

Original Research Article

Simulation of Meteorological Drought of Bankura District, West Bengal: Comparative Study between Exponential Smoothing and Machine Learning Procedures

Abstract

Simulation of drought is needed for proper planning and management of water resources. This study has been developed using the following five key points: a) primarily from rainfall Standard Precipitation Index (SPI), Percentage to Normal (PN), Decile based drought index (DI), Rainfall Anomaly Index (RAI), China Z Index (CZI), and Z-score are estimated on yearly basis (1901-2017), those indices are added and a new index standardized total drought (S_d) has been established. b) Considering S_d as the input parameter a comparative assessment has been made between 4 individual models (3 models from exponential smoothing, 1 model from machine learning) in simulation and prediction of drought status of next 18 time steps (years) in Bankura District and Winexpo model outperforms the other models as it obtains minimized Standard Error (SE), Random Mean Square Error (RMSE), Mean Absolute Error (MAE), and Mean Absolute Percentage Error (MAPE) and highest Correlation coefficient (R^2) value. c) The cumulative drought proneness of the region is also assessed and it is found that the whole district will be drought-prone within the year 2100. This region is historically a drought prone region and agricultural shock is the common issue. In such a circumstances simulation of drought is a good attempt. Though a lot of models already developed in case of simulation of drought but still a perfect, continuous long term prediction is a big issue to solve. Under such a circumstances, this study provides a comparative study between exponential smoothing and machine-learning procedures and also introduces a new combined index standardized total drought. Also the government should take the result seriously and should try seriously to mitigate the effects of drought.

Keywords: Simulation; meteorological drought; Winexpo.

1. Introduction

Drought is one of the natural disasters that human being has been suffering since the ancient era [1, 2, 3,4] and it is the costliest [5,6], long-lasting most severe natural hazard [7,8,9]. It is recurrent natural phenomena associated with the lack of water resources for a prolonged

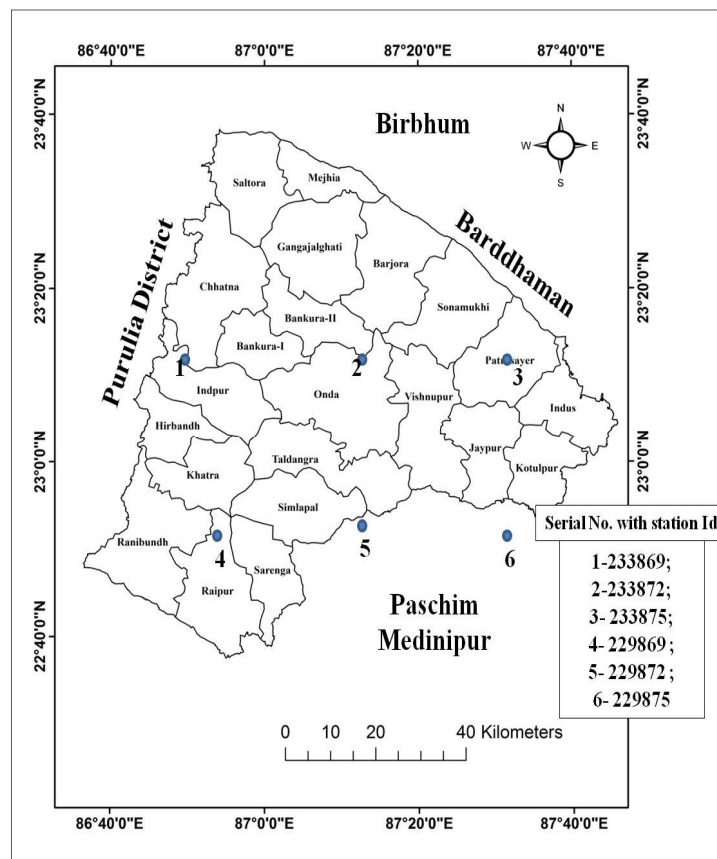
32 period of dryness[10,11,12] can occur in arid, semi-arid and rain-forested region [13,14]
33 however confusion and debates among scholars prove that there are no universal accepted
34 definitions of drought. Drought forecasting is a critical element in drought risk management
35 [15]. Meteorological drought that transforms in a hydrological, agricultural and socio-
36 economic events, onsets with a marked reduction in rainfall sufficient to trigger hydro-
37 meteorological imbalance for a prolonged period [16,17,18,19]. Thus drought monitoring and
38 assessment are hot topics among hydrologists and meteorologists and attract world-wide
39 attention [18,19,20,21]; its' preparedness and mitigation depends upon the large scale drought
40 monitoring and forecasting over a large geographical area [19,20,25]. Many drought
41 forecasting models already develop in the field of civil engineering. Mishra and Desai (2006)
42 [23] developed ARIMA and multiplicative seasonal ARIMA models to forecast drought
43 using SPI series. These models are able to simulate drought up to 2 months lead time. Morid
44 et.al 2007 [17] simulated Effective Drought Index (EDI) and SPI using Artificial Neural
45 Network (ANN). They compared linear stochastic models with recursive multistep neural
46 network model to the 6 months lead time. Barros and Bowden (2008) [25] employed self-
47 organizing maps (SOM) and multivariate linear regression analysis to forecast SPI of Murray
48 Darling basin of Australia in 12 months of forthcoming scenarios. Many scholars worldwide
49 tested SVM in climatological and hydrological applications [26, 27]. There are several
50 scholars used SVM to predict drought . Belayneh and Adamowski in 2012 [28] forecasted
51 meteorological drought using neural network, wavelet neural network and SVM.
52 Exponential smoothing is quite new in this field originally developed in the field of business
53 mathematics in 1960. Exponential smoothing is able to simulate drought in a long term time
54 frame. This study attempts to simulate drought using exponential smoothing in a long-term
55 time frame.

56 **2. Study Area and Background Information**

57 The District Bankura is bounded by 22°38' N to 23°38' N and longitude 86°36' E to 87°47'E
58 covering an area of 6,882 square Kilometers (2,657sq. mile). River Damodar creates the
59 north and north-east boundary of the district [29, 30, 31]. The neighboring districts are
60 Bardhaman in the north, Paschim Medinapore in the south, Hoogly in the east and Purulia in
61 the west (Figure 1). Bankura is a historically a drought prone district and if no supportive
62 action taken quickly in this regard the condition will get much severe in the upcoming
63 periods [32,33,34,35].

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Figure 1 Bankura Location Map and location of Meteorological Stations

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Bankura is located in the south western central part of the State of West Bengal belonging transition zone between the plains of Bengal on the east and Chhota Nagpur plateau on the West [34, 35]. It is a part of Midnapur Division of the State and a part of “Rarh” region thus can be stated as “Rarh in Bengal” [31, 32]. The areas to the east and north-east are rather flat belonging to the low lying alluvial plains, known as rice bowl of Bengal [33,34,35].

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3. Material and Method

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Figure 2 constructively describes the methodological overview of this paper. Monthly rainfall data 1901-2017 has been used for overall analysis and 1901 to 1978 data obtained from Govt. of India water portal website. From 1979 to 2014 daily station wise rainfall data obtained from National Centres for Environmental Protection (NCEP) official website. The rainfall data were collected from Disaster Management Plan of Bankura District 2017 published by District Disaster Management Cell (Table 1) and got 6 individual rainfall stations available for Bankura District and monthly and daily rainfall data have been added to get yearly rainfall trend. Thus 117 years are taken into consideration.

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Table 1 Station list according to the NCEP data set

Id of Stations associated Bankura	Longitude	Latitude	Elevation(m)
229869	86.875	22.9488	133
229872	87.1875	22.9488	61
229875	87.5	22.9488	34
233869	86.875	23.261	127
233872	87.1875	23.261	95
233875	87.5	23.261	46

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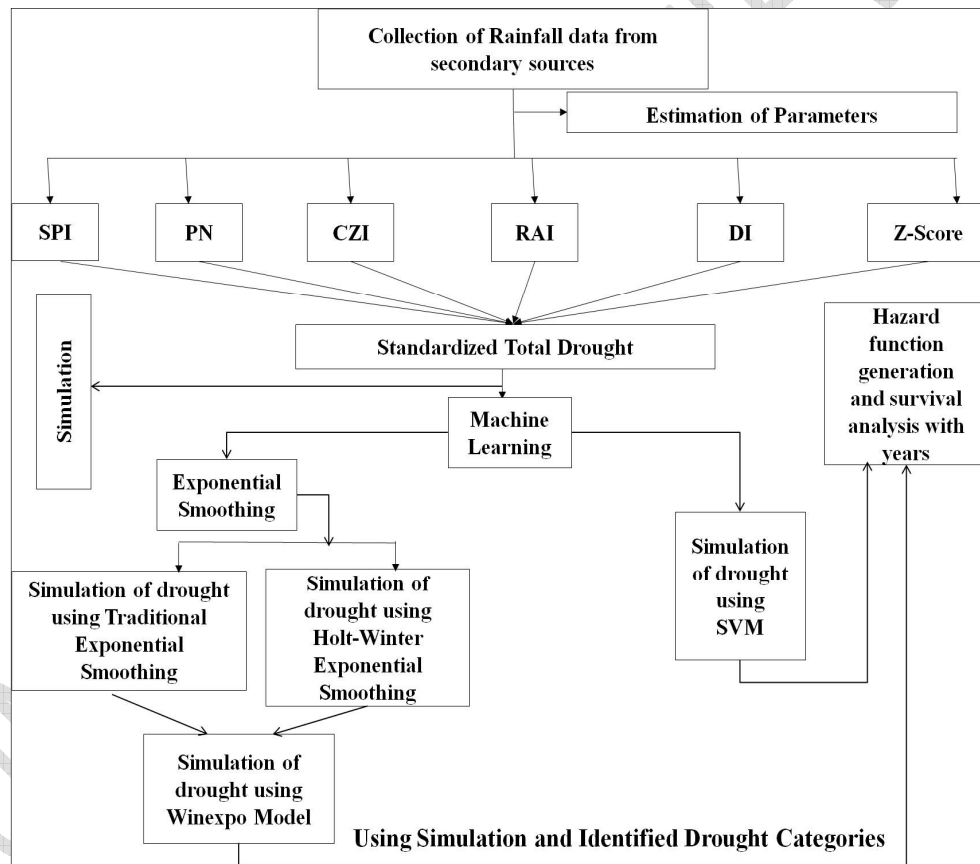
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Figure 2 Methodological Overview

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3.1 Formation of Standardized Total Drought (S_d)

112 There are several indices developed to assess meteorological drought but the most common
 113 are SPI [37,38], DI [39,40], PN [5], Z-Score [5], RAI [41,42] and CZI [43]. First of all, from
 114 the rainfall data, the above mentioned 6 well-known indices i.e. SPI, DI, CZI, PN, Z-score,
 115 and RAI have been estimated on yearly basis and later those are combined and formed a new
 116 Index Standardized Total Drought (S_d). So, those six indices are utilized to estimate the true
 117 nature of meteorological drought and standardized total drought (yearly basis) becomes the
 118 sole input variable for every models of our study.

119 It can be computed as follows:

$$120 \text{ Total Drought}(T_d) = (\text{SPI} + \text{DI} + \text{PN} + \text{ZScore} + \text{RAI} + \text{CZI}) \quad (1)$$

$$121 \text{ Standardized Total Drought}(S_d) = \frac{T_d - \bar{T}_d}{\delta} \quad (2)$$

122 Where, T_d is the total drought.

123 \bar{T}_d is the mean of T_d

124 δ is the standard deviation of the total drought.

125 Based on estimated S_d values the individual drought categories are subdivided into 9 sub-
 126 groups. The whole subgroups are ranging between <-10 to >10 category and <-10 denotes the
 127 most extreme category whereas >10 denotes wet category. Every 9 sub categories are coded
 128 as 1 to 9 (table 2).

129 **Table 2 Probable classes of Standardized Total Drought (S_d)**

Categories of Drought	Code	Ranges of Drought
Most Extreme	1	<-10.00
Extreme	2	-3.00 to -10.00
Severe	3	-2.99 to -2.50
Severe Moderate	4	-2.49 to -2.35
Moderate	5	-2.35 to -1.15
mild drought	6	-1.15 to 1
Normal	7	1-5
Extreme Normal	8	5-10
Wet	9	>10

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131 **3.2 Exponential and Holt-Winter Forecast and Winexpo Method:**

132 Exponential smoothing is the technique to smoothing the time series in exponential window
 133 function. Exponential smoothing assigns decreasing weights over time. Holt in 1957 and
 134 Winter in 1960 developed smoothing technique and later their method was combined and
 135 making Holt-Winter smoothing technique to forecast the recursive trend from the historically
 136 observed data series [44]. Here we use the single exponential smoothing technique as Kaleker
 137 in 2004 [45] used in his thesis:

$$138 \quad S_{t+1} = \alpha * y_t + (1 - \alpha) * S_t \quad 0 < \alpha < 1, t > 0 \quad (3)$$

139 Eq. (3) can be written as

$$140 \quad S_{t+1} - S_t = \alpha * \epsilon_t \quad (4)$$

141 The Holt-Winter method time series can be represented using the following model:

$$142 \quad y_t = (b_1 + b_2 t) * S_t + \epsilon_t \quad (5)$$

143 Where b_1 is the permanent component, b_2 is the linear trend component, S_t is the
 144 multiplicative seasonal factor, ϵ_t is the random error component, t is the time and $t+1$ is the
 145 lead time from t .

146 From the Eq. (5)

$$147 \quad S_t = \frac{y_t}{b_1 + b_2 t} + \epsilon_t \quad (6)$$

148 Sum of all the seasons can be written as

$$149 \quad \sum_{t=1}^{12} S_t = M \quad (7)$$

150 Where M is the length of the year.

151 So, the Eq. (7) can be written as,

$$152 \quad \sum_{t=1}^{12} y_t = (b_1 + b_2 \sum_{t=1}^{12} t) * \sum_{t=1}^{12} S_t + \epsilon_t \quad (8)$$

153 Assuming, $\sum_{t=1}^{12} y_t = Y$, $\sum_{t=1}^{12} t = T$ and $\sum_{t=1}^{12} S_t = M$ we get from Eq. (8)

$$154 \quad Y_t = (b_1 + b_2 T) * M + \epsilon_t \quad (9)$$

155 And Eq. (9) can be written after the sum of all the seasons

$$156 \quad M = \frac{Y_t - \epsilon_t}{b_1 + b_2 T} \quad (10)$$

157 Winexpo method has been developed by us to combine the traditional exponential and Holt-
 158 Winter method. Combining Eq. (4) and Eq. (10) we get,

$$159 \quad \frac{S_{t+1}-S_t}{M} = \frac{\alpha \epsilon_t}{\frac{Y_t - \epsilon_t}{b_1 + b_2 T}} \quad (11)$$

$$160 \quad \text{Or,} \quad \frac{S_{t+1}-S_t}{M} = \frac{\alpha(b_1 + b_2 T)}{(Y_t - \epsilon_t)} + \epsilon_t \quad (12)$$

161 Winexpo is one of the integrative models as it holds the combination of Holt-Winter
 162 exponential smoothing and traditional exponential smoothing.

163 **3.4 Support Vector Machine model (SVM)**

164 Support Vector Machine (SVM) is the supervised learning models that analyse data used for
 165 classification and regression analysis [44, 45, 46, 47, 48,49]. The x related all points can be
 166 mapped in the hyperplane can be defined by the relation $\sum_i \alpha_i k(x_i, x) = \text{constant}$ where $k(x_i,$
 167 $x)$ is the kernel function used to suit the problem. Kernel function becomes small where y
 168 grows further away from x so it becomes the matter of closeness of each point of y to x . With
 169 the kernel function SVM actually use the relative closeness between the each point in the
 170 feature space. The detailed method of analysis can be expressed as following:

171 Suppose our training data is consist of N pairs $(X_1, Y_1), (X_2, Y_2), \dots, (X_n, Y_n)$; where
 172 $X_i \in \mathbb{R}^p$ and $Y_i \in \{-1, 1\}$. Define a hyperplane by, $\{x: f(x) = x^T \beta + \beta_0 = 0\}$, where β is a unit
 173 vector. A classification rule induced by $f(x)$ is $G(x) = \text{sign}\{x^T \beta + \beta_0\}$. Now the signed
 174 distance from the point x to the hyperplane is 0. Here we are able to find the hyperplane that
 175 creates biggest margin between training points for class 1 and -1. So, the optimization
 176 problem just reverses and forms the following dimension:

$$177 \quad \max_{\beta, \beta_0, \|\beta\|=1} = M \quad (13)$$

$$178 \quad \text{subject to, } y_i \{x^T \beta + \beta_0\} \geq M \quad ; \quad i = 1, 2, \dots, N \quad (14)$$

179 Least Square Support Vector Machine is used here based on structural risk minimisation in
 180 the model weight. It counters convex quadratic programming associated with Support Vector
 181 Machine (SVM). The least square version of the SVM classifier is obtained by reformulating
 182 the minimization problem as

$$183 \quad \min J_2(w, b, e) = \frac{\mu}{2} x^T \beta + \frac{\infty}{2} \sum_{i=1}^N e_i^2$$

184 Subject to equality constraints,

$$185 \quad y_i [x^T \beta + \beta_0] = 1 - e_i, \quad i=1,2,\dots,n \quad (15)$$

186 Eq. 15 can be written as

$$187 \quad e_i = 1 - y_i [x^T \beta + \beta_0] \quad (16)$$

188 The eq. 16 hold the case of regression. To solve the eq. 16 we use Lagrangian multiplier by
189 which it can be solved.

$$190 \quad L_2(w, \beta, e, \alpha) = J_2(w, e) - \sum_{i=1}^n \alpha_i \{[\beta + \beta_0] + e_i - y_i\} \quad (17)$$

191 Where, $\alpha_i \in \mathbb{R}$, the Lagrangian multipliers. For evaluation performance test of SVM we use
192 the error estimation and Kappa Coefficient statistic as well as the accuracy. The definition of
193 Cohen's Kappa is as follows:

$$194 \quad k = \frac{p_0 - p_e}{1 - p_e} \quad (18)$$

195 Where, p_0 is the relative observed agreement among variables; p_e is the hypothetical
196 probability of chance agreement. If the rates are in the complete agreement then $k = 1$ and if
197 there is no agreement then $k = 0$.

198 **3.7 Estimation of Cumulative Hazard Proneness**

199 To estimate the cumulative drought-proneness of the study region over the years we took help
200 of the hazard function and survival analysis. Let T be a non-negative random variable
201 representing the waiting time until the occurrence of an event. For simplicity we can adopt
202 the term 'survival analysis' referring to the event of interest as 'hazard proneness' and to the
203 waiting time we state as 'survival time'. We can assume T is a continuous random variable
204 with probability density function (p.d.f.) $f(t)$ and cumulative distribution function (c.d.f.)
205 $\Pr\{k < t\}$ given that probability that the event has occurred by duration t . Complement of
206 c.d.f. the survival function becomes

$$207 \quad S(t) = \Pr\{T \geq t\} = 1 - F(t) = \int_t^{\infty} f(x) dx \quad (19)$$

208 Which gives probability of being 'less drought prone' just before duration t more generally
209 the probability that the event of interest has not occurred by duration t . Here we use the
210 following distribution of T is given by hazard function or instantaneous route of occurrence
211 of the event defined as

$$212 \quad \Omega(t) = \lim_{dt \rightarrow 0} \frac{\Pr \{t \leq T < t+dt, T \geq t\}}{dt} = \frac{f(t)}{S(t)} \quad (20)$$

213 Where $f(t)$ is the derivative of $S(t)$

$$214 \quad S_t = \exp\{-\int_0^t \Omega(x) dx\} \quad (21)$$

215 **3.8 Error Estimation**

216 **3.8.1 Standard Error (SE)**

217 The standard error can be stated as [50, 51]

$$218 \quad SE = \frac{\partial}{\sqrt{n}} \quad (22)$$

219 Where ∂ the standard deviation of the distribution and n is the number of samples.

220 **3.8.2 Root of Mean Squared Error (RMSE)**

221 Root of mean squared deviation can be stated as

$$222 \quad RMSE = \frac{\sqrt{\sum_{t=1}^T (\bar{y}_t - y_t)^2}}{\sqrt{T}} \quad (23)$$

223 Where, the RMSE of predicted values for y_t times t of a regression's dependent
224 variable y_t with variables observed over T times.

225 **3.8.3. Mean Absolute Error (MAE)**

226 MAE measures average magnitude errors in the set of predictions without considering their
227 direction. It is the average over the test sample of the absolute differences between prediction
228 and actual observation where all individual differences have equal weight:

$$229 \quad MAE = 1/n \sum_{j=1}^n |y_j - \bar{y}_j| \quad (24)$$

230 Where y_j is the observed value and \bar{y}_j is the predicted value.

231 **3.8.4. Mean Absolute Percentage Error (MAPE)**

232 Mean Absolute Percentage Error (MAPE) is a measure of prediction accuracy of a
233 forecasting method of accuracy. MAPE can be stated as

$$234 \quad MAPE = \frac{100\%}{n} \sum_{t=1}^n \left| \frac{y_t - F_t}{y_t} \right| \quad (25)$$

235 Where, y_t is the actual value and F_t is the forecasted value.

236 3.10 Significance test

237 3.10.1 Anderson-Darling Test

238 The Anderson-Darling test is the hypothesized distribution is F , and cumulative distribution
239 is F_n and the formula can be written as

$$240 A^2 = n \int_{-\infty}^{\infty} \frac{(F_n(x) - F(x))^2}{F(x)(1 - F(x))} w(x) dF(x) \quad (26)$$

241 3.10.2 Kolmogorov-Smirnov Test

242 Kolmogorov Smirnov test is a nonparametric test of the equality of continuous one
243 dimensional probability distribution with compare of a sample with reference probability
244 distribution [53,54]. Kolmogorov Smirnov test statistic can be expressed as

$$245 F_n(x) = 1/n \sum_{i=1}^n I_{[-\infty, x]}(X_i) \quad (27)$$

246 Where $I_{[-\infty, x]}(X_i)$ is the indicator function, equal 1 if $(X_i) \leq x$ and equal to 0 otherwise.

247 The Kolmogorov-Smirnov statistic of a given cumulative function $F(x)$ is

$$248 D_n = \sup_x (F_n(x) - F(x)) \quad (28)$$

249 Where \sup is the supremum of the set of distance between the $F_n(x)$ and $F(x)$. In our case this
250 model has been run at 95% significance level.

251 3.10.3 Shapiro -Wilk Test

252 Shapiro and Wilk test of the normality formula can be written as,

$$253 W = \frac{(\sum_{i=1}^n a_i x_i)^2}{\sum_{i=1}^n (x_i - \bar{x})^2} \quad (29)$$

254 a_i is the (a_1, \dots, a_n) , \bar{x} is the mean.

255 The constants a_i can be written as $(a_1, \dots, a_n) = \frac{m^T V^{-1}}{(m^T V^{-1} V^{-1} m)^{1/2}}$ here

256 $m = (m_1, \dots, m_n)^T$ and m_1, \dots, m_n are the expected values of the order
257 statistics of independent and identically distributed random variables sampled from the
258 standard normal distribution, and V is the covariance matrix of those order statistics.

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260 4. Results and Discussion

261 Fluctuation of rainfall and a negative exponential trend are specified in Figure 3 ($Y_t =$
262 $1418.88 \times (0.999642^t)$). Rainfalls are more or less normally distributed at 95% confidence
263 interval (Figure 4a). Residuals versus fit plot (Figure 4b) displays that the points are
264 randomly distributed on both sides of zero with no recognisable patterns thus our rainfall data
265 are having a constant variance. Residuals of rainfall are having a mean close to zero and the
266 histogram is symmetric close to around zero (Figure 4c). Residuals versus order fit (Figure
267 4d) shows that the residuals fall randomly around the centre line. Before proceed with rainfall
268 and estimated 6 indices the reliability of those 6 indices are judged using Cronbach's Alpha.
269 The overall value of Cronbach's alpha is 0.9694. Average SPI and Z-score between the time
270 frame 1901-1939 are -0.06 and 0.299, in between 1940-1980, 0.037 and 0.382 respectively
271 and from 1980-2035 the average SPI and Z-score becomes -2.345. Average PN value from
272 1901-1939 is 100.792%, 1940-1980 PN becomes 100.641%; 1980-2035 it is diminished and
273 become 98.967%. In the same way average DI is estimated and from 1901-1939 DI 5.76%,
274 1940 to 1980 5.73% and DI from 1980 to 2035 4.64% value of DI is obtained. CZI and RAI
275 are also decreased from 0.32 (1901-1939) and 0.38 to 0.26 (1940-1980), 0.28 and later 1980-
276 2035 it reaches to 0.14 and 0.19. Overall all the indices attain negative trend. SPI, PN, DI,
277 RAI, CZI and Z-score are added and a new index Standardized Total Drought (S_d) has been
278 formed to estimate overall trend of meteorological drought of Bankura District. Estimation
279 and prediction of the trend of S_d using the traditional exponential smoothing has been done
280 and a slightly negative trend is obtained (Values reach to -0.143 in 2035) (Figure 5a). The
281 residuals of traditional exponential smoothing trend values are ranging between -15 to +5
282 (Figure 5b). In case of traditional exponential smoothing the average value between 1901-
283 1939 experiences -0.170, 1940 to 1980 the value reaches to -0.034 whereas between the 1980
284 to 2035 the average value attains -0.134 thus overall trend is seemed to be more drought
285 prone in recent upcoming periods. Similarly using Holt-Winter exponential smoothing
286 analysis and prediction of drought has been done (Figure 5c) and residuals are fitted
287 randomly as histogram plot based on the centre line (ranging between -2 to +5 range) (Figure
288 5d). In case of Holt-Winter exponential smoothing the average value between 1901-1939
289 achieve -0.163, between the time frame 1940-1980 and 1980 to 1935 it attain 0.061 and -
290 0.261 values respectively. The combined model Winexpo attains 0.423 for 1901-1939, 0.51
291 for 1940-1980 and -1.423 for 1980-2035.

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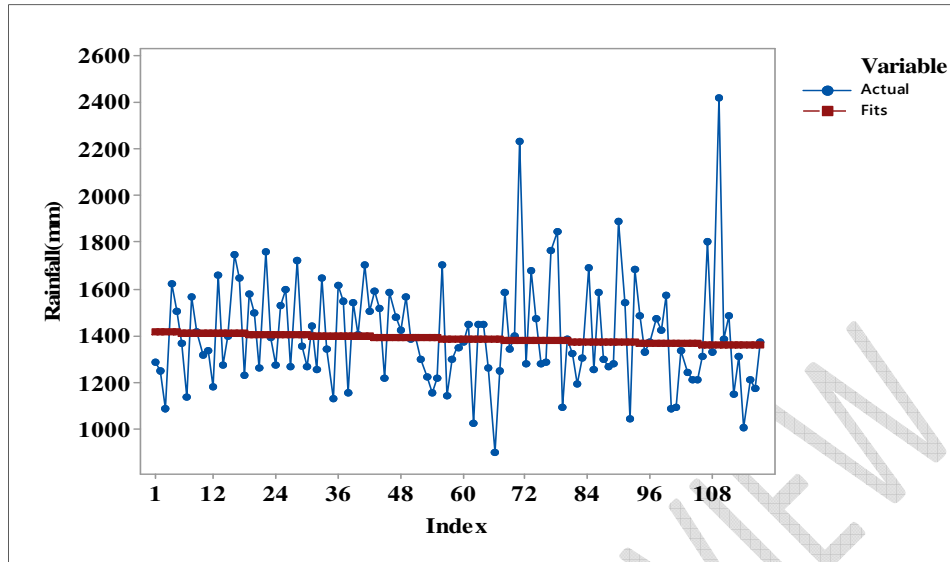
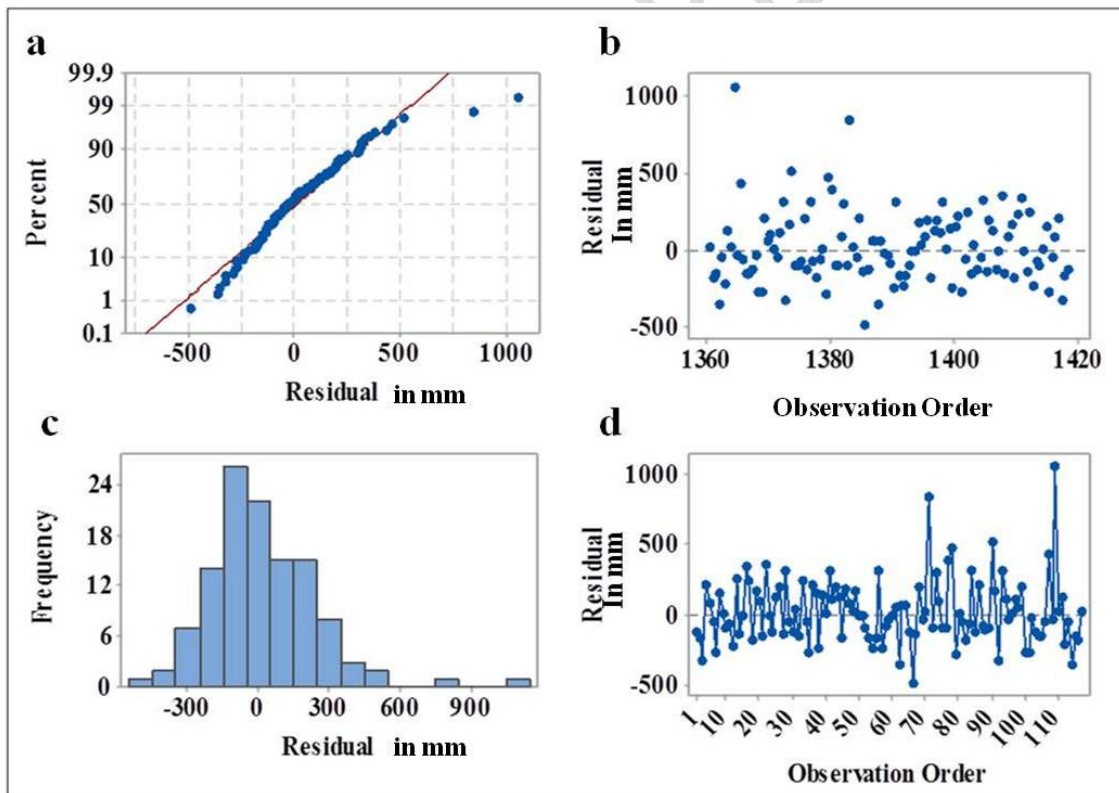


Figure 3 All station accumulated rainfall according to yearly time steps (1901-2017)



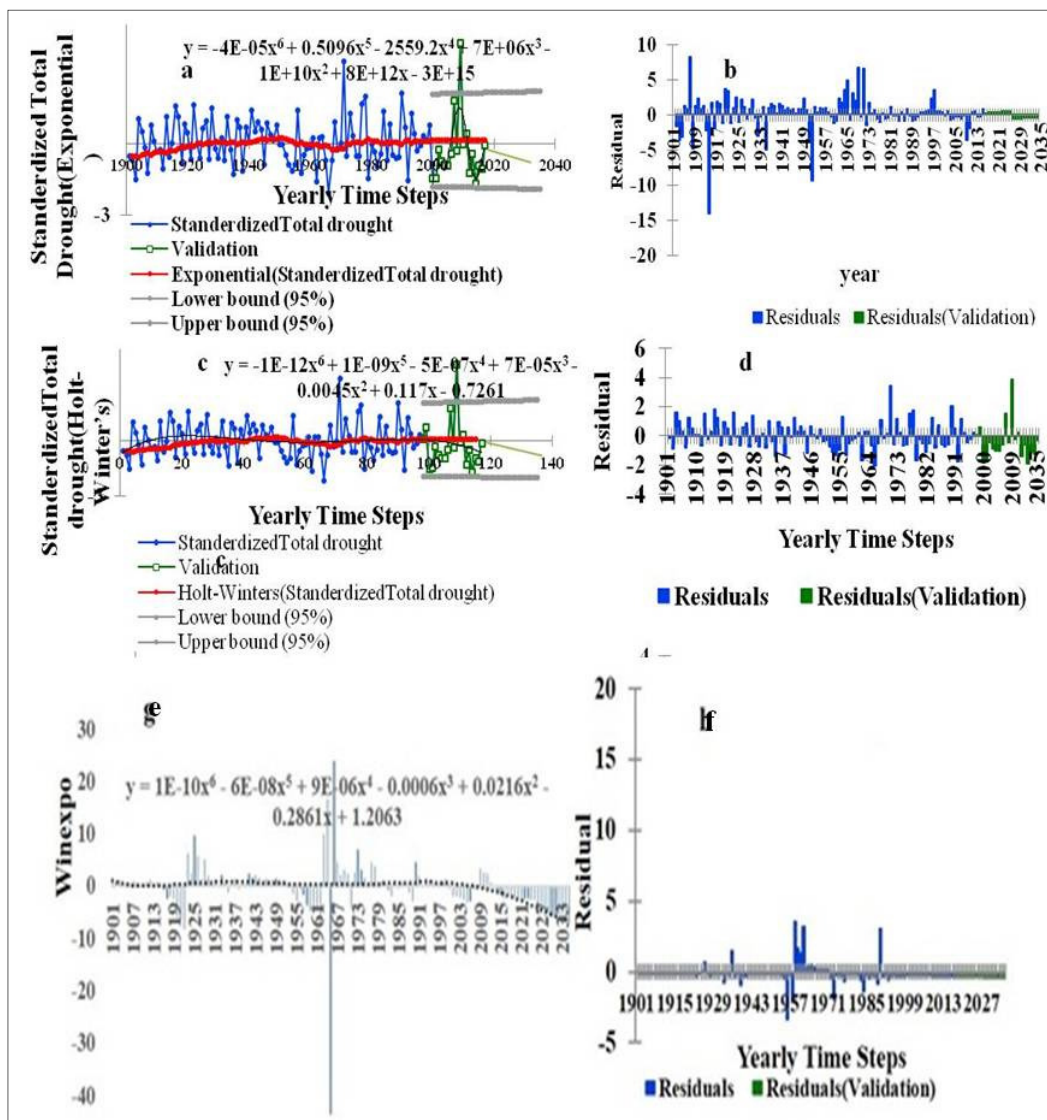
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Figure 4a Normal probability Plot of Rainfall **Figure 4b** Fitted value of rainfall vs. Residual value **Figure 4c** Residual value versus Frequency value **Figure 4d** Observation order vs. Residual value



306 **Figure 5** Exponential Smoothing models and associated Residual Plots a) Exponential
 307 Smoothing c) Holt-Winter Smoothing e) Winexpo Simulation

308 From the true classes determined from the categories of S_d SVM is capable to predict the
 309 nature of drought category. A user friendly SVM tool LSSVM is used to implement the
 310 classification of drought status of Bankura District. At data pre-processing stage raw values
 311 of S_d are linearly rescaled into $[-1, 1]$ using the ranges of their minimums and maximums for
 312 binary distribution of classifiers. Applying the SVM each category against all is estimated in
 313 every case. In case of Extreme vs. others the model is obtained 43 support vectors, for
 314 extreme normal the model is obtained 33 support vectors, for mild drought the model obtains
 315 34 support vectors, most extreme the model obtains 28 support vectors, normal vs. others
 316 obtains 51 support vectors, severe vs. others obtains 8 support vectors and wet vs. others

317 obtains 20 support vectors. From the observed true classes of 135 observations (used
 318 simulated value using Winexpo) drought probability classes are predicted by SVM. SVM
 319 performs with a medium accuracy level. According to SVM identified drought categories
 320 over years over 80% years are concentrated within severe moderate, severe, extreme and
 321 most extreme categories and about 20% years are concentrated within Moderate, Normal, and
 322 Extreme Normal, wet categories (Figure 6a) whereas according to Winexpo identified
 323 drought categories 36% years are mingled with severe moderate, severe, extreme, most
 324 extreme and moderate categories and over 64% are mingled with normal, mild, extreme
 325 normal and wet categories (Figure 6b). The extreme normal versus others, wet versus others,
 326 mild versus others, normal versus others training sample sets achieve over 90% accuracy
 327 whereas extreme and most extreme versus others and severe moderate versus others category
 328 training samples achieve less than 30% accuracy (Table 3). Overall average SVM achieve
 329 0.724 as Cohen's Kappa and overall 60% accuracy has been achieved. So, SVM has
 330 performed moderately well in prediction of drought of our study area.

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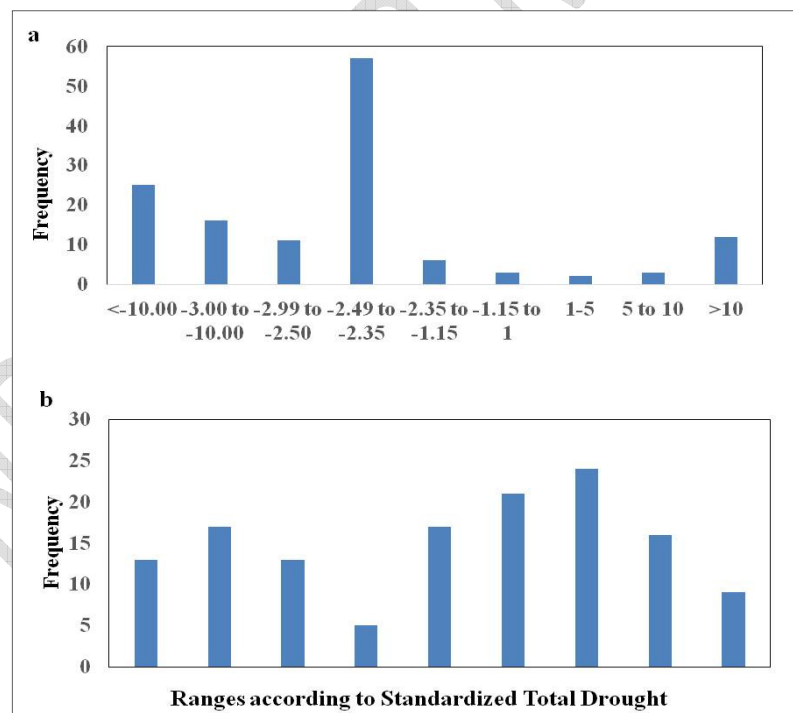
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342 **Figure 6** Frequency of drought under each drought categories a) based on simulation model
 343 of SVM b) based on simulation of Winexpo

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345 **Table 3 Performance matrix of Support Vector Machine (SVM)**

Training set	Accuracy	Cohen's kappa
Extreme versus Others	0.847	0.978
Extreme Normal versus Others	0.187	0.086
Moderate versus Others	0.987	0.987
Most Extreme versus Others	0.847	0.978
Normal versus Others	0.253	0.222
Severe versus Others	0.987	0.998
Severe Moderate versus Others	0.876	0.965
Wet versus Others	0.153	0.042
Mild versus Others	0.165	0.078

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347 The significance test using three individual tests has been run at 95% and 99%
348 confidence interval. The traditional exponential smoothing experiences probability value
349 0.004 for Anderson-Darling test, 0.005 for Shapiro-Wilk test and 0.004 by Kolmogorov-
350 Smirnov test. The Holt-Winter exponential smoothing attains 0.003 probabilities for
351 Anderson-Darling test, 0.004 for Shapiro-Wilk test and 0.001 for Kolmogorov-Smirnov test.
352 Winexpo model also attains probability value 0.002 for Anderson-Darling test, 0.004 for
353 Shapiro-Wilk test and 0.003 for Kolmogorov-Smirnov test. The Bayesian model of LSSVM
354 extreme category vs. other categories experiences 10.275 as Anderson-Darling test statistic
355 value, 0.527 as Shapiro-Wilk test statistic value and 0.435 as KS test statistic value. LSSVM
356 Bayesian most extreme vs. other category is mingled with 5.543 as Anderson-Darling test
357 statistic, 0.727 as Shapiro-Wilk test statistic and 0.316 as KS test statistic. SVM extreme
358 normal vs. other categories achieves 2.165 as Anderson-Darling test statistic, 0.904 as
359 Shapiro-Wilk test statistic and 0.482 as KS test statistic value. Similarly, Mild versus others,
360 severe versus others, severe moderate versus others and wet versus others are also calculated
361 (Table 4). All the Anderson–Darling test is successful and valid at 95% confidence interval as
362 the significance level P-value achieves <0.005 value in all the nine combinations. Shapiro-

363 Wilk and KS test for all the SVM nine possible combinations the probability value is <0.010
 364 that means those values are significant at 99% confidence interval. Overall SVM model is
 365 significant at 95% confidence interval (in case of Anderson-Darling test) and 99%
 366 significance level (in case of Shapiro-Wilk test and KS test). As P values are <0.005 and
 367 <0.010 for all the cases the distribution is not normal here and null hypothesis that there were
 368 no difference between the observed class and predicted class can be rejected and the
 369 alternative hypothesis is accepted. The error estimation and goodness of fit statistics (Table 5)
 370 of the individual models indicate that Winexpo attains the lowest error and highest R-square
 371 value in comparison with the other models altogether.

372 **Table 4 Error Estimation and Goodness of fit statistics (for error estimation 0.001 used**
 373 **as a multiplicative factor)**

Model Name	SE	Adjusted RMSE	Adjusted MAE	Adjusted MAPE	R ² (using Linear kernel)
Traditional exponential smoothing	0.024	0.996	0.790	25.65	0.39
Holt-Winter Smoothing	0.026	1.006	0.654	95.43	0.04
Winexpo Model	0.111	1.64	0.445	49.53	0.35
SVM-Most Extreme versus others	3.080	0.049	0.045	4.559	0.99
SVM-Extreme versus others	1.303	0.038	0.019	2.048	0.94
SVM-Severe versus others	11.180	0.026	0.026	1.915	0.95
SVM-Severe moderate versus others	11.345	0.023	0.045	1.934	0.99
SVM-Moderate versus others	5.533	0.015	0.008	0.833	0.99
SVM-Mild versus others	5.333	0.020	0.013	1.413	0.97
SVM-Normal versus others	1.668	0.033	0.019	2.048	0.52
SVM-Extreme Normal versus others	7.580	0.018	0.014	1.487	0.35

SVM-Wet versus others	83.724	0.001	0.008	0.900	0.34
Overall SVM versus other	0.130	0.02175	0.022	1.904	0.78

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375 **Table 5** Significance test of the models

Standardized Total Drought	Anderson-Darling Test		Shapiro-Wilk Test		Kolmogorov-Smirnov Test		Type of Model
	Test Statistic	Significance Level	Test Statistic	Significance Level	Test Statistic	Significance Level	
Traditional Exponential Smoothing	8.827	0.004 (<0.005)	0.916	0.005 (<0.05)	0.169	0.004 (<0.005)	Exponential Smoothing
Holt-Winter Exponential Smoothing	7.192	0.003 (<0.005)	0.917	0.004 (<0.005)	0.163	0.001 (<0.005)	
Winexpo Model	28.790	0.002 (<0.005)	0.529	0.004 (<0.005)	0.363	0.002 (<0.005)	
SVM-Extreme versus others	10.275	<0.005	0.527	<0.010	0.435	<0.010	Machine Learning
SVM-Extreme normal versus others	2.165	<0.005	0.904	<0.010	0.482	<0.010	
SVM-Mild vs. others	11.598	<0.005	0.482	<0.010	0.419	<0.010	
SVM-Moderate vs. others	10.550	<0.005	0.455	<0.010	0.427	<0.010	
SVM-Most Extreme vs. others	5.543	<0.005	0.727	<0.010	0.316	<0.010	
SVM-Normal vs. others	5.274	<0.005	0.827	<0.010	0.261	<0.010	
SVM-Severe vs. others	5.544	<0.005	0.597	<0.010	0.466	<0.010	
SVM-Severe moderate_vs._others	2.131	<0.005	0.662	<0.010	0.462	<0.010	
SVM-Wet vs. Others	1.108	<0.005	0.935	<0.05	0.236	<0.010	

376 Based on Winexpo and SVM model simulation the hazard prone zones have been estimated
377 (Figure 6). The southern and south-western blocks are extreme drought-prone and northern
378 and north-western blocks are mild to normal mode. The whole regimes form the coherent
379 clusters in space highlighted in figure 7. Most extreme to severe drought categories are
380 clubbed into negative x, y direction and wet categories are clubbed into positive directions of
381 x and y. Based on the whole aspects of meteorological drought the year wise hazard and
382 cumulative failure functions are developed. The most extreme, extreme, severe, severe
383 moderate, moderate and mild categories are included in the category of “hazard prone or
384 failure “whereas normal, extreme normal and wet categories are included in “censored”
385 category. Winexpo attains the best result so this model has been used here. According to
386 simulation of drought category using winexpo, almost 84 observations are fallen into
387 “hazard-prone” category and 51 observations have fallen into the “censored” group. The
388 distribution of yearly censored and failure categories are compared based on Weibull and
389 logistic probability fit but logistic probability fit gave us the better association (Correlation
390 value 0.984 for logistic and 0.678 for Weibull). So, finally the logistic probability fit have
391 been taken for year-wise estimation of cumulative hazard-proneness. The whole logistic
392 model seemed to be more or less normal (Figure 8a and 8b) and it had achieved the 3.223
393 value as the Anderson-Darling test. From the survival function (Figure 8c) fitted based on
394 logistic probability plot encounters the fact that as the time (year) will progress the drought
395 proneness will increase and at the year 2100 the vulnerability will be almost intolerable that
396 will lead to massive disruption over the local community. Reversely, the progression of
397 hazard based on cumulative curve plotting (Figure 9, figure 8d) exhibits the fact that the
398 whole district will be severely affected by drought within 2100. The significance test for
399 hazard function is done in 95% significance level .So, it can be concluded that the district will
400 face extreme to severe drought hazard in the recent future.

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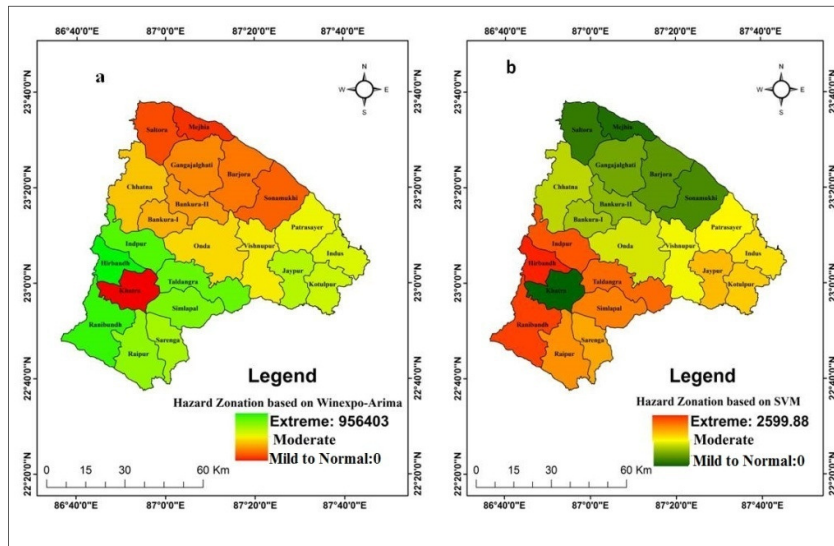
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417 **Figure 7** Drought-prone zone identification (12 month time steps) using a) Winexpo b) SVM

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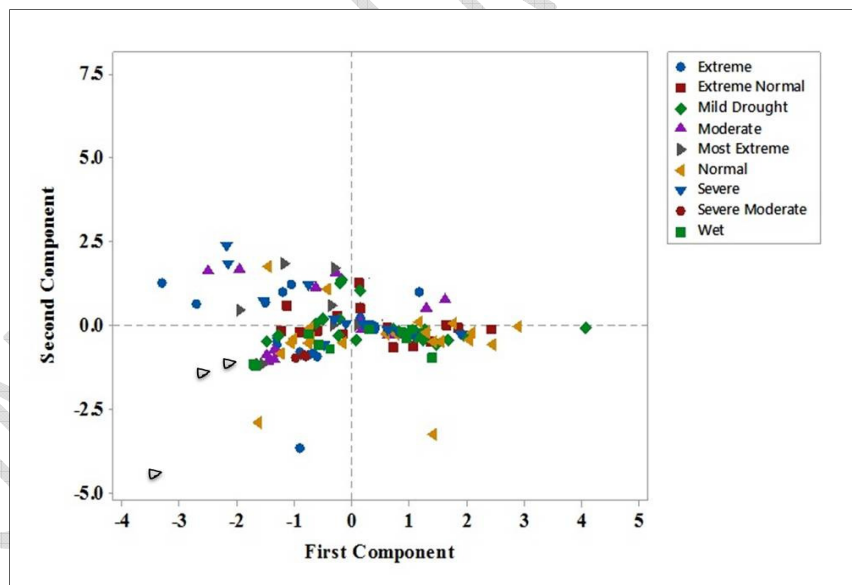
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427 **Figure 8** Plotting of points in the coherent space

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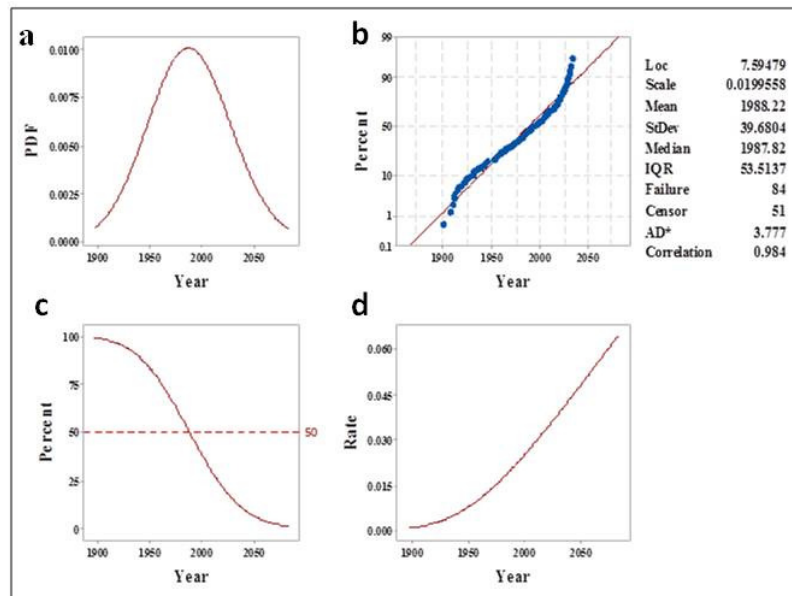
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438 **Figure 9a** Probability density function **b** Logistic probability fit **c** Survival function based on
 439 logistic probability fit **d** Progression of hazard rate with years

440 5. Conclusion

441 The evolution and quantification of drought are necessary for the proper planning and
 442 management of water resources to mitigate the hazard of future occurrences. By far the main
 443 challenge in this field is that a) to identify the correct method to analyze the meteorological
 444 drought b) to identify the spatial dimension over which the drought can be affected c) to
 445 simulate and predict the drought correctly as it is inherently needed for proper planning and
 446 management of water resources. Continuous year wise monitoring and simulation is also an
 447 important issue even seriously neglected in the drought monitoring and assessment. In most
 448 of the cases of drought monitoring and assessment historical rainfall data is one of the input
 449 factors. Our study is also not an exception with the above scenarios. Taking rainfall as the
 450 sole input factor we estimated 6 essential meteorological indices and from those indices we
 451 form a new index Standardized Total Drought (S_d) and simulate it up to 2035 and make a
 452 comparative assessment of exponential smoothing and machine learning procedures.
 453 Cumulative drought-proneness of the region using hazard function has been analysed and we
 454 found that the whole region will be severely drought affected within 2100. The extremities of
 455 rainfall and temperature drive a potential threat to agriculture, food security and socio-
 456 economic vulnerability. Thus a more detailed structural study is required to explore the
 457 synergetic effects of trends and patterns of other climatic variables. However the conclusion

458 reached in this study can be an elementary step to improve the risk management strategy,
459 review of agricultural practices and water use in this counterpart.

460 **Conflict of Interest**

461 There is no conflict of interest regarding the publication of this article.

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