Original Research Article

Model for Mathematics Teachers Development

Abstract

This paper identifies 3 – step model that can be adopted by every mathematics teacher and various training settings to effectively move teaching towards an active learning environment. This model which is built upon existing ideas proposed over the years in education and best practices concerning cognitive development and effective teaching and learning environment tends to equip teachers with very useful skills for classroom instructions. Ultimately, this model can aid teachers to move teaching and learning towards an active learning environment which is more effective and enjoyable for teachers and students for learning.

Keywords: *Teaching model*, *teaching methods*, *critical thinking*, *active learning*

Introduction

The modern day mathematics teacher must not only possess the content knowledge background of the topics in the subject, but must also have the pedagogical content knowledge and adequate classroom management skills to promote active learning. Provision of active learning environment usually makes a subject more enjoyable for both learners and teachers and it also bring about critical thinking among learners. In recent times, mathematics education has been highlighted as a very important subject since almost all domains of human knowledge apply conceptual and computational methods of mathematics (Eshun, 2000). Primarily, Researchers in mathematics education are concerned with the tools, methods and approaches that facilitate practice. Nabie (2004) is with the view that the fundamental objective of mathematics education is to enable children understand, reason and communicate mathematically and solve problems in their daily life. It is believe that teachers in particular and educational planners in general are tasked by society to design practical strategies of teaching and learning that are applicable to the learner's environment and our daily life situations (Sapkova, 2011). The teacher is seen as the center of every education in most communities. It is the core duty of mathematics educators to provide experience that will continue to foster students understanding and appreciation of mathematics to improve on their performance upon the identification of their challenges. Therefore, it is important that the development of a mathematics teacher in his/her work should be of great concern to all.

Granström (2006) is with the view that different teaching approaches in classrooms influence the outcomes for students in different ways. In classroom settings where students are allowed and encouraged to cooperate with classmates, teachers give the students more opportunities to understand and succeed (Andam, Atteh and Obeng – Denteh, 2016). Good teaching involves communication and building relationships with students (Oppendekker & Van Damme, 2006). Reynolds and Muijs (1999) also hold the view that, an effective teaching is signified by a high number of opportunities to learn; where the opportunity to learn consists of factors such as length of school days and year and the number of hours for mathematics

lessons. It also includes the quality of classroom management, especially time – on – task. The teacher‘s achievement is improved when the teacher creates classrooms that include:

- substantial emphasis on academic instruction and students engagement in academic tasks (Cooney, 1994)
- effective question – answer and individual practices (Borich, 1996)
- minimal disruptive behaviour
- high teacher expectations (Clarke, 1997)
- substantial feedback to students (Borich, 1996)

The role of a teacher in the classroom is to guide students in achieving better understanding and not as the only source of knowledge and authority in the classroom (Atteh *et al.,* 2014). In successful teaching, teachers are actively asking a lot of questions and students are involved in class discussions. And in addition to active discussion, students are kept involved in the lesson and the teacher has a chance to continually monitor students' understanding of the concept being taught. In furtherance, teaching should be done in a way of allowing students to wonder why things are, to inquire, to search for solution and to resolve incongruities and not teachers acting as the custodian of knowledge (Andam *et al.,* 2015). Classroom management represents a sizable obstacle to most teachers of today. The issue of ethics in education in general and classroom management in particular must be a concern to all. This research identifies a 3 – step model that can be implemented in all educational settings to effectively help teachers to move towards active learning environment. This model provides mathematics teachers with a very useful assistance which intends to move teaching and learning from lecture – based learning environment towards an active learning environment.

Why teachers development?

The school mathematics curriculum, the teaching and learning of the subject, have become critical issues in most countries over the years. Due to these issues, the school mathematics curricula have been undergoing numerous changes and the evolution of these new school curricula and methods are designed in ways of empowering students to use practical and investigative approaches when learning mathematics (Thomasenia, 2000). In view of this, NCTM (1989) provided a new wave of change affecting how mathematics should be taught and learned in schools. In this agenda, it was noted that there was the need to pay particular attention to *how mathematics is taught* instead of concentrating on *what mathematics was taught* in schools. The sole aim of this agenda was to increase students' participation and engagement in the teaching – learning process by decreasing memorisation of algorithms and reducing teachers' power of being the disseminators of knowledge to their becoming facilitators in the teaching – learning process (NCTM, 1991). However, in the United Kingdom, reforms of mathematics teaching and learning started in the late 1980 with the introduction of a national curriculum and the introduction of new instructional practices (Chambers, 2008). Chambers (2008) further stated that, this new school mathematics

curriculum was therefore aimed at providing a new mathematics classroom environment that promotes conceptual understanding of mathematical concepts and skills through problem solving. The curriculum also aimed at assisting students to develop their own mathematical skills and competencies. In similar situation, the Chinese school mathematics curricula experienced dramatic changes in the late 1990's (Liu and Li, 2010). According to Liu and Li (2010), the changes included "many different aspects of mathematics education ranging from what is valued for all students to learn, how mathematics should be taught and learned, and how the assessment should be viewed and used" (p. 10). They further explain that, the purpose of these dramatic changes was to help and motivate students in learning mathematics through creativity and independent learning which stimulates students' conceptual understanding and interest.

According to Ministry of Education, Science and Sports (2007), Ghana introduced a new mathematics curriculum in 2007 and the aim of this new curriculum was based on the twin premises that all can learn mathematics and that all need to learn mathematics with a view to achieving a curriculum that reflects individual students' needs. The main goal of the new curriculum is to enable all students' acquire the mathematical skills, insight, attitudes and values needed to be successful in their chosen careers and daily lives by increasing their self – oriented learning abilities to the maximum. The curriculum however encourages the acquisition of more skills and use of different teaching methods and resources to help students to develop the mathematical skills that they will need in their daily life activities (MoESS, 2007). The new curriculum further aims at bringing a shift from a teacher – centered approach of teaching and learning to a more participatory teaching and learning methods to help students develop their skills through the application and experimentation of different problem solving skills (MoESS, 2007). The new curriculum advocated for constructivism and the change in teachers' role as custodian of knowledge to facilitators in the teaching and learning process like other school curricula around the world.

However, in the idea of Shulman (1987), to be able to teach all students according to the standards of today, teachers need to understand subject matter deeply and flexibly so they can help students create useful cognitive maps, relate one idea to another, and address misconceptions. Teachers need to see how ideas connect across fields and to everyday activities. In addition, this kind of understanding provides a foundation for Pedagogical Content Knowledge that enables teachers to make ideas accessible to students. This shows that teaching is far more than mere transmitting of concepts and ideas to learners, but it involves bringing out the accumulated ideas and experiences that students come to class with and working on those ideas and experiences together with the students by way of refining, reorganizing, co-constructing and repairing these ideas and experiences into meaningful and comprehensible form for students to assimilate (Shulman, 2000). This therefore indicates that for teachers to teach mathematics effectively, they need to have an in – depth understanding of the mathematical content at hand, the pedagogical principles of the various mathematical topics and curricular materials that inform the scope and direction of teaching and learning mathematics. Shulman (2000) continued that, teaching is about making the internal and external capabilities of an individual and can only be achieved if teachers engage students in

the classroom discourse. It is only when students are engaged in an interactive classroom environment that their ideas, conceptions and experiences are made bare to the teacher to help correct them. The following framework (Figure 1) is a 3 – step model that can be adopted by any mathematics teachers or training setting to help teachers acquire appropriate teaching and learning skills.

The 3 – step model for teacher's development

Step 1: Subject Matter Content Knowledge (SMCK)

Teachers' knowledge must therefore go beyond mere definitions of accepted truths in the subject matter domain and the understanding of mathematical concept should not mean so much to the teacher, but the teacher must further understand why it is so. According to Shulman (1986), Subject Matter Content Knowledge is the amount and organization of knowledge intrinsically in the mind of the teacher. Shulman further argues that teachers' subject matter content knowledge should not be limited to knowledge of facts and procedures; but also an understanding of both the substantive and syntactic structures of the subject matter. The substantive structures comprise the various ways in which the basic concepts and principles of the discipline are organized to incorporate its facts (Shulman, 1986). Teachers will therefore be able to use appropriate materials to teach mathematics well only when they comprehend the network of fundamental concepts and principles of the subject matter at stake. The syntactic structure of a discipline is the set of ways in which truth or falsehood, validity or invalidity are established (Shulman, 1986). And Shulman again explains that, a teacher should be able to explain to his/her students why a particular proposition is deemed justified, the value of knowing it and how it relates to other propositions within or without the discipline and both in theory and in practice. The possession of knowledge on the syntactic and substantive structures of the subject matter assists teachers to teach effectively. The syntactic and substantive structures will enable teachers to clarify and correct students' errors and misconceptions in the teaching and learning process through the process of scrutinizing, analyzing, justifying students' solution.

In the view of Ball, Hill and Bass (2005), they suggested that teachers' use of instructional materials, their ways of assessing students' progress and how they make sound judgments about representations, emphasis and sequencing depend on their mathematical content knowledge for teaching. Therefore in teaching mathematics, the teacher needs to have thorough content knowledge for selecting, designing and using appropriate instructional materials that covers the concepts. And to a large extent, the teachers' ability to choose useful methods and pose appropriate examples to students in a mathematics lesson is dependent on their mathematical content knowledge. Asiedu – Addo and Yidana (2004) hold the view that, in situations where (teachers) knowledge is more explicit, better connected and more integrated, they will tend to teach the subject more dynamically, represent it in more varied ways, encourage and respond fully to students comments and questions. Where their knowledge is limited, they will tend to depend on the text for content, emphasize interactive

discourse in favour of seatwork assignments and in general portray the subject as a collection of static and factual knowledge.

Moreover, knowing that the teaching of mathematics demands a kind of depth and detail knowledge that goes well beyond what is needed to carry out the algorithm reliably to include considerations in choosing good examples for instructional purposes (Ball, Hill and Bass 2005). The teaching of mathematics depends so much on teachers' subject matter knowledge because teachers need to evaluate strategies often used by students to obtain correct solutions, but whose mathematical validity are immediately not clear. In a situation where a teacher is deficient in the subject matter knowledge of mathematics topics it becomes practically impossible for him/her to effectively teach mathematics.

Step 2: Pedagogical Content knowledge (PCK)

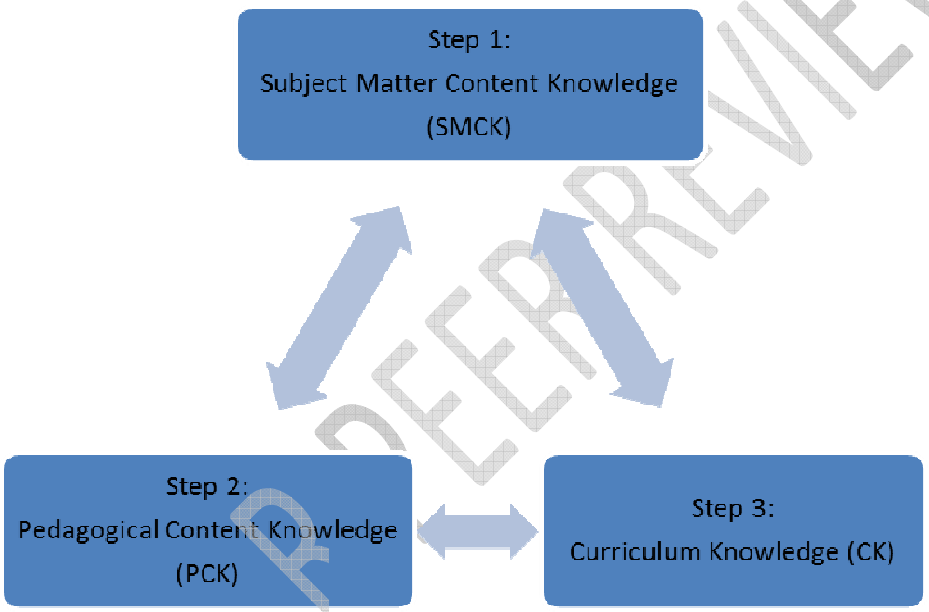
Pedagogical Content Knowledge describes the ways of representing and formulating the subject matter that makes it comprehensible to students with diverse views and understandings. Shulman (1986) is of the view that, pedagogical content knowledge is knowledge about how to combine pedagogy and content effectively. This includes, knowing what approaches fit the content, knowing how elements of content can be arranged for better teaching. It also involves knowledge of teaching strategies that incorporate appropriate conceptual representations to address learner difficulties and misconceptions and foster meaningful understanding and knowledge of what the students bring to the learning situation; knowledge that might be either facilitative or dysfunctional for the particular learning task at hand. Shulman (1986) further explained the pedagogical content knowledge as the combination of the most regular taught topics, the most useful forms of representations of those ideas, the most powerful analogies, examples, illustrations, explanations and demonstrations in the art of teaching. In teaching mathematics through activity oriented base, teachers need to design and present the lesson using appropriate teaching learning materials (TLMs) that can enable the students construct their own knowledge of the concept.

As mathematics teachers, they need to know the pedagogical strategies and techniques most appropriate for reorganizing the understanding of learners who might appear before them as blank slates (Shulman, 2000).

In the view of Harris, Mishra and Koehler (2009), the Pedagogical content knowledge includes generic knowledge about how students learn, teaching approaches, methods of assessment, and knowledge of different theories about learning. Pedagogical content knowledge also entails an understanding of what makes the learning of specific topics difficult, the conceptions and preconceptions that students of different ages and backgrounds often bring with them to the learning environment. Most of these preconceptions are often misconceptions. Pedagogical content knowledge helps teachers to anticipate students' learning difficulties and to provide available alternative models or explanations to mediate those difficulties (Shulman, 1986).

FIGURE 1

The 3-Step Model for Teachers Carrier Development



Ball and Bass (2000) described Pedagogical content knowledge for teaching mathematics as a specialized form of knowledge that combines mathematical knowledge with knowledge of learners, learning and pedagogy. This indicate that teachers need to have control of the subject matter, knowledge about the learners, their strengths and weaknesses as well as resource with varied instructional strategies before they can teach mathematics effectively. And when teachers are prepared to harness all possible pedagogical strategies of teaching and learning and make use of them in the classroom it is likely to improve the teaching of mathematics in the curriculum.

Step 3: Curricular Knowledge (CK)

The *curriculum* is viewed as a composite whole that includes the learner, the teacher, teaching and learning methodologies, anticipated and unanticipated experiences, outputs and outcomes possible within a learning institution. According to Mereku and Agbemaka (2009), Curriculum is the planned and guided learning experiences and intended outcomes, formulated through the systematic reconstruction of knowledge and experience under the auspices of the school, for the learner's continuous and willful growth in personal – social competence. And for Shulman (1986) the mathematics curriculum is represented by a full range of programs designed for the teaching of mathematics topics at a given grade level. It covers a wide variety of instructional materials available in relation to the subject matter to be handled, and the set of characteristics that guides the use of particular curriculum materials in particular circumstances. It is anything and everything that teaches a lesson planned or otherwise. Humans are born learning, thus the learned curriculum actually encompasses a combination of the hidden, null, written, political and societal and so on. Since students learn at all times through exposure and modeled behaviours, it means that they learn important social and emotional lessons from everyone who is in the school.

This highlights the fact that the curriculum must take into account not only established knowledge but also emergent knowledge. This is because curriculum while transmitting the cumulative tradition of knowledge also concerns with the systematic reconstruction of knowledge in relation to the life experience, growth and development of the learner (Mereku and Agbemaka, 2009). Mathematics teachers need to have thorough understandings of the curricular resources available for mathematics instructions so as to make them available to students during teaching. In the view of Ball and Bass (2000), teachers need to think wide about students' mathematical ideas, analyse textbook presentations, and judge the relative value of two different representations in the face of a particular mathematical issue. The theoretical basis on which the concept of teaching mathematics is built on, are the ideas of subject matter content knowledge, pedagogical content knowledge and curricular knowledge. For teachers to teach mathematics effectively, they need to have thorough understanding of the curricular resources available for instruction so as to make them available to students when teaching mathematics for students to make their own meaning of concepts.

Discussion and Conclusion

For the past thirty years, there has been growing concern about falling standards of students' achievements in mathematics at both national and international levels (Blum 2002; Törner and Sriraman 2006). This is why it has been agreed in a broad consensus among mathematics education researchers that the goal of mathematics instructions is not only for students to memorise procedures and acquire reliable methods for producing correct solutions on paper-and-pencil exercises but rather students should learn mathematics with understanding (National Council of Teachers of Mathematics [NCTM], 2000). According to National Mathematics Advisory Panel (2008), the main purpose for teaching and learning mathematics is to develop the ability of the learner to solve a wide variety of both simple and complex mathematics problems in their everyday lives.

It is necessary that teachers earmark considerable time to investigate into current instructional methods and the learning outcomes that drive them to contemplating this particular approach to teaching. Implementing various teaching methods through this model clearly requires a commitment on the part of teachers and the institutional heads as well, at least initially, may be somewhat unfamiliar and uncomfortable to both teachers and head teachers. Through proper planning and creativity, the potential roadblocks to the implementation of this model can be overcomed. Although there is little question that class size and time constraints may influence a particular method of teaching, it is still possible to effectively engage students in large groups.

Specific mathematics topics may also be construed as a limiting factor when considering teaching methods that encourages meaningful learning. With the universally held belief that students need to do more than just listen to learn, a survey of professors in the United States found that 89% of physical scientists and mathematicians use lecturing as their mode of instruction (Chickering and Gamson, 1987). However, considering the subject matter content and the curriculum knowledge in mathematics topics becomes a prerequisite for choosing very effective pedagogical approach that encourages teaching and learning in mathematics classroom. In a mathematics classroom, students are engaged in more activities including debate, dialog, problem solving and writing than just listening (Atteh *et al.*, 2014). This encourages critical thinking among students which can be incorporated into other subject areas as well to solve problems (Atteh, Andam & Obeng – Denteh, 2017).

The effective use of the 3 – step model to help teachers select an appropriate teaching method may lead to change in instructional technique from that of the traditional lecture – based format of teaching, which will likely, brings out a kind of learning experiences that are more enjoyable and interesting to students and teachers.

Competing Interest

Authors have declared that no competing interests exist.

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