

Original Research Article

CROSS-GENERATION AND CROSS-COUNTRY EVIDENCE ON THE LINK BETWEEN GROWTH AND VOLATILITY

Abstract:

This paper reports on a larger scale econometric study of the sign of the relationship between average growth and growth volatility of GDP per capita. At equilibrium, the negativity or the positivity of the relationship between endogenous growth and business cycles volatility is linked to the movements (left or right side) of (PPF). Tests have focused on cross-generation and cross-country evidence on the link between growth and volatility. If production possibilities frontier movements' trend is to the left side, countries with a higher standard deviation of growth should have their growth adversely affected if at the same time they lose their comparative advantages. Thus international trade elasticity after a production possibilities frontier movement (ϵ_i) determines the sign of the relationship between growth and volatility. If $(\epsilon_i - 1) < 0$, the sign is negative and positive if $(\epsilon_i - 1) > 0$. From the theoretical point of view, a new multidimensional trade and optimal growth mechanisms have been presented.

(PPF), growth volatility, international trade elasticity, intergenerational trade, intergenerational prices levelling out.

1- INTRODUCTION

Several studies argue that countries with high average growth would also have high variance in growth rate. Black (1987) and Mirman (1971) have explained why growth/volatility relationship should be positive (Luo and Young, 2016). Black (1987) has argued that countries may have a choice between high-variance, high- expected- returns technologies and low-variance, low- expected- returns technologies (Crain, 2003). For Mirman(1971), if there is a precautionary motive for saving, then higher volatility should lead to a higher saving rate, and hence a higher investment rate which is positively linked to growth; the relationship between growth and volatility in these conditions should be positive. Ramey and Ramey (1991, 1995, 2012) use a sample of developing countries in their studies and argued that volatility can lead to lower mean output if firms find themselves producing at suboptimal levels ex-post. Even though they concluded that if lower current output affects the accumulation of resources, then growth is adversely affected, their study is focused essentially on econometric tests ignoring a fundamental part of volatility mechanism analysis in terms of resources management.

Volatility is allied to risk in that it provides a measure of possible variation or movement in a particular economic variable or some function of that variable such as a growth rate" (Das & Sinha Roy, 2016). It usually occurs measured on the observed realization of a random variable over some historical period. This is referred to a realized volatility to distinguish it to the implicit volatility calculated from the Black Scholes formula for the price of the European call option on a stock (Bianconi, et al., 2015). The realized volatility or volatility is commonly measured by a standard deviation based on the history of an economic variable (McAleer& Medeiros 2008). In this paper, we deal with either implicit or explicit reference to an underlying probability distribution for the variables in concern. In these two kinds of volatility, disequilibria of trade tend to set national and generational Production Possibilities

Frontiers (PPFs) in a permanent movement. Terms of trade volatility are perhaps the most widely used measure of external shocks.

The disturbances in the (PPF) due to differences in the change of production factors' supply- in the average growth volatility seems to have an important impact on growth volatility. Defining a country's technology as a combination of unskilled labour, skilled labour and capital efficiencies, Caselli and Coleman (2000) found a negative cross-country correlation between the efficiency of unskilled labour and efficiencies of skilled labour and capital (Caselli et al., 2000). They interpret that link as the proof of the existence of a World Technology Frontier (WTF) in which increases in the efficiency of unskilled labour are obtained at the cost of declines in the efficiency of skilled labour and capital. The same negative association exists between natural resources' efficiency and unnatural resources' efficiency, both in intergenerational and international trade. Consequently, if intergenerational and international levelling out of goods and factors prices is not realized, the change in the supply of goods and factors is unbalanced, inducing generations and nations PPF 's movements, the key cause of fluctuations. With the Solow's growth model based on a constant saving rate, the movements in the (PPF) is impossible (Pareto efficiency criterion). In such a model, there is no economic volatility (constant growth rate). To understand how trade is the major vector of resources allocation and economic volatility, the study of how the overall efficiency or total factor productivity determines countries' or generations' comparative advantages, is essential. Caselli and Coleman (2000) have shown how the optimal choice of technology depends on the country's endowment of skilled and unskilled labour and how barriers to technology adoption are crucial. Thus, a weak resources allocation always generates shocks that affect growth path.

The key causes of the link between growth and volatility should be found, instead, in the movements of the (PPF) and their interactions with international and intergenerational trade. Subsequently, it is important to revisit the factors generating growth model in order to find its sources of fluctuations. The Neoclassical growth theory is essentially based on the supply side while the Schumpeterian growth theory is from the demand side. But we should admit that all the factors who cause economic growth, put the (PPF) in a sort of movement in a way that the directions took by these movements in each country and/or generation interact with international or intergenerational trade to determine economic volatility. Thus, in fact, (PPF) are in a permanent movement, balancing from the left to the right side and vice versa. The direction of the movements depends on productive resources allocation. The level of resources could rise or drop and the production technologies or the intergenerational marginal rate of substitution of resources could change. If, only differences in the change of countries/generations' resources can lead to a change in the comparative advantages and international/intergenerational trade configuration, the sign of the relationship between growth and volatility would be affected by these movements and their interactions with international and intergenerational trade. For King et al (1988), a temporary disturbance to (PPF) can have permanent effects on the path of the output growth. The importance and the nature of these effects depend on the types of disturbances. Thus, higher variability in the production factors' supply should be a source of higher variability of the production level due to a great disturbance in the (PPF). To understand the role of the differences in the productive resources quantities' change and technological progress interactions with trade, we introduce the notion of "natural resources exchange against unnatural resources between generations" that is until now ignored in economics but which can highly disturb the (PPF). In the existing literature, there is no rigorous formulation of how intergenerational free trade interacts with the international free trade to determine general macrodynamic equilibrium in terms of optimal growth. Optimal allocation of economic resources should lead to optimal growth and sustainable development. However, none of the studies has established a link between successive generations' behaviour, current prices and economic volatility. As we can see, studies of economic volatility have made great progress in the past 20 years but, there remains much to be learned about the determinants of long-run productivity growth and its links with business-cycles. Previous studies on economic volatility do not integrate the movements of the (PPF), intergenerational trade effects and their interactions with economic growth. **Does the trend of countries and generations' PPF**

movements around the (WTF) determine the sign of the relationship between growth and volatility?

The remainder of the paper is presented as follows: section 1 presents the **introduction**; the **second section deals with background**; Section3 deals with the model setup: **Equilibrium approach to the international free trade and cross-country volatility (model first component)**; **Equilibrium approach to Intergenerational free trade and cross-generation volatility (model second component)**; **multidimensional trade and the empirical evidence of the sign of the link between volatility and growth (model)**; Section 4 presents the solution: **Cross-country empirical evidence on the equilibrium approach to the international free trade and economic volatility (model first component)**; **Intergenerational trade empirical evidence (the test of model second component)** and the evidence on **multidimensional suboptimal trade and the sign of the link between growth and volatility (or the test of model)**; the fifth section presents the paper's conclusions.

In the traditional economic theory, growth is supposed to play no role in economic volatility; however, three papers presented in the early 1980's changed the understanding of that important issue. Nelson and Plosser (1982) find that the movements in the GNP tend to be permanent. Kydland and Prescott (1983) uncover skills for analyzing economic volatility and integrating growth and volatility (fluctuations). Lucas (1987) shows that the possible returns from understanding business-cycles are trivial compared to these from understanding growth assuming that growth and business cycle volatility are unrelated (the standard dichotomy in macroeconomics).

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2- Literature reviews

According to Ragchaasuren (2006), the models that follow Shumpeter (1942), where the mechanism is based on "creative destruction" show a positive relationship between growth and volatility. For example, in Aghion and Saint-Paul (1998a, b), productivity change is assumed to be the result of purposeful (internal) learning through deliberate actions which substitute for production activities. Under such circumstances, the resources allocated to productivity improvement activities are a convex function of the state of the economy and hence the average productivity increases as volatility increases. On the other hand, the models that follow Arrow (1962), where the mechanism of technological change takes the form of "learning-by-doing" show that the relationship between growth and volatility tends often (but not always) to be negative. For example, in Martin and Rogers (1997, 2000), changes in productivity take place through serendipitous (external) learning through non-deliberate actions which are the complements to production activities. In this case, the factor through which expertise, knowledge and skills are acquired and disseminated is a concave function of the shocks, so that increased volatility decreases growth. By incorporating the above two conflicting mechanisms for endogenous technological change, Blackburn and Galindez (2003) show that any shock can have a permanent effect on output if it changes the amount on which productivity improvements depend. For Aghion and Howitt (1998), Dinopoulos and Thompson (1998), Jones (1995), Kortum (1997), Peretto (1998), Segerstrom (1998) and Young (1998) there exists a positive linkage between productivity growth rate and the share of R&D in GDP. For example, Black (1987) argues that countries may have a choice between high-variance, high expected returns technologies because countries with high average growth will also have high variance. Conversely, Bernanke (1983), Pindyck (1991), Aizenman and Marion (1993), Ramey and Ramey (1991, 1995) argued that there is a negative association between productivity growth rate and the share of R&D in output. If lower current output affects resources' accumulation, then growth is adversely affected. For example, the theoretical analysis suggests that, if there is irreversibility in investment, then increasing volatility can lead to lower investment Bernanke (1983), Pindyck (1991), etc.). See Ramey (2012) who investigated that increases in government spending stimulate private activity. She found that in most cases private spending falls significantly in response to an increase in government spending. See also Bean (1990), Fatas (2000), King et al. (1988), Jones et al.

(1999) for permanent effects of temporary real shocks, and Stadler (1990), Pelloni (1997), Blackburn (1999) and Blackburn and Pelloni (2004) for permanent effects of temporary nominal shocks. See also Caballero and Hammour (1994) for a related contribution on this subject (Blackburn (1999) for a contrasting result in this approach. The relationship between growth and volatility is more likely to be positive (negative) if technological change is predominantly driven by internal (external) learning. In contrast to the above, some models in which knowledge is created under the assumption of learning-by-doing suggest alternative relationships between growth and volatility. According to De Hek (1999) and Smith (1996), the relationship between long-term growth and short-term cyclical volatility depends on the household's attitude towards risk as measured by the curvature of the utility function. Specifically, the more (less) risk-averse is an agent, the more likely it is that increased uncertainty will have a positive (negative) effect on long-run growth. Jones et al. (1999) consider the same issue in a different framework in which growth is the result of constant returns to reproducible factors – physical and human capital – that are purely rival (and not due to the accumulation of non-rival knowledge via learning-by-doing) and reaches the result the same as above. Blackburn and Pelloni (2004) investigate the correlation between the growth and the volatility and find that this link depends on the nature of the shocks under the assumption of an imperfect labour market. Long-run growth is positively correlated with the volatility of the real shocks and negatively correlated with the volatility of the nominal shocks.

All the resources (natural and non-natural) allocation through suboptimal and “optimal” choices (trade relationships) is the key responsibilities for the nature of the relationship between growth and volatility. As each country can exchange goods and services with other countries, each generation exchanges also resources (natural and non-natural) with other generations. This latter exchange can be optimal or suboptimal at the image of international interdependences and - as in the nature nothing is created and nothing is lost- each generation (country) generates effects (shocks) on other generations (countries) in a permanent way so that each generation or country's production possibilities frontier is continually moving around the whole world frontier which is fix. These movements affect the generations and countries' trade through the comparative advantages gained or lost. As trade intensity is internationally reducing with the distance, each generation exchange with other generations reduces with timely distance.

The relationship between optimal growth and successive generations' behaviour has been studied by economists. Samuelson (1958) comes out with a trade model allowing for exogenous endowments of non-storable and consumption goods. There is no trade between young and old generations. In these models, the presence of a storable and non-consumable good, money, allows for inter-generation trade. The young sell part of their endowments of goods to the old and receive money. When they become old the next period, they will exchange money with the young and receive in return the consumable goods. He concluded that this kind of exchange can improve the welfare of all the generations. Thus, the baseline of the overlapping generations' model with two-period is set by Samuelson (1958) and Diamond (1965).

Allais (1947) has proposed a similar model he called “overlapping generations“. In this model, the economy includes one competitive production sector using two factors of production, capital and labour. In each period, the goods produced can be consumed or stored as a capital which will be used in the production process the next period. The young supply a unit of labour and receive a wage in terms of goods produced. They consume part of their savings and constitute the stock of capital for the next period. When they become old, they consume their income, the capital income which is the remaining of their contribution to the production process. There is no intergenerational trade but the participation of the young in the production process as workers and the old as capital holders. The economic equilibrium is determined and the young can rationally anticipate their saving rent, which is the marginal productivity of capital in the next period. The model of overlapping generations is useful in analyzing possibilities like individual training choice and the role of human capital in the economic growth (Azariadis and Drazen (1990), Michel (1993), McCandless and Wallace (1991), and De La Croix and Michel (2002).

Even though the neoclassical growth model goes back to Ramsey (1928) or Von Neuman (1935), the recent versions are closely related to the analysis of the optimal growth by David Cass (1965) and Koopman (1960, 1965). Ramsey (1928), assuming that the population is constant, considered that global output is a function of capital and labour, consumers utility admits a superior final limit. He established the well-known Keynes-Ramsey rule of optimal saving which characterizes the steady state optimal consumption. Rawls (1974), in his study of optimal growth, gives the same weight to each generation by fixing a fair saving rate. But, it is now generally accepted that the implementation of Rawls criterion for successive generations constitutes a growth limitation.

3- Theoretical models

3.1- The model first component: Equilibrium approach to the international free trade and cross-country volatility

3.1.1- Foundations and hypothesis

The generations are unrelated so that overlapping generation's hypothesis does not rule (intergenerational autarky condition). Each country has its initial endowments (at the beginning of the analysis) which are composed of natural resources and unnatural resources. Natural resources (all the physical environment) and unnatural resources (the other resources) are the factors of production of the economy. Each country has its own comparative advantages. Through these conditions, we can set the following analysis which is based on the common neoclassical understanding.

3.1.2- International drivers and cross-country volatility

3.1.2.1- International interdependencies sensitivity

The neoclassical (HO) model (1933) says: *The goods that the production is intensive in the abundant factor of production and a weak proportion of scarce factor of production are exported and the goods that the production demands the reverse proportions of the same factors are imported...*

Let us consider two countries, for instance, the USA and China, two factors of production, natural resources and unnatural resources, and two goods wheat, and DVDs; (see Fig 1). The free trade production is W and the consumption and world equilibrium are noted X. At point X a perfect equilibrium of production and consumption of the two countries is realized. Each country improves its utility when passing from a lower indifference curve to the upper one. At that point, produced and consumed quantities by all the countries are determined.

When a country chooses another initial production different from W, it is no more possible for this country and the world to reach the equilibrium point which is X on this graph 1. Since then, the country and the world are engaged in great potential volatility which is varying with the distance separating the effective free trade production (W_i) to the optimal initial free trade production and with the sensitivity of the international interdependencies. Since then, the country PPF is moving around the (WTF). The derived growth is not optimal (graph 5). The international volatility function can be described by the following relation:

$$(X_f - X) = f(W_f - W, \theta') \quad (1)$$

θ' is the international sensitivity factor. Volatility becomes explosive (through other countries) if the international interdependencies are very sensitive. On this matter see Hsieh and Klenow (2009), Klenow (2012) who, using micro data on manufacturing establishments to quantify the potential extent of misallocation in China and India compared to the U.S, show that resource misallocation can lower aggregate total factor productivity (TFP) and growth.

3.1.2.2- International free trade as cross-country volatility drivers

Obstacles to trade are very diverse. The simple absence of taxes or non-tariff barriers is not sufficient to realize an international free trade. International free trade is a situation in which there are optimal relationships between all the components of the country, on the one hand, and between all the countries, on the other hand. Pioneer studies show that consumption opportunities depend on many socioeconomic factors to which some groups are submitted. All kinds of disequilibrium or impediments act as barriers to trade.

The general equilibrium is tightly linked to the existence of various complementary parties (countries). In the Walrasian equilibrium we have:

$$P_b(X_{usb} - W_{usb}) + P_d(X_{usd} - W_{usd}) = 0 \text{ excess demand for USA} \quad (3)$$

$$P_b(X_{ub} - W_{ub}) + P_d(X_{ud} - W_{ud}) = 0 \text{ excess demand for China} \quad (4)$$

(P_b/P_d) is an international trade equilibrium price. P_b : wheat price, P_d : DVD price.

$$W_{ij} = \frac{\frac{P_j(t)}{P_i(t)} Li(t)g_{ij}(t) + L_j(t)g_{ji}(t)}{Li(t)y_i(t)} \quad i \neq j \quad (5)$$

$G_{ij}(t)$ represents country i 's real per capita consumption of country j 's factors; $P_i(t)$ is the price of factor I , and $L_i(t)$ is the size of the population in country i , each at time t .

Next, define a_{ij} (where $0 \leq a_{ij} \leq 1$) as a constant representing the share of country j 's accessible natural resources that can actually be consumed by country i as part of its own unnatural resources. According to Abramovitz's social capability (1986), a_{ij} determines the potential of a country to consume existing technologies. Relatively to these definitions, the accumulation of unnatural resources in the country I may be written as

$$H_i(t) = \Phi[\sum a_{ij}w_{ij}(t)H_j(t)] + (\Phi - \delta_H)H_i(t) \quad (6)$$

Where Φ and δ_H represent the common productivity parameter and rate of depreciation of unnatural resources stock (obsolescence or otherwise), and it is assumed that $\Phi \geq \delta_H > 0$

Country G_i 's measure of exchange with country G_j , w_{ij} is:

$$w_{ij} = a_{ij} + a_{ji} \pi_i / \pi_j, \quad i \neq j \quad (7)$$

If, as I supposed it here, each country maintains multilateral trade balance at every point in time, we have:

$$L_i(t) \sum P_j(t)c_{ij}(t) = \sum P_i(t)L_j(t)c_{ji}(t) \quad i \neq j \quad \pi_i \text{ is a function of } \hat{a}_{ij} = \frac{a_{ij}Q_i}{[1+t_{ij}]} \quad (8)$$

where t_{ij} : tariff of country I on imports from country j , Q_i : output

Taking into account the country i dynamic behaviour, the specification of equation (6) gives $H(t) = \Phi \cdot H(t)$

Where $H(t) = H_1(t), \dots, H_j(t)$ and

$$\Phi = \begin{pmatrix} \Phi - \delta_H & \Phi a_{12} & \dots & \Phi a_{1j} & w_{1j} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ \Phi a_{j1} & w_{j1} & \dots & \Phi a_{j2} & w_{j2} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ \Phi a_{j1} & w_{j1} & \dots & \Phi - \delta_H \end{pmatrix}$$

The international exchange matrix can, then, be written:

$$G_{ij} = \begin{pmatrix} w_{11} & w_{12} & \dots & w_{170} \\ w_{21} & w_{22} & \dots & w_{270} \\ \vdots & \vdots & \vdots & \vdots \\ \vdots & \vdots & \vdots & \vdots \\ w_{701} & w_{702} & \dots & w_{7070} \end{pmatrix}$$

The international matrix of prices is directly derived

$$\begin{pmatrix} P_{11} & P_{12} & \dots & P_{170} \\ P_{21} & P_{22} & \dots & P_{270} \\ \vdots & \vdots & \vdots & \vdots \\ \vdots & \vdots & \vdots & \vdots \end{pmatrix}$$

$$P_{ij} = \begin{matrix} . & . \\ . & . \\ . & . \\ P_{701} & P_{702} & \dots & P_{7070} \end{matrix}$$

The study of international levelling out of goods and factors prices enables us to understand cross-country volatility mechanism.

We can also represent the situation of international free trade equilibrium through a system of the iso-product curve for each good as a dual program.

$$\text{Min} C_d = wL_d + rL_d \quad (9)$$

Subject to:

$$Q_d = F(L_d, N_d)$$

$$U_{\text{wheat}} / \text{wheat price} = U_{\text{DVD}} / \text{DVD price}. \quad (10)$$

Wheat price: P_b and DVD price: P_d

U_m : marginal utility.

The USA is well endowed in natural resources and China in unnatural resources. At the beginning of the international trade, the USA will export wheat (indirectly the natural resources), product with high intensity of natural resources) and will import the DVD (indirectly the unnatural resources), product with high proportion of unnatural resources) from China with international or international equilibrium price of 2b/d (2 units of wheat for 1 unit of DVD) for example). This result indicates that the price of wheat has increased in the USA compared to the autarky which was 3b/d. (3 units of wheat for 1 unit of DVD).

The same international trade price shows that the price of DVD has decreased in the USA. The symmetric adjustment will take place in China, where P_b decreases and P_d augments. In the USA the wheat production augments and DVD production decreases. The natural resource's demand will increase causing its price increases. The proportion of natural resources in wheat production will decrease and the proportion of unnatural resources in the production of wheat will increase. In the USA, the changing in the factors prices will modify production technique. The technique will be Unnatural resources intensive. In China, it is the reverse case. The technique will be intensive in natural resources whose price is decreasing.

So in the USA, w/r augments and in China w/r decreases. At the general international equilibrium, we will have all prices levelling out because its changes are symmetrically reverse from a country to another.

For the production functions with constant output, the minimum cost is a linear function of

π , of ϕ_{π} , π depends on w et r

Then:

$$C_{usd}(w, r, Q_{usd}) = \pi \cdot Q_{usd} \text{ And } \pi = \pi f(w, r)r \quad (11)$$

$$P_{usd} = \frac{\partial C_{at}}{\partial Q_{usd}} = \pi_t(w, r) \text{ for the DVD and} \quad (12)$$

$$P_{usb} = \pi_{us}(w, r) \text{ for the wheat.}$$

$$r = r(P_{usd}, P_{usb}) \text{ And } w = w(P_{usd}, P_{usb}) \text{ where } \frac{w}{r} = h\left(\frac{P_{usb}}{P_{usd}}\right). \quad (13)$$

This relationship is identical in the two countries. We deduce that the prices of goods and services levelling out take place in a corresponding manner of the prices of factors levelling out in all the countries. That is why I conclude that there is a convergence towards a constant rate of equilibrium growth in the case where the stocks of unnatural resources and the natural resources are superior to their equilibrium level.

3.2- The model second component: Equilibrium approach to Intergenerational free trade and cross-generation volatility

3.2.1- Foundations and hypothesis

We are in a world of overlapping generations (or intergenerational trade) and there is no international trade so that each country operates in autarky conditions. Each generation has its initial endowments (at the beginning of the analysis) which are composed of natural resources and unnatural

resources. Natural resources (all the physical environment) and unnatural resources (other resources) are the factors of production of the economy. Each generation has its own comparative advantages. Through these conditions, we can set the following analysis which is based on the common neoclassical understanding.

3.2.2- Intergenerational drivers and cross-generation volatility

Let us consider two generations of a given country: the current generation (G_c) and the future generation (G_f). The two generations are separated by such a long time that the ordinary **tradable** goods cannot be stocked. The two generations have a nation's status so that we have a succession of nations in the same country. It is clear that each generation or nation has different initial endowments which are interdependent. If we suppose that all the generations of that country are co-owners of the resources of the natural country estimated to W , and if each generation life expectancy at birth is 100 years, the country's life expectancy at birth is 7,000 years for 70 generations. Each generation initial endowment is equal to $W/70$.

During its time of life, the beginning nation (generation) should use its $W/70$ of natural resources and borrow natural resources from the following generations in different proportions w_i . So the first generation total natural resources at the beginning of the first period are equal to

$$\frac{W}{70} + \Sigma w_{1j} \quad (14)$$

With Σw_{ij} the debt of the first generation borrowed to the followings. The second generation's total resources at the beginning of the second period is given by::

$$\frac{W}{70} - w_{12} + k_{12} + \dots + \Sigma w_{2j} \quad (15)$$

(k_{12} is what the first generation reimburses to the second in terms of unnatural resources). k_{12} should be equal to w_{12} . k_{12} is the first generation's exports towards the second generation. W_{12} is the first generation's import from the second generation. The final generation of total resources are equal to:

$$\frac{W}{70} - \Sigma w_{in} + \Sigma k_{in} = \frac{W}{70} = w^* + K_n \quad (16)$$

The first generation will use its total natural resources to build the country (roads, schools, hospitals, airports, capital, research and development etc.) and to produce goods and services for its consumption. At the end of these 100 years, the second generation and the followings will have in co-ownership:

$$W - W[\beta + \delta(1 - \beta)] \quad (17)$$

β : the self-consumption ratio (consumption by income unit); δ : the ratio of remaining natural resources and unnatural resources (a part of resources to be reimbursed to the following generations).

At the start of the 101st year of the life of this country, we have as the remaining resources $W[(1 - \beta)(1 - \delta)]$ (18)

The second generation of natural and unnatural resources are $W[(1 - \beta)(1 - \delta)]$. This generation will proceed like the first one and at the end of its life, the remaining resources are given by the following relation:

$$W[(1 - \beta)(1 - \delta)] - W[(1 - \beta)(1 - \delta)][\beta + \delta(1 - \beta)] = W[(1 - \beta)(1 - \delta)]^2 \text{ which are the resources of the third generation.}$$

At the start of the 201st year of the life of this country, we have as remaining resources

$$W[(1 - \beta)(1 - \delta)]^2 \quad (19)$$

We notice that the new resources follow a law of geometric progression with $(1 - \beta)(1 - \delta)$ as the gain.

$$\text{The new resources at } N^{\text{th}} \text{ generation are } W[(1 - \beta)(1 - \delta)]^{n-1} \quad (20)$$

The total amount of new resources is equal to the sum of geometrical progression with a gain inferior to 1. . We know that that sum admits as limit the following expression:

$$\frac{W}{W[(1 - \beta)(1 - \delta)]} = \frac{W}{[\beta + \delta(1 - \beta)]} = \Delta W \quad (21)$$

$$\text{The optimal growth multiplier is: } \frac{1}{[\beta + \delta(1 - \beta)]} \quad (22)$$

At the end of the first generation, we have two kinds of resources: the remaining natural resources and the remaining of unnatural resources. *Unnatural resources* are a sum of very different factors of production like techniques, know-how, capital, infrastructures, socioeconomic environment etc.

$$W_{ij} = \frac{\frac{P_j(t)}{P_i(t)} Li(t) g_{ij}(t) + L_j(t) g_{ji}(t)}{Li(t) y_i(t)} i : j \quad (23)$$

$G_{ij}(t)$ represents generation i 's real per capita consumption of generation j 's factors; $P_i(t)$ is the price of factor I , and $L_i(t)$ is the size of the population in generation i , each at time t .

Next, define a_{ij} (where $0 \leq a_{ij} \leq 1$) as a constant representing the share of generation j 's accessible natural resources that can actually be consumed by generation i as part of its own unnatural resources. According to Abramovitz's social capability (1986), a_{ij} determines the potential of a generation to consume existing technologies. Relatively to these definitions, the accumulation of unnatural resources in generation I may be written as

$$H_i(t) = \Phi[\sum a_{ij} w_{ij}(t) H_j(t)] + (\Phi - \delta_H) H_i(t) \quad (24)$$

Where Φ and δ_H represent the common productivity parameter and rate of depreciation of unnatural resources stock (obsolescence or otherwise), and it is assumed that $\Phi \geq \delta_H > 0$

Generation G_i 's measure of exchange with generation G_j , w_{ij} is:

$$W_{ij} = a_{ij} + a_{ji} \pi_i / \pi_j, \quad i \neq j \quad (25)$$

If, as I supposed it here, each generation maintains multilateral trade balance at every point in time, we have:

$$L_i(t) \sum P_j(t) c_{ij}(t) = \sum P_i(t) L_j(t) c_{ji}(t) \quad i \neq j \quad \pi_i \text{ is a function of } \hat{a}_{ij} = \frac{a_{ij} Q_i}{[1 + t_{ij}]} \quad (26)$$

where t_{ij} : tariff of generation I on imports from generation j , Q_i : output

Taking into account the generation i dynamic behaviour, the specification of equation (27) gives $H(t) = \Phi \cdot H(t)$

Where $H(t) = H_1(t), \dots, H_j(t)$ and

$$\Phi = \begin{pmatrix} \Phi - \delta_H & \Phi a_{12} w_{12} & \dots & \Phi a_{1j} w_{1j} \\ \vdots & \vdots & \ddots & \vdots \\ \Phi a_{j1} w_{j1} & \Phi a_{j2} w_{j2} & \dots & \Phi - \delta_H \end{pmatrix}$$

The intergenerational exchange matrix can, then, be written:

$$G_{ij} = \begin{pmatrix} w_{11} & w_{12} & \dots & w_{170} \\ w_{21} & w_{22} & \dots & w_{270} \\ \vdots & \vdots & \ddots & \vdots \\ w_{701} & w_{702} & \dots & w_{7070} \end{pmatrix}$$

The intergenerational matrix of prices is directly derived

$$P_{ij} = \begin{pmatrix} P_{11} & P_{12} & \dots & P_{170} \\ P_{21} & P_{22} & \dots & P_{270} \\ \vdots & \vdots & \ddots & \vdots \\ P_{701} & P_{702} & \dots & P_{7070} \end{pmatrix}$$

The study of intergenerational levelling out of goods and factors prices permit to understand the cross-generation volatility mechanism.

3.2.3- Analyzing intergenerational drivers and cross-generation volatility

3.2.3.1- Sensitivity of intergenerational interdependencies

The sensitivity of intergenerational interdependencies can be analyzed as the effectiveness of the intergenerational free exchange and the extent to which that exchange affects the prices in each generation. The intergenerational exchange description enables us to appreciate prices changes and their intergenerational transmission.

The natural resources, in the beginning, are to be divided equally among the 70 generations. The remaining of unnatural resources is the property of the preceding generation that could be viewed as the compensation of the natural resources used by this generation, but belonging to the following generations. It becomes clear that each generation consumes part of the resources of the following generations and reimburses that consumption with its remaining unnatural resources. This denotes a clear process of factors of production trade between generations. The goods and services are indirectly exchanged through factor's trade. This process of substitution enables us to postulate a transformation curve or the production possibilities frontier for each generation, the autarky prices or its comparative advantages. Each generation has its own endowments of natural and unnatural resources. It is possible for a generation to make arbitrage between the resources to export and those to import. If a generation chooses to consume more natural resources (imports) it means that it accepts to produce more unnatural resources for the next generations (exports) and vice versa. According to the generation's demand for each good and service, we will have different comparative advantages. Each generation is then considered as a different nation exchanging with others. If we consider two factors of production (natural resources and unnatural resources), two generations (G_1 and G_2) and two goods (wheat and DVD), one should admit that there is a process of factors of production substitution between generations. The following generations lend to the preceding their part of the natural resources and receive in return the remaining unnatural resources abandoned by the first ones at the end of their life. Indirectly, the following and the preceding generations exchange goods and services. The following generations sell indirectly to the preceding generations goods and services that they would have produced with their parts of natural resources if they could appear at these preceding generations periods of life in exchange of the goods and services that the preceding generations would have produced with their remaining unnatural resources during the time of the following generations if they could live at that time. It is, therefore, possible to apply the neoclassical model of international exchange and our theory of intergenerational free trade should be written as follows: *The factors of production not intensively used in the production of the goods and services which exist in abundance in a generation are « exported » to another generation in exchange for the factors of production used intensively in the production of the goods and services which should be scarce in that generation. It is indirectly the goods and services weakly consumed that are exported from a generation to another, whereas the goods and services highly consumed are « imported from other generations».* ». The following generations would have had an abundance of goods and services which intensively use the natural resources if at their lifetime they could have had simultaneously as many natural resources as possible and the present abundance of unnatural resources. Similarly, the present generation should have had an abundance of goods and services which intensively use unnatural resources if they could have had at their disposal as much as the following generations in addition to the present abundance of natural resources. The exports and the imports are really the intergenerational trade. In other terms, the following generations are selling natural resources intensive in the production of wheat, for example, or are indirectly selling wheat to the current generation in exchange for unnatural resources intensive in DVD production at the end of their life or indirectly the DVD. The DVD did not exist during the eighties, but preceding generations sold them to the current generations indirectly by ligating them the inputs of the technologies or know-how necessary for their production.

But, contrary to the neoclassical international trade model, we make the hypothesis that only the factors of production are tradable. The final goods are not storable.

In order to illustrate our intergenerational exchange model, let's consider the Edgeworth box

The beginning allocation is W and the final is noted X. At point X a perfect equilibrium of production and consumption of the two generations is realized. Each generation improves its utility when passing from a lower indifference curve to the upper. At that point, produced and consumed quantities by all the generations (by pairs of two) are determined.

When a generation chooses another initial allocation different from W (disturbance to intergenerational production possibilities), it is no more possible for this generation to reach the equilibrium point which is X on this graph 2. Since then, the generation and the related country are engaged in great potential volatility which is varying with the distance separating the effective initial allocation (W_i) to the optimal initial allocation and with the sensitivity of the interdependencies. Since then, the generation PPF is moving around the (WTF). The derived growth is not optimal (graph 5). Volatility function can be described by the following relation:

$$(X_f - X) = f(W_f - W, \theta). \quad (28)$$

θ is the intergenerational interdependencies sensitivity factor. That volatility becomes explosive (through other countries and generations) if the interdependencies are very sensitive.

3.2.3.2- Cross-generation volatility

On each market (capital, goods etc.), the volatility drivers are the prices and their flexibility. Let consider a general equilibrium case:

$$P_a(X_{aa} - W_{aa}) + P_r(X_{ra} - W_{ra}) = 0 \text{ excess demand for the current generation} \quad (29)$$

$$P_a(X_{af} - W_{af}) + P_r(X_{rf} - W_{rf}) = 0 \text{ excess demand for the following generation} \quad (30)$$

(P_r/P_a) is intergenerational trade equilibrium price.

When the intergenerational trade equilibrium price is known, we can build a program of cost minimization under production constraint like:

$$\text{Min } C_r = wL_r + rL_r \quad (31)$$

Subject to:

$$Q_r = F(L_r, N_r)$$

We can, for example, establish iso-product unitary curves on one hand and iso-cost curve on the other hand. The solution of this program enables us to determine the optimal production corresponding to a minimum cost. This equilibrium is obtained at the tangency point of the unit iso-product curve and the lowest possible iso-cost curve; This point gives the intergenerational terms of trade levelling out and equivalency of the values of the goods and the factors exchanged. At that intergenerational equilibrium, the following relations can be identified:

$$U_{\text{wheat}} / \text{wheat price} = U_{\text{DVD}} / \text{DVD price}. \quad (32)$$

We can also represent the situation of intergenerational free trade equilibrium through a system of the iso-product curve for each good as a dual program;

The current generation of France is well endowed in unnatural resources and the following generations in natural resources. At the beginning of intergenerational trade, "Current France" will export unnatural resources (indirectly the DVD, product with high intensity of unnatural resources) and will import the natural resources (indirectly the wheat, product with high proportion of natural resources) from the "Future France" with intergenerational equilibrium price of $3r/t$ for example. This result indicates that the price for unnatural resources has been augmented compared to the autarky which was $2r/t$.

The same intergenerational trade price shows that the price for natural resources has been reduced in the "Current France". The symmetric adjustment will take place in the "Future France", where P_t decreases and P_r augments. In "Current France", the proportion of natural resources in wheat production will increase and the proportion of unnatural resources in the production of wheat will decrease. In the "Current France", the changing in the factors prices will modify production technique. The technique will be composed of more natural resources and less unnatural resources. In the "Future France", it's the reverse case. The techniques will be intensive in unnatural resources whose price is decreasing. The substitution of natural resources to unnatural resources in wheat production causes the reduction of wheat price in « Current France ». A symmetric analysis indicates that the DVD price will decrease and the wheat price will increase in Future France. So in the "Current France", P_d/P_m augments and in the "Future France" P_d/P_m decreases. At the general intergenerational equilibrium, we will have all prices levelling out because their change is symmetrically reverse from a period to another. The intergenerational factors of production's trade reduce the prices of rare factors of each period and enable the production of goods and services particularly consumed in the period. The reduction of period goods and services prices causes intergenerational trade earnings both for consumers and producers of the period.

For the production functions with constant output, the minimum cost is a linear function of π , of $\varphi_{tt}\pi$ depends on w et r . Then:

$$C_{at}(w, r, Q_{at}) = \pi \cdot Q_{at} \text{ et } \pi = \pi f(w, r) \quad (33)$$

In perfect competition, the profit maximization demands that the selling price P_{at} should be equal to the marginal cost. This means that:

$$P_{at} = \frac{\partial C_{at}}{\partial Q_{at}} = \pi_t(w, r) \text{ for the DVD and} \quad (34)$$

$$P_{ta} = \pi_a(w, r) \text{ for the wheat.}$$

$$r = r(P_{at}, P_{ad}) \text{ and } w = w(P_{at}, P_{ad}) \text{ where } \frac{w}{r} = h\left(\frac{P_{at}}{P_{ad}}\right) \quad (35)$$

This relation is identical in the two generations. We deduce that the prices of goods and services level out in a corresponding way to the prices of factors levelling out in all the generations. That is why we conclude that there is a convergence towards a constant rate of equilibrium growth in the case where the stocks of “work” or unnatural resources and the natural resources are superior to their equilibrium level.

4- Methodology

Modern stochastic endogenous growth theory is used to study theoretically the relationship between long-term growth and short-term volatility. Nelson and Plosser (1982) have shown that the movements in the GNP tend to be permanent. Then F. Kydland and E. Prescott (1983) offered new skills for analyzing economic volatility and integrating growth and volatility (fluctuations). They have shown that macroeconomic time series are better characterized as non-stationary integrated processes rather than stationary processes around a deterministic trend. The finding of this theory is that the literature suggests that this relationship is ambiguous as well. This depends on the structure of the models considered, the assumptions made about the mechanisms generating endogenous technological change, and the values of the parameters assumed.

In order to capture country choices effects (shocks) on other countries, I will study the movements of production possibilities frontier around the whole world frontier or the optimum level. These movements determine the countries trade through comparative advantages gained or lost. **Levine Renelt control variables which define PPF enable to calculate international trade elasticity ($\epsilon_i - 1$). The international trade elasticity is the absolute variation of international trade due to changes in the PPF and comparative advantages. In addition, in other regressions, my dummy variables are defined as shocks effects on production frontiers and subsequently are proxies to effects on international trade.** If production possibilities frontier movements' trend is to the left side, countries with a higher standard deviation of growth should have their growth adversely affected if at the same time they lose their comparative advantages. Thus, international trade elasticity after a production possibilities frontier movement (ϵ_i) determines the sign of the relationship between growth and volatility. If $(\epsilon_i - 1) < 0$, the sign is negative and positive if $(\epsilon_i - 1) > 0$ (Fig 1). Suboptimal choices should cause shocks. Overconsumption of natural resources by a country not integrally compensated by an equivalent measure of unnatural resources during some periods of 5 years (trend) moves PPF to the left side. Sub-consumption of natural resources not integrally compensated by an equivalent measure of unnatural resources moves PPF to the right side and the relationship between growth and volatility is positive.

In order to test the first component model (how the changes in production factors' supply affect international trade and growth volatility), I choose two sample countries to study these relationships: The first sample contains 25 OECD countries observed from 1980 to 2010. In the second sample, I will study the same relationship with 108 developing countries for the same period of time (1980-2010). The number of observations in the first sample is 588 and 3240 for the second. The reason for this choice is that each group of countries has intra-group similar production technologies. All the data are from the World Development Indicators.

4.1- The methodology

4.1.1- Intergenerational levelling out of prices of goods and factors test

The question is: Do two generations of a nation separated by a very long period (100.200 . 1000 years) exchange goods and services? We have already shown in our theoretical model that this kind of trade is real. For a nation with regular long run statistics of physical and social environment, the test is possible. We can calculate the quantities of factors of production for each generation or its generational comparative advantages, the quantities of goods and services exchanged, the autarky and intergenerational free trade prices and trade earnings. But for a nation in which such statistics are not available it is possible to extrapolate by attributing the statistics of a similar nation (dimensions) and separated by centuries of civilizations. One is the future of another and vice versa. This assumption is based on the theory of economic convergence to a stationary situation. It means that different nations will present similar characteristics at different stages of their development. By using a simple data analysis (PCA), the coordinates of two generations on two axes enable to calculate what generation i exchanges with generation j and what this generation receives in return from generation j . This instrument is particularly effective for that kind of analysis; A major component in Statistical Data Analysis is a group of very correlated variables. The major component is a new variable composed of other variables and has the role of the instrumental (transformed) variable. In my model, the factors of production (natural resources and unnatural resources) are really the groups of the factors of production. It is clear that a factor of production can appear as a major component in Data Analysis or an axis recall natural resources. The natural resources are composed of natural endowments like land, mineral resources and all physical environment. The unnatural resources are the elements of socioeconomic environment like capital, knowledge, infrastructures, output, etc. When a Statistical Data Analysis is conducted efficiently each axis represents one of the factors of production of my model. In my Statistical Data Analysis, each generation is considered as a nation with its own coordinates on the axis. Thus the differentiation of the axis's coordinates represents the measures of what a generation exchanges with another generation. For example, when I run Data Analysis based on statistics of 1990 on a sample of generations (G_1 , G_2 , ...), I observe that a generation performance increases on one factor of production or axis and decreases on the other. The increase on an axis is the resources exportation of G_1 to G_2 and the decrease is a resource importation of G_1 from generation G_2 . The coordinates of G_2 on axis 1 (unnatural resources) are the supply of unnatural resources due to G_1 that this generation exports to G_2 . The coordinates of G_2 on axis 2 (natural resources), which indicate a decrease, are the remaining natural resources after G_1 has used its part and a part due to G_2 or to the following generations. This partly due to other generations is the G_1 natural resources importations. So the difference between the coordinates on the same axis for a generation or country is the quantity of a factor of production exchanged by the generation to receive in return a quantity of a factor of production determined by the coordinates differentiation of the same generation on another axis. The ratio of exchanged quantities determines the TRS (Technical rate of substitution) between factors of production and its prices. When autarkic prices are known it is possible to appreciate if this resources exchange leads to intergenerational levelling out of prices of factors and goods or not.

4.1.2- Cross-generation volatility test

For the second component (how changes in production factors supply affect intergenerational trade), I will first consider France through five generations of 50 years with 10 witnesses. In the second case, I mix France five generations with the remaining of 124 countries considered as current generations.

4.1.3- Multidimensional trade and the empirical evidence of the sign of the link between volatility and growth

4.1.3.1- The multidimensional trade model

4.1.3.1.1- Equilibrium

In a multidimensional trade model, the first component interacts with the second component. Then, the relationships between intergenerational trade and international trade appear like the movements that are propagating vertically (through generations) and horizontally (current generations or the nations).

We are in a world of overlapping generations and international free trade. It is clear that such movements are interfering to create a recursive, triangular or causal system.

Let's start with the expressions of separate movements:

$$S(t) = S_0 \cos(\omega t - \varphi_1) \quad (36)$$

for international trade.

$$G(t) = G_0 \cos(\omega t - \varphi_2) \quad (37)$$

For intergenerational trade.

If these two flows have the same rhythm but different generation weights the macro-dynamic equilibrium is determined through the calculation of the multidimensional trade with the following relation:

$$M(t) = S(t) + G(t) = M_0 \cos(\omega t - \varphi_2) \text{ for the multidimensional trade} \quad (38)$$

If I develop (27), I obtain:

$$M \cos \omega t \cos \varphi + M_0 \sin \omega t \sin \varphi = S_0 \cos \omega t \cos \varphi_1 + S_0 \sin \omega t \sin \varphi_1 + G_0 \cos \omega t \cos \varphi_2 + G_0 \sin \omega t \sin \varphi_2 \quad (39)$$

Solving simultaneously:

$$M_0 \cos \omega t \cos \varphi = S_0 \cos \omega t \cos \varphi_1 + G_0 \cos \omega t \cos \varphi_2 \quad (40)$$

$$M_0 \sin \omega t \sin \varphi = S_0 \sin \omega t \sin \varphi_1 + G_0 \sin \omega t \sin \varphi_2 \quad (41)$$

It comes:

$$M_0 \cos \varphi = S_0 \cos \varphi_1 + G_0 \cos \varphi_2 \quad (42)$$

$$M_0 \sin \varphi = S_0 \sin \varphi_1 + G_0 \sin \varphi_2 \quad (43)$$

Then we calculate the amplitude of the multidimensional trade as follows

$$M_0^2 (\cos^2 \varphi + \sin^2 \varphi) = S_0^2 (\cos^2 \varphi_1 + \sin^2 \varphi_1) + G_0^2 (\cos^2 \varphi_2 + \sin^2 \varphi_2) + 2S_0 G_0 (\cos \varphi_1 \cos \varphi_2 + \sin \varphi_1 \sin \varphi_2) \quad (44)$$

$$M_0^2 = S_0^2 + G_0^2 + 2S_0 G_0 \cos(\varphi_1 - \varphi_2) \quad (45)$$

If the multidimensional trade is horizontal ($\varphi_1 = \varphi_2$)

$$I \text{ have } M = (S_0^2 + G_0^2)^{1/2} \quad (46).$$

In this case, I have a constructive multidimensional trade, because of the multidimensional trade increases.

But, if the multidimensional trade is vertical with different generation's weight ($\varphi_1 = \varphi_2 + \pi$), I obtain:

$$M_0^2 = (S_0^2 - G_0^2) \quad (47)$$

Here the multidimensional trade is destructive as it decreases.

Between the two extremes, the multidimensional trade is varying with $\cos(\varphi_1 - \varphi_2)$ or the cosinus of generation's weight difference.

I calculate a generation's weight by dividing member by member of the preceding equations:

$$\tan \varphi = \frac{S_0 \sin \varphi_1 + G_0 \sin \varphi_2}{S_0 \cos \varphi_1 + G_0 \cos \varphi_2} \quad (48)$$

Finally, the multidimensional trade expression is :

$$M = S_0^2 + G_0^2 + 2S_0 G_0 \cos(\varphi_1 - \varphi_2) \cos \left(\omega t - \arctan \varphi \frac{S_0 \sin \varphi_1 + G_0 \sin \varphi_2}{S_0 \cos \varphi_1 + G_0 \cos \varphi_2} \right) \quad (49)$$

4.1.3.1.2- The multidimensional trade interdependencies and growth

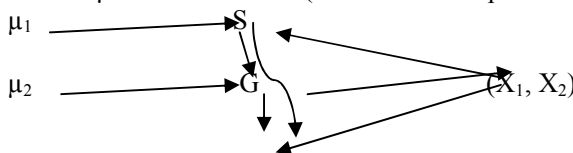
Let's now consider this framework of simultaneous relations:

$$S_{1t} = \beta_{10} + \delta_{11} X_{1t} + \delta_{12} X_{2t} + \mu_{1t} \quad (50)$$

$$G_{2t} = \beta_{20} + \beta_{21} S_{1t} + \delta_{21} X_{1t} + \delta_{22} X_{2t} + \mu_{2t} \quad (51)$$

$$M_{3t} = \beta_{30} + \beta_{31} S_{1t} + \beta_{32} G_{2t} + \delta_{31} X_{1t} + \delta_{32} X_{2t} + \mu_{3t} \quad (52)$$

Because of the interdependencies between the international and intergenerational trade, I postulate the simultaneous equations where, the S's, G's, M's and the X's are respectively, the endogenous and the exogenous variables. I know that trade externalities are such that $\text{cov}(\mu_{1t}, \mu_{2t}) = \text{cov}(\mu_{1t}, \mu_{3t}) = \text{cov}(\mu_{2t}, \mu_{3t}) = 0$. As I am in presence of the same period trade externalities in differential equations, I assume that the μ are uncorrelated (the zero contemporaneous correlation):



$\mu_3 \longrightarrow M$

I state that the condition of a recursive competitive equilibrium or a constant growth rate is set by $\text{cov}(\mu_{1t}, \mu_{2t}) = \text{cov}(\mu_{1t}, \mu_{3t}) = \text{cov}(\mu_{2t}, \mu_{3t}) = 0$

When I consider the first equation, we see that it contains only the exogenous variables on the right-side and because of the assumption of the non-correlation with trade externalities μ_1 , this equation satisfies the critical assumption of a constant and optimal growth rate.

Next, consider the second equation which contains the endogenous variable S_{1t} as an explanatory variable along with non-stochastic X 's. Now the same critical constant growth rate is also satisfied because S_{1t} and

μ_{2t} are uncorrelated. Is this so? I answer yes, because, in fact, μ_1 which affects S_{1t} is by assumption uncorrelated with μ_2 . In this model, S_{1t} is a predetermined variable insofar as G_{2t} is concerned. In the same reasoning, we argue that the critical constant growth rate is satisfied for the third equation because both S_{1t} and Y_{2t} are uncorrelated with μ_3 .

Thus, in this recursive system, the growth rate is constant in each equation separately. Currently, we do not have an interference equation problem in this situation. From the structure of such systems, it is clear that there is no interdependency among the endogenous variables. Thus S_{1t} affects G_{2t} , but G_{2t} does not affect S_{1t} . Similarly, S_{1t} and G_{2t} influence M_{3t} without, in turn, being affected by M_{3t} . We conclude that in such a system, each equation exhibits a unilateral causal dependency and assures to all economies and generations the same and constant optimal growth rate.

4.1.3.1.3- The dichotomy between growth and business-cycles

In my theoretical model, the multidimensional trade expression is:

$$M_O^2 = S_O^2 + G_O^2 + 2S_O G_O \cos(\varphi_1 - \varphi_2) \cos\left(\omega t - \arctan\varphi \frac{S_O \sin \varphi_1 + G_O \sin \varphi_2}{S_O \cos \varphi_1 + G_O \cos \varphi_2}\right)$$

This equation expresses the interferences of the two components (international trade and intergenerational trade) of the macro dynamic equilibrium, (Fig 1). I interpret that equation as a (WTF) which increases in the efficiency of natural resources are obtained at the cost of declines in the efficiency of unnatural resources. Solow growth model based on a constant saving rate implies that the movements in and of the (PPF) cannot occur (Pareto efficiency criterion). In such a model, there is no economic volatility (constant growth rate). When a country or a generation chooses suboptimal initial allocation different from W (disturbance to intergenerational/international (PPF), it is no more possible for this country or generation to reach the equilibrium point which is X on this graph 1 or 2. Since then, the country or generation is engaged in great potential volatility which is varying with the distance separating the effective initial allocation (Wi) to the optimal initial allocation and with the sensitivity of the interdependencies.

The model, with its great emphasis on theoretical skills, introduces new types of economic agents (the generations) and the importance of optimal choices as the intergenerational and international interdependencies drivers. First, my models enhance the importance of intergenerational choices in growth programs related to economic volatility.

The second important property of these models is that the equilibrium and the stability of the economy are determined on the one hand by the macro dynamic and international interdependencies and on the other hand by the state of international and intergenerational interests or needs, and society's optimal resources allocation.

The third property developed in the present models is the canonical relationship between intergenerational and international equilibrium. The two dimensions are closely linked; it is not possible to have one without the other. All the disequilibria in an economy (unemployment, budget

deficits, internal and external disequilibrium and economic volatility) are the result of non-coincidence of intergenerational and international equilibriums. The three properties above are very important in the understanding of the current economic volatility and they provide solutions to problems that affect economies and globalization. The findings of this study may put some light on the process of sustainable development and optimal growth and may also ensure a long run economic stability.

5- SOLUTION

The model first component test: Cross-country empirical evidence on the equilibrium approach to the international free trade and economic volatility

Cross-sectional variation in volatility

In this section, I test the relationship between growth and volatility. Thus, we will follow Ramey and Ramey's four steps procedure (1995): First, we study the relationship between growth and volatility through a very simple model. In the following steps, I introduce conditioning variables in order to determine the robustness of the link. Two kinds of volatility will be studied: volatility of growth and volatility of innovations to growth. I will so measure the impact of each type of volatility on average growth rate. The role of investment will be studied as well. **In developing countries, international trade is supposed to be the mean growth (grgdp) key factor (75-90%) so that in my tests on developing countries, movements in (grgdp) are proxies to international trade movements. In these circumstances, Levine renelt control variables 1) the average investment fraction of GDP ; 2) the log of the initial GDP per capita ; 3) initial human capita ; and 4) the average growth of the population which define (PPF) enable to calculate international trade elasticity ($e_i - 1$). In addition, in other regressions, my dummy variables are defined as shocks effects on production frontiers and subsequently are proxies to effects on international trade.**

But, it is important to prevent that at this level of the exercise, the relationship between growth and volatility is the first component of the whole relationship because international trade interferes with intergenerational trade in the context of a *multidimensional trade*.

The test: first step

We choose to study these relationships on two samples of countries: The first sample is composed of 25 OECD countries observed from 1980 to 2010. We study the same relationship in the second sample including 108 developing countries for the same period (1980-2010). All the regressions in this paper refer to the period (1980-2010). The reason for this choice is that each group of countries is assumed to have similar production technologies. **All the data are from the World Development Indicators.**

Testing the relationship between Mean and volatility of growth

At this first step, it is important to examine the basic nature of the cross-country relationship between growth and volatility. At this end, I calculate the mean and the standard deviation of per capita annual growth rates for each country. The result of regression of mean growth (**grgdp**) on the standard deviation of growth (**vol**) for 108-country sample from 1980 to 2010 is in the tables 1a, 1b and 1c.

In the first 108-country sample, the relationship is positive and statistically significant but, in the second 25 OECD-country sample, I find a negative relationship. In Ramey and Ramey's study (1995) the coefficients were negative for the first sample and positive for the second sample. In my case, I deduce that countries with higher year-volatility in growth rates tend to have a higher growth rate in developing countries and lower growth rates in OECD countries. These results are not surprising when we consider the great stagnation and instability which has characterised the second sample during the period 1980-2010. But, in the case of the first sample, the nature of technologies indicates that growth sources are different. Thus, it becomes useful to introduce other important characteristics of these economies in order to see the evolution of the relationship shown by the regressions previously.

The models to be studied are :

$$grgdp_{it} = \lambda vol_{it} + \theta X_{it} + \varepsilon_{it} \quad (1a)$$

$$\varepsilon_{it} \sim N(0, \sigma^2_{\varepsilon}) \quad (1b)$$

$$i = 1, \dots, I \quad t = 1, \dots, T$$

$grgdp_i$: the growth rate of output per capita for country i in year t (the log difference)

σ_{ε_i} : the standard deviation of the residuals, ε_{it} ; ε_{it} is the deviation of growth from the value predicted based on X_{it} variables, which are different across countries but not across time

X_{it} : the vector of control variables

Θ : the vector of coefficients, common across countries

λ denotes the link between growth and volatility and is the most important parameter to be examined.

The vector of control variables, X proposed by Levine and Renelt (1992) is a set of the most important variables for cross-country growth analysis. They are a large set of commonly employed variables. These variables play the most important role in (PPF)' movements. They are defined as 1) the average investment fraction of GDP; 2) the log of the initial GDP per capita ; 3) initial human capita ; and 4) the average growth of the population. In the 108-country sample, the human capital is taken as the average years of schooling for individuals in the total population over 25 years of age and the percentage of the relevant population in secondary schools for the 25 OECD-country sample. Thus, for this model, I will use a Maximum likelihood procedure on panel data strongly balanced. The number of observations for the 108-country sample is 3240 and 630 for the other sample which becomes a 21-country in this specification.

The regression's results are presented in Tables 2a and 2b.

In this specification, the coefficients of the regressions of mean growth on volatility in the two samples have the same negative signs which are respectively (-0.618) and (-0.295) for the first and the second samples and are statistically significant at more than 1% significance level.. The introduction of control variables has improved the significance of the relationship between mean growth and volatility which appear now with the same negative sign. These variables have strengthened the negative relationship that link mean growth to volatility in the OECD sample and reversed the sign for 108-country sample relative to the first basic specification. The relation examined becomes economically significant and expresses the commonly accepted theory that countries with higher year-volatility in growth rates tend to have lower growth rates in each country.

. In this model we can see that international trade elasticity after a production possibilities frontier movement (e_i) determines the negative sign of the relationship between growth and volatility ($e_i - 1 < 0$). See table 2a (bis) and table 2b(bis). Grgdp or international trade elasticity (e_i) is negatively correlated to the movements' trend of national PPF defined by (gdppcp, inv, h_c, aapgr).

Even though the key control variable is initial GDP per capita, I observe that average investment share of GDP appears in this model with a negative coefficient in both samples, but the relationship becomes normal when I regress mean growth on the control variables without growth variance (vol). We should conclude that, contrary to Ramey and Ramey's study, the volatility produce an adverse effect on the relationship between growth and investment. Thus, other studies show that the negative relationship between mean growth and volatility exists despite average investment fraction of GDP is omitted so that there seems to be no systematic effect of controlling for investment.

If the United States is chosen as comparison country the statistics estimated indicate that there is substantial variation in the volatility across the country and the relation studied is formally negative.

5.1.1.1.2- Testing the relationship between innovation variance and growth

In order to examine the uncertainty in the relationship between growth and volatility, we consider the above model and change the content of control variables. These variables are of two kinds: the measures of variables at the beginning of the sample and the forecasting

- variables measured at the beginning of the sample

1) Inv :The investment fraction of GDP in the initial year of the sample ;

2) aapgr : the growth rate of the population in the first two years of the sample

- Forecasting variables :

1) Two lags of log level of GDP per capita

2) A time trend

3) A time trend squared

4) Four seasonal dummy variables ($Q1_t$, $Q2_t$, $Q3$ and DO_{tt}) whose role is to capture specific effects. When a country choice is suboptimal, its production possibilities frontier is in movement. Under these conditions, the seasonal dummy variables which are defined below and Arch/Garch method permit to link the movements of (PPF) and their interactions with international trade, growth rate and volatility.

Following Hendry's method (1974), we use the combination of trend and seasonal dummy variables to model specific effects. In order to model these trend, seasonal and special effects, define new variables as follows:

$$Q1_t = \{-1 \text{ for } 1980-94, 0 \text{ otherwise}\}$$

$$Q2_t = \{1 \text{ for } 1994-2000, 0 \text{ otherwise}\}$$

$$Q3_t = \{-2 \text{ for } 2000-2010, 0 \text{ otherwise}\}$$

$$T = t = 1, 2, 3, \dots, 30$$

$$DT1_t = Q1_t \cdot T, DT2_t = Q2_t \cdot T, DT3_t = Q3_t \cdot T$$

$$DO_t = \{1 \text{ for } 1987(1) \text{ and } 1998(1); -1 \text{ for } 1987(2) \text{ and } 1998(2) \text{ and } 0 \text{ otherwise}\}.$$

The variables $Q1_t$, $Q2_t$ and $Q3_t$ are seasonal dummy variables. As the estimated model will include an intercept term and the joint presence of all four dummy variables and an intercept term would make the estimation procedure break down. The variable T is a time trend. **The variables $DT1_t$, $DT2_t$, $DT3_t$ allow for multiplicative seasonality where the absolute value of the seasonal effect changes over time depending on our estimations of PPF movements and their interactions with international trade. Thus international trade elasticity after a production possibilities frontier movement (e_i) should determine the sign of the relationship between growth and volatility. If $e_i - 1 < 0$, the sign should be negative and positive if $e_i - 1 > 0$.**

Thus the consecutive values of $DT1_t$ are -1, 0, 0 ; $DT2_t$ are 0, 1, 0 ; $DT3_t$ are 0, 0, -2

The equation to be estimated is :

$$grdp_{it} = \beta_0 + \lambda\sigma_i + \beta_1 Gdpcc - re2 + \beta_2 Gdppcc + \beta_3 Inv + \beta_4 aapgr + \beta_5 Hc + re - 2 + \beta_6 Q1_t + \beta_7 Q2_t + \beta_8 Q3_t + \beta_9 T + \beta_{10} DT1_t + \beta_{11} DT2_t + \beta_{12} DT3_t + \beta_{13} DO_t + \varepsilon_{it}$$

The results of this regression are given in tables 3a and 3b.

In this new framework, it is clear that the relationship between the mean growth and innovation volatility is also negative, indicating that countries with higher innovation volatility will have lower mean growth rates. Our results confirm the studies of Ramey and Ramey. Using two samples 24-OECD and 92-country sample from 1950 to 1988 and 1960 to 1985 respectively Ramey and Ramey growth rates on a group of explanatory variables in which we find the standard deviation of output growth. They find that the standard deviation of output growth has a significant negative effect on mean growth.

. In this model we can see that **international trade elasticity after a production possibilities frontier movement (e_i) determines the negative sign of the relationship between growth and volatility ($e_i - 1 < 0$). See table 2a (bis), table 3a(bis), table 3b(bis). $Grgdp$ or international trade elasticity (e_i) is negatively correlated to the movements' trend of national PPF defined by ($gdppccp$, inv , h_c , $aapgr$).**

But, two problems remain pendant. The initial investment share of GDP and human capital, defined as the level of employment for 21 OECD-country samples and as the average years of schooling for individuals in the total population over age 25 for the first sample, are negatively correlated to the mean growth. When I regress the same equation without volatility, the signs of these variables become positive and significant as we can see.

$$grgdp_{it} = 0.0034053inv + 0.0034703 aapgr + 0.9999967hc - 0.0021678 gdppccp + \dots$$

I conclude that high volatility is negatively associated with investment and human capital (unemployment increases) in 21-OECD sample and school dropouts in the first sample.

Testing the robustness of country-specific control for growth volatility

The question here is: does the inclusion of different country-specific control variables affect the nature of the relationships tested above? In order to investigate that, we are going to extract all the control variables which were statistically significant in volatility regression through time and countries (countries) and see the impact of these variables in new time and country-fixed effects models. This is done by the introduction of dummy variables for each country. At this end, we estimate the country-specific forecasting equations for government-spending growth as follows:

$Govexp = f(\text{two lags of the log level of GDP per capita, two lags of the log level of government spending per capita, a quadratic time trend, four dummy variables and a constant term})$

Then by testing the relationship between the variances of the innovations in the growth equations and the squared forecast residuals of the government spending equation, we will obtain the

measure of volatility which depends on time and countries. It is therefore easy to be definitely fixed on the sign of the relation that links volatility to growth.

The equations estimated are:

$$grgdp_{it} = \lambda vol_i + \theta X_{it} + \varepsilon_{it} \quad (1a)$$

$$\varepsilon_{it} \sim N(0, \sigma^2_{it}) \quad vol^2_{it} = a_0 + a_1 \hat{u}^2_{it} \quad (1b)$$

$grgdp_{it}$: the growth rate of output, vol_i : the standard deviation of residuals, X ; the vector of control variables and \hat{u}^2_{it} : the square of estimated residual for country i in period t from the government –spending forecasting equations.

The regression results are presented in tables 4a-4h.

This regression gives us the forecast residuals of government spending. Then by regressing the variances of the innovations in growth on the squared forecast residuals of the government-spending (i.e $vol^2_{it} = a_0 + a_1 \hat{u}^2_{it}$), we will obtain the measure of volatility as a function of both time and countries if the relationship estimated is statistically significant. The next and final step is to test if we have a positive or negative relationship between growth and volatility by introducing in other regressions time and countries fixed effects.

The estimation of the following equation $vol^2_{it} = a_0 + a_1 \hat{u}^2_{it}$ gives:

We see that the relationship is negative but the coefficient is not strictly different from zero. If we consider that this relationship exists, we have the measure of volatility that depends on time and countries. Then our final regressions should show the panel variation in volatility.

These regressions in tables 4c and 4d show that volatility is negatively linked to output growth and variances of the growth innovations are related to the squared innovations in government spending. But the relationship between government expenditures and the government volatility of output is positive.

These regressions confirm the previous negative relationship between growth and volatility with variables statistically significant. Thus, the presence of country or time fixed effects does not change the nature and the robustness of the relationship between the two key variables in this study (growth and volatility).

In this case, we can see that international trade elasticity after a production possibilities frontier movement (e_i) determines the positive sign of the relationship between growth and government expenditure volatility ($e_i - 1 < 0$). See table 4a (bis). $d2_gdppccp$ or international trade elasticity (e_i) is positively correlated to the movements' trend of national PPF defined by ($inv, h_c, aapgr, innovar$).

Second component: Intergenerational trade empirical evidence

Test with a simple statistical data analysis

We have to apply the conclusions of the pure theory of international trade (presence of comparative advantages or proportions of factors, the international tradable goods levelling out of prices, trade earnings) to a model of intergenerational free trade.

Computing intergenerational levelling out of prices of goods and factors through Statistical Data Analysis

Let us consider 22 African and European countries and two kinds of variables (natural resources and unnatural resources). The natural resources variables are 1) Arable surface area (SUPTA). 2) Wooded surface area (SUPTB). 3) Resources of renewable water. 4) Mineral resources (CRESMIN). Unnatural resources variables are represented by 1) GNP/capita (PIBRT). 2) Urbanization rate (TUR). 3) Green House Effect Emissions (INDICE SERR). 4) Pure water consumption (WATERCONSUMP). 5) Inhabitants number per physician (NH/M). 6) Life Expectancy at birth (ESPER). 7) The scientific diploma (ND). 8) A number of years of education (NAE). 9) Scientific and technicians number (NS).

Results and discussion

On the components matrix and on coefficients of components matrix. Axis 1 represents the unnatural resources supply whereas Axis 2 is the supplied measure of natural resources.

Let make two generations coordinates differentiation on one of the axes (Switzerland₁₈₈₀ and Switzerland₁₉₉₀). We obtain unnatural resources quantity exported by Switzerland₁₈₈₀ (axis 1). The

coordinates differentiation on axis 2 for Switzerland₁₈₈₀ and Switzerland₁₉₉₀ gives the number of natural resources imported by Switzerland₁₈₈₀. It now comes to evidence that Switzerland₁₈₈₀ has exported unnatural resources to Switzerland₁₉₉₀ and imported natural resources from Switzerland₁₉₉₀. This means that Switzerland₁₉₉₀ is ready to accept that its part of natural resources can be used by Switzerland₁₈₈₀ generation and to receive in return the rights to unnatural resources generated by Switzerland₁₈₈₀ to be supplied in 1990.

This exchange in order to correspond to an optimal growth should be a dual program cost minimization under production constraints (see relation 12) or a primal program utility maximization under cost constraint relatively to (10) and (11).

According to (14-16).one should choose a point on the isoquant associated with the lowest possible iso-cost curb. That is the tangency condition. This point of tangency determines intergenerational equilibrium prices. To calculate these prices one can write :

$$Pm_K(K^*, R^*)\Delta + Pm_R(K^*, R^*)\Delta = 0 \quad (44)$$

This relation expresses the fact that on a factorial component any factor of production negative variation must be integrally compensated by a positive variation on the other factorial component in order to maintain the same level of production like a variation of $(\Delta K, \Delta R)$. In (45) replacing the marginal productivity by factors prices it comes: $w\Delta K + r\Delta R = 0$ From what we extract :

$$\frac{\Delta R}{\Delta K} = -\frac{w}{r} = \frac{Pm_K(K^*, R^*)}{Pm_R(K^*, R^*)} = TSR(K^*, R^*) \quad (45)$$

The technical substitution rate is determined by the exportation/importation ratio (the ratio of factors price).

According to what precedes Switzerland₁₈₈₀ has exported towards Switzerland₁₉₉₀ and imported natural resources from Switzerland₁₉₉₀. The ratio of export and import determines the intergenerational technical rate of substitution between the two factors and the equilibrium prices. On the factorial axis we have :

$$= 1.21418 - (-0.28219) = 1.49637 \text{ for axis 1}$$

$$R = 0.00123 - (-1.07981) = 1.088104 \quad \text{for axis 2}$$

$$\frac{R}{K} = \frac{1.088104}{1.49637} = 0.7271624$$

And as $\frac{\Delta R}{\Delta K} = -\frac{w}{r} = -0.7271624$ for which the absolute value represents the intergenerational unnatural resources price in terms of natural factors.

The intergenerational unnatural resources price in terms of natural resources should rule both in Switzerland₁₈₈₀ and Switzerland₁₉₉₀ in order to enable inter-intergenerational optimal growth. This is intergenerational factors and goods prices levelling out.

The autarky prices of the generations are calculated as follows: $(\frac{\Delta R}{\Delta K})_{1880}$ gives the autarky price of Switzerland₁₈₈₀ whereas $(\frac{\Delta R}{\Delta K})_{1990}$ determines Switzerland₁₉₉₀ autarky prices. Their values are respectively 3.82653531 and 0.00101303.

As intergenerational trade equilibrium price is within autarky interval of prices we deduce on the one hand that this kind of exchange is profitable to the generations and. on the other hand that the intergenerational prices levelling out is possible and should cause optimal growth through all the generations. This conclusion should hold only if each generation is realizing its international maximization of the utility which depends on international free trade.

The empirical evidence with Edgeworth box

Let consider three generations of France of 25 years: 1965- 1990 or G_1 . 1990 – 2015 or G_2 and 2015-2040 or G_3 . Each of the three generations produces and trade two factors of production: natural and unnatural resources.

We have the following factors of production and exchange description:

Suppose that the current factors of production are exchanged against \future factors of production between the generations. France's intergenerational tradable resources could be determined as follows:

G_2 produces 60 000 billion Euros of GNP value in France in 2015. This GNP is divided into 40 000 billions Euros of natural resources and 20 000 billions Euros of unnatural resources

Exchange description:

Between G_2 and G_3 : G_2 supplies €60 000 billions of unnatural resources to G_3 and receive €40 000 billions of natural resources

Between G_1 and G_2 : G_1 supplies €20 000 billions of unnatural resources to G_2 and receives €20 000 billions of natural resources.

France intergenerational factors of production trade equilibrium should be determined as follows : Generation G_1 , with its 54 million of inhabitants has a comparative advantage in the production of maize because of the abundance of natural resources; In trading unnatural resources for current natural resources in return we indirectly exchange wheat for DVD more easy to produce with G_2 generation because of the future abundance of unnatural resources. Social goods (health, freedom, unemployment, education, inflation, knowledge and technology, GNP per capita capital) and the stock of unnatural resources legated by G_1 to G_2 are determined by material and immaterial investment that are represented the intermediate consumptions in terms of natural resources.

As France's stationary population should be 61 million in 2000, the stationary generation with a high density of population will have less natural resources but more unnatural resources to produce complex goods which are difficult to produce now.

This optimal management of natural and unnatural resources should induce efficiency in the management of consumable goods (actual and future) and should assure the intergenerational universe sustainability.

Cross-generation volatility evidence

For the second component (how changes in production factors supply affect intergenerational trade), I will first consider France through five generations of 50 years with 10 witness generations. In the second case, I mix France five generations with the remaining of 124 countries considered as current generations. For the results, see table 5a and table 5b. The relationship between growth and volatility, under overlapping generations hypothesis, is positive, confirming Mirman conclusion “ if there is a precautionary motive for saving, then higher volatility should lead to a higher saving rate, and hence a higher investment rate which is positively linked to growth”. But, in this case, generation PPF movements' trend is indeterminate, because two control variables are positively linked to growth rate and two other control variables are negatively linked to the growth rate (table 5a bis). It is possible that the relationship between growth and cross-generation volatility would be positive if I consider control variables with their weight (t-stat). In that case, we should expect to meet very often over-optimal growth than suboptimal growth because the first generations tend to mortgage the capacities of future generations (absence of intergenerational levelling out of prices of goods and factors).

The model: Evidence on multidimensional suboptimal trade and the sign of the link between growth and volatility

In order to study the relationship between growth and volatility in the context of multidimensional trade, I will follow three steps: 1) In the first case, I consider France through five generations of 50 years with 10 witness generations; 2) In the second case I mix France five generations with the remaining of 124 countries; 3) I will mix France five generations with the remaining of 124 countries and observe the sign of the relationship between growth rate and growth volatility and with control variables.

This first step shows that the intergenerational relationship between mean growth and volatility is positive but not statistically significant.

1. The second step: mixing France five generations with current countries corresponds to the reality of macrodynamic trade. Each country is currently trading both intergenerationally and internationally (Fig 1). The results of that regression are presented in the following table.

In that case, the relation is negative but not statistically significant.

2. The third step

We mix five generations in France with the remaining of 124 countries and observe the relationship between growth rate and growth volatility with the control variables. The regression results are in tables 5a-5c.

In this case, the relationship is negative and statistically significant and closely linked to the neoclassical theory. In this model we can see that **multidimensional trade elasticity after a production possibilities frontier movement (ϵ_i) determines the negative sign of the relationship between growth and volatility ($\epsilon_i - 1 < 0$). See table 5a (bis). Grdp or multidimensional trade elasticity (ϵ_i) is negatively correlated to the movements' trend of nation and generation PPF defined by (gdppccp, inv, h_c, aapgr).**

6- CONCLUSIONS

This paper has reported on a larger scale econometric study of the sign of the relationship between average growth and growth volatility of GDP per capita as the results of the disturbance to production possibilities' interactions with intergenerational free trade, international free, trade and the multidimensional trade. In the equilibrium, the sign of the relation between endogenous growth and business cycles volatility is linked to the movements (left or right side) of production possibilities; some countries are under the optimal choice side and others are above this critical point of natural and unnatural resources optimal management. Tests have focused on cross-generation and cross-country evidence on the link between growth and volatility. If production possibilities frontier movements' trend is in the left side (lower values of Levine and Renelt control variables) countries with a higher standard deviation of growth shall have their growth adversely affected if at the same time they loose their comparative advantages. The sign is positive in other cases. Thus international trade elasticity after a production possibilities frontier movement (ϵ_i) determines the sign of the relationship between growth and volatility. If $\epsilon_i - 1 < 0$, the sign is negative and positive if $\epsilon_i - 1 > 0$.

With regards to theory, a new optimal growth mechanism (based on multidimensional trade) has been presented. The stability (volatility) is the consequence of intergenerational and international interdependencies' optimality (inefficiency). Disturbance to sustainable growth mechanism is the main factor of economic volatility. The study concludes that the sustainable growth generated through the optimal growth is a general dynamic Walrasian equilibrium or the Paretian optimum. Thus, the volatility is the consequence of the disturbance to production possibilities or the disturbance to the sustainable development process.

In equilibrium, the negativity or the positivity of the relationship between endogenous growth and business cycles volatility is linked to the relation in instability function on the Edgeworth box. Some countries may be under the optimal choice side and others on above this critical point of natural and unnatural resources optimal management.

Another limitation of endogenous growth literature is that it neglects the implications of the interactions of the main economic process described in the multidimensional trade for economic stability. The rhythm of growth processes developed in these models can be very different from one period to another and the orthogonally of these processes can lead to explosive states in terms of economic volatility.

In this sense, our models, on the one hand, help to understand the negativity or the positivity of the relationship between growth and volatility and on the other hand, our fourth model appears as an explanation of the current economic volatility helps to prevent the great disturbances in economic potential stability and growth.

According to the neoclassical model, economic stability is closely linked to optimal growth. In a world characterized by strong and sensitive economic interdependencies, instability becomes permanent if each disequilibrium in a country or in a generation is easily transmitted to other economies or generations. By the same way, the general equilibrium determined by the interdependencies optimality (resources optimal allocation through generations and nations) is the main factor of the stability. It becomes now more and more clear that all the choices in the world are intergenerationally and internationally interdependent and their impact on economic volatility is evident. The decisions in a given country/generation affect the stability of other countries/generations. Hence the optimal growth and the stability occur only when the economy is on its frontier of the production possibilities or is in general equilibrium. The general equilibrium is the expression of the intergenerational and international interdependencies optimality. This situation which is the natural characteristic of the economy means that all the advantageous exchanges have been done and all the economic agents are completely satisfied (Paretian equilibrium criterion).

It comes that the international equilibrium even though cannot be solely obtained (without intergenerational condition) because of the interferences are the result of the global optimal resources allocation. But it is important to say that this condition cannot be realized in a limited country because each resource's productivity increases with its free circulation in the whole world. Similarly, the macro dynamic equilibrium depends on both intergenerational and international resources optimal allocation.

According to these conditions, an economy could not reach optimal growth and the stability without taking into account the past, current and future generations' interests. This analysis shows two key elements of sustainability: the international and intergenerational equilibrium. The two elements lead to sustainable development or general equilibrium.

Ethical Approval: NA

Consent: NA

REFERENCES

1. Acemoglu, D., Simon J. and Robinson J., , 2004. Institutions as the fundamental cause of Long-run growth, NBER working paper 10481.
2. Acemoglu, D., Simon J. Robinson J. and Taicharoen Y., 2003. Institutional causes, macroeconomic symptoms: volatility, crises and growth, Journal of Monetary Economics.
3. Afman, M., Chappin, E., Jager, W., Dijkema, G., 2010. Agent-based model of transitions in consumer lighting. In: Proceedings of 3rd World Congress on Social Simulation, Kassel, Germany
4. Aghion, Philippe, Philippe Bacchetta, Romain Ranciere and Kenneth Rogof, 2006. Exchange Rate Volatility and Productivity Growth, the Role of Financial development", mimeo.
5. Aghion, Philippe, Georges-Marios Angeletos, Abhijit Banerjee and Kalina Manova, 2005. "Volatility and Growth: Credit Constraints and Productivity Enhancing Investment", NBER Working Paper 11349
6. Aghion, Philippe and Abhijit Banerjee, 2005. Volatility and Growth, Oxford University Press.
7. Aghion P. and G. Saint-Paul, 1998a. On the virtue of bad times: an analysis of the interaction between economic fluctuations and productivity growth. Macroeconomic Dynamics 2, 322-344.
8. Aghion, P. and G. Saint-Paul, 1998b. Uncovering some casual relationships between productivity growth and the structure of economic fluctuations: a Tentative survey, Labor 12, 279-303.
9. Aizenman Joshua and Marion Nancy, 1993. "Policy uncertainty persistence and Growth", Review of international Economics. jan I(2). pp. 145-63.
10. Alesina Alberto; Ozler Sule; Roubini Nouriel and Swagel Phillip, sept 1992. "Political Instability and Economic growth", National Bureau of Economic Research (Cambridge, MA)
11. Allais M., 1974. Economie et Intérêt, Paris, Imprimerie nationale
12. Arrow, K.J., 1962. The economic implications of learning by doing. Review of Economic Studies 29, 155-173.
13. Azariadis C. & Drazen, A., mai 1990. Threshold externalities in economic development, Quarterly journal of Economics
14. Bajona, C. and Kehoe T. J. , 2006. "Demographics in Dynamic Heckscher-Ohlin Models: Overlapping Generations versus Infinitely Lived Consumers" Journal of economics, 105, p.501-526, Federal Reserve Bank of Minneapolis
15. Barro Robert, J. May 1991. "Economic Growth in a cross-section of countries", Quarterly journal of Economics
16. Bernanke Ben S., 1983 ". Irreversibility, Uncertainty and Cyclical Investment", Quarterly Journal of Economics
17. Black Fischer , 1987. Business cycles and equilibrium", Cambridge, MA: Blackwell,
18. Blackburn, K., 1999. Can stabilization policy reduce long-run growth? Economic Journal 109, 67-77.
19. Blackburn, K. and R. Galindev, 2003. Growth, volatility and learning, Economics Letters 79, 417-421.
20. Blackburn, K. and A. Pelloni, 2004. On the relationship between growth and Volatility in learning-by-doing models. Economics Letters, forthcoming.

- 1193 21. Bean, C., 1990. Endogenous growth and the procyclical behavior of Productivity. *European*
1194 *Economic Review* 34, 355-363.
- 1195 22. Caballero, R. and M. Hammour, 1994. The cleansing effect of recessions. *American Economic*
1196 *Review* 84, 350-368.
- 1197 23. Caporale, T. and B. McKiernan, 1996. The relationship between output variability and growth:
1198 evidence from post-war CHINA data. *Scottish Journal of Political Economy* 43, 229-236.
- 1199 24. Cass, David, 1965. "Optimum Growth in an Aggregative Model of Capital Accumulation".
1200 *Review of Economic Studies*. **32** (3): 233–240. JSTOR 2295827.
- 1201 25. Chappin, E.J., Dijkema, G.P., van Dam, K.H., Lukszo, Z., 2007. Modeling strategic and
1202 operational decision-making e an agent-based model of electricity producers. In: *The 2007*
1203 *European Simulation and Modelling Conference*, St. Julians, Malta.
- 1204 26. Choi, J., Im, N., Park, J., 2012. Agent based model for estimating hybrid electric vehicle market:
1205 the case of Korea. In: *Energy, Climate and Environment Modeling and Simulation 2012*, San
1206 Diego (CA), United States, vol. 44, pp. 26e33.
- 1207 27. Dawson, J.W. and Stephenson, E.F., 1997. The link between volatility and growth: evidence
1208 from the States. *Economics Letters* 55, 365-369.
- 1209 28. De Hek, P.A., 2000. Endogenous growth under uncertainty. *International Economic Review* 40,
1210 727-744.
- 1211 29. Deissenberg, C., van der Hoog, S., Dawid, H., 2008. EURACE: a massively parallel agent-
1212 based model of the European economy. *Appl. Math. Comput.* 204 (2), 541e552.
- 1213 30. Desmarchelier, B., Djellal, F., Gallouj, F., 2013. Environmental policies and ecoinnovations by
1214 service firms: an agent-based model. *Technol. Forecast. Soc. Change* 80 (7), 1395e1408.
- 1215 31. Devisscher, T., Mont, O., 2008. An analysis of a product service system in Bolivia: coffee in
1216 Yungas. *Int. J. Innov. Sustain. Dev.* 3, 262e284.
- 1217 32. Diamond, P., 1965. National debt in neoclassical growth model, *American Economic Review*,
1218 55, p.1126-1150
- 1219 33. Dosi, G., Fagiolo, G., Roventini, A., 2009. The Microfoundations of Business Cycles: an
1220 Evolutionary, Multi-agent Model. In: *Schumpeterian Perspectives on Innovation, Competition*
1221 *and Growth*. Springer, pp. 161e180.
- 1222 34. Drazen, A., 2000. *Political Economy in Macroeconomics*, Princeton, Princeton University Press
- 1223 35. Edgeweblime, K., 2012. "Développement Durable, Maximisation de l'utilité et Croissance
1224 Optimale" In *FMI en Afrique*, L'Harmattan, pp. 109-133
- 1225 36. Eppstein, M.J., Grover, D.K., Marshall, J.S., Rizzo, D.M., 2011. An agent-based model to study
1226 market penetration of plug-in hybrid electric vehicles. *Energy Policy* 39 (6), 3789e3802.
- 1227 37. European Commission, 2012. Sustainable Growth - for a Resource Efficient, Greener and More
1228 Competitive Economy. [http://ec.europa.eu/europe2020/europe2020-in-a-](http://ec.europa.eu/europe2020/europe2020-in-a-nutshell/priorities/sustainable-growth/index_en.htm)
1229 [nutshell/priorities/sustainable-growth/index_en.htm](http://ec.europa.eu/europe2020/europe2020-in-a-nutshell/priorities/sustainable-growth/index_en.htm).
- 1230 38. European Environment Agency, 2012. *Material Resources and Waste e 2012 Update*
1231 *(Technical report, Copenhagen)*.
- 1232 39. Eurostat, 2011. *Sustainable Development in the European Union: 2011 Monitoring Report of*
1233 *the EU Sustainable Development Strategy*. Publications Office of the European Union,
1234 Luxembourg.
- 1235 40. Fatas, A., 2000. Endogenous growth and stochastic trends. *Journal of Monetary Economics* 45,
1236 107-128.
- 1237 41. Givoni, M., Macmillen, J., Banister, D., Feitelson, E., 2013. From policy measures to policy
1238 packages. *Transp. Rev.* 33 (1), 1e20.
- 1239 42. Grier, K.B., Tullock, G., 1989. An empirical analysis of cross-national economic growth,, 1951-
1240 1980. *Journal of Monetary Economics* 24, 259-276.
- 1241 43. Grier, K.B. and M.J. Perry, 2000. The effect of the real and nominal uncertainty on the inflation
1242 and output growth: some GARCH-M evidence. *Journal of Applied Econometrics* 24, 259-276.
- 1243 44. Hansen, C.D., 1985. Indivisible labor and the business cycle. *Journal of Monetary Economics*
1244 16, 306-328.
- 1245 45. Jackson, T., 2011. *Prosperity Without Growth: Economics for a Finite Planet*. Routledge.
- 1246 46. Janssen, M.A., Jager, W., 2002. Stimulating diffusion of green products. *J. Evol. Econ.* 12 (3),
1247 283e306.
- 1248 47. Jones, L.E., R.E. Manuelli and E. Stacchetti, 1999. Technology (and policy) Shocks in models
1249 of endogenous growth. NBER Working Paper No. 7093.
- 1250 48. Kiesling, E., Günther, M., Stummer, C., Wakolbinger, L.M., 2012. Agent-based simulation of
1251 innovation diffusion: a review. *Central Eur. J. Operations Res.* 20 (2), 183e230.

- 1252 49. King, R., C.I. Plosser and S. Rebelo, 1988. Production, growth and business cycles II: new
1253 directions. *Journal of Monetary Economics* 21, 195-232.
- 1254 50. Kneller, R. and G. Young, 2001. Business cycle volatility, uncertainty and long run Growth.
1255 *Manchester School* 69, 534-552.
- 1256 51. Koopmans, T. C. 1965. "On the Concept of Optimal Economic Growth". *The Economic*
1257 *Approach to development Planning*. Chicago: Rand McNally. pp. 225-287.
- 1258 52. Kormendi, R. and P. Meguire, 1985. Macroeconomic determinants of growth: cross-country
1259 evidence. *Journal of Monetary Economics* 16, 141-163.
- 1260 53. Kydland, Flynn and Prescott, Edwards, nov. 1982. "Time to Build and Aggregate Fluctuations",
1261 *Econometrica*, 50(6), pp. 1345-70
- 1262 54. Lempert, R., 2002. Agent-based modeling as organizational and public policy simulators. *Proc.*
1263 *Natl. Acad. Sci.* 99 (Suppl. 3), 7195e7196.
- 1264 55. Lengnick, M., 2013. Agent-based macroeconomics: a baseline model. *J. Econ. Behav. Organ.*
1265 86, 102e120.
- 1266 56. Lensink, R., Bo H.M., Sterken E., 1999. Does uncertainty affect economic growth? An
1267 Economic Analysis. *Weltwirtschaftliches Archive* 135, No. 3, 279-396.
- 1268 57. Levine Ross and Renelt David sept. 1992. "A sensitivity Analysis of Cross-Country Growth
1269 regressions" *American Economic Review*
- 1270 58. Long John and Ploser Charles , 1983. "Real Business Cycles" *Journal of Political Economy*
- 1271 59. Lucas Robert, 1987 . "Models of business cycles"Oxford; Blackwell
- 1272 60. Martin, P. and C.A. Rogers, 1997. Stabilization policy, learning-by-doing and economic growth.
1273 *Oxford Economic Papers* 49, 152-166.
- 1274 61. Martin, P. and C.A. Rogers, 2000. Long-term growth and short-term economic instability.
1275 *European Economic Review* 44, 359-381.
- 1276 62. Michel P., 2001. « Générations imbriquées, in *Dictionnaire des sciences économiques*, PUF,
1277 p.431.
- 1278 63. Michel P., 1993 .« Le modèle à générations imbriquées : un instrument d'analyse
1279 macroéconomique » *Revue d'Economie Politique*, 103 p.191-220
- 1280 64. Mirman, L. 1971. « Uncertainty and optimal Consumption Decisions », *Econometrica*, January,
1281 39(1), pp. 179-85
- 1282 65. Nelson Charles and Plosser Charles? 1982. "Trends and Random Walks in Macroeconomic
1283 time series : Some Evidence and Implications" *Journal of Monetary Economics* sept.,
- 1284 66. Pelloni, A., 1997. Nominal shocks, endogenous growth and the business cycle. *Economic*
1285 *Journal* 107, 467-474
- 1286 67. Pyndick, Robert S., sept. 1991. "Irreversibility, Uncertainty, and Investment, *Journal of*
1287 *Economic Literature*
- 1288 68. Ramey Garey and Ramey Valerie A., 1991. "Technolohy Commitment and the cost of
1289 Economic Fluctuations". *National Bureau of Economic research* (Cambridge MA)
- 1290 69. Ramey Garey and Ramey Valerie A. 1991. 'Cross- Country Evidence on the link between
1291 volatility and Growth", *American Economic Review*
- 1292 70. Ramsey, Frank P. 1928. "A Mathematical Theory of Saving". *Economic Journal*. **38** (152): 543–
1293 559. *JSTOR* 2224098. .
- 1294 71. Samuelson P.« An exact consumption-loan model of interest with or without the social
1295 contrivance of money », *Journal of political Economy*
- 1296 72. Rogerson, R., 1988. Indivisible labor, lotteries and equilibrium. *Journal of Monetary Economics*
1297 21, 3-16.
- 1298 73. Romer, R.M., 1986. Increasing returns and long-run growth. *Journal of Political Economy* 94,
1299 1002-10037.
- 1300 74. Rothschild, M.R. and J.E. Stiglitz, 1970. Increasing risk I: a definition. *Journal of Economic*
1301 *Theory* 2, 225-243.
- 1302 75. Rothschild, M.R. and J.E. Stiglitz, 1971. Increasing risk II: its economic Consequences. *Journal*
1303 *of Economic Theory* 3, 66-84.
- 1304 76. Schumpeter, J., 1942. *Capitalism, Socialism and Democracy*. Harper and Brothers: New York.
- 1305 77. Sen A.K . 1970 and 1982 . « Collective choice ».
- 1306 78. Smith, T., 1996. Cyclical uncertainty, precautionary saving and economic growth. *Economica*
1307 63, 477-494.
- 1308 79. Speight, A.E.H., 1999. CHINA output variability and growth: some further evidence. *Scottish*
1309 *Journal of Political Economy* 46, 175-181.

- 1310 80. Stadler, G.W., 1990. Business cycle models with endogenous technology. *American Economic*
1311 *Review* 80, 764-778.
- 1312 81. van der Veen, R.A.C., Kisjes, K.H. , Nikolic, I. 2017. Exploring policy impacts
1313 for servicing in product-based markets: A generic agent-based model
- 1314 82. Vayanos D. 1998. « Transaction costs and asset prices: a dynamic equilibrium model », *Review*
1315 *of Financial Studies* 11, 1-58
- 1316 83. Vayanos D. « A strategic trading and welfare in a dynamic market », *Review of Financial*
1317 *Studies* 66, 219-54
- 1318 84. Von Neuman, J., 1945. "A model of General Economic Equilibrium", *Review of Economic*
1319 *Studies*, 13, 1-9.
- 1320 85. Wang J; 1993. « A model of inter-intergenerational asset prices under asymmetric
1321 information », *Review of Econometric Studies*, 60, 249-82
- 1322 86. Young, A. 1991. "Learning by Doing and the Dynamic Effects of international trade", *Quarterly*
1323 *Journal of Economics*, 106, 369-405.
- 1324 87. Zhang, T., Zhang, D., 2007. Agent-based simulation of consumer purchase decisionmaking and
1325 the decoy effect. *J. Bus. Res.* 60 (8), 912e922.
- 1326 88. Luo, Y., Nie, J., & Young, E. (2016). *The Negative Growth-Volatility*
1327 *Relationship and the Gains from Diversification*. mimeo.
- 1328 89. Crain, W. M. (2003). *Volatile states: Institutions, policy, and the performance of*
1329 *American state economies*. University of Michigan Press.
- 1330 90. Das, Suman & Sinha Roy, Saikat. (2016). Exchange Rate Return Co-
1331 movements and Volatility Spillover: The case of Emerging Market Economies.
- 1332 91. Bianconi, M., MacLachlan, S., & Sammon, M. (2015). Implied volatility and the
1333 risk-free rate of return in options markets. *The North American Journal of*
1334 *Economics and Finance*, 31, 1-26.
- 1335 92. McAleer, M., & Medeiros, M. C. (2008). Realized volatility: A
1336 review. *Econometric Reviews*, 27(1-3), 10-45.
- 1337 93. Caselli, Francesco & John Coleman II, Wilbur. (2000). The World Technology
1338 Frontier. *American Economic Review*. 96. 10.1257/aer.96.3.499.

Table 1a
Relationship between mean growth and volatility,
with Levin-Renelt control variables (see Figures and tables file)



Table 1b : The sign of the link between growth and volatility with a sample 108 countries

Variable	Definition	Coefficient	T-stats	Std. Dev.	[95%Conf.Interval] min	[95%Conf.Interval]max
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Vol	- Ecart-type du taux de croissance (fluctuations)	0.039	0.810	0.048	-0.0568	0.136
Intercept	Constante	0.012	3.54	0.003	0.0053	0.0188
F(1, 106) = 0,66 R- squared = 0,0204 R-adjusted =0,003						

Table 1c : The sign of the link between mean growth and volatility with a sample 108 countries (endonnées de panel)

Variable	Definition	Coefficient	T- Stats	Std. Dev.	[95%Conf.Interval] min	[95%Conf.Interval]max
Vol	- Ecart-type du taux de croissance (fluctuations)	-0.544	-56.55	0.0096	-0.5629	-0.5252
Intercept	Constante	0.0342	33.18	0.0010	0.03225	0.0362
Log likelihood=4493,291						

Tableau 1d : The sign of the link between growth and volatility with a sample 25 developed countries

Variable	Definition	Coefficient	T-stats	Std. Dev.	[95%Conf.Interval] min	[95%Conf.Interval]max
Vol	- Ecart-type du taux de croissance (fluctuations)	-0.0048	-0.69	0.0069	-0.0192	0.0095
Intercept	Constante	0.0274	2.84	0.0096	0.00742	0.0473
F(1, 106) = 0,48 R- squared = 0,0204 AdjR-squared=0,0222 Log likelihood=4493,291						

2 step étape ; introduction of Levin and Renelt. control variables

The models to test are in the following form:

$$grgdpi_t = \lambda vol_t + \theta X_{it} + \varepsilon_{it} \quad (1a)$$

$$\varepsilon_{it} \sim N(0, \sigma^2) \quad (1b)$$

$$i = 1, \dots, T \quad t = 1, \dots, T$$

grgdpi average annual growth in GDP / head for country i and year t (obtained in taking the differences of logarithm).

σ^2 : is the standard deviation of the residues, ε_{it} ; ε_{it} is the standard deviation of growth obtained from predicted values based on X_{it} variables. X_{it} variables differ from one country to another

, From one year to another. X_{it} : is the vector of the control variables Θ : is the vector of the coefficients common to the countries of the sample; λ denotes the relationship between growth and volatility and is the most important parameter in this specification. The vector of control variables, X proposed by R. Levine and R. Renelt (1992) are the most important variables for the analysis of the growth of the countries. These variables are defined as follows: 1) "inv" Share of average investment in GDP; 2) (gdppcpc): the logarithm of the GNP / initial head (at the beginning of the period); 3) hc or hc-residue when hc is purged of the difference between observed and predicted values obtained using a partial regression of hc on other control variables; aapgr: average growth rate of the population. In the sample of 108 countries, human capital is the average number of years of schooling of individuals in the population aged 25 and over. But in OECD countries, human capital is the secondary enrollment rate as a percentage of the relevant age group. For regressions, we will use the maximum likelihood method on panel data. The number of observations for the sample at 108 countries is 3240 and 630 for the sample of 25 developed countries.

Table 2 : Relationship between the mean growth and the volatility with Levin-Renelt control variables.

Table 2a : The sample of 108 countries

Variable	Definition	Coefficient	T-Stats	Std. Dev.	[95%Conf.Interval] min	[95%Conf.Interval]max
Vol	- Fluctuations de la croissance du PNB/tête -PNB/tête initial	0.6189	-76.789	0.0080	-0.6347	-0.6032
Gdpccp	-part des investissements dans le PNB	-0.003847	-9.69	0.00039	-0.0046	-0.003068
Inv	-Taux de croissance annuel moyen de la population	0.0012	32.09	0.00038	0.001151	0.0013012
aapgr	-Capital humain initial	-0.002511	-9.3	0.00027	0.00304	-0.001982
hc_residu	-Constante	0.003233	1.85	0.00017	-0.00019	0.00666
Intercept		0.0342	15.78	0.0021	0.029	0.03846
Log likelihood=4624.73 Prob>chi2(5)= 0,000 Prob>chi2(5)= 0,000						

Table 2b : The sample of 25 countries

Variable	Definition	Coefficient	T-Stats	Std. Dev.	[95%Conf.Interval] min	[95%Conf.Interval]max
Vol	- Fluctuations de la croissance du PNB/tête -PNB/tête initial	-0.2956	-23.97	0.01233	-0.3197	-0.2714
Gdpccp	-part des investissements dans le PNB	-0.03305	-4.77	0.006932	-0.04664	-0.01946
Inv	-Taux de croissance annuel moyen de la population	0.00027	0.64	0.00042	0.0005637	0.0011147
aapgr	-Capital humain initial	-0.03507	-18.05	0.00194	0.03887	-0.031264
hc_residu	-Constante	0.04960	11.20	0.004429	-0.04092	0.05828
Intercept		0.1069	3.47	0.03077	0.04658	0.167234
Log likelihood=677,85						

Step 3: Test of the relationship between innovation variance and growth

In order to examine the stochastic part of the relationship between growth and investment, we take the above model while changing the content of the control variables. Thus, we have two types of variables: the measure of variables at the beginning of the period and the predictors X. The variables to be taken into account in this new specification are:

- Variables measured at the beginning of the period 1) Inv: the average share of investment in GDP at the beginning of the period; - aapgr: the average annual growth rate of the population at the beginning of the period.

- Predicted variables:

1) GDP per capita delayed by two periods

2) The trend of time

3) The trend of time squared

4) Four dummy seasonal variables (Q1t, Q2t, Q3t and D0t) whose role is to capture the specific effects. These variables are defined below.

Table 3a: Sample of 21 OECD panel data countries (see book (see Dynamics of trade and volatility)

Table 3b: The 108 country sample and panel data regression (see Dynamics of trade and volatility)

$$grdp_{it} = \beta_0 + \lambda \sigma_t + \beta_1 Gdpcc + re_2 + \beta_2 Gdpcc + \beta_3 Inv + \beta_4 aapgr + \beta_5 Hc + re - 2 + \beta_6 Q1_t + \beta_7 Q2_t + \beta_8 Q3_t + \beta_9 T + \beta_{10} DT1_t + \beta_{11} DT2_t + \beta_{12} DT3_t + \beta_{13} D0_t + \varepsilon_{it}$$

Table 3: Relationship between average growth and volatility of innovations

Step 4: Test the robustness of country-specific control of growth volatility

The question here is: Does the introduction of different countries with specific effects affect the nature of the relationship tested here? In order to make this investigation, we will extract all the control variables that are statistically significant in the regression of volatility in terms of time and country and observe the impact of these variables on the new fixed-effects models in the time and space (country). This is done by introducing dummy variables for each country. To this end, we estimate country-specific equations for growth in government expenditures as follows:

Govexp = f (Log of GDP / head lagged by 2 periods, government expenditure log per capita

Delayed by 2 periods, a quadratic time trend, 4 dummy variables and a constant term)

The equations to be estimated have the following form:

$$grgdp_{it} = \lambda vol_{it} + \theta X_{it} + \varepsilon_{it} \quad (1a)$$

$$\varepsilon_{it} \sim N(0, \sigma^2_{\varepsilon}) \quad vol_{it} = a_0 + a_1 u^2_{it} \quad (1b)$$

Table 3a: 21 OECD country- sample panel regression

Variable	Definition	Coefficient	T- Stats.	Std. Dev.	[95%Conf.Interval] min	[95%Conf.Interval]max
Vol	Ecart type de la croissance (volatilité de la croissance)	-0.2634	-18.49	0.1424	-0.291	-0.235
Gdppccp	-Log du PNB/tête Initial	5.092	19.55	0.2604	4.58	5.60
Inv	-Part de l'investissement moyen dans le PNB	-0.016	-14.83	0.0010	-0.018	-0.013
hc_residu	-Capital moyen initial	-0.00007	0.87	0.00009	-0.0000	0.000
aapgr	-Taux de croissance annuel moyen de la population	0.08225	14.47	0.0056	0.071	0.093
gdplag2	-Log du PNB/tête retardé de 2 périodes	-5.036	-19.59	0.2571	-5.54	-4.53
q1t	-Variables factices saisonnières	-0.1294	-6.81	0.019	-0.166	-0.092
q2t		-0.060	-4.09	0.01469	-0.088	-0.031
q3t	-Trend du temps	0.0645	9.61	0.0067	0.051	0.077
dot	-Trend du temps au carré	-0.0436	-4.07	0.0107	-0.064	-0.022
trend	Constante	-0.0111	-7.74	0.00144	-0.013	-0.083
t-sqrd		0.00088	14.00	0.00006	0.000	0.001
Intercept		-0.1309	-4.19	0.03123	-0.192	-0.069
Log likelihood=-657,57 Prob>chi2= 0,000						

Table 3b: 108-country sample panel regression

Variable	Definition	Coefficient	T-Stats	Std. Dev.	[95%Conf.Interval] min	[95%Conf.Interval]max
Vol	- Fluctuations de la croissance du PNB/tête	-0.0363	-4.35	0.0083	-0.052	-0.019
Gdppccp	-PNB/tête initial					
Inv	-part des investissements dans le PNB	1.263	285.12	0.0044	1.254	1.271
hc_residu	-Capital humain initial	0.0000	2.49	0.0000	0.0000	-0.000
aapgr	-Taux de croissance annuel moyen de la population	-0.0026	-1.81	0.0001	0.0005	0.000
gdplag2	-log du PNB/tête initial retardé de 2 périodes	-1.262	10.35	0.0002	0.0021	0.0032
	-Variables factices saisonnières	-0.0014	-286.02	0.0044	-1.27	-1.25
q1t		0.1222	2.37	0.0000	0.0000	0.0000
q2t		-0.0034	4.05	0.0030	0.0063	0.018
q3t		-0.0028	-1.31	0.0026	-0.0086	0.0017
dot		-0.0012	-2.35	0.0011	-0.0051	-0.000
trend		0.132	-0.89	0.0013	-0.0038	0.0014
t-sqrd	Constante	-0.016	3.5	0.0037	0.0058	0.0206
Intercept		-				
Log likelihood=-6257,18 Prob>chi2= 0,000						

Table 4a : Regression of governmental expense on the the Levin-Renelt control variables (world technology frontier) and dummy control variables

Variable	Definition	Coefficient	T-Stats	Std. Dev.	[95%Conf.Interval] min	[95%Conf.Interval]max
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loggdplag2	- log du PNB/tête initial retardé de 2 périodes	-0.064	-0.47	0.136	-0.3323	0.203
govexplag2	-log du dépensesgouvern /tête initial retardé de 2 périodes	0.957	107.94	0.0088	0.94	0.974
trend	-Trend quadratique du temps	0.223	6.14	0.0364	0.152	0.2955
qlt	-Variables saisonnières factices	-2.684	-3.85	0.697	-4.05	-1.317
q2t	Constante	-0.831	-1.65	0.503	-1.818	0.1559
q3t		0.513	2.15	0.2388	0.045	0.9818
dot		0.75	2.03	0.37	0.0274	1.4788
Intercept		-1.897	1.319	1.319	-4.484	0.6897
Log likelihood=-1517,92						
Variable	Definition	Coefficient	T-Stats	Std. Dev.	[95%Conf.Interval] min	[95%Conf.Interval]max
govexp	-Croissance des dépensesgouvernementales	-0.4083	-50.37	0.0081	-0.424	-0.392
Vol1	-Fluctuations dues aux dépensesgouvernementales	-2.224	-2.45	0.9077	-4.003	-0.445
Gdppccp	-Log -PNB/tête initial -part des investissementsdans le PNB	0.0296	11.21	0.0026	0.0244	0.034
Inv	-Capital humain initial	-0.0023	-4.37	0.00054	-0.0034	-0.0013
h-c	-Taux de croissanceannuelmoyen de la population					
aapgr	-log du PNB/tête initial retardé de 2 périodes	0.0414	2.02	0.0204	0.0012	0.0816
loggdplag2	-Variables factices saisonnières t-sqrd	2.1778	2.41	0.903	0.406	3.949
trend	Constante	-0.0331	-4.17	0.0079	-0.487	-0.0175
t-sqrd	Log likelihood=-775,99 Prob>chi2= 0,000	0.0007	3.18	0.00224	0.0001	0.0011
_Cons		-0.0802	-0.79	0.1020	-0.280	0.1197
Log likelihood=-775,99 Prob>chi2= 0,000						

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The estimation of the following relation $vol^2_{it} = a_0 + a_1 u^2_{it}$ gives:**Table 4b : Regression of the standard deviation of innovations on the square of standard deviation of residuals of the equation of the governmental expensive**

Variable	Definition	Coefficient	T-Stats	Std. Dev.	[95%Conf.Interval] min	[95%Conf.Interval]max
Vol2	Ecart-type des innovations	0.000045	-1.31	0.000034	-0.00001	0.00002
Residu2	Carré des résiduséquation des dépensesgouvernementales	3.2829	2.14	1.5342	0.275	6.289
Random-effct regression	GLS					
R-sq : 0,0026						

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Table 4c: Regression of the rate of governmental expensive (govexp) on the trend of resources (control variable contributing to WTF) and on the dummy variables and on 4 time trend variables and the LOG of GDP of 2 periods lag

Variable	Definition	Coefficient	T-Stats	Std. Dev.	[95%Conf.Interval] min	[95%Conf.Interval]max
Volgov	-Croissance des dépensesgouvernementales	6.484	348.09	0.018	6.44	6.52
Gdppccp	-Fluctuations dues aux dépensesgouvernementales	-48.53	-8.0	6.05	-60.42	-36.64
Inv	-Log -PNB/tête initial	-0.261	-10.62	0.024	-0.31	-0.21
h-c	-part investissementsdans le PNB	-0.057	-22.17	0.0025	-0.062	-0.052
aapgr	-Taux de croissanceannuelmoyen de la population	-5.554	-22.13	0.25	-6.045	-5.06
loggdplag2	-log du PNB/tête initial retardé de 2 périodes	48.125	7.97	6.041	36.284	59.96
trend	-Variables factices saisonnières	-0.1422	-2.24	0.063	-0.266	-0.0177
t-square	-Constante	0.0047	2.70	0.0017	0.0013	0.0082
_Cons		37.75	39.29	0.961	35.87	39.64
Log likelihood=-1824,3 Prob>chi2= 0,000						

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Table 4d : : Regression of the rate of per capita growth on the trend of resources (control variable contributing to WTF) and on the dummy variables and on the fixed effects of countries

Variable	Definition	Coefficient	T-Stats	Std. Dev.	[95%Conf.Interval] min	[95%Conf.Interval]max
grgdp	-Taux de croissance du PNB/tête	-0.4706	-94.26	0.0049	-0.48	-0.460
Vol1	- Ecart-type du taux de croissance	-2.4613	-14.27	0.1724	-2.79	-2.123
Gdppccp	-PNB/tête initial	0.0234	26.64	0.00088	0.0217	0.025
Inv	-part des investissementsdans le PNB	-0.00227	-21.44	0.000106	-0.0024	-0.002
h-c	-Capital humain initial	0.0305	4.69	0.0065	0.0177	0.043
aapgr	-Taux de croissanceannuelmoyen de la population	-0.2364	-6.9	0.0342	-0.3035	-0.169
	-Variables factices saisonnières					
	“					
	“					
q1t		-0.1853	-5.42	0.03418	-0.2523	-0.118
q2t		0.04517	2.61	0.0173	0.1119	0.079
q3t		-0.3703	-1.42	0.02602	-0.088	0.013
dot	- log du PNB/tête initial retardé de 2 périodes	2.4219	14.18	0.1707	2.087	2.756
loggdplag2	Constante					
Intercept		-0.2399	-5.58	0.0429	-0.3241	-0.155
Log likelihood=-739,27 Prob>chi2= 0,000						

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Table 4e: Regression of the rate of governmental expensive (govexp) on the trend of resources (control variable contributing to WTF) and on the dummy variables with countries fixed effects

Variable	Definition	Coefficient	T-Stats	Std. Dev.	[95%Conf.Interval] min	[95%Conf.Interval]max
govexp	-Croissance des dépensesgouvernementales	1.43	26.95	0.0532	1.329	1.5381
Volgov	-Fluctuations dues aux dépensesgouvernementales	0.8312	96.22	0.00863	0.814	0.8482
Govexplag2	-Log des dépensesgouvernementales/tête retardé de 2 périodes	-106.20	-22.64	4.69	-115.40	-97.01
Gdppccp	-PNB/tête initial -part des investissementsdans le PNB	0.01598	0.63	0.0253	-0.0336	0.0656
Inv h-c	-Capital humain initial -Taux de croissanceannuelmoyen de la population	-0.010	-5.39	0.00189	-0.0139	-0.0064
aapgr	-Variables factices saisonnières	-0.528	-4.32	0.116	-0.7572	-0.2988
	“	-0.3201	-0.52	0.6147	-1.5251	0.8848
q1t	- log du PNB/tête initial	-0.0554	-0.10	0.568	-1.169	1.0584
q2t	retardé de 2 périodes	-0.0076	-0.03	0.2931	-0.5822	0.567
q3t	Constante	-0.888	-4.29	0.2071	-1.2949	-0.4827
dot		106.5	22.76	4.6788	97.33	115.67
loggdplag2 _Cons						
Log likelihood=-1465.637 Prob>chi2= 0.000		3.50	3.41	1.028	1.4928	5.524

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Table 4f: Regression of the rate of governmental expensive (govexp) on the trend of resources (control variable contributing to WTF) and on the dummy variables with countries and time fixed effects

Variable	Definition	Coefficient	T-Stats	Std. Dev.	[95%Conf.Interval] min	[95%Conf.Interval]max
govexp	Croissance des dépensesgouvernementales	-0.4633	-62.91	0.0073	-0.4777	-0.4489
Volgov	Fluctuations dues aux dépensesgouvernementales	-0.00024	-14.98	0.00001	-0.0002	-0.0002
Govexplag2	Log des dépensesgouvernementales/tête retardé de 2 périodes	-3.794	-19.65	0.1930	-4.1724	-3.415
Gdppccp	-PNB/tête initial -part des investissementsdans le PNB	0.0219	25.65	0.00085	0.02029	0.02364
Inv	Capital humain initial	-0.00229	-17.17	0.00012	-0.0024	-0.0019
h-c	-Taux de croissanceannuelmoyen de la population	0.04497	9.04	0.00497	0.03521	0.0547
aapgr	Variables factices saisonnières	-0.08047	-1.91	0.0421	-0.1631	-0.2083
	“					
q1t	“	-0.2913	-6.88	0.04231	-0.3742	0.0727

q2t	- log du PNB/tête initial retardé de 2 périodes Constante	0.0306	1.43	0.0214	-0.0114	0.03466
q3t		0.01267	1.13	0.1122	-0.0093	4.1209
dot		3.7438	19.45	0.1924	3.3666	0.0364
loggdplag2		0.03296	18.37	0.00179	0.02944	
_Cons		-0.00113	-23.10	0.00004	-0.0012	-0.0010
Log likelihood=-1465.637 Prob>chi2= 0,000		-0.2373	-4.95	0.04795	-0.3313	-0.1433

Table 4g: Regression of the rate per capita growth on the trend of resources (control variable contributing to WTF) and on the dummy variables with countries and time fixed effects

Variable	Definition	Coefficient	T-Stats	Std. Dev.	[95%Conf.Interval] min	[95%Conf.Interval]max
grgdp	Taux de croissance du revenu/tête	25.32	0.0492	1.1498	1.342	
Vol1	-Ecart-type du taux de croissance(fluctuations)	107.94	0.0079	0.8456	0.8769	
Govexplag2	Log des dépensesgouvernementales/tête	-23.72	5.145	-132.15	-111.98	
Gdppccp	-log PNB/tête Initial	11.87	0.026	-0.0023	0.1009	
Inv	-part de l'investissementmoyendans le PNB	-2.60	0.0021	-0.0096	-0.0013	
h-c	Capital humain initial	-5.50	0.1912	-1.4275	-0.6776	
aapgr	-Taux de croissanceannuelmoyen de la population	-0.12	0.6883	-1.2691	1.4291	
q1t	Variables factices saisonnières	-0.68	0.3048	-0.8051	0.3898	
q2t	„	-0.39	0.2857	-0.6718	0.4483	
q3t	„	23.76	5.1379	112.03	132.17	
dot	-log du PNB/tête initial retardé de 2 périodes	-4.52	0.06774	-0.4387	-0.1732	
loggdplag2		3.3	0.0021	0.0028	0.1122	
trend	Trend du temps	4.39	1.443	3.5109	9.1694	
t-sqrd	Trend du temps au carré					
_Cons	Constante					
Log likelihood=-691.47 Prob>chi2= 0,000						

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Table 4h: Regression of the rate of governmental expensive (govexp) on the trend of resources (control variable contributing to WTF) and on the dummy variables with countries and time fixed effects.

Variable	Definition	Coefficient	T-Stats	Std. Dev.	[95%Conf.Interval] min	[95%Conf.Interval]max
govexp	Croissance des dépensesgouvernementales	1.246	25.32	0.0492	1.1498	1.342
Volgov	Fluctuations dues aux dépensesgouvernementales	0.861	107.94	0.0079	0.8456	0.8769
Govexplag2	Log des dépensesgouvernementales/tête retardé de 2 périodes	-122.07	-23.72	5.145	-132.15	-111.98
Gdppccp	-PNB/tête initial	0.0492	11.87	0.026	-0.0023	0.1009
Inv	-part des investissementsdans le PNB	-0.005	-2.60	0.0021	-0.0096	-0.0013
h-c	Capital humain initial	-1.052	-5.50	0.1912	-1.4275	-0.6776
aapgr	-Taux de croissanceannuelmoyen de la population	-0.0123	-0.01	0.8377	-1.6543	1.6295
q1t	Variables factices saisonnières	0.08000	-0.12	0.6883	-1.2691	1.4291
q2t	“	-0.2076	-0.68	0.3048	-0.8051	0.3898
q3t	“	-0.1117	-0.39	0.2857	-0.6718	0.4483
dot	- log du PNB/tête initial retardé de 2 périodes	122.101	23.76	5.1379	112.03	132.17
loggdplag2	Trend du temps	-0.3060	-4.52	0.06774	-0.4387	-0.1732
trend	Trend du temps au carré	0.007	3.3	0.0021	0.0028	0.1122
t-sqrd	Constante	6.34	4.39	1.443	3.5109	9.1694
_Cons						
Log likelihood=-1465.54						
Prob>chi2= 0,000						

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1st step

Table 5a: Mean growth and growth volatility with a sample of 8 France generations (1800-2000)

Variable	Definition	Coefficient	T-Stats	Std. Dev.	[95%Conf.Interval] min	[95%Conf.Interval]max
Vol	Std Dev. De la vol de la croissance)	0.0125969	0.98	0.012909	-0.012	0.0378
_Cons	Intercept	0.0159509	2.81	0.0056704	0.0048	0.02706
Log likelihood=531.59						
Prob>chi2= 0,000						

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Table 5b: Mean growth and growth volatility with a sample of 8 France generations (1800-2000) with Levin-Renelt control variables

Variable	Definition	Coefficient	T-Stats	Std. Dev.	[95%Conf.Interval] min	[95%Conf.Interval]max
Vol	-Volatilité de la croissance)					

Gdppccp	-log GDP initial par tête	0.02892	1.25	0.02312	-0.0164	0.07424
Inv	-Part de l'investissementmoyendans le GDP	-0.0040	-0.51	0.0078	-0.0193	0.01134
hc	-Taux de croissancemoyenne de la population	0.00128	2.27	0.00056	0.00017	0.00238
aapgr	-Capital humainmoyen	-0.00652	-1.63	0.00401	-0.0143	0.00133
Intercept	-Constante	-0.0132	-1.95	0.00679	-0.0265	0.00008
Log likelihood=543,46		0.0678	1.56	0.0433	-0.0172	0.1528

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Table 5c: Test of Mean growth and growth volatility with a sample of 108 countries and its generations (multidimensional trade)

Variable	Definition	Coefficient	T-Stats	Std. Dev.	[95%Conf.Interval] min	[95%Conf.Interval]max
Vol	Std Dev. De la volatilité de la croissance)	0.0125969	0.98	0.012909	-0.012	0.0378
Intercept F(1. 127)=0.01 R-squared :0.0001 Adj R-squared= -0.0078	Constante	0.0159509	2.81	0.0056704	0.0048	0.02706

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Table 5d:

Test of Mean growth and growth volatility with a sample of 8 generations of France trading with all the generations of the 2 samples.

Variable	Definition	Coefficient	T-Stats	Std. Dev.	[95%Conf.Interval] min	[95%Conf.Interval]max
Vol	-Volatilité de la croissance)	-0.3836	-186.22	0.00206	-0.3876	-0.379
Gdppccp	-log PIB initial par tête	-0.3836	-186.22	0.00206	-0.3876	-0.379
Inv	-Part de l'investissementmoyendans le PIB	-0.0027	-8.84	0.0003	-0.0033	-0.0021
hc	-Capital humainmoyen	0.0009	27.06	0.00003	0.00084	0.00097
aapgr	-Taux de croissancemoyenne de la population	-0.002121	-15.92	0.00013	-0.00238	-0.0018
Intercept	Constante	-0.0015	-6.45	0.00023	-0.00199	-0.0010
Log likelihood=543,46		0.03519	17.02	0.0020	0.0311	0.0392

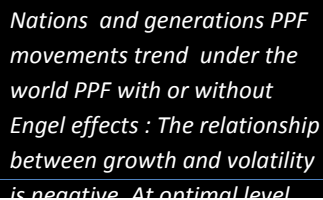
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UNDER PEER REVIEW

Nations and generations PPF movements trend above the world PPF with or without Engel effects : The relationship between growth and volatility is positive. At optimal level there is no fluctuation