## British Journal of Mathematics & Computer Science X(X): XX-XX, 20XX

#### **SCIENCEDOMAIN** international

www.sciencedomain.org

## A measure for evaluating the degree of slope rotatability in three level Second order slope rotatable designs

Chepkemoi Ednah \*1, John Muindi Mutiso<sup>2</sup> and Fredrick Oluoch Nyamwala<sup>3</sup>

<sup>1</sup>Department of Statistics and Computer science, Moi University, P.O Box 3900, Eldoret, Kenya
 <sup>2</sup>Department of Statistics and Computer science, Moi University, P.O Box 3900, Eldoret, Kenya
 <sup>3</sup>Department of Mathematics and Physics, Moi University, P.O Box 3900, Eldoret, Kenya

Original Research Article

> Received: XX December 20XX Accepted: XX December 20XX Online Ready: XX December 20XX

## Abstract

Response surface methodology (RSM) often deals with a natural and desirable property rotatability, which requires that, the variance of the predicted response at a point remains constant at all such points that are equidistant from the design center. To achieve stability in prediction variance, this important property of rotatability was developed. Analogous to rotatability, the concept of sloperotatability has been progressed. The idea of slope - rotatability is an important design criterion for response surface design. Recently, in the design of experiments for response surface analysis, attention has been focused on the estimation of differences in response rather than absolute value of the response mean itself. The slope-rotatable design is that of which the variance of partial derivative is only a functions of  $\rho$ : distance from the design center. If circumstances are such that exact slope rotatability is unattainable because of more cost and time, and more important restrictions such as orthogonal blocking it is still a good idea to make the design as slope rotatable as possible. Thus, it is important to measure the extent of deviation from slope rotatability. In this study, a new measure of the degree of slope-rotatability for three level second-order slope rotatable designs using a pair of a partially balanced incomplete block design is suggested that enables us to assess the degree of slope-rotatability for a given response surface design. This determines the degree slope rotatability for the design when subjected to existing conditions of measure. The measure takes the value zero when the design is exact slope-rotatable, and becomes larger as the design deviates from being slope-rotatable design.

Keywords: slope- rotatability;second order slope rotatable designs(SOSRD); measure; partially balanced incomplete block designs(PBIBD).

2010 Mathematics Subject Classification: 53C25; 83C05; 57N16

\*Corresponding author: E-mail: ednahkiprop@outlook.com

## 1 Introduction

Response surface methodology (RSM) is used in a situation where the form of the relationship between the response and independent variables is unknown. Therefore, the first step in RSM is to find a suitable approximation for the true functional relationship between the response variable and the set of independent variables. The technique to be used is to fit a low order polynomial to the response and if it is inadequate, then we graduate it to higher order polynomial. If the response is well demonstrated by a linear function of the independent variables, then the approximating function is the first order model. We use a second-order model when the portion of the response surface that we are describing has curvature. Response surface methodology is a gathering of mathematical and statistical techniques that are suitable for the demonstrating and analysis of problems in which a response of interest is impacted by several variables and the objective is to optimize this reaction. The study of rotatable designs mainly emphasized the estimation of absolute responses. The property of rotatability as a desirable quality of an experimental design was first advanced by [1]. A design is assumed to be rotatable if the variance of the response estimate is a function only of the distance of the point from the design center. In many applications of Response Surface Methodology, noble estimation of the derivatives of the response function may be as significant as or possibly more significant than the estimation of mean response [3]. Certainly, the computation of a stationary point in a second-order analysis or the use of gradient methods, for example, steepest ascent or ridge analysis depends heavily on the partial derivatives of the estimated response function with respect to the design variables. Since designs that achieve certain properties in Y (estimated response) do not delight in the same properties for the estimated derivatives (slopes), it is vital for the user to ponder experimental designs that are constructed with the derivatives in mind. The study of slope rotatable designs is mainly stressed on the estimation of differences of yields and their precision. Estimation of variances in responses at two different points in the factor space will often be of great importance. If variances in responses at two points close together are of interest then estimation of local slope (rate of change) of the response is essential. Several studies have been done on this aspect pertaining to the development of experimental designs. [3] presented slope rotatability for central composite designs. For the central composite designs, they altered [1] rotatability to slope rotatability essentially by altering the axial point distance (a), so that the variance of the assessed unadulterated quadratic coefficients is one-fourth the variance of the assessed mixed second order coefficients. [8] examined in detail the conditions to be satisfied by a common second-order slope rotatable designs (SOSRD) and developed SOSRD using balanced incomplete block designs (BIBD). [8] constructed SOSRD through a pair of incomplete block designs. The slope-rotatable design is that of which the variance of partial derivative is only a functions of  $\rho$ : distance from the design center. If circumstances are such that exact slope rotatability is unattainable because of more cost and time, and more important restrictions such as orthogonal blocking it is still a good idea to make the design as slope rotatable as possible. Thus, it is important to measure the extent of deviation from slope rotatability. [5], [7] proposed a measure of slope rotatability for second-order response experimental designs. [4] recommended a measure and graphical method for assessing slope rotatability in response surface designs. [13] examined a measure of SOSRD utilizing BIBD. [11] examined a measure of SOSRD utilizing pairwise balanced designs, [10] studied a measure of SOSRD utilizing symmetrical unequal block arrangements with two unequal block sizes. [12] examined the degree of SOSRD utilizing partially balanced incomplete block designs. [14] examined a measure of SOSRD utilizing a pair of balanced incomplete block design. These measures are valuable to empower us to survey the degree of slope rotatability for a given second-order response surface designs. In this study, we propose a method of construction of second-order slope rotatable designs using a pair of partially balanced incomplete block design and their measure which leads to designs with a lesser number of design points than what is available in the existing designs.

#### 1.1 Conditions for second order slope rotatable designs

This section presents briefly the conditions for slope rotatability to be satisfied by a symmetric secondorder response surface design by [3] and Victorbabu [9]. Suppose there are v factors denoted by  $(x_1, x_2, ..., x_v)$  and the design point  $(x_{u1}, x_{u2}, ..., x_{uv})$ ,  $1 \le u \le N$  yields a response  $y_u$  on the study variable y. Assuming that the response surface is of second- order, we adopt the model

$$y_u = b_0 + \sum_{i=1}^{v} b_i x_{iu} + \sum_{i=1}^{v} b_{ii} x_{iu}^2 + \sum_{i \neq j} \sum_{i \neq j} b_{ij} x_{iu} x_{iu} + e_u,$$
(1.1)

where  $x_{iu}$  denotes the level of the  $i^{th}$  factor (i = 1, 2, ..., v) in the  $u^{th}$  run (u = 1, 2, ..., N) of the experiment, where  $e_u's$  are uncorrelated random errors with same mean zero and variance  $\sigma^2$ . The parameters of the model  $b_0, b_i, b_{ii}$  and  $b_{ij}$  are estimated by the least squares estimation to provide  $\hat{b}_0, \hat{b}_i, \hat{b}_{ii}$  and  $\hat{b}_{ij}$ . The design is said to be SOSRD if the variance of the estimate of first order partial derivative of  $y_u$  with respect to each of independent variables  $(x_i)$  is only a function of the distance  $d^2 = \sum x_i^2$  of the point  $(x_1, x_2, ..., x_v)$  from the origin (center) of the design. Such a spherical variance function for estimation of slopes in the Second Order Response Surface is achieved if the design points satisfy the following conditions [3]:

$$A. \qquad \Sigma x_{iu} = 0, \qquad \Sigma x_{iu} x_{ju} = 0, \qquad \Sigma x_{iu}^3 = 0, \qquad \Sigma x_{iu} x_{ju} x_{ku} = 0,$$
  
$$\Sigma x_{iu}^2 x_{ju} x_{ku} = 0, \qquad \Sigma x_{iu}^3 x_{ju} = 0 \qquad \Sigma x_{iu} x_{ju} x_{ku} x_{lu} = 0; \qquad \text{for} \quad i \neq j \neq k \neq l$$

B. 
$$(i)\Sigma x_{iu}^2 = \text{constant} = N\lambda_2$$

(*ii*) 
$$\Sigma x_{iu}^4 = \text{constant} = cN\lambda_2$$
 for all i

C. 
$$\Sigma x_{iu}^2 x_{ju}^2 = \text{constant} = N\lambda_4$$
 for  $i \neq j$  (1.2)

 $D. \qquad \frac{\lambda_4}{\lambda_2^2} > \frac{v}{(c+v-1)}$ 

E. 
$$\lambda_4[v(5-c) - (c-3)^2] + \lambda_2^2[v(c-5) + 4] = 0$$

where  $c, \lambda_2, \lambda_4$  are constants and v denotes the number of factors. Using these symmetry conditions, we can obtained the following estimates of the parameters as [6]

$$\hat{b_0} = \frac{\lambda_4(c+v-1)\Sigma y - \lambda_2 \Sigma(\Sigma x_i^2 y)}{N[\lambda_4(c+v-1) - v\lambda_2^2]}$$
$$\hat{b_i} = \frac{\Sigma x_i y}{N\lambda_2}$$
$$\hat{b_{ij}} = \frac{\Sigma x_i x_j y}{N\lambda_4}$$
$$\hat{b_{ii}} = \frac{\Sigma x_i x_j y}{(c-1)N\lambda_4} - \frac{\lambda_2 \lambda_4(c-1)\Sigma y - \Sigma(\Sigma x_i^2 y)(\lambda_2^2 - \lambda_4)}{(c-1)N\lambda_4[\lambda_4(c+v-1) - v\lambda_2^2]}$$

102

The variances and covariances of the evaluated parameters are [6]:

$$V(\hat{b_0}) = \frac{\lambda_4(c+v-1)\sigma_2}{N[\lambda_4(c+v-1)-v\lambda_2^2]}$$

$$V(\hat{b_i}) = \frac{\sigma_2}{N\lambda_2}$$

$$V(\hat{b_{ij}}) = \frac{\sigma_2}{N\lambda_4}$$

$$V(\hat{b_{ii}}) = \frac{\sigma_2}{(c-1)N\lambda_4} \left(\frac{\lambda_4(c+v-2)-(v-1)\lambda_2^2}{\lambda_4(c+v-1)-v\lambda_2^2}\right)$$

$$Cov(\hat{b_0}, \hat{b_{ii}}) = \frac{-\lambda_2\sigma_2}{N[\lambda_4(c+v-1)-v\lambda_2^2]}$$

$$Cov(\hat{b_{ii}}, \hat{b_{jj}}) = \frac{(\lambda_2^2 - \lambda_4)\sigma_2}{(c-1)N\lambda_4[\lambda_4(c+v-1)-v\lambda_2^2]}$$
(1.3)

and other covariances are zero. A necessary condition for the existence of a second order design

$$\frac{\lambda_4}{\lambda_2^2} > \frac{v}{(c+v-1)} \tag{1.4}$$

(non -singularity condition) From equation (1.1), we have:

$$\frac{\partial \hat{y}}{\partial x_i} = \hat{b}_i + 2\hat{b}_{ii}x_i + \sum \hat{b}_{ij}x_j$$

$$V\left(\frac{\partial \hat{y}}{\partial x_i}\right) = V(\hat{b}_i) + 4x_i^2 V(\hat{b}_{ii}) + \sum x_j^2 V(\hat{b}_{ij})$$
(1.5)

For slope rotatability (1.5) has to be function of  $d^2 = \sum x_i^2$ . This leads to the condition that

$$4V(\hat{b_{ii}}) = (1/4)V(\hat{b_{ij}}) \tag{1.6}$$

Simplifying equation (1.6) becomes:

$$\lambda_4[v(5-c) - (c-3)^2] + \lambda_2^2[v(c-5) + 4] = 0$$
(1.7)

(slope rotatability condition)

is

## 2 Second order slope rotatable designs using a pair of PBIBDs

Let  $D_1 = (v, b_1, r_1, k_1, \lambda_{11} \neq 0, \lambda_{12}=0)$  and  $D_2 = (v, b_2, r_2, k_2 = 2, \lambda_{21} = 0, \lambda_{22} = 1)$  are two PBIBDs. [*a*-(*v*, *b*<sub>1</sub>, *r*<sub>1</sub>, *k*<sub>1</sub>,  $\lambda_{11} \neq 0, \lambda_{12}=0$ )] denote the design points generated from the transpose of the incidence matrix of the design  $D_1$ .

 $[a - (v, b_1, r_1, k_1, \lambda_{11}, \lambda_{12} = 0)]2^{t(k_1)}$  are the  $2^{t(k_1)}$  design points generated from  $D_1$  by multiplication (see[2]). $[a - (v, b_2, r_2, k_2 = 2, \lambda_{21} = 0, \lambda_{22} = 1)]2^2$  are the  $b_2 2^2$  design points generated from  $D_2$  by multiplication. The set of  $b_2 2^2$  design points was repeated  $m_2$  times. Let  $n_0$  be the number of central points.

#### **Result:**

The design points,

 $[a - (v, b_1, r_1, k_1, \lambda_{11}, \lambda_{12} = 0]2^{t(k_1)} \cup [a - (v, b_2, r_2, k_2 = 2, \lambda_{21} = 0, \lambda_{22} = 1)]2^2 \cup n_0$  give a v-dimensional three level SOSRD in  $N = b_1 2^{t(k_1)} + m_2 b_2 2^2 + n_0$  design points where  $m_2$  is

$$m_2 = \left\{ \frac{(r_1 - c\lambda_{11})2^{t(k_1) - 2}}{(c\lambda_{21} - r_2)} \right\}^{1/4}.$$
(2.1)

#### 2.1 Conditions of measure of second order slope rotatable designs

Using the equation below

$$\frac{\partial \hat{y}}{\partial x_i} = \hat{b}_i + 2\hat{b}_{ii}x_i + \sum_{\substack{j=1, j\neq i}}^{v} \hat{b}_{ij}x_j,$$

the variance of this derivative is written as

$$var[\frac{\partial \hat{y}}{\partial x_{v}}] = var(b_{i}) + 4x_{i}^{2}var(b_{ii}) + \sum_{j=1, j \neq i}^{v} x_{j}^{2}var(b_{ij}) + 4x_{i}cov(b_{ii}, b_{ij}) + 2\sum_{j=1, j \neq i}^{v} x_{j}cov(b_{i}, b_{ij}) + 4x_{i}\sum_{j=1, j \neq i}^{v} x_{j}cov(b_{i}, b_{ij}) + 2\sum_{j<1, l \neq i}^{v} x_{j}x_{i}cov(b_{ij}, b_{ij})$$
(2.2)

Following [3],[8], [7], equations (2-3) give the necessary and sufficient conditions for a measure of slope rotatability for any general second order response surface designs. Further, from the above equation (2.2), it can be seen that the necessary and sufficient conditions are:

- $var(b_i)$  are equal for *i*
- $var(b_{ii})$  are equal for i
- $4var(b_{ii}) = var(b_{ij})$  are equal for i, j where  $i \neq j$
- $cov(b_i, b_{ii}) = cov(b_i, b_{ij}) = cov(b_{ii}, b_{ij}) = cov(b_{ij}, b_{il}) = 0$  for all  $i \neq j \neq l$ ,

[7] proposed that, if the following conditions below are met, that is

i. All odd-order moments up-to order 4 are zero,

$$\begin{split} & \text{ii. } \quad \frac{1}{N}\sum_{u=1}^N x_{iu}^2 \text{ are equal for all } i, \\ & \text{iii. } \quad \frac{1}{N}\sum_{u=1}^N x_{iu}^4 \text{ are equal for all } i, \\ & \text{iv. } \quad \sum_{u=1}^N x_{iu}^2 x_{ju}^2 \text{ are equal for all } i \neq j, \end{split}$$

Then the following measure assesses the degree of slope-rotatability for a design  ${\bf D}$  with  ${\bf v}$  independent variables.

$$Q_{v}(D) = \frac{1}{2(v-1)\sigma^{4}} \left\{ (v+2)(v+4) \sum_{i=1}^{v} \left[ (var(b_{i}) - \tilde{v}) + \frac{a_{i} - \tilde{a}}{v+2} \right]^{2} + \frac{4}{v(v+2)} \sum_{i=1}^{v} (a_{i} - \tilde{a})^{2} + 2 \sum_{i=1}^{v} \left[ (4var(b_{ii}) - \frac{a_{i}}{v})^{2} + \sum_{j=l,j\neq i}^{v} var(b_{ij}) - \frac{a_{i}}{v})^{2} \right] + 4(v+4) \left[ 4cov(b_{i}, b_{ii})^{2} + \sum_{j=l,j\neq i}^{v} cov(b_{i}, b_{ij})^{2} \right] + 4 \sum_{i=1}^{v} \left[ \sum_{j=l,j\neq i}^{v} cov(b_{ii}, b_{ij})^{2} + \sum_{j

$$(2.3)$$$$

where

$$\tilde{v} = \frac{1}{v} \sum_{i=1}^{v} var(b_i)$$
  
 $a_i = 4var(b_{ii}) + \sum_{j=l, j \neq i}^{v} var(b_{ij}) \ (i = 1, 2, 3, ...v)$   
 $\tilde{a} = \frac{1}{v} \sum_{i=1}^{v} a_i.$ 

where  $Q_v(D)$  is the proposed measure of slope rotatability. Further [7] went ahead and simplified the above equation (2.3) to

$$Q_v(D) = \frac{1}{\sigma^4} \left[ 4V(b_{ii}) - V(b_{ij}) \right]^2$$
(2.4)

 $Q_v(D)$  becomes zero if and only if the necessary and sufficient conditions hold. If the measure is zero, the design is slope-rotatable. If it becomes larger, it deviates from being slope-rotatable.

## 3 Construction of measure of slope rotatability of three level second-order response surface designs using a pair of PBIBDs

The proposed measure of slope rotatability of three level second-order response surface designs using a pair of PBIBDs is suggested in this section. Let  $D_1 = (v, b_1, r_1, k_1, \lambda_{11} \neq 0, \lambda_{12}=0)$  and  $D_2 = (v, b_2, r_2, k_2 = 2, \lambda_{21} = 0, \lambda_{22} = 1)$  are two PBIBDs. [a-  $(v, b_1, r_1, k_1, \lambda_{11} \neq 0, \lambda_{12}=0)$ ] denote the design points generated from the transpose of the incidence matrix of the design  $D_1$ .

 $[1 - (v, b_1, r_1, k_1, \lambda_{11}, \lambda_{12} = 0)]2^{t(k_1)}$  are the  $2^{t(k_1)}$  design points generated from  $D_1$  by multiplication (see[2]).  $[a - (v, b_2, r_2, k_2 = 2, \lambda_{21} = 0, \lambda_{22} = 1)]2^2$  are the  $b_2 2^2$  design points generated from  $D_2$  by multiplication. The set of  $b_2 2^2$  design points was repeated  $m_2$  times. Let  $n_0$  be the number of central points. Then with the above design points, we can obtain measure of slope rotatability for second order slope rotatable designs as given in the theorem below.

#### Result:

The design points,

 $[a - (v, b_1, r_1, k_1, \lambda_{11}, \lambda_{12} = 0]2^{t(k_1)} \cup [a - (v, b_2, r_2, k_2 = 2, \lambda_{21} = 0, \lambda_{22} = 1)]2^2 \cup n_0$  give a *v*-dimensional measure of slope rotatability of three level second order response surface designs using

a pair of PBIBDs in  $N = b_1 2^{t(k_1)} + m_2 b_2 2^2 + n_0$  design points with 'c' pre-fixed,  $n_0$  chosen and the design levels, suitably such that the design points satisfy the conditions of SOSRD that is

$$m_2 = \left\{ \frac{(r_1 - c\lambda_{11})2^{t(k_1) - 2}}{(c\lambda_{21} - r_2)} \right\}^{1/4},$$
(3.1)

$$n_0 = \left[ \frac{[v(c-5)+4][r_1 2^{t(k_1)} + r_2 2^2]^2}{[v(c-5)+(c-3)^2][\lambda_{11} 2^{t(k_1)} + \lambda_{21} 2^2]} \right] - b_1 2^{t(k_1)} - m_2 b_2 2^2$$
(3.2)

### Proof

For the design points generated from a pair of PBIBDs, conditions (A) to (C) are true. Conditions in (A) are true obviously. Conditions (B) to (C) of equation 1.2 are true as follows:

$$\sum_{u=1}^{N} x_{iu}^2 = r_1 2^{t(k_1)} a^2 + r_2 2^2 a^2 = N\lambda_2$$
(3.3)

$$\sum_{u=1}^{N} x_{iu}^4 = r_1 2^{t(k_1)} a^4 + r_2 2^2 a^4 = cN\lambda_4$$
(3.4)

$$\sum_{u=1}^{N} x_{iu}^2 x_{ju}^2 = \lambda_{11} 2^{t(k_1)} a^4 + \lambda_{21} 2^2 a^4 = N\lambda_4$$
(3.5)

From equation (3.4) and (3.5), we get  $m_2$ ,

$$a = \left[\frac{(r_1 - c\lambda_{11})2^{t(k_1) - 2}}{(c\lambda_{21} - r_2)}\right],$$

The value of **a** can be obtained from equation (3.3) by taking the scaling condition, that is  $\lambda_2 = 1$ .

$$a = \left[\frac{N}{r_1 2^{t(k_1)} + m_2 r_2 2^2}\right]^{\frac{1}{2}}$$
(3.6)

Measure of slope rotatability of three level second order designs using a pair of PBIBDs can be obtained by solving the given simplified equation by [7]

$$Q_{v}(D) = \frac{1}{\sigma^{4}} \left[ 4V(b_{ii}) - V(b_{ij}) \right]^{2}$$
(3.7)  
where  $V(\hat{b}_{ii}) = \frac{\sigma^{2}}{(c-1)N\lambda_{4}} \left( \frac{\lambda_{4}(c+v-2)-(v-1)\lambda_{2}^{2}}{\lambda_{4}(c+v-1)-v\lambda_{2}^{2}} \right) \text{ and } V(\hat{b}_{ij}) = \frac{\sigma^{2}}{N\lambda_{4}}$   
Therefore  
 $Q_{v}(D) = \frac{1}{\sigma^{4}} \left[ \frac{4\sigma^{2}}{(c-1)N\lambda_{4}} \left( \frac{\lambda_{4}(c+v-2)-(v-1)\lambda_{2}^{2}}{\lambda_{4}(c+v-1)-v\lambda_{2}^{2}} \right) - \frac{\sigma^{2}}{N\lambda_{4}} \right]^{2}$   
 $Q_{v}(D) = \frac{1}{\sigma^{4}} \left[ \frac{4\sigma^{2} \left( \lambda_{4}(c+v-2)-(v-1)\lambda_{2}^{2} \right) - \sigma^{2} \left( (c-1)(c+v-2)\lambda_{4}-v\lambda_{2}^{2} \right)}{N(c-1)\lambda_{4} \left[ (c+v-1)\lambda_{4}-v\lambda_{2}^{2} \right]} \right]^{2}$   
 $Q_{v}(D) = \frac{\sigma^{4}}{\sigma^{4}} \left[ \frac{4 \left( \lambda_{4}(c+v-2)-(v-1)\lambda_{2}^{2} \right) - \left( (c-1)(c+v-2)\lambda_{4}-v\lambda_{2}^{2} \right)}{N(c-1)\lambda_{4} \left[ (c+v-1)\lambda_{4}-v\lambda_{2}^{2} \right]} \right]^{2}$   
After simplification  $Q_{v}(D)$  becomes:

A ,(D)

$$Q_{v}(D) = \left[\frac{\lambda_{4}[v(5-c) - (c-3)^{2}] + \lambda_{2}^{2}[v(c-5) + 4]}{N(c-1)\lambda_{4}[(c+v-1)\lambda_{4} - v\lambda_{2}^{2}]}\right]^{2}$$
(3.8)

106

where  $\lambda_4, \lambda_2, a, m_2$  and N are as shown below

$$\lambda_{2} = \left[\frac{r_{1}2^{t(k_{1})}a^{2} + r_{2}2^{2}a^{2}}{N}\right]$$
$$\lambda_{4} = \left[\frac{\lambda_{11}2^{t(k_{1})}a^{4} + \lambda_{21}2^{2}a^{4}}{N}\right]$$
$$m_{2} = \left[\frac{(r_{1} - c\lambda_{11})2^{t(k_{1}) - 2}}{(c\lambda_{21} - r_{2})}\right],$$
$$a = \left[\frac{N}{r_{1}2^{t(k_{1})} + m_{2}r_{2}2^{2}}\right]^{\frac{1}{2}}$$
$$N = b_{1}2^{t(k_{1})} + m_{2}b_{2}2^{2} + n_{0}$$

and

$$n_0 = \left[\frac{[v(c-5)+4][r_12^tk_1+r_22^2a^2]^2}{[v(c-5)+(c-3)^2][\lambda_{11}2^tk_1+\lambda_{21}2^2a^4]}\right] - b_12^{k_1} - m_2b_22^2$$

The computation of measure of slope rotatability  $Q_v(D)$  of three level second order response surface designs using various parametres of PBIBDs for varied values of c ranging from 3 to 16 and level a are tabulated below in the appendix.

## 4 CONCLUSIONS

In this study, a measure of slope rotatability for second-order response surface designs using a pair of PBIBDs is suggested which enables us to assess the degree of slope rotatability of a given three level second-order response surface design. It can be verified that measure of slope rotatability is zero if and only if a design D is a second order slope-rotatable design. Measure of slope rotatability becomes larger as D deviates from a second order slope rotatable design. The method can be used to compare the degree of slope rotatability of the same v. We may point out here that the measure of slope rotatability for second order response surface designs using a pair of PBIBDs with parameters  $D_1 = (v = 16, b_1 = 20, r_1 = 5, k_1 = 4, \lambda_{11} = 1, \lambda_{12} = 0), D_2 = (v = 16, b_2 = 8, r_2 = 1, k_2 = 2, \lambda_{21} = 0, \lambda_{22} = 1)$  has only 400 design points for 16– factors, whereas the corresponding measure of slope rotatability for second order response surface designs using a pair of BIBDs with parameters  $D_1 = (v = 16, b_1 = 16, r_1 = 6, k_1 = 6, \lambda_1 = 2), D_2 = (v = 16, b_2 = 80, r_2 = 15, k_2 = 3, \lambda_2 = 2)$  of [14] needs 1154 design points. Thus this new method leads to 16-factor measure of SOSRD with less number of design points than the existing measure of slope rotatable designs using a pair of BIBDs. When c = 5 the design is exact slope rotatability that is the measure of slope rotatability is zero.

# Appendices

	$a^2$	2.3	2.5	2.5	-	വ	4.3	4.3	4.4	4.5	4.7	4.8	4.9	ъ	5.1	5.2	5.3	5.4	5.5	5.6	5.7	5.7	5.8	5.9	9	9	6.1	6.1
= 1),	$Q_v(D)$	5.86E-08	1.45E-07	4.05E-10	8.25E-12	0	8.01E-11	2.01E-11	2.69E-10	6.51E-09	6.18E-12	8.3E-09	2.68E-09	2.53E-09	4.36E-12	1E-10	3.61E-10	2.71E-10	9.25E-10	6.66E-09	1.74E-09	8.2E-09	4.28E-09	1.14E-09	6.81E-09	2.26E-09	6.21E-09	1.04E-08
$\frac{1}{1, \lambda_{12}} = 0, \lambda_{22} = 0$	$(\lambda_2)^2$	0.929	1.049	1.049	-	0.937	1.032	1.036	0.994	0.945	1.084	1.032	0.982	0.937	1.073	1.03	0.992	0.956	0.925	1.065	1.032	1.006	0.982	0.955	0.935	0.914	1.053	1.034
$3,\lambda_{11}=$	$\lambda_2$	0.964	1.024	1.024	-	0.968	1.016	1.018	0.997	0.972	1.041	1.016	0.991	0.968	1.036	1.015	0.996	0.978	0.962	1.032	1.016	1.003	0.991	0.977	0.967	0.956	1.026	1.017
$= 3, k_1 =$ = 2, $k_2 =$	$\lambda_4$	0.723	0.749	0.655	0.222	0.937	0.815	0.748	0.676	0.613	0.672	0.614	0.564	0.521	0.577	0.537	0.502	0.47	0.442	0.495	0.468	0.445	0.423	0.402	0.384	0.367	0.414	0.397
$\frac{9, r_1}{9, r_2} =$		56	70	80	36	200	191	208	230	254	279	305	332	360	388	417	446	476	506	536	567	597	628	660	691	723	755	787
$0, b_1 = 0, b_2 = 0, b_2 = 0$	$n_0$	-16	-20	-28	06-	56	29	28	32	38	45	53	62	72	82	93	104	116	128	140	153	165	178	192	205	219	233	247
$\frac{(v)}{(v)} = \frac{1}{2}$	a	1.5	1.6	1.6	-	2.2	2.1	2.1	2.1	2.1	2.2	2.2	2.2	2.2	2.3	2.3	2.3	2.3	2.3	2.4	2.4	2.4	2.4	2.4	2.4	2.4	2.5	2.5
$D_1 = D_2 = D_2$	$m_1$	0	0.5	-	1.5	2	2.5	ო	3.5	4	4.5	2	5.5	9	6.5	7	7.5	ω	8.5	<b>б</b>	9.5	10	10.5	11	11.5	12	12.5	13
	c	ო	3.5	4	4.5	Ŋ	5.5	9	6.5	~	7.5	ω	8.5	6	9.5	10	10.5	<del>-</del>	11.5	12 12	12.5	13 13	13.5	14 4	14.5	15	15.5	16

Table 1: Measure of 9 factors three level SOSRD(PBIBD)

British Journal of Mathematics and Computer Science X(X), XX–XX, 20XX

	$D_1 =$	v = v	$16, b_1$	$= 20, r_1$	$= 5, k_1$	$= 4, \lambda_{11} =$	$= 1, \lambda_{12} =$	= 0),			$D_1 =$	(v = 1	$4, b_1 =$	$= 28, r_1$	$= 6, k_1 =$	$= 3, \lambda_{11} =$	$= 1, \lambda_{12}$	= 0),	
	$D_2 =$	=(v =	$= 16, \bar{b}_2$	$2 = 8, r_2$	$= 1, k_2$	$= 2, \lambda_{21} =$	$= 0, \lambda_{22} =$	= 1)			$D_2 =$	(v =	$14, b_2$	$= 7, r_2 =$	$= 1, k_2 =$	= 2, $\lambda_{21}$ =	$= 0, \lambda_{22} =$	= 1)	
0	$m_1$	a	$n_0$	N	$\lambda_4$	$\lambda_2$	$(\lambda_2)^2$	$Q_v(D)$	$a^2$	c	$m_1$	a	$n_0$	N	$\lambda_4$	$\lambda_2$	$(\lambda_2)^2$	$Q_v(D)$	$a^2$
6	ထု	1.6	62	126	0.832	0.9752	0.951	6.32E-09	2.63	e	မှ	1.6	9	62	0.846	0.991	0.982	2.91E-05	2.6
3.5	9	1.7	37	165	0.81	0.9808	0.962	3.38E-10	2.95	3.5	ц	1.7	4-	80	0.835	1.012	1.024	7.68E-07	2.9
4	4	1.8	13	205	0.819	1.0115	1.023	4.9E-10	3.2	4	4-	1.8	-14	98	0.857	1.058	1.119	3.14E-08	3.1
4.5	Ņ	1.8	မ်	225	0.746	1.0368	1.075	4.54E-11	3.13	4.5	ကု	1.7	-38	102	0.655	1.02	1.04	1.46E-10	2.8
2 2	0	2.2	80	400	0.937	0.968	0.937	0	Ŋ	2	Ņ	2.2	32	200	0.937	0.968	0.937	0	5
5.5	2	2.2	24	408	0.919	1.0439	1.09	4.45E-11	4.64	5.5	Ţ	2.1	5	201	0.774	0.965	0.931	2.6E-10	4.6
9	4	2.2	<del>1</del> 3	461	0.813	1.0079	1.016	1.07E-11	4.8	9	0	2.2	-	225	0.833	1.033	1.067	3.18E-09	4.7
6.5	9	2.2	9	522	0.718	0.9643	0.93	1.19E-10	5.02	6.5	-	2.2	2	254	0.738	0.991	0.982	3.01E-10	4.9
7	ω	2.3	42	588	0.761	1.0076	1.015	7.93E-11	5.25	7	2	2.3	S	285	0.786	1.039	1.08	8.86E-10	5.1
7.5	10	2.3	17	657	0.682	0.9662	0.934	2.22E-11	5.48	7.5	ო	2.3	10	318	0.704	0.998	0.996	5.36E-09	5.3
8	12	2.4	25	729	0.728	1.0114	1.023	1.39E-09	5.7	ω	4	2.3	16	352	0.636	0.962	0.925	9.5E-09	5.5
8.5	14	2.4	36	804	0.66	0.9743	0.949	1.34E-10	5.91	8.5	ß	2.4	23	387	0.686	1.012	1.024	1.29E-08	5.7
6	16	2.5	49	881	0.709	1.0216	1.044	3.48E-09	6.12	6	9	2.4	31	423	0.627	0.98	0.96	1.25E-08	5.9
9.5	18	2.5	65	961	0.65	0.9886	0.977	4.56E-11	6.32	9.5	2	2.5	40	460	0.679	1.033	1.067	7.24E-10	6.1
10	20	2.6	82	1042	0.702	1.038	1.077	1.85E-09	6.51	10	ω	2.5	49	497	0.629	1.006	1.012	1.93E-09	6.2
10.5	22	2.6	101	1125	0.65	1.0095	1.019	5.93E-11	6.7	10.5	ი	2.5	60	536	0.583	0.979	0.958	1.03E-08	6.4
11	24	2.6	122	1210	0.604	0.9833	0.967	1.25E-09	6.88	1	10	2.5	72	576	0.543	0.955	0.912	1.71E-08	6.5
11.5	26	2.7	145	1297	0.656	1.0342	1.07	6.47E-10	7.05	11.5	÷	2.6	84	616	0.593	1.01	1.02	9.47E-09	6.7
12	28	2.7	169	1385	0.614	1.0106	1.021	8.5E-10	7.21	12	12	2.6	96	656	0.557	0.989	0.978	3.2E-09	6.8
12.5	30	2.7	194	1474	0.577	0.9891	0.978	3.87E-10	7.37	12.5	13	2.6	110	698	0.524	0.968	0.937	3.01E-08	7
13	32	2.7	221	1565	0.543	0.9689	0.939	6.15E-09	7.52	13	1 4	2.7	124	740	0.575	1.025	1.051	1.76E-10	7.1
13.5	34	2.8	250	1658	0.593	1.0214	1.043	5.73E-12	7.68	13.5	15	2.7	138	782	0.544	1.007	1.014	7.31E-11	7.2
14	36	2.8	279	1751	0.562	1.0029	1.006	2.64E-10	7.82	14	16	2.7	153	825	0.515	0.99	0.98	3.64E-08	7.4
14.5	38	2.8	310	1846	0.533	0.9853	0.971	1.38E-10	7.96	14.5	17	2.7	169	869	0.489	0.973	0.947	8.53E-10	7.5
15	40	2.8	342	1942	0.506	0.9689	0.939	6.42E-09	8.09	15	18	2.8	185	913	0.539	1.03	1.061	3.17E-08	7.6
15.5	42	2.9	10 325 10 22	2039	0.555	1.0229	1.046	2.21E-10	8.22	15.5	19	2.8	201	957	0.514	1.016	1.032	7.21E-10	7.7
16	44	2.9	409	2137	0.53	1.0075	1.015	2.16E-09	8.35	16	20	2.8	218	1002	0.491	1.002	1.004	8.34E-10	7.8
	Table	e 5: N	leasu	re of 16	factors t	hree level	SOSRD	(PBIBD)			Table	e 4: M	easure	e of 14 f	actors th	iree leve	SOSR	D(PBIBD)	

British Journal of Mathematics and Computer Science X(X), XX–XX, 20XX

### References

- George EP Box and J Stuart Hunter. Multi-factor experimental designs for exploring response surfaces. *The Annals of Mathematical Statistics*, pages 195–241, 1957.
- [2] MN Das and VL Narasimham. Construction of rotatable designs through balanced incomplete block designs. *The Annals of Mathematical Statistics*, pages 1421–1439, 1962.
- [3] RJ Hader and Sung H Park. Slope-rotatable central composite designs. *Technometrics*, 20(4):413–417, 1978.
- [4] Dae H Jang and Sung H Park. A measure and a graphical method for evaluating slope rotatability in response surface designs. *Communications in Statistics-Theory and methods*, 22(7):1849– 1863, 1993.
- [5] AI Khuri. A measure of rotatability for response-surface designs. *Technometrics*, 30(1):95–104, 1988.
- [6] Das M.N and N.C Giri. Design and analysis of experiments. Wiley Eastern Ltd, 1986.
- [7] Sung H Park and Hyuk J Kim. A measure of slope-rotatability for second order response surface experimental designs. *Journal of Applied Statistics*, 19(3):391–404, 1992.
- [8] B Re Victor Babu and VL Narasimhant. Construction of second order slope rotatable designs through balanced incomplete block designs. *Communications in statistics-theory and methods*, 20(8):2467–2478, 1991.
- [9] B. Re Victorbabu. *some contributions to Second order slope rotatable designs*. PhD thesis, 1989.
- [10] B Re Victorbabu and Ch VVS Surekha. Construction of measure of second order slope rotatable designs using symmetrical unequal block arrangements with two unequal block sizes. *Journal* of Statistics and Management Systems, 15(4-5):569–579, 2012b.
- [11] B Re Victorbabu and Ch VVS Surekha. Construction of measure of second-order slope-rotatable designs using pairwise balanced designs. *International Journal of Statistics and Systems*, 2(2):97–106, 2012c.
- [12] Bejjam R Victorbabu and Chandaluri VVS Surekha. On measure of second order slope rotatable designs using partially balanced incomplete block type designs. *Thailand Statistician*, 11(2):159–171, 2013.
- [13] Bejjam Re Victorbabu and Chandaluri Valli Venkata Siva Surekha. Construction of measure of second order slope rotatable designs using balanced incomplete block designs. *Journal of Statistics*, 19(1), 2012a.
- [14] Bejjam Re Victorbabu and Chandaluri Valli Venkata Siva Surekha. A note on measure of slope rotatability for second order response surface designs using a pair of balanced incomplete block esigns. *Thailand Statistician*, 2016.

©2011 Author1 & Author2; This is an Open Access article distributed under the terms of the Creative Commons Attribution License http://creativecommons.org/licenses/by/2.0, which permits unrestricted use, distribution, and reproduction in any medium, provided the original work is properly cited.