Original Research Article

Face Recognition using 2D-FLDA based on Approximate SVD obtained with Kronecker Products with one Training Sample

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ABSTRACT

Aims: In a face recognition task, it is a challenging problem to find lots of images for a person. Even, sometimes there can be only one image, available for a person. In these cases many of the methods are exposed to serious performance drops even some of these fail to work. Recently this problem has become remarkable for researchers. In some of these studies the database is extended using a synthesized image which is constructed from the singular value decomposition (SVD) of the single training image. In this paper, for such a method, SVD based 2 Dimensional Fisher Linear Discriminant Analysis (2D-FLDA), it is proposed a new approach to find the SVD of the image matrix with the aim of to increase the recognition performance.
Study design: In this paper, in a face recognition task with 2D-FLDA, in one training sample case, instead of original SVD of the image matrix, the approximate SVD of its based on multiple kronecker product sums is used. In order to obtain it, image matrix is first reshaped thus it is to be lower dimensional matrices and, then the sum of multiple kronecker products (MKPS) is applied in this lower dimensional space.
Methodology: Experiments are performed on two known databases Ar-Face and ORL face databases. The performance of the proposed method is evaluated when there are facial expression, lightning conditions and pose variations.
Results: In each experiment, the approximate SVD approach based on multiple kronecker product sum gets approximately 3% better results when compared with the original SVD.
Conclusion: Experimental results verify that the proposed method achieves better recognition performance over the traditional one. The reason for this is the proposed approximate SVD has the advantages of simplicity, and also as the kronecker factors possess additional linear structure, kronecker product can capture potential self-similarity.

Keywords: Face recognition, Singular Value Decomposition (SVD), Approximate SVD, Multiple Kronecker product sum, Single training image per person

1. INTRODUCTION

In a face recognition task, the aim is to identify or verify a person from its stored face images and many areas can be seen such as person identification, law enforcement, security and etc. In many of these researches it is focused on the basis of the algorithm, to improve the recognition system accuracy [1] but, they ignore the face database at hand. However in order to obtain high recognition rates in many methods, there must be sufficient number of images for a person. Nevertheless, it cannot be found many image samples in many applications. Some of these applications are passport identification, law enforcement, drive license and etc. In this case most well-known methods such as Eigen face [2, 3] and fisher face [4-6] will suffer and some of them even fail to work. In a recognition task, for instance the methods that require the intra class scatter matrix fail to work if there is only one image sample as the training set for each person. Because in this case the intra class scatter matrix would be zero and the method will fail to work. Recently this problem has become remarkable for researchers. In [7], an algorithm which makes Fisher linear discriminant analysis (FLDA) to be applicable in the case where there is only one training sample in the training set is proposed. In their work they propose a method which evaluates the intra class variation from the available single training image. The database is extended using a synthesized image which is constructed from the SVD of the single training image. Thus FLDA has become applicable on this extended database. In this paper for face recognition, the efficiency of, the approximate SVD that is based kronecker products and the traditional SVD are evaluated, in the case of there is only one image sample per person. Thus the effectiveness of the approximate SVD over the traditional SVD is investigated. This analysis is performed via approximate SVD which is a better and more intuitive way based on kronecker products. The proposed approximate SVD has the advantages of simplicity, and also as the kronecker factors possess additional linear structure, kronecker product can capture potential self-similarity. These make the performance of the proposed method [8] better than the other methods given especially when there are changes in illumination, facial expression and pose variations in the face images.

In the rest of paper, in case of one training sample, the procedure of traditional 2D-FLDA and, the procedure of 2D-FLDA are briefly given. Section three reviews the representation of images using the multiple kronecker product sums. Section four performs experiments on the well-known databases and then conclusion is given.

2. SVM-BASED 2D-FLDA IN ONE TRAINING SAMPLE CASE

FLDA has emerged as a popular 1D feature extraction approach for facial recognition problems. This approach tries to find, a series of projection vectors in order to project the face images onto a space, in which different classes are separated as possible and the similar classes are gathered together. For 2D case, Ye et al. [9] proposed 2D-FLDA, in which 2D reflection matrices are estimated directly from 2D images. 2D-FLDA can theoretically be formulated as follows. It is assumed that there are C classes and each class has K training image samples with the size $A_k \in R^{mxn}$ $(k = 1,2, \ldots, K)$. In 2D-FLDA it is tried to found a vector set w_j $(j = 1,..,d)$ to construct a transformation matrix $W = [w_1 \quad w_2 \quad \cdot \quad \cdot \quad w_d\,]$ by maximizing the following criteria

$$J(W) = \frac{tr(W^T S_b W)}{tr(W^T S_w W)} \tag{1}$$

here tr is used to denote the trace of a matrix and the superscript "T" is used to denote matrix transpose. S_b and S_w, that are the inter class and the intra class scatter matrices respectively, are given as follows

$$S_b = \sum_{i=1}^{C} \frac{K_i}{K} (\bar{A}_i - \bar{A})^T (\bar{A}_i - \bar{A})$$

$$S_w = \frac{1}{C} \sum_{k=1}^{K} \sum_{A_k \in C_i} \frac{1}{K_i} (A_k - \bar{A}_i)^T (A_k - \bar{A}_i) \quad (2)$$

where $K = \sum_{i=1}^{C} K_i$, $\bar{A}$ is the overall image mean of all samples in the set of training, and $\bar{A}_i$ gives the i.th class's (C_i) mean image.

2.1 SVD based image decomposition

Maximizing Equation 1 corresponds the generalized eigenvalue solution problem: $S_b W = \Lambda S_w W$, where Λ is a diagonal matrix, and eigenvalues are placed on the main diagonal. In the SVD-based FLDA application in the case where there is only one sample as training using SVD the image is decomposed as to be two supplementary sections, thus the first section of the image is constructed using the SVD basis images obtained by the several singular values that are the largest, and the second section of the image is constructed using the SVD basis images obtained by the rest of the singular values. Image matrix's first section represents the general appearance of that image, while second section represents the difference between the original image and the first part. The difference image can reflect the variations between the images in the same class images. In this method in order to evaluate the intra class scatter matrix, the second part of the image matrix is used, while the first part is used to evaluate the inter class scatter matrix.
Image decomposition using SVD is given in the following expression. Let $A \in R^{mxn}$and suppose$m \geq n$,

$$A = \sum_{i=1}^{n} \sigma_i u_i v_i^T \quad (3)$$

where u_i is the ith column vector of $U \in R^{mxm}$which is constructed from the eigenvectors of AA^T, $v_i$is the ith column vector of $V \in R^{nxn}$ which is constructed from the eigenvectors of $A^T A$ and $\sigma_i$is the ith singular value of image matrix A. According to the Equation 3 A can be decomposed into the basis images $F_i = \sigma_i u_i v_i^T$, $i = 1, \ldots, n$ and a large amount of energy from image A is concentrated on images $F_i$obtained using large singular values.

3. APPROXIMATE SVD BASED ON KRONECKER PRODUCTS

As a mathematical tool kronecker product [10] of matrices is very important and recently several researchers began to use this technique in image processing such as image restoration [11, 12], fast transform generation [13] and image quantization and coding [14]. Kronecker product provides separating a large matrix into smaller factor matrices [15] which are independent of each other [16].These factor matrices, especially when dealing with multidimensional matrices in a classification task, represents the intra- class and inter-class variations between image samples [17]. In [8] it is used for image representation and processing. To get a lower dimension representation the image matrix is decomposed into small simpler matrices by using the multiple kronecker product sum (MKPS). Thus the original high dimension image matrix is represented with lower dimension matrices. Than they applied singular value decomposition (SVD) to the lower dimension matrices so as to find the SVD of the original high dimension image matrix and it can be seen kronecker product based SVD analyses is a better and more intuitive way to analyze an image matrix.

In general SVD any mxn matrix can be factored as

$$A = U \sum V^T \tag{4}$$

where U is an mxm orthogonal matrix whose columns are the eigenvectors of AA^T, V is an nxn orthogonal matrix whose columns are the eigenvectors of $A^T A$ and $\sum$ is an mxn diagonal matrix of the form

$$\sum = \begin{bmatrix} \sigma_1 & 0 & 00 & 0 & 0 \\ 0 & . & 00 & 0 & 0 \\ 0 & 0 & .0 & 0 & 0 \\ 0 & 0 & 0\sigma_r & 0 & 0 \\ 0 & 0 & 0\,0 & 0 & 0 \\ 0 & 0 & 0\,0 & 0 & 0 \end{bmatrix} \tag{5}$$

$\sigma_1 \geq \sigma_2 \geq \cdots \geq \sigma_r > 0$ and $r = rank(A)$. Here $\sigma_1, \sigma_2, \cdots, \sigma_r$ are the square roots of the eigenvalues of $A^T A$. They are called the singular values of A.

The theory of the approximate SVD is as given below:

The kronecker product of two matrices B and C of $m_1 x n_1$and $m_2 x n_2$respectively is an $m_1 m_2 x n_1 n_2$matrix (say A) defined by

$$A = B \otimes C = \begin{bmatrix} b_{11}C & b_{12}C & . & b_{1n_1}C \\ b_{21}C & b_{22}C & . & b_{2n_1}C \\ . & . & . & . \\ . & . & . & . \\ b_{m_1 1}C & b_{m_1 2}C & . & b_{m_1 n_1}C \end{bmatrix} \tag{6}$$

here B is the left, and C is right factor matrix of A. The kronecker factor C in Equation 6 can be interpreted as an image filter that finds the boundary of the regions in which the most of the structural information concentrates [18]. In Zhang et al. [8] as $A_i \;\; i = 1,2, \dots, k$ to be matrices of size $m_i x n_i$

$$\bigotimes_{i=1}^{k} A_i = (\bigotimes_{i=1}^{k-1} A_i) \otimes A_k = A_1 \otimes A_2 \otimes .... \otimes A_k \tag{7}$$

this is called a multiple kronecker product. They approximate a given matrix A by the sum of several multi kronecker products. They called it in their study multiple kronecker product sum approximation (MKPS) of matrix A.

A matrix A of dimension mxn with $m = m_1 x m_2$ and $n = n_1 x n_2$, matrices B and C of size $m_1 x n_1$ and $m_2 x n_2$ respectively that minimize the following error

$$f_A(B,C) = \left\| res(A) - vec(B)vec(C)^T \right\|_F \quad (9)$$

where $vec(X) = (X(1:m,1) \quad X(1:m,2) \quad . \quad . \quad X(1:m,n))^T \in R^{mnx1}$ and reshape operation $res(A) = (A_1, A_2, ..., A_{m_1})^T A_j = (vec(A_{1,j}^T), vec(A_{2,j}^T), ..., vec(A_{m_1,j}^T))$, where

$A_{ij} = A((i-1)m_2 + 1 : im_2, (j-1)n_2 + 1 : in_2$ are blocks of A with dimension $m_2 x n_2$. It follows that

$$f_A(B,C) = \left\| res(A) - vec(B)vec(C)^T \right\|_F \quad (10)$$

Then the following theorem is obtained.

According to Theorem1 that $A \in R^{mxn}$ with $m = m_1 x m_2$ and $n = n_1 x n_2$. Assume the SVD of $\tilde{A}$ which corresponds to $res(A)$ is as follows

$$U^T \tilde{A} V = S = diag(\sigma_1, ..., \sigma_n) \quad (11)$$

Here $(\sigma_1, ..., \sigma_n)$ the singular value sequence is given in descending order and $diag(.)$ means to generate the diagonal matrix using the elements of this sequence. Let $U1, V1$ are the corresponding singular vectors of the largest singular value (σ_1) , the matrices $B \in R^{m_1 x n_1}$ and $C \in R^{m_2 x n_2}$ are defined by

$$vec(B) = \sqrt{\sigma_1} U_1, \quad vec(C) = \sqrt{\sigma_1} V_1 \quad (12)$$

minimize $f_A(B,C)$.

Theorem 2. Same as the assumption as Theorem 1 and let Ui, Vi be the corresponding singular vectors of (σ_i) , then the matrix $B \in R^{m_1 x n_1}$ and $C \in R^{m_2 x n_2}$ defined by $vec(B_i) = \sqrt{\sigma_i} U_i, \quad vec(C_i) = \sqrt{\sigma_i} V_i$ minimize $\left\| A - \sum_{i=1}^{k} B_i \otimes C_i \right\|_F$. Accordingly, when the k is equal to the matrix rank, it is evident that the matrix can be fully represented by the kronecker product sum, since the approach will not fail theoretically.

In the application of this method to a face recognition task, face image is divided into small blocks and by using reshape process is reshaped so as to be a row of a new matrix. Thus, each of the small blocks of the face image is considered to be an example of a cluster. With this approximate factorization, the decomposition process is transformed into finding the most important block sample among these blocks. Thus, the factors represented by B and C are respectively considered the down sampling version of the A matrix, and the measure of similarity between these block samples. Here the size of B and C can be chosen freely this makes the procedure framework much flexible.

4. EXPERIMENTAL STUDIES

In the experimental studies, the performance of the proposed method is evaluated using the AR-Face database [19] and ORL face database [20]. In order to see, the performance of the approximate SVD based on kronecker products over the traditional SVD, experiments are performed for both, traditional SVD-based FLDA [7] and kronecker product based approximate-SVD FLDA.

Using the Ar-Face database the method's performance proposed in this study is evaluated. This evaluation is made for expression variations among the face images. 37 individuals as to be 17 females and 20 males were taken and used, that are corresponds to only the non-occluded images. The images were cropped as to be 50x40 pixels. An example from AR-Face for a person is shown in Figure1.

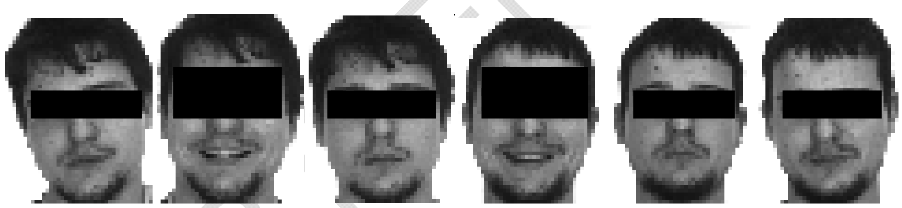

Fig. 1. A sample from AR-Face Database

Two different experiments are performed on this database. In the first one, the first three images are used for each person as the database. Then, each of the three images is used as the training image once and the remaining two images are used as the test images. The average of the recognition results obtained from these three experiments is given in Table 1. In the second experiment, database is constructed from the all six images of a person given in Figure 1. As it is in the first experiment each of the six images are used as the training image once and the remaining five images are used as the test images then the average recognition result that belongs to the the six experiment, is given in Figure 1.
ORL face database contains 10 grayscale images from 40 subjects. In Figure 2 a sample from the database is given. The size of the images is 112x92 and they contain different lighting conditions and facial expressions. They also were taken at dark background and subjects are in the frontal position with tolerance to some side movement. In the experimental study ORL face database is used in its original size.

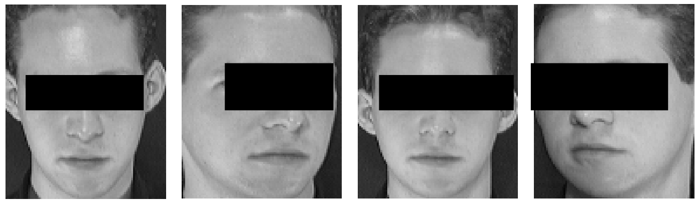

Fig. 2. A sample from ORL Database

Table 1. Recognition results of the experimental studies.

Databases	**Recognition Results**	
	Approximate SVD-based FLDA (%) (proposed)	**Traditional SVD-based FLDA (%)**
AR_Face 1	90.5405	87.3873
AR_Face 2	71.3738	68.5586
ORL	77.0834	74.3229

5. RESULTS AND DISCUSSION

In this study, the face recognition method based on 2D-FLDA given in [7] is used as the basic method. The 2D-FLDA normally requires at least two or more images in the training set, otherwise it cannot be used as it needs intra class and inter class variations for the recognition process. In [7], it is given the adaptation of this method, in the case of one training sample per person. SVD is applied to the single training image and then while SVD basis images obtained by using several largest singular values are using for finding the intra class variations, SVD basis images obtained by using the rest of the singular values are used to find the inter class variations. Thus, in the proposed method in this paper, taking the face recognition task given in [7] so as to be the basic method, it is advanced due to obtain reasonably better recognition results. In the proposed method, an approximate factorization for the traditional SVD is obtained using the MKPS approach and this approximate SVD is used in order to obtain the SVD basis images which will be used to represent the intra class and the inter class variations.

The main motivation in this study is to present a method which will increase the recognition accuracy and, which is more robust to changes in illumination conditions, pose and resolution, in a face recognition task in case of one training sample per person problem. With this motivation, using multiple sums of kronecker products an approximate factorization is obtained for SVD of an image matrix. This approximate factorization method for a wide variety of structures ,in this proposed study ,the approximate factorization of the SVD of an image matrix, obtains quite good approximation results for especially the largest singular values [21].In [21] a similar approximation manner for the truncated SVD is given. Garvey et all. computed an approximate truncated SVD obtained by using kronecker product summations due to its computational speed and its more accurate results, compared to kronecker based methods in the literature. Then this decomposition is used in image restoration and image reconstruction problems. The use of the proposed method in a face recognition task provides better recognition results even in a one training sample case because of the method can handle with the changes in illumination, resolution and pose [22]. The face databases used in this study are consisting of the images having the variations in illumination conditions, pose and resolution. The performance of the proposed method over the traditional SVD can also be seen from the experimental results. In each of the experimental study, using approximate SVD of the image matrix instead of traditional SVD of the image matrix increases the recognition results about 3% that is reasonably accurate.

6. CONCLUSION

In this paper, a novel usage of approximate SVD decomposition of an image matrix is performed, that is a face recognition application. This decomposition is used in order to extend the face database at hand when there is only one sample image per person in the training set. The performance of the proposed method is evaluated with the experimental studies. To see the performance of the method experiments are performed for both approximate SVD decomposition based 2D-FLDA face recognition algorithm and the traditional SVD based 2D-FLDA face recognition algorithm. Experimental results verify that the proposed method achieves better recognition performance over the traditional one. The reason for this is the proposed approximate SVD has the advantages of simplicity, and also as the kronecker factors possess additional linear structure, kronecker product can capture potential self-similarity.

The main contributions of the proposed study are as follows. First a novel method for a face recognition task in case of a one training sample case is proposed. Second as it can be seen from the related studies given in the literature, this method have been applied to different image application areas such as image restoration, image reconstruction, image compression [8, 21, 23] and etc. But, however in this proposed study, the method, is applied into a different and new image application area (face recognition). Third the method gets better recognition results according to its traditional one (about 3% more). Fourth it proposes a face recognition method that is robust to illumination, pose and resolution variations. Beside these main contributions, at the same time the method also has the advantages of the kronecker products. These are, the computational speed, obtaining quite good approximated singular values especially for the largest singular values, more accurate results than the other kronecker based methods in the literature.

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