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A measure for evaluating the degree of slope rotatability in three level Second order slope rotatable designs

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Abstract

Response surface methodology (RSM) often deals with a natural and desirable property rotatability, which requires that, the variance of the predicted response at a point remains constant at all such points that are equidistant from the design center. To achieve stability in prediction variance, this important property of rotatability was developed. Analogous to rotatability, the concept of slope-rotatability has been progressed. The idea of slope - rotatability is an important design criterion for response surface design. Recently, in the design of experiments for response surface analysis, attention has been focused on the estimation of differences in response rather than absolute value of the response mean itself. The slope-rotatable design is that of which the variance of partial derivative is only a functions of ρ: distance from the design center. If circumstances are such that exact slope rotatability is unattainable because of more cost and time, and more important restrictions such as orthogonal blocking it is still a good idea to make the design as slope rotatable as possible. Thus, it is important to measure the extent of deviation from slope rotatability. In this study, a new measure of the degree of slope-rotatability for three level second-order slope rotatable designs using a pair of a partially balanced incomplete block design is suggested that enables us to assess the degree of slope-rotatability for a given response surface design. This determines the degree slope rotatability for the design when subjected to existing conditions of measure. The measure takes the value zero when the design is exact slope-rotatable, and becomes larger as the design deviates from being slope-rotatable design.

Keywords: slope- rotatability;second order slope rotatable designs(SOSRD); measure; partially balanced incomplete block designs(PBIBD).

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1 Introduction

Response surface methodology (RSM) is used in a situation where the form of the relationship between the response and independent variables is unknown. Therefore, the first step in RSM is to find a suitable approximation for the true functional relationship between the response variable and the set of independent variables. The technique to be used is to fit a low order polynomial to the response and if it is inadequate, then we graduate it to higher order polynomial. If the response is well demonstrated by a linear function of the independent variables, then the approximating function is the first order model. We use a second-order model when the portion of the response surface that we are describing has curvature. Response surface methodology is a gathering of mathematical and statistical techniques that are suitable for the demonstrating and analysis of problems in which a response of interest is impacted by several variables and the objective is to optimize this reaction. The study of rotatable designs mainly emphasized the estimation of absolute responses. The property of rotatability as a desirable quality of an experimental design was first advanced by [1]. A design is assumed to be rotatable if the variance of the response estimate is a function only of the distance of the point from the design center. In many applications of Response Surface Methodology, noble estimation of the derivatives of the response function may be as significant as or possibly more significant than the estimation of mean response [3]. Certainly, the computation of a stationary point in a second-order analysis or the use of gradient methods, for example, steepest ascent or ridge analysis depends heavily on the partial derivatives of the estimated response function with respect to the design variables. Since designs that achieve certain properties in Y (estimated response) do not delight in the same properties for the estimated derivatives (slopes), it is vital for the user to ponder experimental designs that are constructed with the derivatives in mind. The study of slope rotatable designs is mainly stressed on the estimation of differences of yields and their precision. Estimation of variances in responses at two different points in the factor space will often be of great importance. If variances in responses at two points close together are of interest then estimation of local slope (rate of change) of the response is essential. Several studies have been done on this aspect pertaining to the development of experimental designs. [3] presented slope rotatability for central composite designs. For the central composite designs, they altered [1] rotatability to slope rotatability essentially by altering the axial point distance (a), so that the variance of the assessed unadulterated quadratic coefficients is one-fourth the variance of the assessed mixed second order coefficients. [7] examined in detail the conditions to be satisfied by a common second-order slope rotatable designs (SOSRD) and developed SOSRD using balanced incomplete block designs (BIBD). [7] constructed SOSRD through a pair of incomplete block designs. The slope-rotatable design is that of which the variance of partial derivative is only a functions of ρ: distance from the design center. If circumstances are such that exact slope rotatability is unattainable because of more cost and time, and more important restrictions such as orthogonal blocking it is still a good idea to make the design as slope rotatable as possible. Thus, it is important to measure the extent of deviation from slope rotatability. [5], [6] proposed a measure of slope rotatability for second-order response experimental designs. [4] recommended a measure and graphical method for assessing slope rotatability in response surface designs. [12] examined a measure of SOSRD utilizing BIBD. [10] examined a measure of SOSRD utilizing pairwise balanced designs, [9] studied a measure of SOSRD utilizing symmetrical unequal block arrangements with two unequal block sizes. [11] examined the degree of SOSRD utilizing partially balanced incomplete block designs. [13] examined a measure of SOSRD utilizing a pair of balanced incomplete block design. These measures are valuable to empower us to survey the degree of slope rotatability for a given second-order response surface designs. In this study, we propose a method of construction of second-order slope rotatable designs using a pair of partially balanced incomplete block designs and their measure which leads to designs with a lesser number of design points than what is available in the existing designs.

1.1 Conditions for second order slope rotatable designs

This section presents briefly the conditions for slope rotatability to be satisfied by a symmetric second-order response surface design by [3] and Victorbabu [8]. Let consider the general second order response surface

$$y(x) = b_0 + \sum b_i x_i + \sum b_{ii} x^2{}_i + \sum \sum_{i \neq j} b_{ij} x_i x_j + e_u, \tag{1.1}$$

where x_{iu} denotes the level of the i^{th} factor $(i = 1, 2, , v)$ in the u^{th} run $(u = 1, 2, , N)$ of the experiment, where $e'_u s, u = 1, 2, ..$ are uncorrelated random errors with same mean zero and variance σ^2. the parameters of the model b_0, b_i, b_{ii} and b_{ij} are estimated by the least squares estimation to provide $\hat{b_0}, \hat{b_i}, \hat{b_{ii}}$ and $\hat{b_{ij}}$. The design is said to be SOSRD if the variance of the estimate of first order partial derivative of Y_u with respect to each of independent variables (x_i) is only a function of the distance $d^2 = \sum x_i^2$ of the point $(x_1, x_2, ..., x_v)$ from the origin (center) of the design. Such a spherical variance function for estimation of slopes in the Second Order Response Surface is achieved if the design points satisfy the following conditions [3]:

$A.$ $\quad \Sigma x_{ui} = 0, \quad \Sigma x_{ui} x_{uj} = 0, \quad \Sigma x_{ui}^3 = 0, \quad \Sigma x_{ui} x_{uj} x_{uk} = 0,$
$\quad \Sigma x_{ui}^2 x_{uj} x_{uk} = 0, \quad \Sigma x_{ui}^3 x_{uj} = 0 \quad \Sigma x_{ui} x_{uj} x_{uk} x_{ul} = 0;$ for $\quad i \neq j \neq k \neq l$

$B.$ $\quad (i) \Sigma x_{ui}^2 =$ constant $= N\lambda_2$

$(ii) \quad \Sigma x_{ui}^4 =$ constant $= cN\lambda_2$ for all i

$C.$ $\quad \Sigma x_{ui}^2 x_{uj}^2 =$ constant $= N\lambda_4$ for $i \neq j$ (1.2)

$D.$ $\quad \dfrac{\lambda_4}{\lambda_2^2} > \dfrac{v}{(c + v - 1)}$

$E.$ $\quad \lambda_4[v(5 - c) - (c - 3)^2] + \lambda_2^2[v(c - 5) + 4] = 0$

where c, λ_2, λ_4 are constants and v denotes the number of factors. Using these symmetry conditions, we can obtained the following estimates

$$\hat{b_0} = \frac{\lambda_4(c + v - 1)\Sigma y - \lambda_2 \Sigma(\Sigma x_i^2 y)}{N[\lambda_4(c + v - 1) - v\lambda_2^2]}$$

$$\hat{b_i} = \frac{\Sigma x_i y}{N\lambda_2}$$

$$\hat{b_{ij}} = \frac{\Sigma x_i x_j y}{N\lambda_4}$$

$$\hat{b_{ii}} = \frac{\Sigma x_i x_j y}{(c - 1)N\lambda_4} - \frac{\lambda_2 \lambda_4 (c - 1)\Sigma y - \Sigma(\Sigma x_i^2 y)(\lambda_2^2 - \lambda_4)}{(c - 1)N\lambda_4[\lambda_4(c + v - 1) - v\lambda_2^2]}$$

The variances and covariances of the evaluated parameters are;

$$V(\hat{b_0}) = \frac{\lambda_4(c+v-1)\sigma_2}{N[\lambda_4(c+v-1)-v\lambda_2^2]}$$

$$V(\hat{b_i}) = \frac{\sigma_2}{N\lambda_2}$$

$$V(\hat{b_{ij}}) = \frac{\sigma_2}{N\lambda_4}$$

$$V(\hat{b_{ii}}) = \frac{\sigma_2}{(c-1)N\lambda_4}\left(\frac{\lambda_4(c+v-2)-(v-1)\lambda_2^2}{\lambda_4(c+v-1)-v\lambda_2^2}\right)$$

$$Cov(\hat{b_0},\hat{b_{ii}}) = \frac{-\lambda_2\sigma_2}{N[\lambda_4(c+v-1)-v\lambda_2^2]}$$

$$Cov(\hat{b_{ii}},\hat{b_{jj}}) = \frac{(\lambda_2^2-\lambda_4)\sigma_2}{(c-1)N\lambda_4[\lambda_4(c+v-1)-v\lambda_2^2]} \tag{1.3}$$

and other covariances are zero.

$$\frac{\partial\hat{y}}{\partial x_i} = \hat{b}_i + 2\hat{b}_{ii}x_i + \sum \hat{b}_{ij}x_j$$
$$V\left(\frac{\partial\hat{y}}{\partial x_i}\right) = V(\hat{b_i}) + 4x_i^2V(\hat{b_{ii}}) + \sum x_j^2V(\hat{b_{ij}})$$

2 Second order slope roratable designs using a pair of PBIBDs

Let D_1 =$(v, b_1, r_1, k_1, \lambda_{11} \neq 0, \lambda_{12}$=0) and D_2= $(v, b_2, r_2, k_2 = 2, \lambda_{21} = 0, \lambda_{22} = 1)$ are two PBIBDs. [a-$(v, b_1, r_1, k_1, \lambda_{11} \neq 0, \lambda_{12}$=0)] denote the design points generated from the transpose of the incidence matrix of the design D_1.
$[a - (v, b_1, r_1, k_1, \lambda_{11}, \lambda_{12} = 0)]2^{t(k_1)}$ are the $2^{t(k_1)}$ design points generated from D_1 by multiplication (see[2]).$[a - (v, b_2, r_2, k_2 = 2, \lambda_{21} = 0, \lambda_{22} = 1)]2^2$ are the $b_2 2^2$ design points generated from D_2 by multiplication.The set of $b_2 2^2$ design points was repeated m_2 times. Let n_0 be the number of central points.

Result:

The design points,
$[a - (v, b_1, r_1, k_1, \lambda_{11}, \lambda_{12} = 0]2^{t(k_1)} \cup [a - (v, b_2, r_2, k_2 = 2, \lambda_{21} = 0, \lambda_{22} = 1)]2^2 \cup n_0$ give a v-dimensional three level SOSRD in N = $b_1 2^{t(k_1)}$ +$m_2 b_2 2^2$ + n_0 design points where m_2 is

$$m_2 = \left\{\frac{(r_1 - c\lambda_{11})2^{t(k_1)-2}}{(c\lambda_{21} - r_2)}\right\}^{1/4}, \tag{2.1}$$

2.1 Conditions of measure of second order slope rotatable designs

Using the equation below

$$\frac{\partial \hat{y}}{\partial x_i} = \hat{b}_i + 2\hat{b}_{ii}x_i + \sum \hat{b}_{ij}x_j,$$

the variance of this derivative is written as

$$\begin{aligned} var[\frac{\partial \hat{y}}{\partial x_v}] &= var(b_i) + 4x_i^2 var(b_{ii}) + \sum_{j=1,j=i}^{v} x_j^2 var(b_{ij}) \\ &+ 4x_i cov(b_{ii}, b_{ij}) + 2\sum_{j=1,j=i}^{v} x_j cov(b_i, b_{ij}) \\ &+ 4x_i \sum_{j=1,j=i}^{v} x_j cov(b_i, b_{ij}) + 2\sum \sum_{j=1,j=i}^{v} x_j x_i cov(b_{ij}, b_i) \end{aligned} \tag{2.2}$$

Following [3],[7], [6], equations $(2-3)$ give the necessary and sufficient conditions for a measure of slope rotatability for any general second order response surface designs. Further, from the above equation (2.2), it can be seen that the necessary and sufficient conditions are:

- $var(b_i)$ are equal for *i*
- $var(b_{ii})$ are equal for *i*
- $4var(b_{ii})$ = $var(b_{ij})$ are equal for *i*, j where *i* $\neq j$
- $cov(b_i, b_{ii}) = cov(b_i, b_{ij}) = cov(b_{ii}, b_{ij}) = cov(b_{ij}, b_{il}) = 0$ for all *i* $\neq j \neq l,$

[6] proposed that, if the following conditions below are met, that is

i. All odd-order moments up-to order 4 are zero,

ii. $\frac{1}{N}\sum_{u=1}^{N} x_{iu}^2$ are equal for all *i*,

iii. $\frac{1}{N}\sum_{u=1}^{N} x_{iu}^4$ are equal for all *i*,

iv. $\sum_{u=1}^{N} x_{iu}^2 x_{ju}^2$ are equal for all $i \neq j$,

Then the following measure assesses the degree of slope-rotatability for a design **D** with **v** independent variables.

$$\begin{aligned} Q_v(D) = \frac{1}{2(v-1)\sigma^4} \Bigg\{ &(v+2)(v+4)\sum_{i=1}^{v}\left[(var(b_i) - \tilde{v}) + \frac{a_i - \tilde{a}}{v+2}\right]^2 \\ &+ \frac{4}{v(v+2)}\sum_{i=1}^{v}(a_i - \tilde{a})^2 + 2\sum_{i=1}^{v}\left[(4var(b_{ii}) - \frac{\tilde{a}}{v})^2 + \sum_{j=l,j\neq i}^{v} var(b_{ij}) - \frac{\tilde{a}}{v})^2\right] \\ &+ 4(v+4)\left[4cov(b_i, b_{ii})^2 + \sum_{j=l,j\neq i}^{v} cov(b_i, b_{ij})^2\right] + 4\sum_{i=1}^{v}\Bigg[\sum_{j=l,j\neq i}^{v} cov(b_{ii}, b_{ij})^2 \\ &+ \sum^{v} \sum_{j<l,j,l\neq i}^{v} cov(b_{ij}, b_{il})^2\Bigg]\Bigg\} \end{aligned} \tag{2.3}$$

where
$\tilde{v} = \frac{1}{v}\sum_{i=1}^{v} v_i$
$a_i = 4v_{ii} + \sum_{j=l, j\neq i}^{v} v_{ij}\ (i = 1, 2, 3, ...v)$
$\tilde{a} = \frac{1}{v}\sum_{i=1}^{a} v_i.$
where $Q_v(D)$ is the proposed measure of slope rotatability. Further [6] went ahead and simplified the above equation (2.3) to

$$Q_v(D) = \frac{1}{\sigma^4}\Bigg[4V(b_{ii}) - V(b_{ij})\Bigg]^2 \tag{2.4}$$

$Q_v(D)$ becomes zero if and only if the necessary and sufficient conditions hold. If the measure is zero, the design is slope-rotatable. If it becomes larger, it deviates from being slope-rotatable.

3 Construction of measure of slope rotatability of three level second-order response surface designs using a pair of PBIBDs

The proposed measure of slope rotatability of three level second-order response surface designs using a pair of PBIBDs is suggested in this section. Let D_1 =$(v, b_1, r_1, k_1, \lambda_{11} \neq 0, \lambda_{12}$=0) and D_2= $(v, b_2, r_2, k_2 = 2, \lambda_{21} = 0, \lambda_{22} = 1)$ are two PBIBDs. [a- $(v, b_1, r_1, k_1, \lambda_{11} \neq 0, \lambda_{12}$=0)] denote the design points generated from the transpose of the incidence matrix of the design D_1.
$[1 - (v, b_1, r_1, k_1, \lambda_{11}, \lambda_{12} = 0)]2^{t(k_1)}$ are the $2^{t(k_1)}$ design points generated from D_1 by multiplication (see[2]). $[a - (v, b_2, r_2, k_2 = 2, \lambda_{21} = 0, \lambda_{22} = 1)]2^2$ are the $b_2 2^2$ design points generated from D_2 by multiplication.The set of $b_2 2^2$ design points was repeated m_2 times. Let n_0 be the number of central points.Then with the above design points, we can obtain measure of slope rotatability for second order slope rotatable designs as given in the theorem below.

Result:

The design points,
$[a - (v, b_1, r_1, k_1, \lambda_{11}, \lambda_{12} = 0]2^{t(k_1)} \cup [a - (v, b_2, r_2, k_2 = 2, \lambda_{21} = 0, \lambda_{22} = 1)]2^2 \cup n_0$ give a v-dimensional measure of slope rotatability of three level second order response surface designs using a pair of PBIBDs in N = $b_1 2^{t(k_1)}$ +$m_2 b_2 2^2$ + n_0 design points with 'c'' pre-fixed, n_0 chosen and the design levels,suitably such that the design points satisfy the conditions of SOSRD that is

$$m_2 = \left\{\frac{(r_1 - c\lambda_{11})2^{t(k_1)-2}}{(c\lambda_{21} - r_2)}\right\}^{1/4}, \tag{3.1}$$

$$n_0 = \left[\frac{[v(c-5)+4][r_1 2^{t(k_1)} + r_2 2^2]^2}{[v(c-5)+(c-3)^2][\lambda_{11}2^{t(k_1)} + \lambda_{21}2^2]}\right] - b_1 2^{t(k_1)} - m_2 b_2 2^2 \tag{3.2}$$

Proof

For the design points generated from a pair of PBIBDs, conditions (A) to (C) are true. Conditions in (A) are true obviously. Conditions (B) to (C) of equation 1.2 are true as follows:

$$\sum_{u=1}^{N} x_{iu}^2 = r_1 2^{t(k_1)} a^2 + r_2 2^2 a^2 = N\lambda_2 \tag{3.3}$$

$$\sum_{u=1}^{N} x_{iu}^4 = r_1 2^{t(k_1)} a^4 + r_2 2^2 a^4 = cN\lambda_4 \tag{3.4}$$

$$\sum_{u=1}^{N} x_{iu}^2 x_{ju}^2 = \lambda_{11} 2^{t(k_1)} a^4 + \lambda_{21} 2^2 a^4 = N\lambda_4 \tag{3.5}$$

From equation (3.4) and (3.5), we get m_2,

$$a = \left[\frac{(r_1 - c\lambda_{11})2^{t(k_1)-2}}{(c\lambda_{21} - r_2)}\right],$$

The value of **a** can be obtained from equation (3.3) by taking the scaling condition,that is $\lambda_2 = 1$.

$$a = \left[\frac{N}{r_1 2^{t(k_1)} + m_2 r_2 2^2}\right]^{\frac{1}{2}} \tag{3.6}$$

Measure of slope rotatability of three level second order designs using a pair of PBIBDs can be obtained by solving the given simplified equation by [6]

$$Q_v(D) = \frac{1}{\sigma^4}\left[4V(b_{ii}) - V(b_{ij})\right]^2 \tag{3.7}$$

where $V(\hat{b_{ii}}) = \frac{\sigma^2}{(c-1)N\lambda_4}\left(\frac{\lambda_4(c+v-2)-(v-1)\lambda_2^2}{\lambda_4(c+v-1)-v\lambda_2^2}\right)$ and $V(\hat{b_{ij}}) = \frac{\sigma^2}{N\lambda_4}$

Therefore

$$Q_v(D) = \frac{1}{\sigma^4}\left[\frac{4\sigma^2}{(c-1)N\lambda_4}\left(\frac{\lambda_4(c+v-2)-(v-1)\lambda_2^2}{\lambda_4(c+v-1)-v\lambda_2^2}\right) - \frac{\sigma^2}{N\lambda_4}\right]^2$$

$$Q_v(D) = \frac{1}{\sigma^4}\left[\frac{4\sigma^2\left(\lambda_4(c+v-2)-(v-1)\lambda_2^2\right) - \sigma^2\left((c-1)(c+v-2)\lambda_4 - v\lambda_2^2\right)}{N(c-1)\lambda_4\left[(c+v-1)\lambda_4 - v\lambda_2^2\right]}\right]^2$$

$$Q_v(D) = \frac{\sigma^4}{\sigma^4}\left[\frac{4\left(\lambda_4(c+v-2)-(v-1)\lambda_2^2\right) - \left((c-1)(c+v-2)\lambda_4 - v\lambda_2^2\right)}{N(c-1)\lambda_4\left[(c+v-1)\lambda_4 - v\lambda_2^2\right]}\right]^2$$

After simplification $Q_v(D)$ becomes;

$$Q_v(D) = \left[\frac{\lambda_4[v(5-c) - (c-3)^2] + \lambda_2^2[v(c-5)+4]}{N(c-1)\lambda_4\left[(c+v-1)\lambda_4 - v\lambda_2^2\right]}\right]^2 \tag{3.8}$$

Where λ_4,λ_2, a,m_2 and N are as shown below

$$\lambda_2 = \left[\frac{r_1 2^{t(k_1)} a^2 + r_2 2^2 a^2}{N}\right]$$

$$\lambda_4 = \left[\frac{\lambda_{11} 2^{t(k_1)} a^4 + \lambda_{21} 2^2 a^4}{N}\right]$$

$$m_2 = \left[\frac{(r_1 - c\lambda_{11})2^{t(k_1)-2}}{(c\lambda_{21} - r_2)}\right],$$

$$a = \left[\frac{N}{r_1 2^{t(k_1)} + m_2 r_2 2^2}\right]^{\frac{1}{2}}$$

$$N = b_1 2^{t(k_1)} + m_2 b_2 2^2 + n_0$$

and

$$n_0 = \left[\frac{[v(c-5)+4][r_1 2^t k_1 + r_2 2^2 a^2]^2}{[v(c-5)+(c-3)^2][\lambda_{11} 2^t k_1 + \lambda_{21} 2^2 a^4]}\right] - b_1 2^{k_1} - m_2 b_2 2^2$$

The computation of measure of slope rotatability $Q_v(D)$ of three level second order response surface designs using various parametres of PBIBDs for varied values of c ranging from 3 to 16 and level a are tabulated below in the appendix.

4 CONCLUSIONS

In this study, a measure of slope rotatability for second-order response surface designs using a pair of PBIBDs is suggested which enables us to assess the degree of slope rotatability of a given three level second-order response surface design. It can be verified that measure of slope rotatability is zero if and only if a design D is a second order slope-rotatable design. Measure of slope rotatability becomes larger as D deviates from a second order slope rotatable design. The method can be used to compare the degree of slope rotatability of the same v. We may point out here that the measure of slope rotatability for second order response surface designs using a pair of PBIBDs with parameters $D_1 = (v = 16, b_1 = 20, r_1 = 5, k_1 = 4, \lambda_{11} = 1, \lambda_{12} = 0)$, $D_2 = (v = 16, b_2 = 8, r_2 = 1, k_2 = 2, \lambda_{21} = 0, \lambda_{22} = 1)$ has only 400 design points for $16-$ factors, whereas the corresponding measure of slope rotatability for second order response surface designs using a pair of BIBDs with parameters $D_1 = (v = 16, b_1 = 16, r_1 = 6, k_1 = 6, \lambda_1 = 2)$, $D_2 = (v = 16, b_2 = 80, r_2 = 15, k_2 = 3, \lambda_2 = 2)$ of [13] needs 1154 design points. Thus this new method leads to 16-factor measure of SOSRD with less number of design points than the existing measure of slope rotatable designs using a pair of BIBDs. **When $c = 5$ the design is exact slope rotatability that is the measure of slope rotatability is zero.**

Appendices

$D_1 = (v = 9, b_1 = 9, r_1 = 3, k_1 = 3, \lambda_{11} = 1, \lambda_{12} = 0)$,
$D_2 = (v = 9, b_2 = 9, r_2 = 2, k_2 = 2, \lambda_{21} = 0, \lambda_{22} = 1)$

c	m_1	a	n_0	N	λ_4	λ_2	$(\lambda_2)^2$	$Q_v(D)$	a^2
3	0	1.5	-16	56	0.723	0.964	0.929	5.86E-08	2.3
3.5	0.5	1.6	-20	70	0.749	1.024	1.049	1.45E-07	2.5
4	1	1.6	-28	80	0.655	1.024	1.049	4.05E-10	2.5
4.5	1.5	1	-90	36	0.222	1	1	8.25E-12	1
5	2	2.2	56	200	0.937	0.968	0.937	0	5
5.5	2.5	2.1	29	191	0.815	1.016	1.032	8.01E-11	4.3
6	3	2.1	28	208	0.748	1.018	1.036	2.01E-11	4.3
6.5	3.5	2.1	32	230	0.676	0.997	0.994	2.69E-10	4.4
7	4	2.1	38	254	0.613	0.972	0.945	6.51E-09	4.5
7.5	4.5	2.2	45	279	0.672	1.041	1.084	6.18E-12	4.7
8	5	2.2	53	305	0.614	1.016	1.032	8.3E-09	4.8
8.5	5.5	2.2	62	332	0.564	0.991	0.982	2.68E-09	4.9
9	6	2.2	72	360	0.521	0.968	0.937	2.53E-09	5
9.5	6.5	2.3	82	388	0.577	1.036	1.073	4.36E-12	5.1
10	7	2.3	93	417	0.537	1.015	1.03	1E-10	5.2
10.5	7.5	2.3	104	446	0.502	0.996	0.992	3.61E-10	5.3
11	8	2.3	116	476	0.47	0.978	0.956	2.71E-10	5.4
11.5	8.5	2.3	128	506	0.442	0.962	0.925	9.25E-10	5.5
12	9	2.4	140	536	0.495	1.032	1.065	6.66E-09	5.6
12.5	9.5	2.4	153	567	0.468	1.016	1.032	1.74E-09	5.7
13	10	2.4	165	597	0.445	1.003	1.006	8.2E-09	5.7
13.5	10.5	2.4	178	628	0.423	0.991	0.982	4.28E-09	5.8
14	11	2.4	192	660	0.402	0.977	0.955	1.14E-09	5.9
14.5	11.5	2.4	205	691	0.384	0.967	0.935	6.81E-09	6
15	12	2.4	219	723	0.367	0.956	0.914	2.26E-09	6
15.5	12.5	2.5	233	755	0.414	1.026	1.053	6.21E-09	6.1
16	13	2.5	247	787	0.397	1.017	1.034	1.04E-08	6.1

Table 1: Measure of 9 factors three level SOSRD(PBIBD)

$D_1 = (v = 12, b_1 = 9, r_1 = 3, k_1 = 4, \lambda_{11} = 1, \lambda_{12} = 0)$, $D_2 = (v = 12, b_2 = 12, r_2 = 2, k_2 = 2, \lambda_{21} = 0, \lambda_{22} = 1)$									
c	m_1	a	n_0	N	λ_4	λ_2	$(\lambda_2)^2$	$Q_v(D)$	a^2
3	0	1.6	-24	120	0.874	1.024	1.049	2.93E-09	2.5
3.5	1	1.7	-37	155	0.862	1.044	1.09	4.36E-08	2.8
4	2	1.7	-54	186	0.718	0.994	0.988	1.9E-10	2.9
4.5	3	1.5	-115	173	0.468	0.936	0.876	1.06E-11	2.4
5	4	2.2	64	400	0.937	0.968	0.937	0	5
5.5	5	2.1	11	395	0.788	0.982	0.964	4.19E-11	4.5
6	6	2.1	7	439	0.709	0.964	0.929	3.15E-10	4.6
6.5	7	2.2	12	492	0.762	1.023	1.047	1.08E-10	4.7
7	8	2.2	21	549	0.683	0.987	0.974	1.24E-09	4.9
7.5	9	2.3	33	609	0.735	1.042	1.086	3.51E-11	5.1
8	10	2.3	47	671	0.667	1.009	1.018	5.31E-10	5.2
8.5	11	2.3	64	736	0.608	0.978	0.956	1.39E-09	5.4
9	12	2.4	82	802	0.662	1.034	1.069	1.31E-10	5.6
9.5	13	2.4	102	870	0.61	1.006	1.012	1.03E-10	5.7
10	14	2.4	123	939	0.565	0.981	0.962	1.23E-10	5.9
10.5	15	2.4	146	1010	0.526	0.958	0.918	8.65E-10	6
11	16	2.5	170	1082	0.578	1.017	1.034	1.55E-10	6.1
11.5	17	2.5	195	1155	0.541	0.996	0.992	1.11E-09	6.3
12	18	2.5	221	1229	0.509	0.976	0.953	4.21E-09	6.4
12.5	19	2.5	248	1304	0.479	0.959	0.92	8.13E-09	6.5
13	20	2.6	276	1380	0.53	1.019	1.038	1.19E-09	6.6
13.5	21	2.6	304	1456	0.502	1.003	1.006	1.74E-09	6.7
14	22	2.6	334	1534	0.477	0.987	0.974	4.14E-09	6.8
14.5	23	2.6	364	1612	0.454	0.973	0.947	5.99E-10	6.9
15	24	2.6	395	1691	0.432	0.959	0.92	2.66E-10	7
15.5	25	2.7	426	1770	0.48	1.021	1.042	5.65E-11	7.1
16	26	2.7	459	1851	0.459	1.008	1.016	4.4E-11	7.2

Table 3: Measure of 12 factors three level SOSRD(PBIBD)

$D_1 = (v = 10, b_1 = 15, r_1 = 6, k_1 = 4, \lambda_{11} = 2, \lambda_{12} = 0)$, $D_2 = (v = 10, b_2 = 10, r_2 = 2, k_2 = 2, \lambda_{21} = 0, \lambda_{22} = 1)$									
c	m_1	a	n_0	N	λ_4	λ_2	$(\lambda_2)^2$	$Q_v(D)$	a^2
3	0	1.5	-10	230	0.704	0.939	0.882	7.06E-08	2.4
3.5	2	1.6	-28	292	0.718	0.982	0.964	1.5E-09	2.6
4	4	1.6	-59	341	0.615	0.961	0.924	1.32E-10	2.7
4.5	6	1.3	-244	236	0.387	1.031	1.063	5.23E-13	1.6
5	8	2.2	240	800	0.937	0.968	0.937	0	5
5.5	10	2.1	134	774	0.804	1.003	1.006	4.04E-12	4.4
6	12	2.1	129	849	0.733	0.997	0.994	1.12E-11	4.4
6.5	14	2.1	143	943	0.66	0.973	0.947	9.46E-12	4.5
7	16	2.2	165	1045	0.717	1.037	1.075	1.37E-11	4.7
7.5	18	2.2	194	1154	0.65	1.007	1.014	5.19E-12	4.8
8	20	2.2	226	1266	0.592	0.979	0.958	2.23E-11	4.9
8.5	22	2.3	262	1382	0.648	1.041	1.084	3.19E-12	5.1
9	24	2.3	301	1501	0.597	1.015	1.03	2.65E-10	5.2
9.5	26	2.3	342	1622	0.552	0.991	0.982	2.18E-10	5.3
10	28	2.3	385	1745	0.513	0.97	0.941	9.89E-11	5.5
10.5	30	2.4	431	1871	0.567	1.034	1.069	4.4E-12	5.6
11	32	2.4	478	1998	0.531	1.015	1.03	5.01E-10	5.7
11.5	34	2.4	528	2128	0.499	0.996	0.992	1.33E-10	5.8
12	36	2.4	578	2258	0.47	0.98	0.96	4.93E-10	5.9
12.5	38	2.4	630	2390	0.444	0.964	0.929	4.1E-11	6
13	40	2.5	684	2524	0.495	1.03	1.061	3.27E-11	6.1
13.5	42	2.5	738	2658	0.47	1.016	1.032	4.09E-10	6.2
14	44	2.5	794	2794	0.447	1.002	1.004	2.27E-10	6.2
14.5	46	2.5	851	2931	0.426	0.989	0.978	1.2E-11	6.3
15	48	2.5	909	3069	0.407	0.978	0.956	1.02E-09	6.4
15.5	50	2.5	968	3208	0.39	0.966	0.933	2.18E-09	6.5
16	52	2.5	1027	3347	0.373	0.956	0.914	1.39E-09	6.5

Table 2: Measure of 10 factors three level SOSRD(PBIBD)

$D_1 = (v = 16, b_1 = 20, r_1 = 5, k_1 = 4, \lambda_{11} = 1, \lambda_{12} = 0)$, $D_2 = (v = 16, b_2 = 8, r_2 = 1, k_2 = 2, \lambda_{21} = 0, \lambda_{22} = 1)$									
c	m_1	a	n_0	N	λ_4	λ_2	$(\lambda_2)^2$	$Q_v(D)$	a^2
3	-8	1.6	62	126	0.832	0.9752	0.951	6.32E-09	2.63
3.5	-6	1.7	37	165	0.81	0.9808	0.962	3.38E-10	2.95
4	-4	1.8	13	205	0.819	1.0115	1.023	4.9E-10	3.2
4.5	-2	1.8	-31	225	0.746	1.0368	1.075	4.54E-11	3.13
5	0	2.2	80	400	0.937	0.968	0.937	0	5
5.5	2	2.2	24	408	0.919	1.0439	1.09	4.45E-11	4.64
6	4	2.2	13	461	0.813	1.0079	1.016	1.07E-11	4.8
6.5	6	2.2	10	522	0.718	0.9643	0.93	1.19E-10	5.02
7	8	2.3	12	588	0.761	1.0076	1.015	7.93E-11	5.25
7.5	10	2.3	17	657	0.682	0.9662	0.934	2.22E-11	5.48
8	12	2.4	25	729	0.728	1.0114	1.023	1.39E-09	5.7
8.5	14	2.4	36	804	0.66	0.9743	0.949	1.34E-10	5.91
9	16	2.5	49	881	0.709	1.0216	1.044	3.48E-09	6.12
9.5	18	2.5	65	961	0.65	0.9886	0.977	4.56E-11	6.32
10	20	2.6	82	1042	0.702	1.038	1.077	1.85E-09	6.51
10.5	22	2.6	101	1125	0.65	1.0095	1.019	5.93E-11	6.7
11	24	2.6	122	1210	0.604	0.9833	0.967	1.25E-09	6.88
11.5	26	2.7	145	1297	0.656	1.0342	1.07	6.47E-10	7.05
12	28	2.7	169	1385	0.614	1.0106	1.021	8.5E-10	7.21
12.5	30	2.7	194	1474	0.577	0.9891	0.978	3.87E-10	7.37
13	32	2.7	221	1565	0.543	0.9689	0.939	6.15E-09	7.52
13.5	34	2.8	250	1658	0.593	1.0214	1.043	5.73E-12	7.68
14	36	2.8	279	1751	0.562	1.0029	1.006	2.64E-10	7.82
14.5	38	2.8	310	1846	0.533	0.9853	0.971	1.38E-10	7.96
15	40	2.8	342	1942	0.506	0.9689	0.939	6.42E-09	8.09
15.5	42	2.9	375	2039	0.555	1.0229	1.046	2.21E-10	8.22
16	44	2.9	409	2137	0.53	1.0075	1.015	2.16E-09	8.35

Table 5: Measure of 16 factors three level SOSRD(PBIBD)

$D_1 = (v = 14, b_1 = 28, r_1 = 6, k_1 = 3, \lambda_{11} = 1, \lambda_{12} = 0)$, $D_2 = (v = 14, b_2 = 7, r_2 = 1, k_2 = 2, \lambda_{21} = 0, \lambda_{22} = 1)$									
c	m_1	a	n_0	N	λ_4	λ_2	$(\lambda_2)^2$	$Q_v(D)$	a^2
3	-6	1.6	6	62	0.846	0.991	0.982	2.91E-05	2.6
3.5	-5	1.7	-4	80	0.835	1.012	1.024	7.68E-07	2.9
4	-4	1.8	-14	98	0.857	1.058	1.119	3.14E-08	3.1
4.5	-3	1.7	-38	102	0.655	1.02	1.04	1.46E-10	2.8
5	-2	2.2	32	200	0.937	0.968	0.937	0	5
5.5	-1	2.1	5	201	0.774	0.965	0.931	2.6E-10	4.6
6	0	2.2	1	225	0.833	1.033	1.067	3.18E-09	4.7
6.5	1	2.2	2	254	0.738	0.991	0.982	3.01E-10	4.9
7	2	2.3	5	285	0.786	1.039	1.08	8.86E-10	5.1
7.5	3	2.3	10	318	0.704	0.998	0.996	5.36E-09	5.3
8	4	2.3	16	352	0.636	0.962	0.925	9.5E-09	5.5
8.5	5	2.4	23	387	0.686	1.012	1.024	1.29E-08	5.7
9	6	2.4	31	423	0.627	0.98	0.96	1.25E-08	5.9
9.5	7	2.5	40	460	0.679	1.033	1.067	7.24E-10	6.1
10	8	2.5	49	497	0.629	1.006	1.012	1.93E-09	6.2
10.5	9	2.5	60	536	0.583	0.979	0.958	1.03E-08	6.4
11	10	2.5	72	576	0.543	0.955	0.912	1.71E-08	6.5
11.5	11	2.6	84	616	0.593	1.01	1.02	9.47E-09	6.7
12	12	2.6	96	656	0.557	0.989	0.978	3.2E-09	6.8
12.5	13	2.6	110	698	0.524	0.968	0.937	3.01E-08	7
13	14	2.7	124	740	0.575	1.025	1.051	1.76E-10	7.1
13.5	15	2.7	138	782	0.544	1.007	1.014	7.31E-11	7.2
14	16	2.7	153	825	0.515	0.99	0.98	3.64E-08	7.4
14.5	17	2.7	169	869	0.489	0.973	0.947	8.53E-10	7.5
15	18	2.8	185	913	0.539	1.03	1.061	3.17E-08	7.6
15.5	19	2.8	201	957	0.514	1.016	1.032	7.21E-10	7.7
16	20	2.8	218	1002	0.491	1.002	1.004	8.34E-10	7.8

Table 4: Measure of 14 factors three level SOSRD(PBIBD)

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