Japan's Demographic-Momentum on a Deflationary Spiral Tracing a Path toward Extinction: Ecological Economics of a Declining Population

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ABSTRACT

In the future, Japan will suffer an accelerating decrease of the population, which is predicted by a demographic report of the National Institute of Population and Social Security Research (IPSS). The authors have reanalyzed this issue from a viewpoint of ecological economics and resource statistics and have come to confirm that the results of IPSS indicate Japan's population to have entered an "extinction mode".

An economic situation called "deflationary spiral" means a vicious cycle in which a decline in income levels causes a general price reduction and this reduction causes a further decline in income levels. In Japan, there is an ongoing cycle of this kind that a population decrease leads to a socio-economic contraction, which in turn will cause a further decrease in population. Such a situation can be called a "deflationary spiral of the population". This research treats this population down-spiral in Japan with an analytical approach considerably different from the usual demography.

In the meantime, some UN statistics show that Asian countries are highly liable to face a population decline, which may realize, following Japan, after some decades toward the end of the 21st century. We would like to show that Japan's case can provide an effective and forward-looking suggestion for these countries.

1.THE BACKGROUND AND THE PURPOSE OF THIS RESEARCH

Japan *'is entering an uncharted societal stage of perpetual population declining, after achieving the world's longest life expectancy'*. [1]

We do not know how far this fact is recognized worldwide; however, the population-dynamical prediction of the National Institute of Population and Social Security Research (IPSS) shows an objective estimation of harsh Japanese population decline. (Fig.1)

The IPSS says that *"As in the previous projections, the cohort component method is used for the Population Projections for Japan. This is a method of projecting the future population of each age- and sex-specific group according to assumptions about three components of population change, namely fertility, mortality, and migration"* [2] ; where 'cohort' means, in their estimation, a birth cohort born in the same year (a cohort in general means a group that shares demographic events such as birth or marriage at the same time).
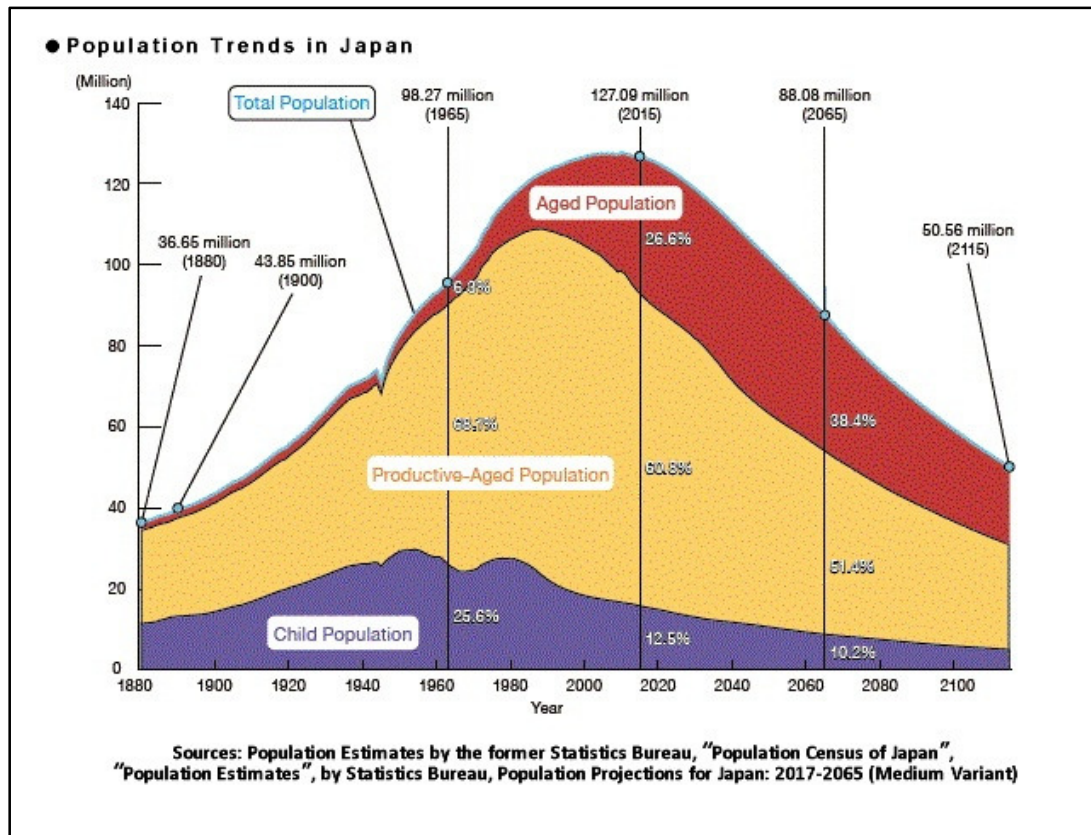
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Figure 1. Population Trends in Japan Source: [3]

Estimation of future populations is also called "population projection" because it calculates population trends from the past to the present and 'projects' the results for the future, by setting certain adequate assumptions. The simplest way of population projection is a "function fitting method" which applies a mathematical function to express the trend of the population in the past. However, IPSS does not adopt this method seemingly because they cannot find an appropriate function to fit the hill-shaped curve. Indeed, the population curve of Japan does not conform either to the exponential function (to rise ever acceleratingly) or to the logistic function (to saturate asymptotically).

On dealing with a function curve to show a hill-like shape, what comes first to mind would be a normal distribution curve; however, there is another important case, "the Hubbert function" especially in the field of resource statistics and ecological economics.
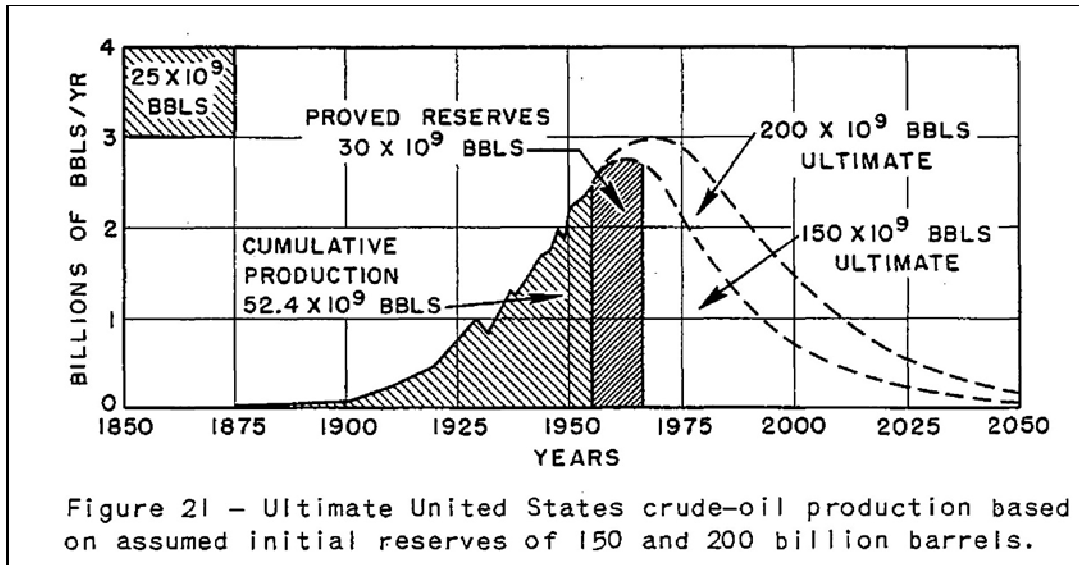
Our research question is very simple: is it possible to find a proper analytical function to successfully describe Japan's population dynamics.

The "Hubbert function" is therefore applied to a sequence of <the past population transition and the population projection> by IPSS (on the cohort factor method). Let us examine this assumption (Fig. 1).

2.JAPAN'S DEMOGRAPHIC DYNAMICS TO REGRESS TO A HUBBERT CURVE

The Hubbert function is a formula originally used by M. King Hubbert to describe the production of exhaustible resources, specifically to show trends of oil and coal production in the mainland United States. Although this mathematical equation itself has been known as "the first derivative of logistic function" from a long time ago, it is Hubbert who utilized this

54 function systematically in the context of growth theory or growth limit theory. The naming as
55 "Hubbert curve or function" comes from his theoretical achievement.
56 The Hubbert curve basically represents a transition of "annual productions" of exhaustible
57 resources. He predicted in 1956 that the crude oil production in the southern US would
58 follow a bell-shaped curve and that it would reach a peak in the late 1960s (Fig.2). The
59 subsequent production of crude oil in the United States had nearly reconfirmed Hubbert's
60 prediction and his analysis was accepted as the first theoretical accomplishment for
61 forecasting resource productions.
62 He assumed that cumulative production of fossil fuels ($\leq$ the ultimate reserves) should
63 generally follow the logistic function, and therefore their annual production could be
64 estimated by its differential.
65

Figure 2I — Ultimate United States crude-oil production based on assumed initial reserves of 150 and 200 billion barrels.

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67
68 **Figure 2. Estimation of M. K. Hubbert**Source: [4]
69
70 If the time and the height of the vertex and the point corresponding to the half of the vertex
71 height can be determined, these points can designate the Hubbert function parameters.
72 The population of Japan reached its peak at 128,084 thousand in 2008. The half of the peak
73 value was 64,400 thousand. This population lies halfway between in 1929 (63,461 thousand)
74 and in1930 (64,450 thousand) and the half-peak point can be assigned approximately to
75 1929.5. The time interval between 1929.5 and 2008 is 78.5 years.
76 We tried to derive the exponential coefficient of the Hubbert function under this condition
77 (see Appendix for details). Together with Japan's population dynamics, we draw the curve at
78 the top of in 2008, between in 2115 (the IPSS's research forecast, it is 107 years ahead from
79 2008) and in 1901 (107 years before from 2008).
80 Next, the real population from 1901 to 2015 is plotted as it is; and after that, the closest
81 values among the estimated values of the nine types denoted by the IPSS (Table 1) are
82 plotted we extended it until 2115.
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90 **Table 1.   Nine Types Suggested by the IPSS**

91

	Fertility		
(Mortality)	Low-fertility (Low-mortality)	Medium-fertility (Low-mortality)	High-fertility (Low-mortality)
	Low-fertility (Medium-mortality)	Medium-fertility (Medium-mortality)	High-fertility (Medium-mortality)
	Low-fertility (High-mortality)	Medium-fertility (High-mortality)	High-fertility (High-mortality)

Projection	Low Variant	Medium Variant	High Variant

92
93
94 Figure 3 shows the superimposition of the regressed Hubbert function and the IPSS data
95 close to the former.
96 The demographic dynamics of our country is confirmed clearly to have regressed to the
97 Hubbert curve from the past to the future. It may be said that the future part of the Hubbert
98 curve approximately suggests the most probable values (maximum likelihood) out of the nine
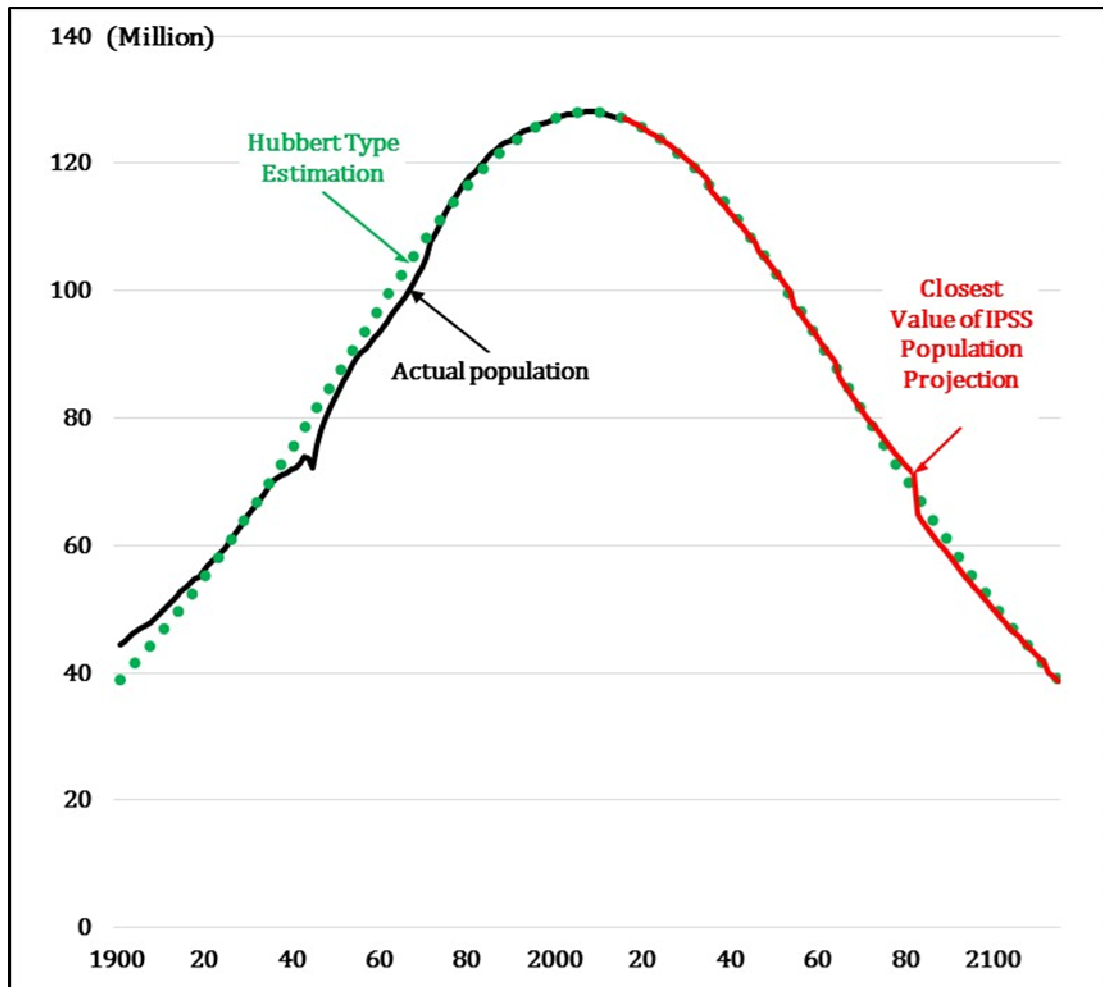99 estimates by IPSS.

Figure 3.   Regression of the Population Trends to the Hubert Type Estimation

3. THE HUBBERT THEORY

The Hubbert function and its differentiation from the conventional growth function have been depicted in Fig. 4 below.
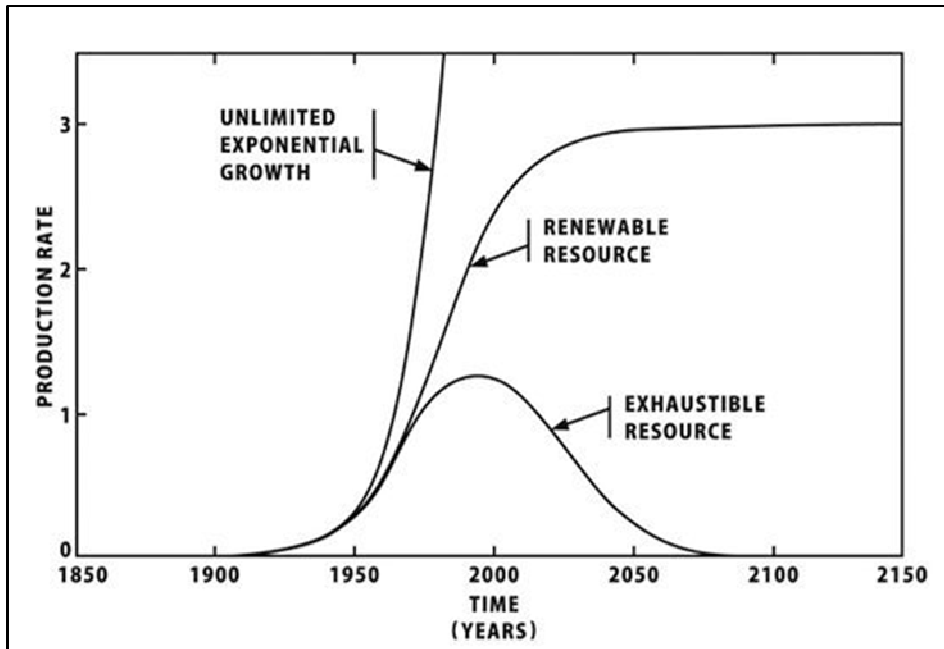
Figure 4. The type of growth Source: [5]

In Fig. 4, the curve of "unlimited exponential growth" is an exponential function, the curve of "renewable resource" is a logistic function, and the curve of "exhaustible resource" is the time derivative of the logistic function (the Hubbert function).

As seen clearly from this figure, both the exponential function (the first type) and the logistic function (the second type) are monotonically increasing, and essentially express an aspect of perpetual growth. Only the function of the third kind can express a trend toward a decrease through the peak.

Growth = increase had been predominant in the past human culture. Therefore, the first and the second type alone are recognized, and discussion of growth rate has been deemed sufficient to argue, where the highest rate corresponds to an exponential function and the lowest rate, to a logistic function.

In contrast, Hubbert insisted that the third type, an <increase/peak/decrease> mode should also be included in the group of "growth patterns" (secular changes). And he proved that infinite growth (exponential type) is only a temporary/transient phenomenon in human history.

As far as we deal with a process of growth, we can presuppose that the growth curve will neither exceed the exponential function nor fall below the logistic function. If all the growth patterns that could happen have to fall in between the two curves, to deal with both sides would be sufficient. By the way, even a Hubbert curve of the third kind shows a sharp increase for a considerable period (e.g., in Japan's population, until the latter half of 20th century), like the first and the second type. In other words, during this period we cannot tell which of these three courses represent the actual growth trajectory. Hubbert had recognized this complex situation from the beginning. Therefore, he pointed out that, transitionally, an exponential growth would seem to be a general phenomenon.

However, Hubbert's initial indication was released at the heyday of the "infinite growth" theory. As a result, his pioneering research was severely reproached or neglected by the then mainstream economists; his theory had been pushed into a corner of an academic journal.

141 Reevaluation of Hubbert in earnest began when the "peak oil" was recognized as a real
142 problem in the 21st century.
143
4. WHY DOES JAPAN'S POPULATION DYNAMICS DRAW THE HUBBERT CURVE?

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147 Then why do Japan's population dynamics follow the Hubbert curve? What mechanism has
148 caused this phenomenon? These questionnaires are very difficult in dynamic perspectives.
149 Tentatively, it might be considered as follows.
150 First, consider one birth cohort, for example, the generation born in 1955 (population group).
151 Perhaps the cumulative number of children whose would-be mothers were born that year will
152 start to rise from around 15 years later (1970), will peak at around 30 years later (1985), and
153 will draw a logistic curve with a ceiling at about 40 years later. If so, the number of annual
154 births will be on the Hubbert curve. On the other hand, the cumulative number of deaths of
155 the people who were born in the same 1955 will begin to rise from the age of 70 (i.e., 2025),
156 will be accelerated around the age 80 to 85, and will draw a logistic curve ending at about 95
157 years old. If so, the annual number of deaths will also follow the Hubbert curve. Naturally,
158 every cohort will show the same trend.
159 However, if and only if a cohort can reproduce a next generation greater than itself, the birth
160 trend and the mortality trend will be offset, and exhaustion will not occur. But, since this
161 depletion offsetting has been lost in present Japan, the trend of the total population
162 comprising all the generations also seems to follow the Hubbert curve.
163 Here, the number of newborn babies has a decisive significance for either case of the
164 increase or the decrease. On the left side of the Hubbert curve (increasing phase), it will
165 determine an increasing trend. And next, the vertex becomes broad because of the
166 prolonged lifetime. After that, on the right side of the Hubbert curve (decreasing phase), the
167 increasing death of the generation, who once took up the population increase, dominates the
168 decreasing trend.
169
5. THE POPULATION DYNAMICS AND THE HUBBERT CURVE

171
172 The Logistic curve to express the logistic function of Verhulst and the Hubbert curve, the
173 derivative of the logistic function, have different meanings for renewable resources versus
174 for exhaustible resources.
175 In the case of renewable resources, the [Logistic curve] shows the "proliferation potential"
176 under a constant reproductive condition, therefore its derivative, [Hubbert curve], represents
177 "annual proliferation amount possible". However, it does not mean that the proliferation is
178 unilateral along the time axis. The proliferation never goes out of the [Logistic curve]
179 trajectory, but the movement is reversible, and it is possible to go backward. When collecting
180 biological resources such as bluefin tuna, as far as the growth (supply) and the collection
181 (consumption) are balanced, the state can be sustained (steady state) and will remain at a
182 certain point on the logistic curve. Although the curve from under constraints should bring a
183 growth saturation (a gradual decrease in growth), it will not result in reproduction shrinkage.
184 This situation should apply also to the human population, which in principle belongs to a
185 "renewable resource" as a kind of biological resources.
186 In contrast, in the case of exhaustible resources, there can be no "proliferation"; there is only
187 "withdrawal - attenuation/depletion" of resource reserve. Therefore, the Logistic curve shows
188 a unilateral time transition of the "cumulative output" (under a given state of technology). The
189 differential [Hubbert curve] represents "an amount that can be redeemed annually" and
190 proceeds unilaterally and irreversibly to the right side on its track as far as the withdrawal
191 (mining) continues. For this reason, unilateral contraction is inevitable after the peak. Even if
192 the pace might slow down, the reducing trend of annual output will remain firmly rigid.

193 The population of Japan should be, intrinsically, a "renewable resource" (an attribute
194 common to all living things). Therefore, the growth trajectory of the population should be on
195 the [Logistic curve]. However, the actual population of Japan deviates far below this orbit;
196 and it fits perfectly for the [Hubbert curve] that peaked at 128,084 thousand people in 2008.
197 What does the [Hubbert curve] mean at the stage of beginning and acceleration of the
198 resource decrease? It represents, for example, an annual output of an ageing oil field or a
199 coal field past one's heyday. It is a stage where the annual output declines steadily as the
200 residual reserve decreases. Ultimately, the "Hubbert curve" at this stage represents the
201 route leading to resources depletion/extinction.
202 The fact that the population transition of Japan matches this function means that the total
203 population runs on an "orbit to depletion". Unless the population trend can escape from this
204 route, the end will be "extinction."
205 Furthermore, the Hubbert curve is determined mechanically from the statistics of population
206 transition. In other words, it does not allow arbitrary operations such as "optimistic outlook"
207 by taking into consideration factors for example "improving birthrate".
208 Even if the total fertility rate of the current 0-year-old children can recover the replacement
209 level immediately, yet it will be around 20 to 40 years later that the actual decrease trend
210 becomes moderate and the total population move toward a steady state: the time when the
211 newborn women will reach childbearing ages.
212 Anyway, the fact that the population dynamics of Japan is on the Hubbert function is
213 'astonishing' at all events. This cruel fact compels us again to recognize that each cohort is
214 an "exhaustible resource" in the sense that everyone dies sooner or later.
215 Moreover, unlike underground resources such as fossil fuels or minerals, the "exhaustible
216 resources" of the population does not accept any artificial control like "resource
217 conservation". After passing the peak, it will proceed irreversibly toward shrinkage.
218 Population decline is a "confirmed future".
219 This population trend seems not limited to Japan alone. Many other countries are falling into
220 the same situation. Let us see it in the following.
221
222 6. JAPAN'S POPULATION PROBLEMS TO PRECEDE THE WORLDWIDE
223 CIRCUMSTANCES
224
225 Inaba [1] introduced that, according to C. Wilson and G. Pison [6]half of the world's
226 population by 2003 lived in countries where the average birthrate per woman is less than
227 2.1. Considering this, Inaba [2008] said that'*The level of Total Fertility Rate (TFR) of 2.1 is*
228 *the population replacement level that allows for simple reproduction of the population in*
229 *developed countries. In higher death rate countries, it is impossible to simple reproduction*
230 *on the levels. Therefore, now more than half of the world people lives in a low-birthrate*
231 *society that cannot hold a simple reproduction". Furthermore, he commented that "we have*
232 *an estimate that the probability that the world population will reach a negative growth rate by*
233 *the end of the 21st century is about 80%*[7].
234 The United Nations has released estimates that will back it up. First, let's see what the
235 population trends of the world's largest population from 1st to 3rd, China, India and the
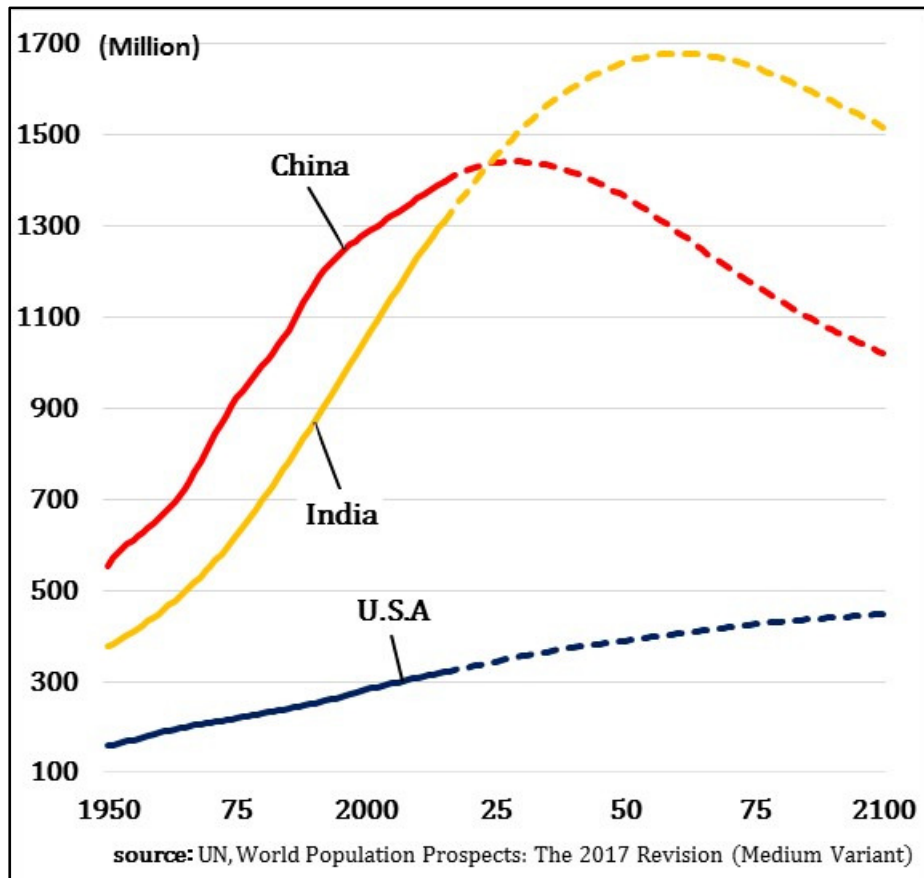236 United States. (FIG. 5)
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238
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Figure 5. Demographic Dynamics Forecast for China, India, and U.S.A Source: UN data [8] adopted by the authors.

It is estimated that China will reach the population peak (1.44 billion people) by 2030 when China will give the top position to India, and the population will shrink down to 1.02 billion in 2100. Also, it is estimated that India will reach 1.68 billion people by around 2060 but will decline to 1.52 billion at the end of the century.

In contrast to both countries, the US is estimated that its population of 320 million in 2015 will continue to grow steadily to 450 million in end of 21st century.

Next, let's look at the trends of the countries of about 100 million to 250 million people as of 2015. (FIG. 6)
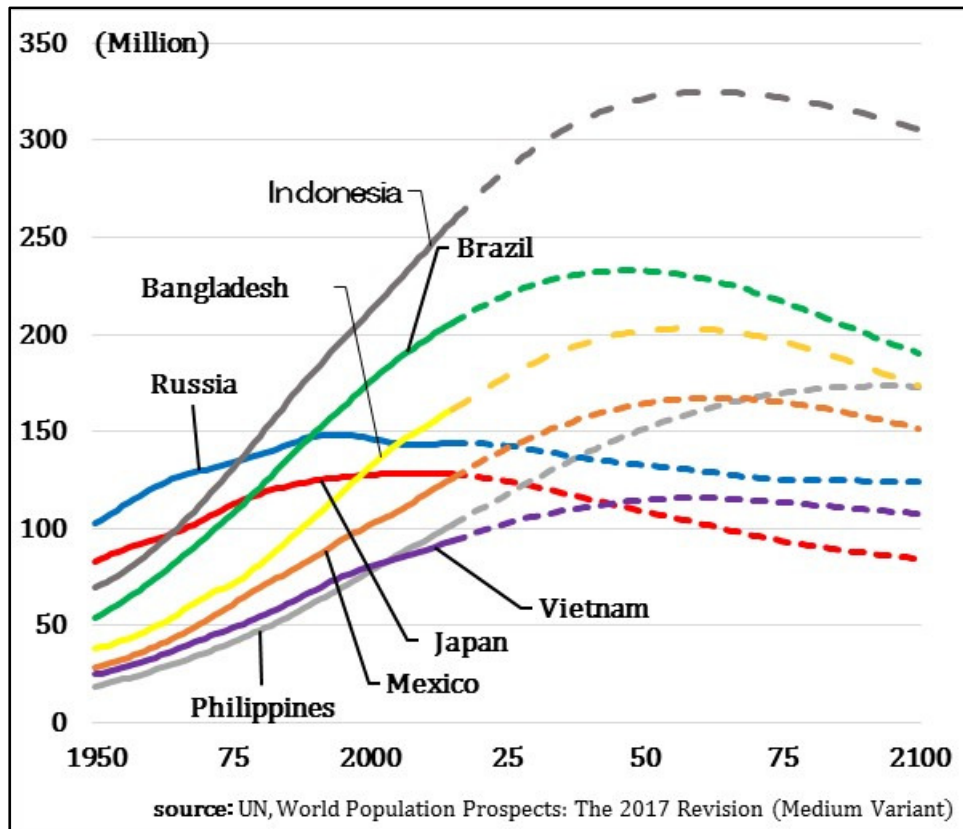
Figure 6. Demographic Dynamics Forecast for Countries with Population over 100 Million Source: UN data [8] adopted by the authors.

The sharp decline in Japan stands out, but also other countries' population will begin to decrease after reaching a peak, similar to Japan.

Finally, we will list the trends of the countries with a population of less than 100 million as of 2015. (FIG. 7)
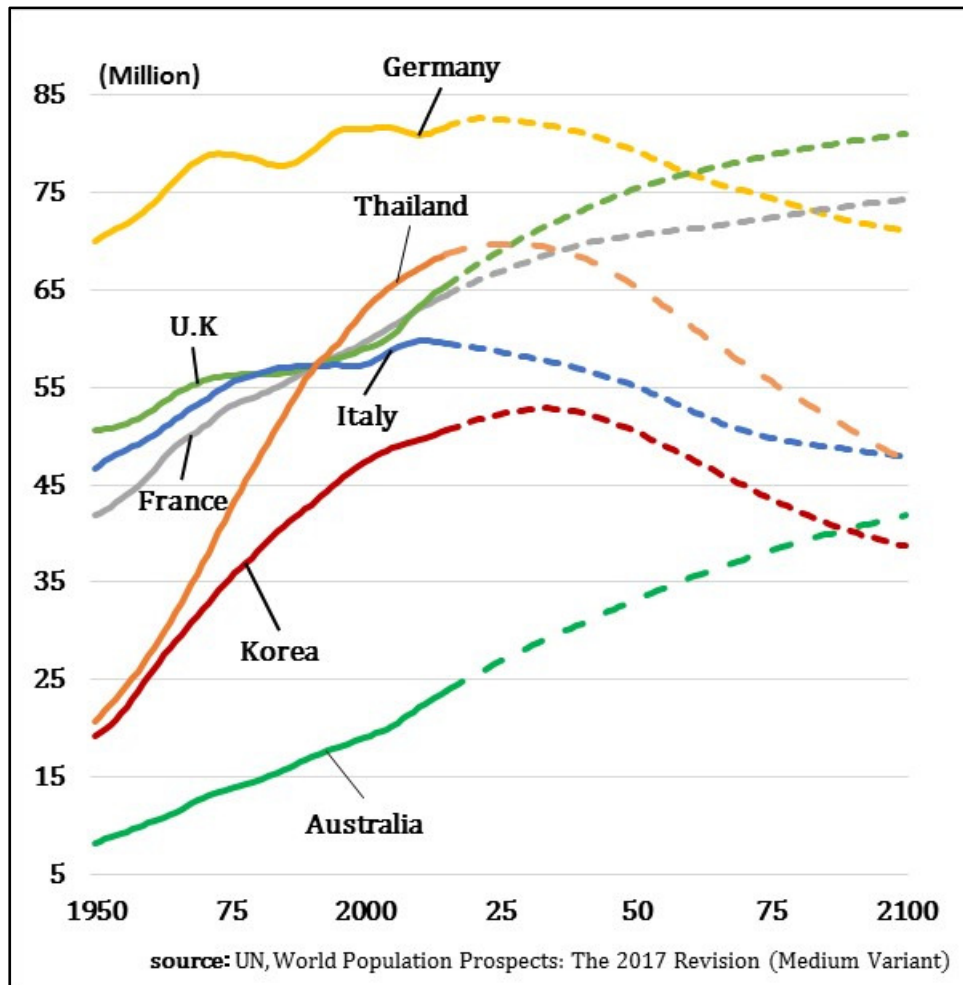
Figure 7. Demographic Dynamics Forecast for Countries with Population under 100 Million Source: UN data [8] adopted by the authors.

It apparently turns out that Germany, Italy, Thailand and Korea (south)are on the same population transition curve as that of Japan.

By comparison, it is definite that a certain exceptional country which actively accepts immigrants and aims for a multiethnic state, such as the United States, France, the UK and Australia, can alone maintain population growth.

We can learn, from these countries, one countermeasure against population reduction: i.e. "social increase" by immigration, however, cannot hold forever because emigrating countries are also facing population decline. After all, to keep the population steady, it is essential for each cohort including immigrants and their successors to achieve population-replacing fertility rate. In addition, the factor of "social increase" by immigration sustains basically the population increase of immigrant countries. When the population decline begins in the supplying countries, the immigrant population will also decrease greatly.

Anyway, it is highly possible that many countries, including China, India and other Asian countries, will follow Japan's population transition. Japan's population problem should be considered as a clear and grave precursor.

8. CONCLUSION

In the field of demography, the concept of [population momentum] is generally accepted. To be brief, it is <a virtual equilibrium population ÷ the current population>. It is defined as: "the ratio of the population level to which the [virtual population transition] ultimately quiesces to the current population level on the assumption that the birthrate be immediately recovered to population replacement level (death rate is constant, international migration movement is zero)". In other words, it is understood that "it represents a kind of inertia that a population structure of a certain point keeps increasing/decreasing its total population". [9]

In this research, we analyzed past trends of the population of our country and forecasted the future by using the ecological economics approach. As a result, we confirmed what the population decline rate in Japan will increase from the current 0.1% or 0.2% to 0.5% in the 2030s, 0.7% to 0.8% in the 2040s and over 1% in the 2050s, and we can regard that "population momentum in Japan is entering extinction mode".

The purpose of this research has been to *verify*, *by mathematical analysis*, the IPSS's "nearly confirmed future" of the population *conjectured by a statistical/demographic approach*. This purpose is considered to be successfully achieved by our mathematical approach to *apply the Hubburt function to the population dynamics*, which would be of a universal applicability world-wide and could provide the demography with an effectual analytical method.

REFERENCES

1. Inaba Hisashi "JinkoMondai–JinkogakutekiApurochi–(Population Problem - Demographic Approach -)" (Graduate School of Mathematical Sciences, the University of Tokyo)JapaneseAccessed 29 September 2018.
   Available:http://www.ms.u-tokyo.ac.jp/~inaba/inaba_gendaishakai_2008.pdf
2. National Institute of Population and Social Security Research "Population Projections for Japan (2017): 2016 to 2065", p.5Accessed 29 September 2018.
   Available:http://www.ipss.go.jp/pp-zenkoku/e/zenkoku_e2017/pp29_summary.pdf
3. National Institute of Population and Social Security Research "Population Trends in Japan"Accessed 29 September 2018.
   Available: http://www.ipss.go.jp/pr-ad/e/eng/03.html
4. M. King Hubbert "Nuclear Energy and the Fossil Fuels" Accessed 29 September 2018.
   Available: http://www.hubbertpeak.com/hubbert/1956/1956.pdf
5. M. King Hubbert "Exponential Growth as a Transient Phenomenon in Human History" World Wildlife Fund, Fourth International Congress, The Fragile Erath: Toward Strategies for Survival, San Francisco 1976Accessed 29 September 2018.
   Available:http://www.hubbertpeak.com/hubbert/wwf1976/
6. C. Wilson and G. Pison, 'More than half of the global population lives where fertility is below replacement level', *Population and Societies* No. 405, October 2004, INED.
7. W. Lutz, W. C. Sanderson and S. Scherbov, 'The end of world population growth', *Nature* 412: 543-545, 2001.
8. United Nations Population Division World Population Prospects 2017Accessed 29 September 2018.
   Available: https://population.un.org/wpp/Download/Standard/Population/
9. Ishi Futoshi 'Keishiki Jinkogaku Kara Mita Jinkogen Jidai (Population decay term as seen from formal demography)' "21 Seiki No Tokei Kagaku" Dai 1 Kan Dai 1 Bu 'Jinko Mondai To Tokei Kagaku' Dai 2 ShooJapan Statistical Society HP Version, November 2011 Japanese Accessed 29 September 2018.

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337   **APPENDIX**
338   **Logistic Function and Hubbert Function**
339   As mentioned in the main text, the Hubbert function is the derivative of the logistic function,
340   which was devised by P. F. Verhulst as follows. In 1798, Thomas Robert Malthus pointed out
341   in his population theory that population would increase exponentially in principle. However,
342   there are constraints by the environment and resources, which would prevent any infinite
343   increase: as the population increases, the rate of increase would decrease, resulting finally
344   in population saturation. Verhulst treated this problem analytically and came to devise his
345   "logistic equation" to model a population growth of creatures.
346   If N is the number of current individuals; K, the number of individuals ultimately sustainable
347   in the given environment; and a, the growth rate coefficient; then, the rate of population
348   increases, dN/dt, is to be expressed by the following equation.

$$\frac{dN}{dt} = aN\left(\frac{K-N}{K}\right)$$

349

350   The right side = the rate of increase consists of [aN : a growth rate proportional to N, a
351   positive feedback term] and [$(K-N)/K$: an indicator of the environmental allowance to
352   afford the population to grow up to the breeding age, a negative feedback term].
353   If one divides the both sides of this equation by K, set$y = N/K$, and$a = 1$, one obtains a
354   normalized form of the logistic equation:
355

$$\frac{dy}{dt} = y(1-y) = y - y^2$$

356   The solution will be:

$$y = \frac{1}{(1 + \exp(-t))}$$

357   If this expression is multiplied by $\exp(t)/\exp(t)$, the result is:

$$y = \frac{\exp(t)}{(1 + \exp(t))}$$

358   This equation will be illustrated as follows:
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361
362   **Figure A.**
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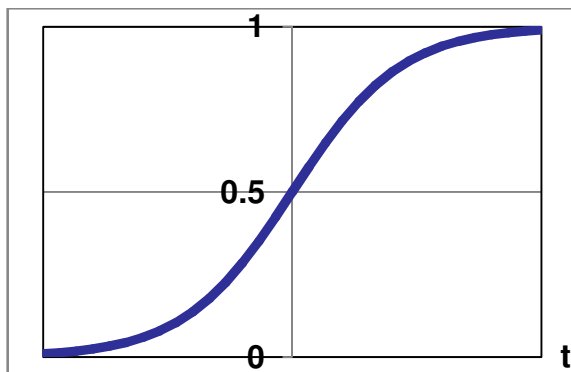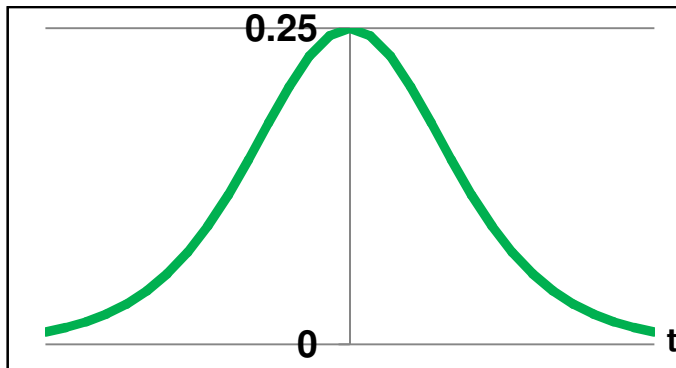
364 This is like the curve (logistic function) in the middle of Fig. 4 of the main text, in its
365 normalized form.
366 Further, the derivative of this equation with time t is the normalized Hubbert function:
367

$$\frac{dy}{dt} = \frac{\exp(t)}{\left(1 + \exp(t)\right)^2}$$

368 This equation will be illustrated as seen below:
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373 **Figure B**
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375 This is like the curve (Hubbert function) in Fig. 2 of the main text, in the normalized form.
376
377 Since Hubbert's theory has been explained in the main text, here we supplement a
378 mathematical interpretation of the Hubbert function.
379 As already mentioned, the Hubert function is a derivative of the logistic function:

$$h(t) = \frac{K\exp(-\alpha t)}{\left(1 + \exp(-\alpha t)\right)^2}$$

380
381 The parameters K and α can be directly read from the actual data (tables and graphs). Let
382 us regress these actual demographic dynamics of Japan to the Hubbert curve in reference to
383 the above formula.
384 The population of Japan reached its peaked 128,084 thousand people in 2008.
385 This corresponds to the peak value h (0) of the Hubbert function h (t). This is,

$$h(0) = \frac{K}{4} = 1.28[\text{Billion}] \colon K = 1.28 \times 4[\text{Billion}]$$

386 This coefficient K is determined by the function value at $t = 0$, independently of the
387 coefficient α, the time factor; and corresponds to the height of the Hubbert curve (the size in
388 the vertical direction).
389 On the other hand, the time coefficient α determines the breadth of the curve (the horizontal
390 size). There is a simple method for estimating the α; of which the first step is to find a time
391 point t at which the function value h(t) becomes the half of the peak value (= the half height
392 value).
393 The half of the maximum population, 128,084 thousand people, is 64,042 thousand: this
394 population lies just midway between 63,461 thousand in 1929 and 64,450 thousand in 1930,
395 therefore, the half maximum population may correspond to the population in 1929.5. Thus,
396 the distance u (= half value width) from the peak year can be estimated as:

$$u = 1929.5[\text{year}] - 2008[\text{the peak year}] = -78.5[\text{years}]$$

$$h(-78.5) = {}^{h(0)}\!/_2 = {}^{K}\!/_8 = Kh_0(\tau)$$

397 Here,

$$\frac{\exp(\tau)}{\left(1 + \exp(\tau)\right)^2} = \frac{1}{8}$$

398 To solve $\exp(\tau)$, one can substitute x for $\exp(\tau)$ to obtain:

$$\frac{x}{(1+x)^2} = \frac{1}{8}$$
$$\therefore 8x = (1+x)^2 = 1 + 2x + x^2$$
$$\therefore x^2 - 6x + 1 = (x-3)^2 - 8 = 0$$

399 $\quad (x-3)^2 = 8 = \left(\pm\sqrt{8}\right)^2 \rightarrow \quad (x-3) = \pm\sqrt{8} = 2\sqrt{2}$

$$x = 3 \pm 2\sqrt{2} = \exp(\tau)$$

400 This τ must satisfy the formula: $\tau = \ln(3\pm2\sqrt{2})$
401 (It is enough to take one of + or -)
402 Here, this constant τ for $^1\!/_8 = h_0(\tau)$ is an informative factor to represent the half width of the
403 normalized Hubbert curve.
404 By directly assigning $t = -78.5$ to the Hubbert function $h(t)$,

$$h(-78.5) = \frac{K\exp(78.5\alpha)}{\left(1 + \exp(78.5\alpha)\right)^2} = {}^{K}\!/_8$$

405 On the other hand, the function value on the normalized Hubbert curve at $t = \tau$ is,

$$h_0(\tau) = \frac{\exp(\tau)}{\left(1 + \exp(\tau)\right)^2} = {}^1\!/_8$$

406 A comparison of the numerator and the denominator of the above two formulas will give,

$$78.4\alpha = \tau \quad \Rightarrow \quad \tau = \ln(3 + 2\sqrt{2}), \quad \text{i.e.,} \quad 78.4\alpha = \ln(3 + 2\sqrt{2})$$

407 Therefore,

$$\alpha = \frac{\ln(3 + 2\sqrt{2})}{78.5} = 0.022455$$

408 Although this value of α is estimated by a simplified method by using the half-value width,
409 the h (t) curve (calculated value) using this α correlates to the actual demographic dynamics
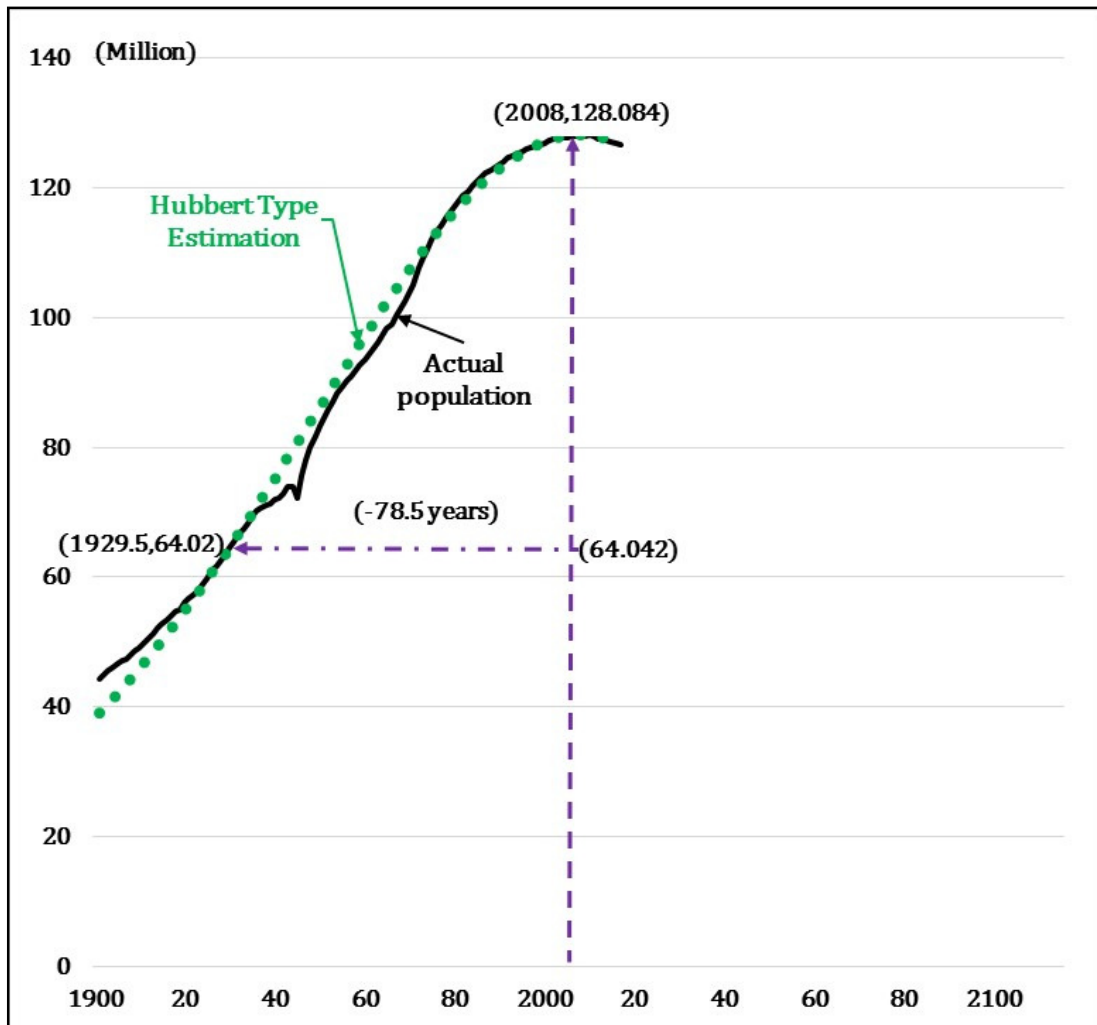410 with high accuracy (Fig. C).
411

412
413
414 **Figure C**

415
416 Some supplements to Figure C:
417 The observation period: 1901 - 2015, Data points: $n = 114$
418 The correlation coefficient between the data and the calculated values is: $r = 0.99478$
419 The correlation coefficient after the correction for the degree of freedom (= 2) will be:
420 $Q = 0.99469$
421 Formula for correcting the degree of freedom is:

$$Q^2 = \frac{1 - (1 - r^2)(n - 1)}{(n - 2)} = \frac{r^2 - (1 - r^2)}{(n - 2)}$$

422 The correlation coefficient (> 0.99) is highly accurate in the descriptive statistics. Therefore,
423 further adjustment (optimization) of α value will be unnecessary. In fact, if we try to minimize
424 the squared deviation of (actual value - computed value) for optimization, it must bring the
425 anomalous phenomenon of population decline during World War II into the theoretical
426 equation, which should rather be arrested.
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