

In this study, we suggest ananalytical model andderivea formula for the diffuse field incidence of acoustic load through multi-layer panels. Based on the developed theoretical model[21], the influence of diffuse incidence, layer number and layer space onthe sound transmission lossthrough multi-walled panels were investigated and analyzed in detail. The analytical results can be applied to design parameters to improve the performance characteristics of sound insulation.

2. Theoretical analytical model

2.1 Governing equation

The progressive impedance modelof multi-walled plates under scattering incidenceis present to predict the sound transmission loss. Fig. 1 shows an analytical model of three-walledplatesseparated byair spaces or decompressed airlayers. The dimension of plate is taken as infinite in the *x*-direction, andthethickness direction of plateis the *z*-axis. w_j denotes the transverse displacement of different plates with $j = 1, 2, 3$. p_i, p_r and p_t are the incident, the reflected and the transmitted acoustic pressures, respectively. ρ_0 and c_0 are the air density and the sound speed at atmosphere, respectively. ρ' and c' are the decompressed density and the sound under thedecompressed air, respectively.

The sound transmission is caused by the bending wave propagation in walls.According toFig. 1, we obtain theequations of motion interms of the transverse deflection, given by

$$D_1 w_1^{(4)} + \rho_1 h_1 \ddot{w}_1 = \left[p_{1i} + p_{1r} - p_{1t} - p_{tr} \right]_{z=0} \tag{1}$$

$$D_2 w_2^{(4)} + \rho_2 h_2 \ddot{w}_2 = [p_{2i} + p_{2r} - p_{2t} - p'_{tr}]_{z=0} \tag{2}$$

$$D_3 w_3^{(4)} + \rho_3 h_3 \ddot{w}_3 = [p_{3i} + p_{3r} - p_{3t}]_{z=0} \tag{3}$$

and

$$D_j = \frac{E_j h_j^3}{12(1-\nu_j^2)}, \qquad j = 1, 2, 3 \tag{4}$$

where ρ and h are the mass density and the thickness of plates, respectively. D,E and ν are the flexural rigidity, the elastic modulus, and the Poisson's ratioof plates, respectively.

2.2 Analytical solution of sound transmission loss

Consideringharmonic wave vibration, the acoustic pressures of the incident, the reflected and the transmitted sound waves in Eqs. (1)-(3) can be expressed as

$$p_i = P_i e^{i\omega t - ik(x\sin\theta + z\cos\theta)} \tag{5}$$

$$p_r = P_r e^{i\omega t - ik(x\sin\theta - z\cos\theta)} \tag{6}$$

$$p_t = P_t e^{i\omega t - ik(x\sin\theta + z\cos\theta)} \tag{7}$$

and, the transversedisplacement of platescan be written as

$$w_j = W_j e^{i(\omega t - kx\sin\theta)}, \quad j = 1, 2, 3 \tag{8}$$

where θ is the angle of incidence, ω is the angular frequency and $k = \omega/c$ is the wave number of the sound velocity in air. W, P_i, P_r and P_t are the amplitudes of the vertical displacement of plates, the incident, reflected and transmitted acoustic pressures, respectively.

SubstitutingEqs.(5)-(8) into Eqs.(1)-(3), we obtain

$$D_1 k^4 \sin^4\theta W_1 - \rho_1 h_1 \omega^2 W_1 = P_{1i} + P_{1r} - P_{1t} - P_{tr} \tag{9}$$

$$D_2 k^4 \sin^4 \theta W_2 - \rho_2 h_2 \omega^2 W_2 = P_{2i} + P_{2r} - P_{2t} - P'_{tr} \tag{10}$$

$$D_3 k^4 \sin^4 \theta W_3 - \rho_3 h_3 \omega^2 W_3 = P_{3i} + P_{3r} - P_{3t} \tag{11}$$

According to boundary conditions ofplates in the coupled analysis,the wave velocities at the two sides of each panel are continuous, we have

$$P_{1t} - P_{tr} = \frac{iZ'\omega W_1}{\cos\theta} \; P_{2i} - P_{2r} = \frac{iZ'\omega W_2}{\cos\theta} \tag{12}$$

$$P_{2t} - P'_{tr} = \frac{iZ'\omega W_2}{\cos\theta} \; P_{3i} - P_{3r} = \frac{iZ'\omega W_3}{\cos\theta} \tag{13}$$

$$P_{1i} - P_{1r} = \frac{iZ\omega W_1}{\cos\theta} \; P_{3t} = \frac{iZ\omega W_3}{\cos\theta} \tag{14}$$

where Z' is the impedance of decompression air space, and Z is airimpedance at atmospheric pressure.Next, the relationship of sound pressure between the plateswithair space*d*, is given as

$$P_{1t} = e^{ikd\cos\theta} P_{2i} \; P_{tr} = e^{-ikd\cos\theta} P_{2r} \tag{15}$$

$$P_{2t} = e^{ikd\cos\theta} P_{3i} \; P'_{tr} = e^{-ikd\cos\theta} P_{3r} \tag{16}$$

Substituting Eq. (15) into Eq. (12), and Eq. (16) into Eq. (13),we obtain

$$P_{2i} = \frac{iZ'\omega}{\cos\theta} \frac{W_2 e^{-ikd\cos\theta} - W_1}{e^{-ikd\cos\theta} - e^{ikd\cos\theta}} \qquad P_{2r} = \frac{iZ'\omega}{\cos\theta} \frac{W_2 e^{ikd\cos\theta} - W_1}{e^{-ikd\cos\theta} - e^{ikd\cos\theta}} \tag{17}$$

$$P_{3i} = \frac{iZ'\omega}{\cos\theta} \frac{W_3 e^{-ikd\cos\theta} - W_2}{e^{-ikd\cos\theta} - e^{ikd\cos\theta}} \; P_{3r} = \frac{iZ'\omega}{\cos\theta} \frac{W_3 e^{ikd\cos\theta} - W_2}{e^{-ikd\cos\theta} - e^{ikd\cos\theta}} \tag{18}$$

Furthermore, substituting Eqs. (17)-(18) into Eqs. (9)-(11), the motion equations can be expressed as amatrix form

$$\begin{bmatrix} s_{11} & s_{12} & 0 \\ s_{21} & s_{22} & s_{23} \\ 0 & s_{32} & s_{33} \end{bmatrix} \begin{Bmatrix} W_1 \\ W_2 \\ W_3 \end{Bmatrix} = \begin{Bmatrix} 2P_{1i} \\ 0 \\ 0 \end{Bmatrix} \tag{19}$$

where

$$s_{11} = D_1 (k \sin\theta)^4 - \rho_1 h_1 \omega^2 + \frac{iZ\omega}{\cos\theta} - \frac{iZ'\omega}{\cos\theta} \cdot \frac{1 + e^{i2kd\cos\theta}}{1 - e^{i2kd\cos\theta}} \qquad s_{12} = \frac{2iZ'\omega}{\cos\theta} \cdot \frac{e^{ikd\cos\theta}}{1 - e^{i2kd\cos\theta}} \qquad s_{13} = 0 \tag{20}$$

$$s_{21} = s_{12} \qquad s_{22} = D_2 (k \sin\theta)^4 - \rho_2 h_2 \omega^2 - \frac{i2Z'\omega}{\cos\theta} \cdot \frac{1 + e^{i2kd\cos\theta}}{1 - e^{i2kd\cos\theta}} \qquad s_{23} = \frac{i2Z'\omega}{\cos\theta} \cdot \frac{e^{ikd\cos\theta}}{1 - e^{i2kd\cos\theta}} \tag{21}$$

$$s_{31} = 0 \qquad s_{32} = s_{23} \qquad s_{33} = D_3 (k \sin\theta)^4 - \rho_3 h_3 \omega^2 + \frac{iZ\omega}{\cos\theta} - \frac{iZ'\omega}{\cos\theta} \cdot \frac{1 + e^{i2kd\cos\theta}}{1 - e^{i2kd\cos\theta}} \tag{22}$$

FromEq. (14) and Eq. (19), the transmission coefficient is defined as the power ratio of the transmitted to incident sound, which is

$$\tau(f, \theta) = \left| \frac{P_{3t}}{P_{1i}} \right|^2 = \left| \frac{i2Z\omega s_{21} s_{32} / \cos\theta}{s_{11}s_{22}s_{33} - s_{11}s_{32}s_{23} - s_{33}s_{12}s_{21}} \right|^2 \tag{23}$$

The transmission coefficient is a function of the incident angle θ.For a diffuse sound field, the averaged form of the transmission coefficient $\tau(f, \theta)$ can be obtainedfromintegrating a semi-circular surfaceover all the angles. The averaged transmission coefficient over all angles of incidence θis given by

$$\bar{\tau} = \frac{\int_0^{\theta_0} \tau(f, \theta) \sin\theta \cos\theta \, d\theta}{\int_0^{\theta_0} \sin\theta \cos\theta \, d\theta} \tag{24}$$

where the limiting θ_0 isusually taken to be 78°, above which it is assumed that no sound is received. Thus, the sound transmission loss (STL) with random incidence is obtained as

$$STL = -10 \log_{10} |\tau| \tag{25}$$

3. Analytical results and discussion

In thestudy, the multi-walled platesare considered to be two- or three-layered glass panels with an air layer between them. The Young's modulus ofa glass plate is taken as 72GPa, the density is 2200 kg/m^3, and the Poisson's ratio is 0.3. The air density (ρ) and the sound velocity (c) can be calculated from the atmospheric pressure (p) and the absolute temperature (T), given by

$$\rho = \frac{p}{RT}, c = \sqrt{\frac{\kappa p}{\rho}} \tag{25}$$

where $R = 287\, J/kg \cdot K$ is the specific gas constant for air, and $\kappa = 1.403$ is the heat capacity ratio of theair.

Figure 2 shows the sound transmission loss through two-walledglass platessubjected to the incident anglesof 0 and 45 degrees, and thescattering diffuse angle. The simulated result shows that the sound transmission loss depends largely onthe incidenceangles. Comparing the result of scattering angle with oblique incidences,the sound transmission loss is the maximumfor the case of normal incidence. The dips shown inFig. 2givethe mass-air-mass resonance frequencies, called coincidence critical frequency,ofglass panels separated by air space. The coincidence criticalfrequency is clearly observed to be large foran identical incidence, butthe dipsbecome weak for thescattering incidence.According to the present prediction, the critical frequency is about 900 Hz for the glass panels subjected to a normal incidence, and shifts to higher frequencywith increasing incidence angle. The dips of the critical frequencyfor the scattering incidence are found to be lower than that for the identical incidence, and the sound insulationperformanceof the two-walled glass platesdecreases.

In the following simulation, we considered general forms of the sound propagation and analyzed the sound transmission losses through panels based on scattering incidence of sound.The influence of layer numbers on the sound transmission loss are shown in Fig. 3 as a function of vibration frequency. The two- or three-walled panels are considered to have the same total thickness of 6 mm and the air space of 2 mm thick.The dip numbers of coincidence critical frequency increases with the increase of air layers because of interference vibration between adjacent plates.At low frequency less than 400 Hz (coincidence frequency), the multi-walled glass panel is observed to provide more high sound transmission loss, compared with single panel due to internal air space. A single glass panel hasthe largest sound insulation when the frequency is over 500 Hz.The influences of air space on the sound insulation of double-walled panels are shown in Fig.4, when the air interval between the two plates are 2 mm, 5mm, and 10 mm, and the thickness of each glass panel is 3 mm.It can be seen that thecoincidence critical frequencyof the glass panelsdecreases with the increase of air space. The influence of the air space on the sound transmission loss is very small for high frequency, except close to thecoincidence frequency value.

Figure 5 shows the sound transmission loss of the double-walled panels with separated by air space of 2.0 mm. The total panels are 6.0 mm thick, in which the thickness ratio $(t_1 : t_2)$are $(1,1)$, $(1,1.2)$ and $(1,2)$, respectively.It is observed that the thickness ratio ofthe double-walledpanelaffects the sound transmission loss, especially for the coincidencefrequency larger than1000 Hz.The sound insulation of double-walled panels can be largely improved just introducing a little variation. This can be explained that the difference in plate thickness cause more energy loss of sound pressure due to the mismatch in the resonance frequency between two plates.

Figures 6shows the influence of the decompressed pressure on the sound transmission loss of double-walled panelsseparated by air space of 2.0 mm. In the simulation, both layers between glass plates have the same pressure value, taken as the standard atmospheric pressure (1.0 atm), 1/5 atm and 1/10 atm with decompressed air. The sound insulationloss was precitedfor identical wall thickness of 3.0 mm (Fig. 6), and different wall thickness ratio (Fig. 7).The results suggest that both sound transmission loss and critical frequency are affected largely by decompressed air.The critical frequency decreases with decreasing air pressure, which resultsin improvement of the sound insulation ability in high frequency range. The sound transmission loss increases by about 5 dB for the vibration frequency over 3000 Hz, when the air pressure decreases tobelow 1/5 atm. For adifferent thickness ratio of 1:2, the influence of the decompressed pressure on the sound insulation is obtained fromFig. 7. It is observed that the effect of decompressed pressure onthe sound insulation clearly increase for the double-walled panel for different wall thickness ratio.

In summary, the mass-air-mass resonance frequencies(coincidence critical frequency) can be moved out of interest frequency,by adjusting such as the decompressed air, the air space and the different thickness ratio of panels. Therefore, the sound transmission performance of multi-walled panels can be improved by shifting resonance frequencies.

4. Conclusions

A theoretical model and an exact solution procedure areproposed to predictthe sound transmission loss of multi-walled structures with air layer. The motion equationsof the multi-walledplate areobtained based on the bending vibration mode and acoustic pressure method, and thesolution of the motion equation is derived. According to the proposed theoretical model and analysis, we investigate the influences of various parameters on the sound insulation loss of multi-walled panels.The sound insulation characteristics of the multi-walled panels is largely affected by air-space interval, plate thickness ratio and decompressed pressure.The result suggests that the sound transmission performanceof panels with air space can be improved through design parameters optimization.

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Figure captions

Figure 1Schematic illustration showingathree-layered construction

Figure 2Effect of the incidence angle on the sound transmission loss through two layers of panels with 2 mm thick and separated by an air space of 2 mm.

Figure3 Sound transmission loss through different panels, having the same total thickness but different layer numbers

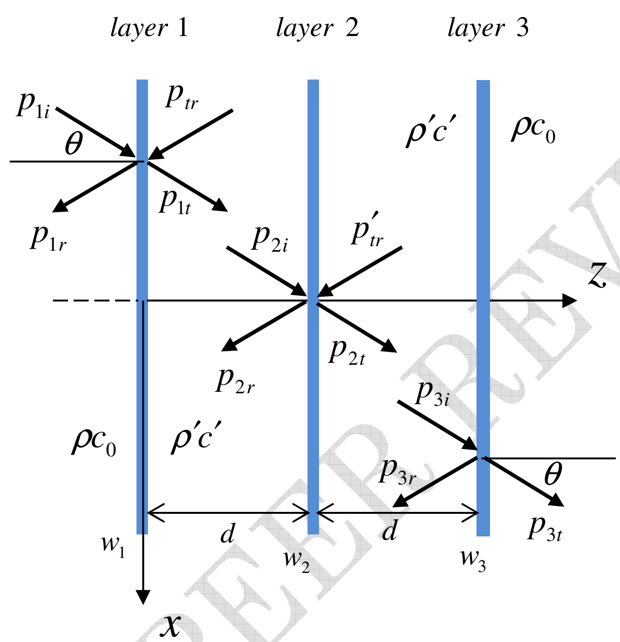
Figure 4 Influences of different air space on the sound insulation of panelswith 3 mm thick

Figure 5 Sound transmission loss of double-walled panels with a different thickness ratio and an air space of 2 mm

Figure 6 Influence of the decompressed air on the sound transmission loss of double-walled panels with each 3 mm thick and separated by 2 mm air layer

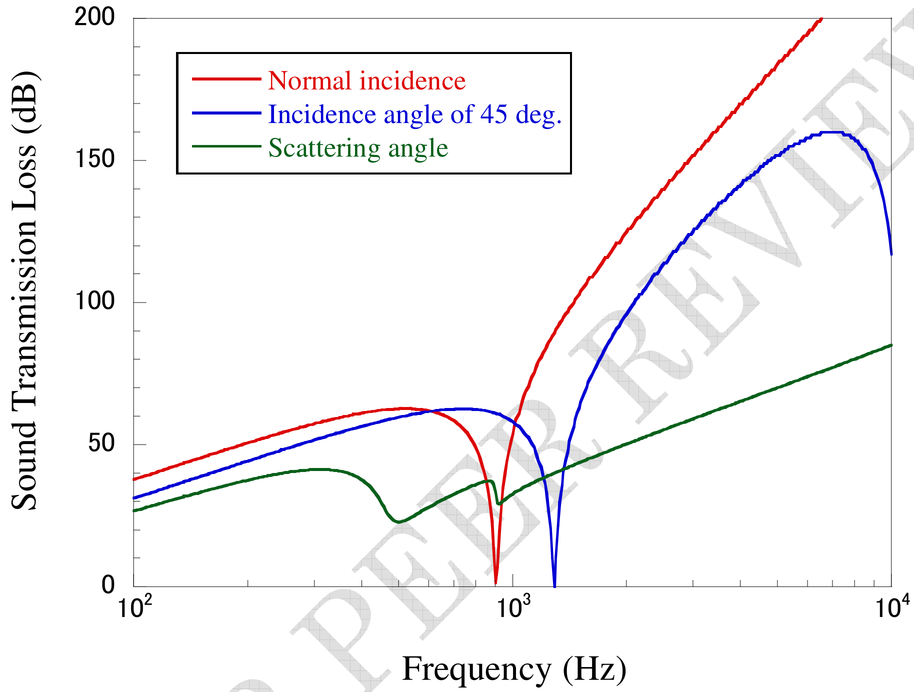
Figure 7 Influence of the decompressed air on the sound transmission loss of double-walled panels with thickness ratios $(t_1 : t_2)$ of $(1, 1.2)$ and separated by 2 mm air space

Figure1



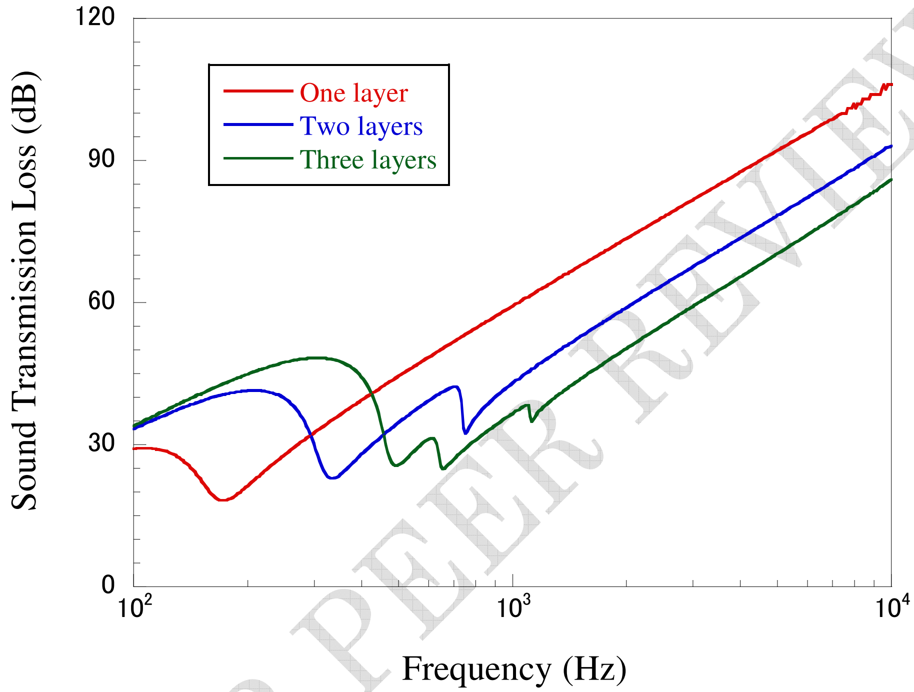
layer 1
layer 2
layer 3
p_{1i}
p_{tr}
θ
p_{1t}
p_{1r}
p_{2i}
p'_{tr}
$\rho' c'$
ρc_0
z
p_{2t}
p_{2r}
p_{3i}
ρc_0
$\rho' c'$
p_{3r}
θ
d
d
p_{3t}
w_1
w_2
w_3
x

Figure2



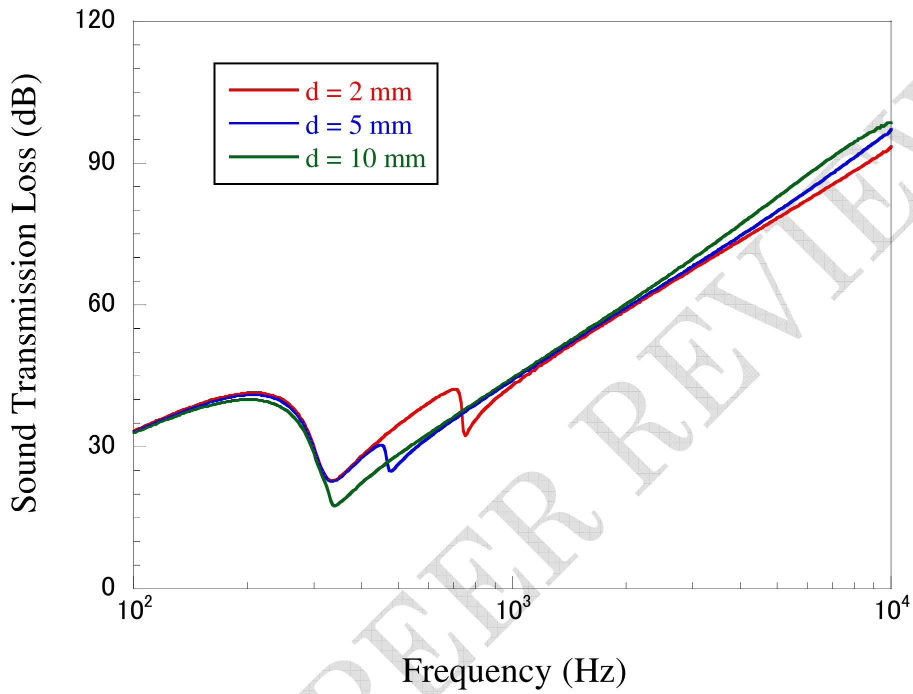
200
150
100
50
0
Sound Transmission Loss (dB)
Normal incidence
Incidence angle of 45 deg.
Scattering angle
10^2
10^3
10^4
Frequency (Hz)

Figure3



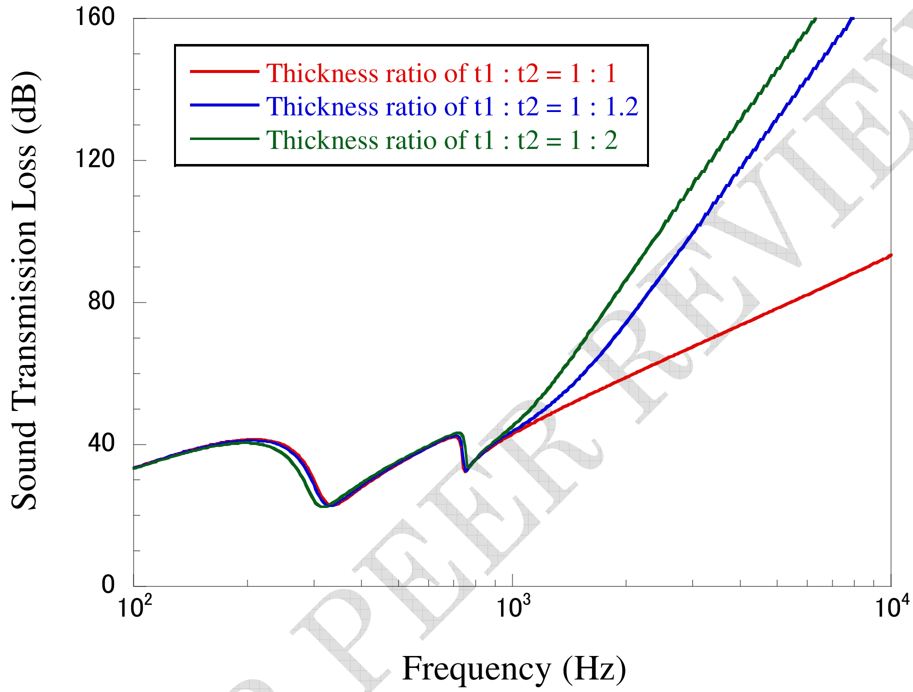
120
90
60
30
0
Sound Transmission Loss (dB)
10^2
10^3
10^4
Frequency (Hz)
One layer
Two layers
Three layers

Figure4



d = 2 mm
d = 5 mm
d = 10 mm
Sound Transmission Loss (dB)
Frequency (Hz)
0
30
60
90
120
10^2
10^3
10^4

Figure 5



Thickness ratio of t1 : t2 = 1 : 1
Thickness ratio of t1 : t2 = 1 : 1.2
Thickness ratio of t1 : t2 = 1 : 2
Sound Transmission Loss (dB)
Frequency (Hz)
160
120
80
40
0
10^2
10^3
10^4

Figure6

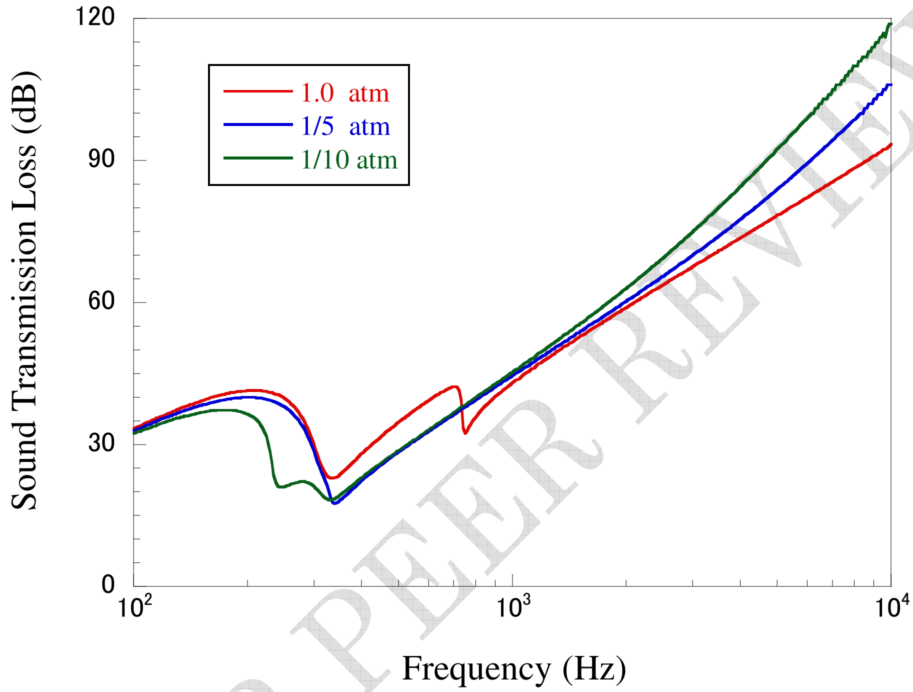
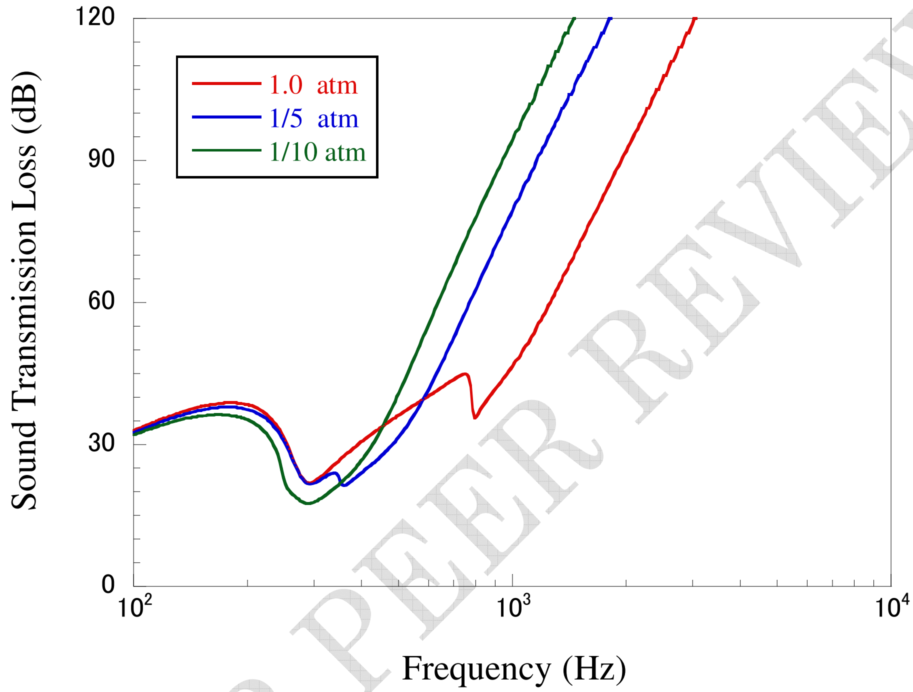

120
90
60
30
0
Sound Transmission Loss (dB)
1.0 atm
1/5 atm
1/10 atm
10^2
10^3
10^4
Frequency (Hz)

Figure 7


120
90
60
30
0
Sound Transmission Loss (dB)
1.0 atm
1/5 atm
1/10 atm
10^2
10^3
10^4
Frequency (Hz)