Original Research Article

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- 3 Simulation of Meteorological Drought of Bankura District, West Bengal: Comparative
- 4 Study between Exponential Smoothing and Machine Learning Procedures

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- Abstract
- Simulation of drought is needed for proper planning and management of water resources. 7
- 8 This study has been developed using the following five key points: a) primarily from rainfall
- 9 Standard Precipitation Index (SPI), Percentage to Normal (PN), Decile based drought index
- (DI), Rainfall Anomaly Index (RAI), China Z Index (CZI), and Z-score are estimated on 10
- yearly basis (1901-2017), those indices are added and a new index standardized total drought 11
- 12 (S_d) has been established. b) Considering S_d as the input parameter a comparative assessment
- has been made between 4 individual models (3 models from exponential smoothing, 1 model 13
- from machine learning) in simulation and prediction of drought status of next 18 time steps 14
- 15 (years) in Bankura District and Winexpo model outperforms the other models as it obtains
- 16 minimized Standard Error (SE), Random Mean Square Error (RMSE), Mean Absolute Error
- (MAE), and Mean Absolute Percentage Error (MAPE) and highest Correlation coefficient 17
- 18 (R²) value. c) The cumulative drought proneness of the region is also assessed and it is found
- that the whole district will be drought-prone within the year 2100. This region is historically a 19
- drought prone region and agricultural shock is the common issue. In such a circumstances 20
- 21 simulation of drought is a good attempt. Though a lot of models already developed in case of
- 22
- simulation of drought but still a perfect, continuous long term prediction is a big issue to
- 23 solve. Under such a circumstances, this study provides a comparative study between
- exponential smoothing and machine-learning procedures and also introduces a new combined 24
- index standardized total drought. Also the government should take the result seriously and 25
- should try seriously to mitigate the effects of drought. 26
- **Keywords**: Simulation; meteorological drought; winexpo. 27

28 1. Introduction

- 29 Drought is one of the natural disasters that human being has been suffering since the ancient
- 30 era [1, 2, 3,4] and it is the costliest [5,6], long-lasting most severe natural hazard [7,8,9]. It is
- 31 recurrent natural phenomena associated with the lack of water resources for a prolonged

period of dryness[10,11,12] can occur in arid, semi-arid and rain-forested region [13,14] however confusion and debates among scholars prove that there are no universal accepted definitions of drought. Drought forecasting is a critical element in drought risk management [15]. Meteorological drought that transforms in a hydrological, agricultural and socioeconomic events, onsets with a marked reduction in rainfall sufficient to trigger hydrometeorological imbalance for a prolonged period [16,17,18,19]. Thus drought monitoring and assessment are hot topics among hydrologists and meteorologists and attract world-wide attention [18,19,20,21]; its' preparedness and mitigation depends upon the large scale drought monitoring and forecasting over a large geographical area [19,20,25]. Many drought forecasting models already develop in the field of civil engineering. Mishra and Desai (2006) [23] developed ARIMA and multiplicative seasonal ARIMA models to forecast drought using SPI series. These models are able to simulate drought up to 2 months lead time. Morid et.al 2007 [17] simulated Effective Drought Index (EDI) and SPI using Artificial Neural Network (ANN). They compared linear stochastic models with recursive multistep neural network model to the 6 months lead time. Barros and Bowden (2008) [25] employed selforganizing maps (SOM) and multivariate linear regression analysis to forecast SPI of Murray Darling basin of Australia in 12 months of forthcoming scenarios. Many scholars worldwide tested SVM in climatological and hydrological applications [26, 27]. There are several scholars used SVM to predict drought. Belayneh and Adamowski in 2012 [28] forecasted meteorological drought using neural network, wavelet neural network and SVM. Exponential smoothing is quite new in this field originally developed in the field of business mathematics in 1960. Exponential smoothing is able to simulate drought in a long term time frame. This study attempts to simulate drought using exponential smoothing in a long-term time frame.

2. Study Area and Background Information

The District Bankura is bounded by 22°38' N to 23°38' N and longitude 86°36' E to 87°47'E covering an area of 6,882 square Kilometers (2,657sq. mile). River Damodar creates the north and north-east boundary of the district [29, 30, 31]. The neighboring districts are Bardhaman in the north, Paschim Medinapore in the south, Hoogly in the east and Purulia in the west (Figure 1). Bankura is a historically a drought prone district and if no supportive action taken quickly in this regard the condition will get much severe in the upcoming periods [32,33,34,35].

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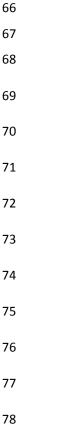
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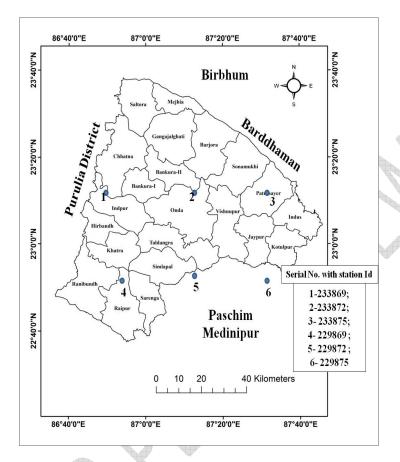


Figure 1 Bankura Location Map and location of Meteorological Stations

Bankura is located in the south western central part of the State of West Bengal belonging transition zone between the plains of Bengal on the east and Chhota Nagpur plateau on the West [34, 35]. It is a part of Midnapur Division of the State and a part of "Rarh" region thus can be stated as "Rarh in Bengal' [31, 32]. The areas to the east and north-east are rather flat belonging to the low lying alluvial plains, known as rice bowl of Bengal [33,34,35].

3. Material and Method

Figure 2 constructively describes the methodological overview of this paper. Monthly rainfall data 1901-2017 has been used for overall analysis and 1901 to 1978 data obtained from Govt. of India water portal website. From 1979 to 2014 daily station wise rainfall data obtained from National Centres for Environmental Protection (NCEP) official website. The rainfall data were collected from Disaster Management Plan of Bankura District 2017 published by District Disaster Management Cell (Table 1) and got 6 individual rainfall stations available

for Bankura District and monthly and daily rainfall data have been added to get yearly rainfall trend. Thus 117 years are taken into consideration.

Table 1 Station list according to the NCEP data set

| Id of | Longitude | Latitude | Elevation(m) |
|------------|-----------|----------|--------------|
| Stations | | | |
| associated | | | |
| Bankura | | | |
| 229869 | 86.875 | 22.9488 | 133 |
| 229872 | 87.1875 | 22.9488 | 61 |
| 229875 | 87.5 | 22.9488 | 34 |
| 233869 | 86.875 | 23.261 | 127 |
| 233872 | 87.1875 | 23.261 | 95 |
| 233875 | 87.5 | 23.261 | 46 |



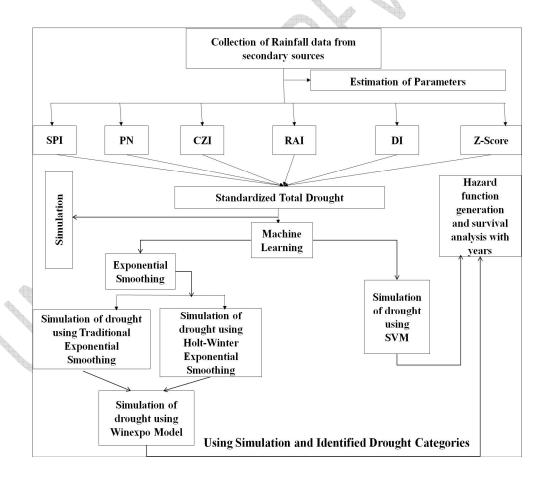


Figure 2 Methodological Overview

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3.1 Formation of Standardized Total Drought (S_d)

- There are several indices developed to assess meteorological drought but the most common
- are SPI [37,38], DI [39,40], PN [5], Z-Score [5], RAI [41,42] and CZI [43]. First of all, from
- the rainfall data, the above mentioned 6 well-known indices i.e. SPI, DI, CZI, PN, Z-score,
- and RAI have been estimated on yearly basis and later those are combined and formed a new
- 126 Index Standardized Total Drought (S_d). So, those six indices are utilized to estimate the true
- nature of meteorological drought and standardized total drought (yearly basis) becomes the
- sole input variable for every models of our study.
- 129 It can be computed as follows:

Total Drought(
$$T_d$$
) = (SPI + DI + PN + ZScore + RAI + CZI) (1)

131 Standardized Total Drought(
$$S_d$$
) = $\frac{T_d - \overline{T_d}}{\delta}$ (2)

- Where, T_d is the total drought.
- 133 $\overline{T_d}$ is the mean of T_d
- 134 δ is the standard deviation of the total drought.
- Based on estimated S_d values the individual drought categories are subdivided into 9 sub-
- groups. he whole subgroups are ranging between <-10 to >10 category and <-10 denotes the

most extreme category whereas >10 denotes wet category. Every 9 sub categories are coded as 1 to 9 (table 2).

Table 3 Probable classes of Standardized Total Drought (S_d)

| Categories of Drought | Code | Ranges of Drought |
|-----------------------|------|-------------------|
| Most Extreme | 1 | <-10.00 |
| Extreme | 2 | -3.00 to -10.00 |
| Severe | 3 | -2.99 to -2.50 |
| Severe Moderate | 4 | -2.49 to -2.35 |
| Moderate | 5 | -2.35 to -1.15 |
| mild drought | 6 | -1.15 to 1 |
| Normal | 7 | 1-5 |
| Extreme Normal | 8 | 5-10 |
| Wet | 9 | >10 |

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3.2 Exponential and Holt-Winter Forecast and Winexpo Method:

Exponential smoothing is the technique to smoothing the time series in exponential window function. Exponential smoothing assigns decreasing weights over time. Holt in 1957 and Winter in 1960 developed smoothing technique and later their method was combined and making Holt-Winter smoothing technique to forecast the recursive trend from the historically observed data series [44]. Here we use the single exponential smoothing technique as Kaleker in 2004 [45] used in his thesis:

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$$S_{t+1} = \alpha * y_t + (1 - \alpha) * S_t$$
 $0 < \alpha < 1, t > 0$ (3)

Eq. (11) can be written as

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$$S_{t+1} - S_t = \alpha * \mathcal{E}_t$$
 (4)

151 The Holt-Winter method time series can be represented using the following model:

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$$y_t = (b_1 + b_2 t) * S_t + \mathcal{E}_t$$
 (5)

Where b_1 is the permanent component, b_2 is the linear trend component, S_t is the multiplicative seasonal factor, \mathcal{E}_t is the random error component, t is the time and t+1 is the lead time from t.

156 From the Eq. (13)

157
$$S_t = \frac{y_t}{b_1 + b_2 t} + \mathcal{E}_t$$
 (6)

158 Sum of all the seasons can be written as

159
$$\sum_{t=12} S_t = M$$
 (7)

- 160 Where L is the length of the year.
- 161 So, the Eq. (7) can be written as,

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$$\sum_{t=1}^{12} y_t = (b_1 + b_2 \sum_{t=1}^{12} t) * \sum_{t=1}^{12} S_t + \mathcal{E}_t$$
 (8)

Assuming, $\sum_{t=1}^{12} y_t = Y$, $\sum_{t=1}^{12} t = T$ and $\sum_{t=1}^{12} S_t = M$ we get from Eq. (16)

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$$Y_t = (b_1 + b_2 T) * M + \mathcal{E}_t$$
 (9)

And Eq. (14) can be written after the sum of all the seasons

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$$M = \frac{Y_t - \epsilon_t}{b_1 + b_2 T}$$
 (10)

- Winexpo method has been developed by us to combine the traditional exponential and Holt-
- Winter method. Combining Eq. (12) and Eq. (18) we get,

$$\frac{S_{t+1}-S_t}{M} = \frac{\alpha * \epsilon_t}{\frac{Y_t - \epsilon_t}{b_1 + b_2 T}}$$
(11)

170 Or,
$$\frac{S_{t+1}-S_t}{M} = \frac{\alpha * (b_1+b_2T)}{(Y_t-\epsilon_t)} + \epsilon_t$$
 (12)

- 171 Winexpo is one of the integrative models as it holds the combination of Holt-Winter
- exponential smoothing and traditional exponential smoothing.

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3.4 Support Vector Machine model (SVM)

- Support Vector Machine (SVM) is the supervised learning models that analyse data used for
- classification and regression analysis [44, 45, 46, 47, 48,49]. The x related all points can be
- mapped in the hyperplane can be defined by the relation $\sum_i \alpha_i k(x_{i,} x) = \text{constant}$ where $k(x_i, x) = \text{constant}$
- 179 x) is the kernel function used to suit the problem. Kernel function becomes small where y
- grows further away from x so it becomes the matter of closeness of each point of y to x. With

- the kernel function SVM actually use the relative closeness between the each point in the
- feature space. The detailed method of analysis can be expressed as following:
- Suppose our training data is consist of N pairs (X_1, Y_1) , (X_2, Y_2) (X_n, Y_n) ; where
- 184 Xi $\in \mathbb{R}^p$ and $Y_i \in \{-1, 1\}$. Define a hyperplane by, $\{x: f(x) = x^T \beta + \beta_0 = 0\}$, where β is a unit
- vector. A classification rule induced by f(x) is $G(x) = \text{sign } \{x^T\beta + \beta_0\}$. Now the signed
- distance from the point x to the hyperplane is 0. Here we are able to find the hyperplane that
- creates biggest margin between training points for class 1 and -1. So, the optimization
- problem just reverses and forms the following dimension:

$$\max_{\beta, \beta_{0,||\beta||}=1} = M \tag{13}$$

190 Subject to,

191 subject to,
$$y_i\{x^T\beta + \beta_0\} \ge M$$
; $i = 1, 2, ..., N$ (14)

- 192 We have used here Least Square Support Vector Machine is based on structural risk
- minimisation in the model weight. It counters convex quadratic programming associated with
- Support Vector Machine (SVM) .The least square version of the SVM classifier is obtained
- by reformulating the minimization problem as

196
$$\min J_2(w, b, e) = \frac{\mu}{2} x^T \beta + \frac{\infty}{2} \sum_{i=1}^{N} e_i^2$$

197 Subject to equality constraints,

198
$$y_i[x^T\beta + \beta_0] = 1 - e_i, i=1,2,.....$$
 (15)

199 Eq. 36 can be written as

200
$$e_i = 1 - y_i [x^T \beta + \beta_0]$$
 (16)

- The eq. 37 hold the case of regression. To solve the eq. 37 we use Lagrangian multiplier by
- which it can be solved.

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$$L_2(w, \beta, e, \alpha) = J_2(w, e) - \sum_{i=1}^{n} \alpha_i \{ [\beta + \beta_0] + e_i - y_i \}$$
 (17)

- Where, $\alpha_i \in \mathbb{R}$, the Lagrangian multipliers. For evaluation performance test of SVM we use
- the error estimation and Kappa Coefficient statistic as well as the accuracy. The definition of
- 206 Cohen's Kappa is as follows:

$$207 k = \frac{p_0 - p_e}{1 - p_o} (18)$$

Where, P_0 is the relative observed agreement among variables; Pe is the hypothetical probability of chance agreement. If the rates are in the complete agreement then k = 1 and if there is no agreement then k = 0.

3.7 Estimation of Cumulative Hazard Proneness

To estimate the cumulative drought-proneness of the study region over the years we took help of the hazard function and survival analysis. Let T be a non-negative random variable representing the waiting time until the occurrence of an event. For simplicity we can adopt the term 'survival analysis' referring to the event of interest as 'hazard proneness' and to the waiting time we state as 'survival time'. We can assume T is a continuous random variable with probability density function (p.d.f.) f(t) and cumulative distribution function (c.d.f.) $Pr\{k < t\}$ given that probability that the event has occurred by duration t. Complement of c.d.f. the survival function becomes

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$$S(t) = Pr\{T \ge t\} = 1 - F(t) = \int_{t}^{\infty} f(x) dx$$
 (19)

Which gives probability of being 'less drought prone' just before duration t more generally the probability that the event of interest has not occurred by duration t. Here we use the following distribution of T is given by hazard function or instantaneous route of occurrence of the event defined as

$$225 \qquad \Omega(t) = \lim_{dt \to 0} \frac{\Pr\left\{t \le T < t + dt, T \ge t\right\}}{dt} = \frac{f(t)}{S(t)}$$
(20)

Where f(t) is the derivative of S(t)

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$$S_t = \exp\{-\int_0^t \Omega(x)\} dx$$
 (21)

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230 3.8 Error Estimation

231 3.8.1 Standard Error (SE)

The standard error can be stated as [50, 51]

$$SE = \frac{\partial}{\sqrt{n}}$$
 (22)

Where ∂ the standard deviation of the distribution and n is is the number of samples.

3.8.2 Root of Mean Squared Error (RMSE)

236 Root of mean squared deviation can be stated as

237 RMSE =
$$\frac{\sqrt{\sum_{t=1}^{T} (\overline{y_t} - y_t)^2}}{\sqrt{T}}$$
 (23)

- Where, The RMSD of predicted values for $\overline{y_t}$ times t of a regression's dependent
- variable y_t with variables observed over T times.
- 3.8.3. Mean Absolute Error (MAE)
- 241 MAE measures average magnitude errors in the set of predictions without considering their
- direction. It is the average over the test sample of the absolute differences between prediction
- and actual observation where all individual differences have equal weight:

244 MAE =
$$1/n \sum_{i=1}^{n} |y_i - \overline{y_i}|$$
 (24)

Where y_j is the observed value and $\overline{y_j}$ is the predicted value.

246 3.9.4. Mean Absolute Percentage Error (MAPE)

- 247 Mean Absolute Percentage Error (MAPE) is a measure of prediction accuracy of a
- 248 forecasting method of accuracy. MAPE can be stated as

249 MAPE =
$$\frac{100\%}{n} \sum_{t=1}^{n} \left| \frac{y_t - F_t}{y_t} \right|$$
 (25)

Where, y_t is the actual value and F_t is the forecasted value.

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3.10 Significance test

255 **3.10.1 Anderson-Darling Test**

- 256 The Anderson-Darling test is the hypothesized distribution is F, and cumulative distribution
- is F_n and the formula can be written as

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$$A^{2} = n \int_{-\infty}^{\infty} \frac{(F_{n}(x) - F(x)^{2})}{F(x)(1 - F(x))} dF(x)$$
 (26)

3.10.2 Kolmogorov-Smirnov Test

- 260 Kolmogorov Smirnov test is a nonparametric test of the equality of continuous one
- 261 dimensional probability distribution with compare of a sample with reference probability
- 262 distribution [53,54]. Kolmogorov Smirnov test statistic can be expressed as

263
$$F_n(x) = 1/n \sum_{i=1}^n I_{[-\infty,x]}(X_i)$$
 (27)

- Where $I_{[-\infty,x]}(X_i)$ is the indicator function, equal 1 if $(X_i) \le x$ and equal to 0 otherwise.
- The Kolmogorov-Smirnov statistic of a given cumulative function F(x) is

266
$$D_n = \sup_x (F_n x - F_x)$$
 (28)

- Where sup is the supremum of the set of distance between the F_nx and F_x . In our case this
- model has been run at 95% significance level.

269 3.10.3 Shapiro -Wilk Test

270 Shapiro and Wilk test of the normality formula can be written as,

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$$W = \frac{(\sum_{i=1}^{n} a_i x_i)^2}{\sum_{i=1}^{n} (x_i - \bar{x})^2}$$
 (29)

- 272 a_i is the (a_1, \dots, a_n) , \bar{x} is the mean.
- 273 The constants a_i can be written as $(a_1 \dots a_n) = \frac{m^T V^{-1}}{(m^T V^{-1} V^{-1} m)^{1/2}}$ here
- 274 $m = (m_1 m_n)^T$
- and $m_1 \dots m_n$ are the expected values of the order statistics of independent and
- 276 identically distributed random variables sampled from the standard normal distribution,
- and V is the covariance matrix of those order statistics.

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4. Results and Discussion

- Fluctuation of rainfall and a negative exponential trend are specified in Figure 3 (Y_t =
- $1418.88 \times (0.999642^{\circ}t)$. Rainfalls are more or less normally distributed at 95% confidence
- interval (Figure 4a). Residuals versus fit plot (Figure 4b) displays that the points are
- randomly distributed on both sides of zero with no recognisable patterns thus our rainfall data
- are having a constant variance. Residuals of rainfall are having a mean close to zero and the

histogram is symmetric close to around zero (Figure 4c). Residuals versus order fit (Figure 4d) shows that the residuals fall randomly around the centre line. Before proceed with rainfall and estimated 6 indices the reliability of those 6 indices are judged using Cronbach's Alpha. The overall value of Cronbach's alpha is 0.9694. Average SPI and Z-score between the time frame 1901-1939 are -0.06 and 0.299, in between 1940-1980, 0.037 and 0.382 respectively and from 1980-2035 the average SPI and Z-score becomes -2.345. Average PN value from 1901-1939 is 100.792 %, 1940-1980 PN becomes 100.641%; 1980-2035 it is diminished and become 98.967%. In the same way average DI is estimated and from 1901-1939 DI 5.76%, 1940 to 1980 5.73% and DI from 1980 to 2035 4.64% value of DI is obtained. CZI and RAI are also decreased from 0.32 (1901-1939) and 0.38 to 0.26 (1940-1980), 0.28 and later 1980-2035 it reaches to 0.14 and 0.19. Overall all the indices attain negative trend. SPI, PN, DI, RAI, CZI and Z-score are added and a new index Standardized Total Drought (S_d) has been formed to estimate overall trend of meteorological drought of Bankura District. Estimation and prediction of the trend of S_d using the traditional exponential smoothing has been done and a slightly negative trend is obtained (Values reach to -0.143 in 2035) (Figure 5a). The residuals of traditional exponential smoothing trend values are ranging between -15 to +5 (Figure 5b). In case of traditional exponential smoothing the average value between 1901-1939 experiences -0.170, 1940 to 1980 the value reaches to -0.034 whereas between the 1980 to 2035 the average value attains -0.134 thus overall trend is seemed to be more drought prone in recent upcoming periods. Similarly using Holt-Winter exponential smoothing analysis and prediction of drought has been done (Figure 5c) and residuals are fitted randomly as histogram plot based on the centre line (ranging between -2 to +5 range) (Figure 5d). In case of Holt-Winter exponential smoothing the average value between 1901-1939 achieve -0.163, between the time frame 1940-1980 and 1980 to 1935 it attain 0.061 and -0.261 values respectively. The combined model Winexpo attains 0.423 for 1901-1939, 0.51 for 1940-1980 and -1.423 for 1980-2035.

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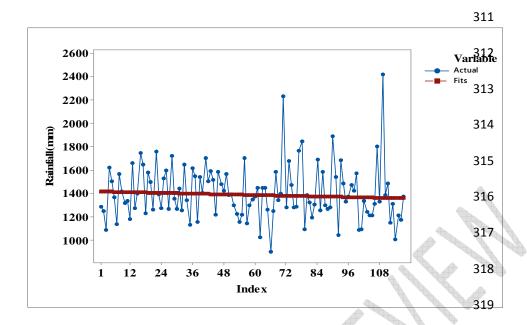


Figure 3 All station accumulated rainfall according to yearly time steps (1901-2017)

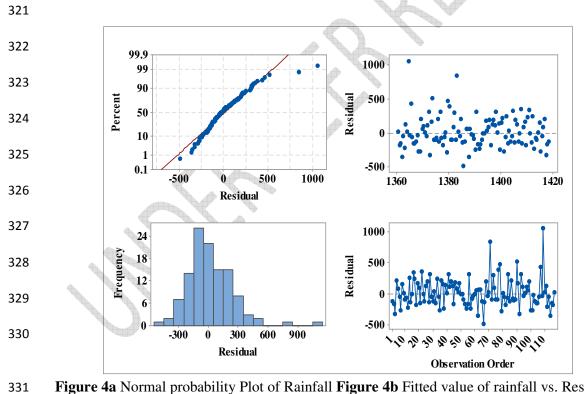


Figure 4a Normal probability Plot of Rainfall **Figure 4b** Fitted value of rainfall vs. Residual value **Figure 4c** Residual value versus Frequency value **Figure 4d** Observation order vs.

Residual value

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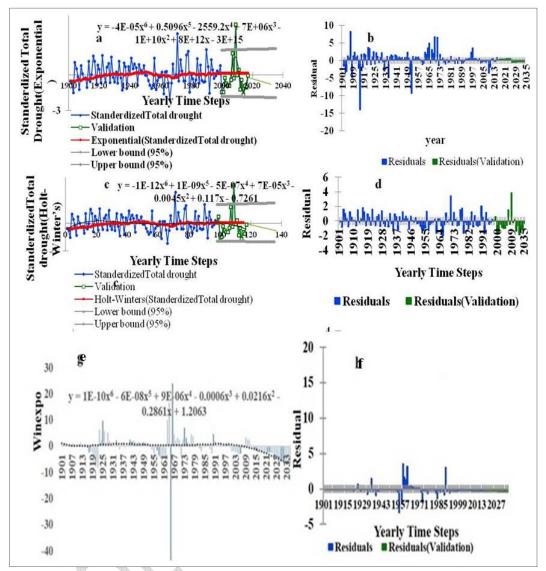


Figure 5 Exponential Smoothing models and associated Residual Plots a) Exponential Smoothing c) Holt-Winter Smoothing e) Winexpo Simulation

From the true classes determined from the categories of S_d SVM is capable to predict the nature of drought category. A user friendly SVM tool LSSVM is used to implement the classification of drought status of Bankura District. At data pre-processing stage raw values of S_d are linearly rescaled into [-1, 1] using the ranges of their minimums and maximums for binary distribution of classifiers. Applying the SVM each category against all is estimated in every case. In case of Extreme vs. others the model is obtained 43 support vectors, for extreme normal the model is obtained 33 support vectors, for mild drought the model obtains 34 support vectors, most extreme the model obtains 28 support vectors, normal vs. others obtains 51 support vectors, severe vs. others obtains 8 support vectors and wet vs. others

obtains 20 support vectors. From the observed true classes of 135 observations (used simulated value using Winexpo) drought probability classes are predicted by SVM. SVM performs with a medium accuracy level. According to SVM identified drought categories over years over 80% years are concentrated within severe moderate, severe, extreme and most extreme categories and about 20% years are concentrated within Moderate, Normal, and Extreme Normal, wet categories (Figure 6a) whereas according to Winexpo identified drought categories 36% years are mingled with severe moderate, severe, extreme, most extreme and moderate categories and over 64% are mingled with normal, mild, extreme normal and wet categories (Figure 6b). The extreme normal versus others, wet versus others, mild versus others, normal versus others training sample sets achieve over 90% accuracy whereas extreme and most extreme versus others and severe moderate versus others category training samples achieve less than 30% accuracy (Table 4). Overall average SVM achieve 0.724 as Cohen's Kappa and overall 60% accuracy has been achieved. So, SVM has performed moderately well in prediction of drought of our study area.

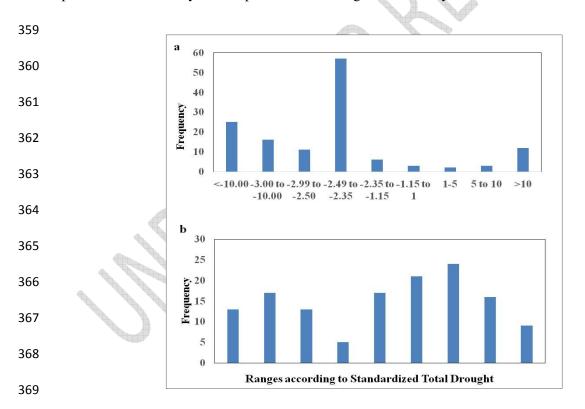


Figure 6 Frequency of drought under each drought categories a) based on simulation model of SVM b) based on simulation of Winexpo

Table 4 Performance matrix of Support Vector Machine (SVM)

| Training set | Accuracy | Cohen's kappa |
|----------------|----------|---------------|
| Extreme versus | 0.847 | 0.978 |
| Others | | |
| Extreme | 0.187 | 0.086 |
| Normal versus | | |
| Others | | |
| Moderate | 0.987 | 0.987 |
| versus Others | | |
| Most Extreme | 0.847 | 0.978 |
| versus Others | | |
| Normal versus | 0.253 | 0.222 |
| Others | | |
| Severe versus | 0.987 | 0.998 |
| Others | | |
| Severe | 0.876 | 0.965 |
| Moderate | | |
| versus Others | | |
| Wet versus | 0.153 | 0.042 |
| Others | | |
| Mild versus | 0.165 | 0.078 |
| Others | | |

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The significance test using three individual tests has been run at 95% and 99% confidence interval (Table 5). The traditional exponential smoothing experiences probability value 0.004 for Anderson-Darling test, 0.005 for Shapiro-Wilk test and 0.004 by Kolmogorov-Smirnov test. The Holt-Winter exponential smoothing attains 0.003 probabilities for Anderson-Darling test, 0.004 for Shapiro-Wilk test and 0.001 for Kolmogorov-Smirnov test. Winexpo model also attains probability value 0.002 for Anderson-Darling test, 0.004 for Shapiro-Wilk test and 0.003 for Kolmogorov-Smirnov test. The Bayesian model of LSSVM extreme category vs. other categories experiences 10.275 as Anderson-Darling test statistic value, 0.527 as Shapiro-Wilk test statistic value and 0.435 as KS test statistic value. LSSVM Bayesian most extreme vs. other category is mingled with 5.543 as Anderson-Darling test statistic, 0.727 as Shapiro-Wilk test statistic and 0.316 as KS test statistic. SVM extreme normal vs. other categories achieves 2.165 as Anderson-Darling test statistic, 0.904 as Shapiro-Wilk test statistic and 0.482 as KS test statistic value. Similarly, Mild versus others, severe versus others, severe moderate versus others and wet versus others are also calculated (Table 5). All the Anderson-Darling test is successful and valid at 95% confidence interval as the significance level P-value achieves <0.005 value in all the nine combinations. Shapiro-Wilk and KS test for all the SVM nine possible combinations the probability value is <0.010 that means those values are significant at 99% confidence interval. Overall SVM model is significant at 95% confidence interval (in case of Anderson-Darling test) and 99% significance level (in case of Shapiro-Wilk test and KS test). As P values are <0.005 and <0.010 for all the cases the distribution is not normal here and null hypothesis that there were no difference between the observed class and predicted class can be rejected and the alternative hypothesis is accepted. The error estimation and goodness of fit statistics (Table 6) of the individual models indicate that Winexpo attains the lowest error and highest R-square value in comparison with the other models altogether.

Table 5 Error Estimation and Goodness of fit statistics (for error estimation 0.001 used as a multiplicative factor)

| Model Name | SE | Adjusted | Adjusted | Adjusted | R ² (using Linear |
|-----------------|--------|----------|----------|----------|------------------------------|
| | | RMSE | MAE | MAPE | kernel) |
| Traditional | 0.024 | 0.996 | 0.790 | 25.65 | 0.39 |
| exponential | | | | | |
| smoothing | | | | • | |
| Holt-Winter | 0.026 | 1.006 | 0.654 | 95.43 | 0.04 |
| Smoothing | | | | | |
| Winexpo Model | 0.111 | 1.64 | 0.445 | 49.53 | 0.35 |
| SVM-Most | 3.080 | 0.049 | 0.045 | 4.559 | 0.99 |
| Extreme versus | | | | | |
| others | | | | | |
| SVM-Extreme | 1.303 | 0.038 | 0.019 | 2.048 | 0.94 |
| versus others | | | | | |
| SVM-Severe | 11.180 | 0.026 | 0.026 | 1.915 | 0.95 |
| versus others | | | | | |
| SVM-Severe | 11.345 | 0.023 | 0.045 | 1.934 | 0.99 |
| moderate versus | | | | | |
| others | | | | | |
| SVM-Moderate | 5.533 | 0.015 | 0.008 | 0.833 | 0.99 |
| versus others | | | | | |
| SVM-Mild versus | 5.333 | 0.020 | 0.013 | 1.413 | 0.97 |
| others | | | | | |
| SVM-Normal | 1.668 | 0.033 | 0.019 | 2.048 | 0.52 |
| versus others | | | | | |
| SVM-Extreme | 7.580 | 0.018 | 0.014 | 1.487 | 0.35 |
| Normal versus | | | | | |
| others | | | | | |

| SVM-Wet versus | 83.724 | 0.001 | 0.008 | 0.900 | 0.34 | |
|----------------|--------|---------|-------|-------|------|--|
| others | | | | | | |
| Overall SVM | 0.130 | 0.02175 | 0.022 | 1.904 | 0.78 | |
| versus other | | | | | | |

Table 6 Significance test of the models

| Standardized | Anderson-Darling Test | | Shapiro-Wilk Test | | Kolmogorov- Smirnov Test | | Type of Model |
|---------------|-----------------------|-------------|-------------------|----------|-----------------------------|----------|------------------|
| Total | Test | Significanc | Test | Signific | Test | Signific | Model |
| Drought | Statistic | e Level | Statistic | ance | Statistic. | ance | |
| Drought | Statistic | C Ecver | Statistic | Level | Statistic | Level | |
| Traditional | 8.827 | 0.004 | 0.916 | 0.005 | 0.169 | 0.004 | |
| Exponential | | (<0.005) | | (<0.05) | # # | (<0.005) | Exponen |
| Smoothing | | | | | | | tial |
| Holt-Winter | 7.192 | 0.003 | 0.917 | 0.004 | 0.163 | 0.001 | Smoothi |
| Exponential | | (<0.005) | | (<0.005) | | (<0.005) | ng |
| Smoothing | | | | | | | |
| Winexpo | 28.790 | 0.002 | 0.529 | 0.004 | 0.363 | 0.002 | Combine |
| Model | | (<0.005) | | (<0.005) | | (<0.005) | d model |
| SVM- | 10.275 | < 0.005 | 0.527 | <0.010 | 0.435 | < 0.010 | |
| Extreme | | | | | | | |
| versus others | | 4 | | | | | |
| SVM- | 2.165 | < 0.005 | 0.904 | < 0.010 | 0.482 | < 0.010 | |
| Extreme | | | | | | | |
| normal | | | | | | | |
| versus others | | | | | | | |
| SVM-Mild | 11.598 | < 0.005 | 0.482 | < 0.010 | 0.419 | < 0.010 | |
| vs. others | | | | | | | |
| SVM- | 10.550 | < 0.005 | 0.455 | < 0.010 | 0.427 | < 0.010 | Machine |
| Moderate vs. | | | | | | | Learning |
| others | | 0.005 | 0 - 4 - | 0.040 | 0.016 | 0.010 | |
| SVM-Most | 5.543 | < 0.005 | 0.727 | < 0.010 | 0.316 | < 0.010 | |
| Extreme vs. | | | | | | | |
| others | 5 0 5 4 | 0.007 | 0.007 | 0.010 | 0.261 | 0.010 | |
| SVM- | 5.274 | < 0.005 | 0.827 | < 0.010 | 0.261 | < 0.010 | |
| Normal vs. | | | | | | | |
| others | 5 5 4 4 | <0.005 | 0.507 | <0.010 | 0.466 | <0.010 | |
| SVM-Severe | 5.544 | < 0.005 | 0.597 | <0.010 | 0.466 | <0.010 | |
| vs. others | | | | | | | |
| SVM-Severe | 2.131 | < 0.005 | 0.662 | < 0.010 | 0.462 | < 0.010 | |
| moderate_vs. | | | | | | | |
| _others | | | | | | | |
| SVM-Wet vs. | 1.108 | < 0.005 | 0.935 | < 0.05 | 0.236 | < 0.010 | |
| Others | | | | | | | |
| | | | | | | | |

Based on Winexpo and SVM model simulation the hazard prone zones have been estimated (Figure 6). The southern and south-western blocks are extreme drought-prone and northern and north-western blocks are mild to normal mode. The whole regimes form the coherent clusters in space highlighted in figure 7. Most extreme to severe drought categories are clubbed into negative x, y direction and wet categories are clubbed into positive directions of x and y. Based on the whole aspects of meteorological drought the year wise hazard and cumulative failure functions are developed. The most extreme, extreme, severe, severe moderate, moderate and mild categories are included in the category of "hazard prone or failure "whereas normal, extreme normal and wet categories are included in "censored" category. Winexpo attains the best result so this model has been used here. According to simulation of drought category using winexpo, almost 84 observations are fallen into "hazard-prone" category and 51 observations have fallen into the "censored" group. The distribution of yearly censored and failure categories are compared based on Weibull and logistic probability fit but logistic probability fit gave us the better association (Correlation value 0.984 for logistic and 0.678 for Weibull). So, finally the logistic probability fit have been taken for year-wise estimation of cumulative hazard-proneness. The whole logistic model seemed to be more or less normal (Figure 8a and 8b) and it had achieved the 3.223 value as the Anderson-Darling test. From the survival function (Figure 8c) fitted based on logistic probability plot encounters the fact that as the time (year) will progress the drought proneness will increase and at the year 2100 the vulnerability will be almost intolerable that will lead to massive disruption over the local community. Reversely, the progression of hazard based on cumulative curve plotting (Figure 9, figure 8d) exhibits the fact that the whole district will be severely affected by drought within 2100. The significance test for hazard function is done in 95% significance level .So, it can be concluded that the district will face extreme to severe drought hazard in the recent future.

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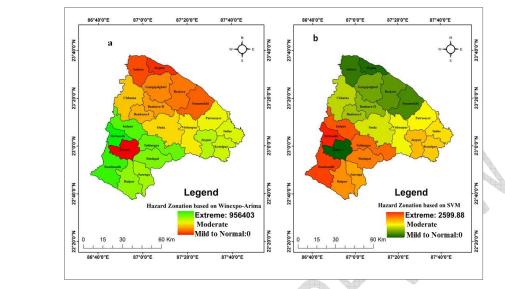


Figure 6 Drought-prone zone identification (12 month time steps) using a) Winexpo b) SVM

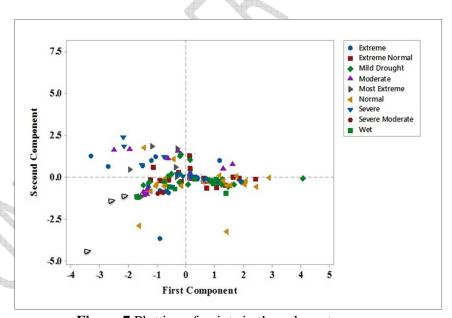


Figure 7 Plotting of points in the coherent space

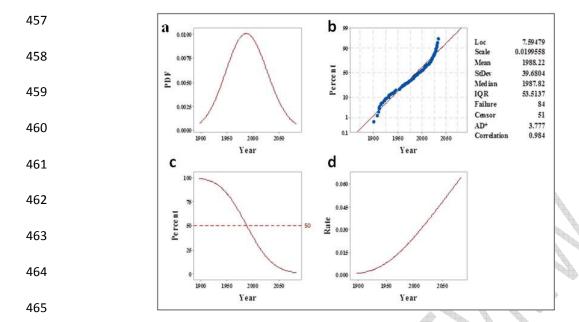


Figure 8a Probability density function **b** Logistic probability fit **c** Survival function based on logistic probability fit **d** Progression of hazard rate with years

5. Conclusion

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The evolution and quantification of drought are necessary for the proper planning and management of water resources to mitigate the hazard of future occurrences. By far the main challenge in this field is that a) to identify the correct method to analyze the meteorological drought b) to identify the spatial dimension over which the drought can be affected c) to simulate and predict the drought correctly as it is inherently needed for proper planning and management of water resources. Continuous year wise monitoring and simulation is also an important issue even seriously neglected in the drought monitoring and assessment. In most of the cases of drought monitoring and assessment historical rainfall data is one of the input factors. Our study is also not an exception with the above scenarios. Taking rainfall as the sole input factor we estimated 6 essential meteorological indices and from those indices we form a new index Standardized Total Drought (S_d) and simulate it up to 2035 and make a comparative assessment of exponential smoothing and machine learning procedures. Cumulative drought-proneness of the region using hazard function has been analysed and we found that the whole region will be severely drought affected within 2100. The extremities of rainfall and temperature drive a potential threat to agriculture, food security and socioeconomic vulnerability. Thus a more detailed structural study is required to explore the synergetic effects of trends and patterns of other climatic variables. However the conclusion

- 486 reached in this study can be an elementary step to improve the risk management strategy,
- review of agricultural practices and water use in this counterpart.

488 Conflict of Interest

There is no conflict of interest regarding the publication of this article.

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