Forecasting Maternal Mortality with Modified Gompertz Model

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Abstract

The Gompertz model is one of the earliest most influential mortality models. The model dominated for more than 100 years and is still one of the most important models in the field of mortality. Even though the model was designed exclusively for human mortality, it has found its application in many fields. However, the model has not solely been applied to maternal mortality. The work, therefore, fit a modified form of Gompertz model to Ghana' maternal mortality data (2016-2018) and the fit looks quite good. We also forecast with the model and the result shows that Ghana is far from achieving the Sustainable Development Goal (SDG) objective of reducing maternal mortality by 68 percent in the next 11 years. That is, the work shows that there will be an annual reduction of 2.9 percent in maternal mortality rate for the next 11 years. This reduction, however, is not enough to achieve the Sustainable Development Goal (SDG) objective of a 6 percent annual reduction. To make the SDG objective a reality, Ghana needs a further reduction of 3.1 percent annually in its maternal mortality rate. This calls for intensifying programmes that improve maternal health and reduces maternal mortality.

Forecasting, Gompertz Model, Maternal Mortality, Sustainable Development Goal.

1 Introduction

Mortality modelling goes as far as the 17th century when Abraham De Moivre suggests a linear relationship between survival rate and age. It, however, took Benjamin Gompertz to make the greatest impact in the 18th century when he proposed a mortality model that suggests an exponential relationship between mortality and age. His model formed the basis on which other mortality models were built and dominated for more than 100 years. Even though the model was originally designed exclusively for human mortality, it has also been applied in many fields. For example, Hedau and Soni(2016) used the Gompertz model to determine the growth pattern of mobile subscribers in India. Rzadkowski, Glazewskaet and Sawinska (2015). on the other hand, applied the Gompertz model in economics and management in which they investigated the Gompertz model as a diffusion model and predict the saturation level of a given phenomenon under investigation using only the early values of time series. Rossi, S.,Deslaurier, A., and H.Morin, H. (2003) applied the Gompertz model to the study of xylem cell development in which the Gompertz model was used to calculate cell number increase and to estimate both the rates and periods of the differentiation phases on a daily scale during the growing season. Bi-Huei also applied the Gompertz model to foreign direct investment in Taiwan. And there is much more literature that have applied the Gompertz model in one way or the other. The model has however not been applied to maternal mortality modelling and the first contribution of this paper is to fit a modified version of the Gompertz model to age-specific maternal mortality data in Ghana.
On the other hand, the World Health Organisation (WHO) defined maternal mortality as "the death of a woman while pregnant or within 42 days of termination of pregnancy, irrespective of the duration and site of the pregnancy, from any cause related to or aggravated by the pregnancy or its management but not from accidental or incidental causes. According to the same organisation, the global maternal deaths fell from 385 in 1990 to 216 in 2015, an estimated fall of 44 percent in Maternal Mortality Ratio (MMR). Of these, the greatest decline over the period was observed in Eastern Asia with an approximate decline of 72 percent. The decline in maternal mortality rate over the period was mostly due to the impact of the Millennium Development Goal(MDG) initiative which called for a 75 percent reduction in the maternal mortality by 2015. And even though the MDG initiative made a significant impact in reducing maternal mortality, it was not able to meet its target of

75 percent reduction in maternal mortality. As a result, maternal mortality is still high in some countries especially those in Sub-Saharan Africa. For example, according to MDG report, Ghana currently loses 144 mothers to every 1000 live birth and the story is not very different from other countries in the sub region. It is against this background that the Sustainable Development Goal (SDG) initiative was lunch to replace MDGs initiative. On maternal mortality, the SDGs initiative aims at reducing global maternal mortality to less than 70 deaths per 100000 live-births by 2030, a 68 percent reduction in only 11 years. It has been almost four years now since the SDGs initiative was introduced and this work seeks to forecast maternal mortality based on the 3 year maternal mortality data available since the SDG was introduced. This will help organisers of the programme to ascertain whether Ghana is on course of achieving the 68 percent reduction in maternal mortality by 2030 or something more has to be done in terms of improving maternal health care in order to achieve the 68 percent target. This work therefore developed a model that fit the age pattern of maternal mortality over the three year period and use the model to forecast maternal mortality to 2030.

2 The model

Abraham De Moivre proposed the first mathematical mortality model when he tried to establish the relationship between man's survival and his age. His model suggests a linear relationship between survival rate and age. That is, the probability of man's survival decreases linearly as he ages. In terms of mortality function, his model can be stated as

$$\mu(x) = \frac{1}{\omega - x} \tag{2.1}$$

where μ(x) refers to the force of mortality at age x, ω is the highest attainable age and x is the current age. In the paper, De Moivre assume the highest attainable age to be 86. Though he applied his model to various actuarial calculations, his model was not able to represent human survival accurately across ages (De Moivre, 1725).

In 1825, a British actuary by the name Benjamin Gompertz introduced one of the most successful mortality models. His model was widely used and dominated for more than 100 years. It also served as the basis on which most other mortality models evolved. Fitting his model to the age profile of mortality, Gompertz realized that the force of mortality increases with age as suggested by De Moivre. However, unlike De Moivre, Gompertz's model shows an exponential relationship between mortality and age. His model showed that mortality increases exponentially with age and man's resistance to death declines exponentially as he ages. Explaining his model, Gompertz stated that humans are endowed with a special ability which he called the power to oppose destruction (death) (Gompertz,1825). According to Gompertz, this power decreases exponentially with age and for each additional year, a person loses a constant fraction of his/her power to oppose destruction (death). His model can be stated as

$$\mu(x) = \alpha\beta^{kx} \tag{2.2}$$

where a positive α represents the overall level of adult mortality and β determines how the risk of death accelerate with increases in age, μ is the force of mortality and x represent age (Gompertz, 1872).
Even though Gompertz's model was designed exclusively for human mortality, it has also been used extensively in other fields especially by evolutionary biologists. In applying his model to human mortality, Gompertz stated that his model is not able to fit perfectly the entire age range and suggested the model fit better the 40-60 age category well.
As the aim of the paper is to forecast maternal mortality who are usually between the ages 15-35, it means the Gompertz model will not fit this age bracket and there is a need for further modification. After various modifications designed to accommodate age departures, we finally fit a model of the form:

$$\mu(x) = \frac{\alpha e^{\beta(t)x}}{1 + \alpha e^{\beta(t)(x)^2}} + \gamma(t)(x)^3 \tag{2.3}$$

where x is the age, t is time and α, β, and γ are parameters to be estimated. Also, $\mu(x)$ represent the force of mortality, the numerator of the first term represent the Gompterz model while the term in the denominator helps to level off the high mortality rate at higher ages modeled by the Gompertz model. The second term helps improve the fit.
Of course if we have data at only few time point as our case (3 years), then we may not use time series and thus remove the dependence on t with the model taken the form:

$$\mu(x) = \frac{\alpha e^{\beta(x)}}{1 + \alpha e^{\beta(x)^2}} + \gamma(x)^3 \tag{2.4}$$

where the parameters and terms have the same meaning as previously defined.

Fitting the Data

After defining a model, we next consider how to fit it. In fitting the model, we use an approach similar to that used by Assabil and Don(2019, in review). It is important to note that as the focus of the work is to consider the possibility of achieving the 68 percent reduction in maternal mortality under the SDG objective, we only consider maternal mortality data after the introduction of SDG initiative (2016-2018).
Suppose there are $N_{x,i}$ individuals in age cohort x in region i. Then the number of deaths D_x at time t, $t+\Delta t$ in age group x can be modeled as a binomial (since an individual is alive or dead, such a process could be considered as Bernouli process and thus modeled with Binomial model) with parameters $N_x, \mu(x,t)\Delta t$ where N_x is the number alive at the beginning of this interval. Given that the number alive is large with a small probability of dying in this cohort over a short time interval of length Δt, then the Binomial can be approximated by Poisson(as the Poisson distribution is just a special case of the binomial when the number of trials is large and the probability of sucess in any given one is small) with mean $(N_{x,i}, \mu(x,t)\Delta t)$. The log-likelihood for a poisson $(\lambda = E(D))$ random variable D is given as

$$In(\lambda^D e^{-\lambda}) = D\ln(\lambda) - \lambda. \tag{2.5}$$

Also, the score function for a poisson random variable D is of the form

$$\frac{D - E(D)}{E(D)} = \frac{D}{E(D)} - 1 \tag{2.6}$$

Therefore the score function takes the form of a system of equations

$$\sum_i \sum_x \frac{D_{x,i} - N_{x,i}\mu(x)\Delta t}{N_{x,i}\mu(x)\Delta t}\big(N_{x,i}\Delta t\frac{\partial \mu(x)}{\partial \theta}\big) = 0 \tag{2.7}$$

where θ runs through the parameters α, β, and γ . Maximize the log likelihood, we obtain:

$$\max_{\alpha,\beta,\gamma} \sum_i \sum_x D_{x,i}\ln(N_{x,i}\mu(x)\Delta t - (N_{x,i}\mu(x)\Delta t \tag{2.8}$$

or substituting for $\mu(x)$ and assuming $\Delta t = 1$, this becomes

$$\max_{\alpha,\beta,\gamma,\rho} \sum_i \sum_x D_{x,i} \ln \big(N_{x,i}\big(\frac{\alpha e^{\beta x}}{1+\alpha e^{\beta x^2}} + \gamma(x)^3\big) - (N_{x,i}\big(\frac{\alpha e^{\beta x}}{1+\alpha e^{\beta x^2}} + \gamma(x)^3\big) \tag{2.9}$$

$$\max_{\alpha,\beta,\gamma,\rho} \sum_i \sum_x D_{x,i} \ln \big(N_{x,i}\big(\frac{\alpha e^{\beta x}}{1+\alpha e^{\beta x^2}} + \gamma(x)^3\big) + \sum_i \sum_x D_{x,i} \ln(N_{x,i}) \tag{2.10}$$
$$-(N_{x,i}\big(\frac{\alpha e^{\beta x}}{1+\alpha e^{\beta x^2}} + \gamma(x)^3\big)$$

and since the middle term does not depend on the parameters, it suffices to maximize

(2.11)

The above maximization is then implemented in MATLAB and we show the result in the next section.

3 Results

The above model was fitted to the Ghanian female maternal mortality data, and the result is shown below:

Figure 1, represents our model fit to Ghana's maternal mortality between the ages of 15-35. The x-axis measures age while the y-axis measures mortality. The dash black curve is the data plot while the straight black curve is the model fit. From the plot, it can be seen that the model fit the data quite well. The curve is almost level between the ages of 15-20 and upward sloping above the ages of 20 years. That is, maternal mortality is low at early adulthood and increases as one age. This is to be expected as explained by Gompertz, that the power to oppose destruction decreases with age even though in the case of maternal mortality the decreases are not exponential as suggested by Gompertz.

4 Forecasting with the Model

In other to forecast with our model, we need to introduce time as a variable which affects the parameters of the model. That is, to assume that time

Figure 1: Model fit to maternal mortality rate in Ghana.

affects all parameters multiplicatively so that the model can be expressed:

$$\mu(x,t) = k(t)b(x) \tag{4.1}$$

where $k(t) = 1 + \theta t$ and $b(x)$ is our model given by $\mu(x) = \frac{\alpha e^{\beta(x)}}{1+\alpha e^{\beta(x)^2}} + \gamma(x)^3$. At time t =0, the value of $k(t)b(x) = b(x)$ (negative) and at time t = 1, the value is $(1+\theta t)b(x)$ which is smaller than $b(x)$ if $\theta > 0$ since b(x) is negative so the mortality at time 1 is lower and expected lifespans higher.

It must also be noted that the parameter θ in this case represents the rate of change in mortality from time 0 to time 1. To estimate the value of θ, we use maternal mortality data provided by demographic health survey (Ghana) after the introduction of Sustainable Development Goals. That is, we use maternal mortality data in Ghana between the years 2016-2018 to calculate the rate of change and use it together with our model to forecast over the desired period. Since we have three years of mortality data we estimate the average rate of change over the three year period as the value for θ. And this gave us a θ value of 0.029 percent and the resulting forecast over the 11 year period is shown in figure 2.

The forecast shows a decreasing trend with an average reduction of 0.029 or 2.9 percent per year. That is, since the introduction of SDG, Ghana's maternal mortality has further reduced by 2.9 percent annum. This reduction

Figure 2: Model fit to maternal mortality rate in Ghana.

however is not enough to achieve the sustainable development goal objective of reducing maternal mortality by 66 percent over 11 year period, which translate to a 6 percent annual maternal mortality reduction. With the current reduction of 2.9 percent per annum, Ghana needs an additional 3.1 percent annual reduction to achieve the sustainable development goal initiative. (that is, $6 - 2.9 = 3.1$, the desired 6 percent reduction needed to achieve the SDG, less the current annual reduction of 2.9 percent) This calls for intensifying programmes that improves maternal health and reduces maternal mortality such as long-term investment in community education and family planning, empowerment and strengthening of women's socioeconomic status, skilled birth attendance, access to skilled care before, during and after they give birth, providing incentives to health providers to motivate them to do their job effectively, etc. This will go a long way to help achieve the sustainable development goal on reducing the maternal mortality rate.

5 Standard Errors in Estimating Model Parameters

We now consider estimating the uncertainty associated with model parameters. There are various methods of estimating model standard errors and in this work we choose to estimate the standard errors by using simulation(Assabil and Don,2019, in review). This is a simple procedure that begins with estimating the number of individuals alive and the number of deaths in each age cohort: N_i and D_i. These values have already been estimated from our model.

From these estimates, we defined the probability of death in this cohort as $p_i = \frac{D_i}{N_i}$ and then generate a random number of deaths $\hat{D}$ from Binomial (N_i, p_i). We then re-run the estimation of the parameters with the same population size but with the new generated number of deaths several times till convergence (in this case we run 100 simulations), with the data N_i and $\hat{D}_i$. Record the values of these new estimated parameters. This whole process is repeated many times (100 in our case). Now we obtain the standard errors of all of these parameter estimates and the covariance matrix. The results after going through the procedure are shown in Table 1.

Table 1: Standard Error of Model Parameters

	α	β	γ
values	1.3342	-0.5477	0.1591
Std. Error	0.025	0.017	0.012

From the table, it can be seen that the error associated with model parameters are quite small indicating that our forecast is quite accurate.

6 Conclusion and Recommendation

In this work, we have developed a modified Gompertz model that fits the age pattern of maternal mortality in Ghana. We tried the model on maternal mortality data from Ghana between 2016 to 2018 and the fit looks quite good. We also incorporate time into the model that allows forecasting and we forecasted maternal mortality up to 2030. The results show that even though maternal mortality in Ghana has decreased over the 3-year period since the inception of SDG initiative, the rate of decrease is not enough to achieve the objective of reducing maternal mortality by 68 percent in 2030. To make that objective a reality, the work shows that Ghana needs to reduce its maternal mortality by a further 3.1 percent annually. This calls for intensifying programmes that reduce maternal mortality.

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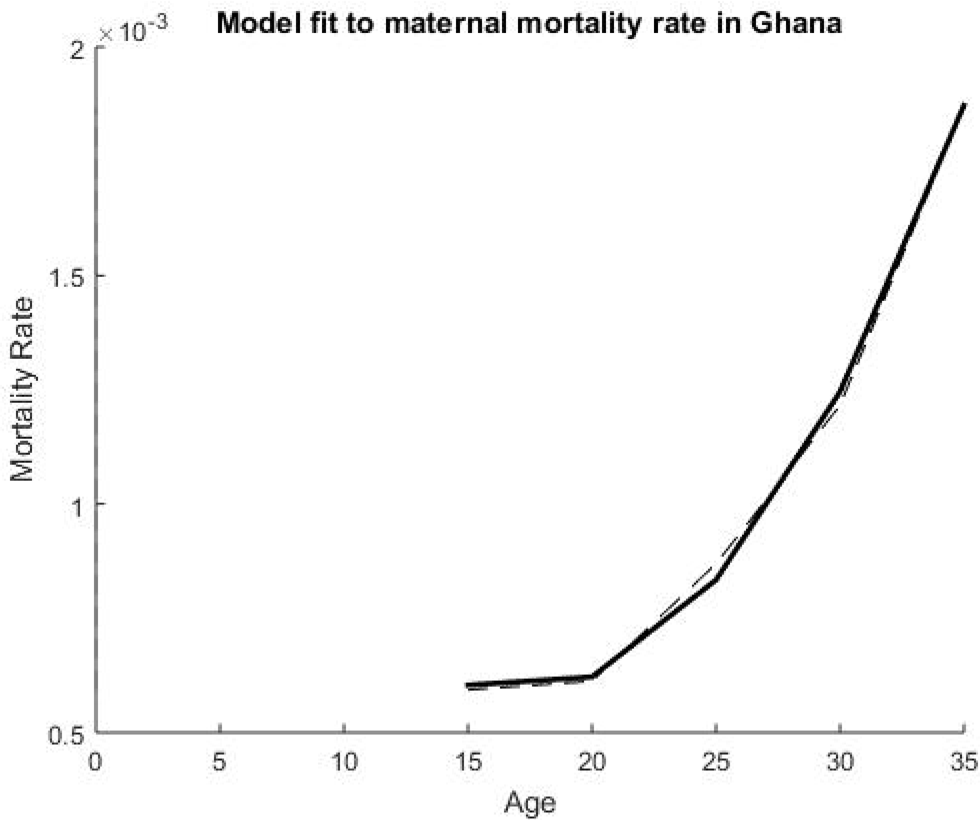
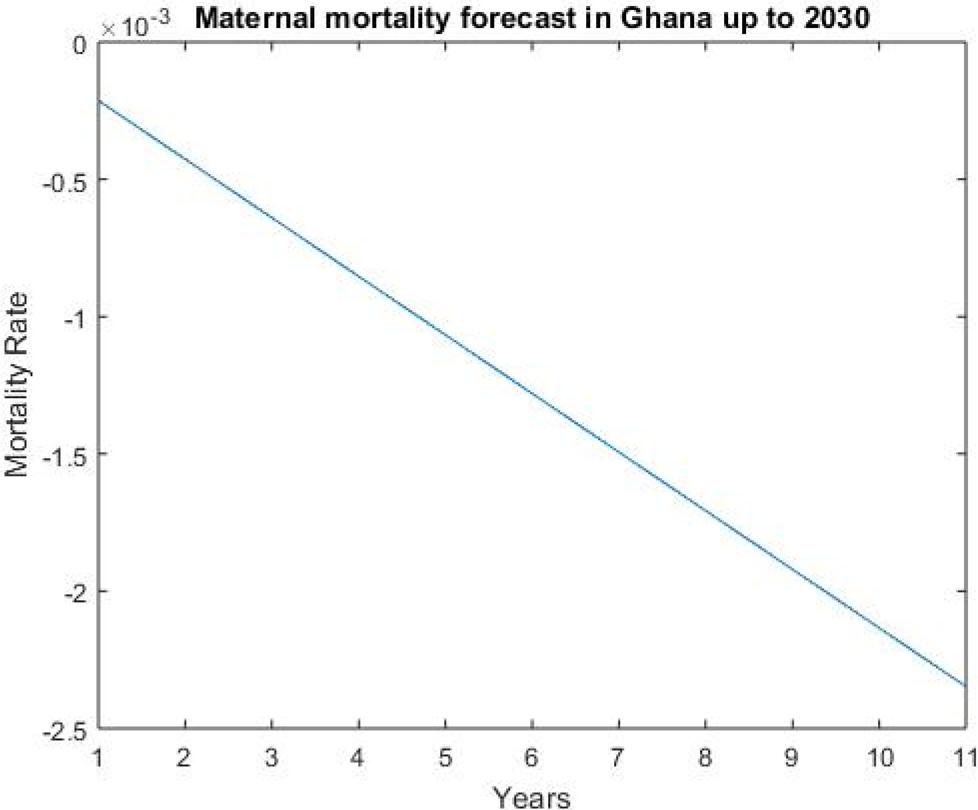


Model fit to maternal mortality rate in Ghana
×10-3
2
1.5
1
0.5
Mortality Rate
0
5
10
15
20
25
30
35
Age


Maternal mortality forecast in Ghana up to 2030
×10^-3
0
-0.5
-1
-1.5
-2
-2.5
Mortality Rate
1
2
3
4
5
6
7
8
9
10
11
Years