Differences Between Two Weak Interaction Theories

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Abstract

This paper analyzes differences between theoretical elements of the Standard Model electroweak theory and corresponding properties of a dipole-dipole weak interaction theory. The analysis relies on a number of self-evident criteria that are valid for quantum theories. The results demonstrate the existence of fundamental errors in the electroweak theory and the advantage of the dipole-dipole weak interaction theory.

Keywords: The Variational Principle, Quantum Theories, Weak Interactions, Consistency Test

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1 Introduction

Like every physical theory, a theory of weak interactions should be consistent with experimental data that are included in its validity domain. These data can be divided into two sets. The first set comprises the (not too many) data that serve as cornerstones of the theory. The second set comprises all other data and a coherent theory is expected to provide an adequate explanation for each of them.

The neutrino is an important particle that takes part in weak interaction processes. Therefore, a knowledge of its inherent properties plays a crucial role in a construction of a weak interaction theory. The problem of whether the neutrino is a massless particle or a massive particle was settled about twenty years ago. It is now recognized that "neutrinos can no longer be considered as massless particles" [1, 2].

Massive and massless particles are completely different physical objects. For example, a massless spin-1/2 particle is described by a two-component Weyl spinor whereas a massive spin-1/2 particle is described by a four-component Dirac (or Majorana) spinor. Obviously, the number of components of a spinor is a crucial element of a theory of a spin-1/2 particle. At the time of the electroweak construction, the neutrino was regarded as a massless particle which is described by a two-component Weyl spinor. For example: "in 1957, Weyl's theory was triumphantly vindicated" (see [3], p. 139).

The established evidence of a massive neutrino might entails modifications in the structure of a weak interaction theory that uses a Weyl neutrino. This issue is irrelevant to the dipole-dipole weak interaction theory which is examined herein because this theory takes a Dirac neutrino as one of its cornerstones [4, 5]. Other elements of this theory are described later in this work. Properties of this theory are compared with corresponding properties of the electroweak theory which was constructed at the time when a massless neutrino of Weyl's theory was the dominant concept.

In this work units where $\hbar = c = 1$ are used. Therefore, just one dimension is required and the dimension of length $[L]$ is used. The Minkowski metric is diagonal and its entries are (1,-1,-1,-1). Relativistic expressions are written in the standard notation. The second section describes acceptability criteria that are valid for a quantum theory of a massive particle. The third section describes known problems of the electroweak theory. The fourth section describes elements of the dipole-dipole weak interaction theory. The fifth section shows the impressive superiority of the dipole-dipole weak interaction theory over the electroweak theory. The last section contains concluding remarks.

2 Acceptability Criteria for a Quantum Theory

Like any other physical theory, a quantum theory of a massive particle should explain and predict experimental data that are included in its validity domain. On the other hand, physics is a mature science and it already has well-established theoretical requirements. Some of these requirements which are relevant to quantum theories of a massive particle are listed below together with a very short explanation. These requirements are later used in this work as acceptability criteria for a quantum theory.

C.1 Accelerators provide a tremendous amount of data that are consistent with special relativity. Therefore, a physical theory should take a relativistic covariant form. The non-relativistic low velocity limit is acceptable.

C.2 A quantum theory that is derived from an application of the variational principle that uses an appropriate Lagrangian density satisfies many physical requirements. The key role of this issue is described in the following textbook which states that this principle provides "the foundation on which virtually all modern theories are predicated" (see [3], p. 353). Therefore, a consistent Lagrangian density that yields the theory's equations of motion is required for every specific quantum theory.

C.3 Correspondence relationships exist between the four theories:

$$QFT \leftrightarrow RQM \leftrightarrow QM \leftrightarrow NRCM. \tag{1}$$

Here QFT denotes Quantum Field Theory; RQM denotes Relativistic Quantum Mechanics; QM denotes Quantum Mechanics; NRCM denotes Non-Relativistic Classical Mechanics. The following well-known textbook states loud and clear the correspondence between the first three theories of (1). "First, some good news: quantum field theory is based on the same quantum mechanics that was invented by Schroedinger, Heisenberg, Pauli, Born, and others in 1925-26, and has been used ever since in atomic, molecular, nuclear and condensed matter physics" (see [6], p. 49). Below, these relationships are called the *Weinberg correspondence principle.*

C.4 Another important correspondence principle states that the classical limit of QM should fit corresponding quantities of classical physics (see e.g. [7], pp. 25-27, 137, 138; [8]). This principle is called below the *classical limit*.

C.5 QM has a Hilbert space and "observables are represented by Hermitian operators" that operate on this space (see [6], pp. 49, 50). A Hilbert space comprises normalizable functions (see [9], p. 164), and the standard normalization is $\int \psi^* \psi d\tau = 1$, where $d\tau$ denotes volume element. It means that a consistent expression for density is required for a construction of a Hilbert space of quantum functions. Furthermore, energy is defined in the classical theory. Therefore, the arguments presented above together with the Weinberg correspondence principle and the classical limit entail that *a quantum theory should define density* *and a Hamiltonian operator, and that the expectation values of this Hamiltonian should agree with the system's energy states.*

C.6 The superposition principle is a fundamental property of quantum mechanics (see [10], p. 7, [11], p. 12). This principle imposes conditions on the equations of an acceptable quantum theory. "The equation must be linear and homogeneous; the wave thus possesses the property of superposition, characteristic of waves in general" (see [9], p. 61). The Weinberg correspondence principle means that also the QFT equations should take this form.

These criteria are used below in a comparison between the electroweak theory and the dipole-dipole weak interaction theory.

3 Problems with the Electroweak Theory

Section 2 of [4] describes several errors in the mathematical structure of the electroweak theory. A logical point of view indicates that just one mathematical error is a sufficient condition for a disqualification of a physical theory. However, a general rule states that if a given mathematical theory has one error then it is very likely that this theory has other kinds of errors. And indeed, section 2 of [4] shows several independent errors of the electroweak theory. Evidently, a presentation of several kinds of errors increases one's confidence concerning his opinion on the theory's structure.

For the completeness of the present discussion, the electroweak errors shown in section 2 of [4] are briefly mentioned herein. Later, other electroweak problematic points are discussed in appropriate places.

1. The electroweak theory uses the factor $(1 \pm \gamma^5)$ as an operator that projects a quantum function onto a parity violating form. Evidently, the parity of the pure number 1 is even whereas the parity of γ^5 is odd (see [14], p. 26). Hence,

$(1 \pm \gamma^5)$ represents a maximal parity violation. This operation is unacceptable for a massive spinor. The following analysis extends the validity of this claim. Some QFT textbooks use the factor $(1 \pm \lambda\gamma^5)$ $(|\lambda| > 0)$ in a discussion of the electroweak Lagrangian function (see e.g. [12], p. 220, [13], p. 550).

Let us see what happens after applying the factor $(1 \pm \lambda\gamma^5)$ to the spinor of a free motionless spin-up Dirac particle

$$\begin{pmatrix} 1 & 0 & \pm\lambda & 0 \\ 0 & 1 & 0 & \pm\lambda \\ \pm\lambda & 0 & 1 & 0 \\ 0 & \pm\lambda & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \\ \pm\lambda \\ 0 \end{pmatrix}. \tag{2}$$

Here the notation of the γ matrices is that of [14], p. 17 and the Dirac spinor is presented in [14], p. 30.

The three cases where $|\lambda| > 1, |\lambda| = 1$ and $0 < |\lambda| < 1$ are examined below.

- If $|\lambda| > 1$ then the right-hand side of (2) is a negative-energy Dirac spinor (see [14], pp. 28-30). It means that in this case the operator $(1 \pm \lambda\gamma^5)$ projects a massive motionless Dirac particle into an unphysical state. This state is not included in the particle's Hilbert space and expectation values of observables cannot be calculated.
- If $|\lambda| = 1$ then the right-hand side of (2) is a Dirac spinor that has an infinite energy-momentum (see [14], p. 30). It means that the operator $(1 \pm \gamma^5)$ projects a massive motionless Dirac particle into an unphysical state and the previous result holds.
- $0 < |\lambda| < 1$ then the right-hand side of (2) is a Dirac spinor that moves in the z-direction. The energy of this spinor is greater than that of the original motionless spinor. It follows that in this case the operator $(1 \pm \lambda\gamma^5)$ violates energy conservation. The same is true for the conservation of the linear momentum.

Conclusion: The electroweak factor $(1 \pm \lambda\gamma^5)$ is unacceptable.

2. In spite of the fact that the electroweak theory is about 50 years old, it still has no consistent expression for the interaction of the electrically charged $W^\pm$ particles with an electromagnetic 4-potential.

3. Similarly, the electroweak theory of the $W^\pm$, Z particles still has no consistent expression for density. This property means that it has no Hilbert space and the acceptability criterion C.5 is violated.

Figure 1 illustrates the significance of the last requirement. Here a decay mode of the W^-, Z comprises Dirac particles. These particles are detected by devices that measure their space-time position and their energy-momentum. The measured data of each case show that the two particles were emitted from the same very small space-time region, and that the combined energy-momentum of the particles is consistent with the mass of the decaying particle. Therefore, it is concluded that they are decay products of the respective particle.

The Dirac theory of a lepton provides a consistent expression for density. It is the 0-component of $\bar{\psi}\gamma^\mu\psi$ (see [14], pp. 23, 24). Furthermore, the Dirac theory provides a consistent expression for the Hamiltonian, which means that the particles' energy is appropriately defined. Therefore, the Dirac theory of the leptons of figure 1 is consistent with the decay measurements. On the other hand, the electroweak theory of the $W^\pm$, Z particles should also provide a consistent expression for density. This requirement indicates a discrepancy of the electroweak theory because it provides no

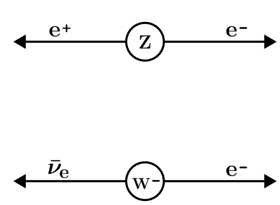

Figure 1: *Electronic decay channels of the Z and the W^- (see text.)*

consistent expression for density. A fortiori, a Hilbert space cannot be constructed and one wonders about the meaning of operators like the Hamiltonian.

Other errors of the electroweak theory are discussed below.

4 The Dipole-Dipole Weak Interaction Theory

Elements of the dipole-dipole weak interaction theory are discussed in [4, 5]. The main points of this theory are briefly presented in the following lines.

A Lagrangian density plays a key role in quantum theories. This issue and other general principles are used in the construction of the weak dipole-dipole interaction theory. This assignment uses two specific assumptions which are based on weak interactions experimental evidence: The Fermi coupling constant has the dimension $[L^2]$ (see [12], p. 19), and “neutrinos can no longer be considered as massless particles” [1]. The combined Lagrangian density of this theory and of electrodynamics is [4]

$$\mathcal{L}_{EMW} = \bar{\psi}(\gamma^\mu i\partial_\mu - m)\psi - \frac{1}{16\pi}F^{\mu\nu}F_{\mu\nu} - e\bar{\psi}\gamma^\mu A_\mu\psi + d\bar{\psi}\sigma_{\mu\nu}\mathcal{F}^{\mu\nu}\psi. \qquad (3)$$

Here the first term represents a free Dirac particle, the second term represents free electromagnetic fields and the third term represents electromagnetic interaction of a charged Dirac particle. These terms are the fundamental elements of quantum electrodynamics (QED) (see [15], p. 84; [16], p. 78). The last term of (3) represents the interaction of a Dirac particle with the external weak field tensor $\mathcal{F}^{\mu\nu}$ which is associated with an external weak dipole. $\sigma_{\mu\nu} = i(\gamma^\mu\gamma^\nu - \gamma^\nu\gamma^\mu)/2$ is the standard tensor obtained from the product of two γ matrices (see [14], p. 21), and d denotes the strength of the weak dipole which has the dimension of length.

The third and the last terms of the Lagrangian density (3) represent interaction, and they have the following similarities: Both are a contraction of γ matrices with an external field that carries the interaction, and both are free of derivatives. Here derivatives are regarded as troublesome elements of interaction terms (see e.g. [15], p.

87). In particular, the Noether theorem proves that an interaction term that contains a derivative alters the 4-current (see e.g. [6], p. 309, [15], p. 20). Hence, density is destroyed.

It means that an interaction term that contains derivatives is unacceptable because it destroys the Hilbert space which is based on the particle's density. The corresponding Hamiltonian that is obtained from (3) is the equation of motion of a Dirac particle in electromagnetic and weak fields

$$i\frac{\partial\psi}{\partial t} = H\psi = [\boldsymbol{\alpha}\cdot(\boldsymbol{p} - e\boldsymbol{A}) + \beta m - d\gamma^0\sigma_{\mu\nu}\mathcal{F}^{\mu\nu}]\psi \tag{4}$$

The product $\gamma^0\sigma_{\mu\nu}$ yields two terms, a vector and an axial vector.

It is interesting to note that the formal structure of the last term of (3) has been examined in the literature as a term that may contributes to electromagnetic interactions (see e.g. [6], p. 14). It is now recognized that such a term is irrelevant to electromagnetic interactions. A theoretical reason for this conclusion is that the strength of electromagnetic interaction of a charged particle is proportional to its charge e which is a dimensionless Lorentz scalar whereas the dimension of a dipole is $[L]$. Therefore, unlike the electric charge, which is a dimensionless quantity, the coefficient d of the last term of (3) has the dimension of length.

The following lines describe important properties of the last term of (3). The Lagrangian density is a Lorentz scalar and its terms are written in the form $\bar{\psi}\hat{O}\psi$, where $\bar{\psi} \equiv \psi^\dagger\gamma^0$ and $\hat{O}$ denotes an appropriate operator. On the other hand, the Hamiltonian is written in the following form $\psi^\dagger\hat{O}\psi$. Here the Dirac $\boldsymbol{\alpha}$, β matrices replace the four γ matrices (see [14], p. 48). Therefore, the Legendre transformation that is applied to the Lagrangian and yields a Hamiltonian, adds an extra γ^0 factor (see e.g. [17], p. 123). In the case of the operator $\sigma^{\mu\nu}$, the additional γ^0 yields two quantities, a vector and an axial vector [4]

$$\mathrm{d}\psi^\dagger\gamma^0\sigma_{\mu\nu}\mathcal{F}^{\mu\nu}\psi = 2\mathrm{d}\psi^\dagger(i\gamma_i\mathcal{E}^i - \gamma^5\gamma_i\mathcal{B}^i)\psi, \tag{5}$$

where $\mathcal{F}^{\mu\nu}$ denotes the external 4×4 antisymmetric tensor of the weak field and $\mathcal{E}$, $\mathcal{B}$ denote its vector and axial vectors entries, respectively. The running index i takes the values 1,2,3. This outcome *proves* that the parity violation weak interaction property $\boldsymbol{V} - \boldsymbol{A}$ (see [12], p. 217) is an inherent attribute of the dipole-dipole weak interaction theory.

Another result of the dipole-dipole weak interaction theory is that its fields do not contain radiation [5].

5 Discussion

Several aspects of the electroweak theory and of the dipole-dipole weak interaction theory are discussed below.

5.1. Parity violation is a unique property of weak interactions. The electroweak theory does not prove this issue but invokes the factor $(1 \pm \gamma^5)$ as an operator that projects the quantum function into a parity violating form. It is proved above that an application of this operator is an error that justifies the disqualification of the electroweak theory. Besides this erroneous attribute, it is pointed out here that the introduction of the $(1 \pm \gamma^5)$ factor means that the electroweak theory *postulates* parity violation.

By contrast, the discussion presented in the previous section shows that the dipole-dipole weak interaction theory *proves* that parity violation is an intrinsic property of the theory that is derived from the Lagrangian density (3). Therefore, the dipole-dipole weak interaction theory certainly takes a much better logical status.

5.2. The Occam razor principle regards simplicity as an argument used for a selection between two different theories that otherwise have the same merits. The dipole-dipole theory is still restricted to cases like the scattering experiments $\nu_e e \rightarrow \nu_e e$

or $\bar{\nu}_e e \to \bar{\nu}_e e$. The electroweak theory uses Feynman diagrams for an evaluation of these processes. It turns out that these processes depend on the $W^\pm$ and on the Z particles as mediators of this interaction (see [12], p. 327). Therefore, the electroweak Lagrangian density of these particles and of the photon, as well as their interaction with fermions is compared with the corresponding Lagrangian density of the dipole-dipole weak interaction theory (3). These Lagrangian densities comprise terms of the specific particles and other terms that represent interaction between particles.

The electroweak Lagrangian density comprises more than 20 terms (see e.g. [18], p. 518; [19]), and note that in [18] $\bar{D}_\mu \equiv \partial_\mu + ig(Z_\mu \cos\theta_W + A_\mu \sin\theta_W)$ represents 3 terms. Therefore, for the sake of simplicity of this discussion, its explicit form is not presented here. It is just stated here that this Lagrangian density contains a very large number of terms *but* it omits a term that represents the interaction of the electrically charged $W^\pm$ particles with the electromagnetic 4-potential A_μ. By contrast, the dipole-dipole weak interaction theory uses a combined electromagnetic and weak interactions Lagrangian density that takes the form of (3). This Lagrangian density comprises just four terms.

A comparison between the extreme complexity of the electroweak Lagrangian density (see e.g. [18], p. 518; [19]) and (3) indicates that the relative simplicity of the dipole-dipole weak interaction Lagrangian density (3) is quite amazing. Therefore, the Occam razor principle favors the dipole-dipole weak interaction theory.

5.3. The general form of the Euler-Lagrange equation is (see [15], p. 15; [16], p. 16)

$$\frac{\partial \mathcal{L}}{\partial \varphi_r} - \frac{\partial}{\partial x^\mu} \frac{\partial \mathcal{L}}{\partial(\partial \varphi_r / \partial x^\mu)} = 0. \tag{6}$$

Here $\varphi_r,\ r = 1, ..., N$ denotes the rth independent quantum field of the system and the equations of motion of the system are obtained "by independently varying each field, $\delta\varphi_r(x)$" (see [15], p. 15).

An application of the Euler-Lagrange equation (6) to a Lagrangian density $\mathcal{L}$ proves that if $\mathcal{L}$ is a quadratic function of the fields then the corresponding equations of motion are linear partial differential equations. This property holds for Maxwell equations and for Dirac equation. These celebrated equations are experimentally successful and provide the theoretical basis for modern technology. Evidently, also the dipole-dipole weak interaction term of (3) is a quadratic function of the quantum field. These equations are consistent with the acceptability requirement C.6.

By contrast, the electroweak Lagrangian density contains terms of the *third and the fourth power* of the $W^{\pm}$, Z functions (see e.g. [18], p. 518; [19]). Therefore, the Euler-Lagrange equation of the $W^{\pm}$, Z are *nonlinear inhomogeneous third order partial differential equation.* Hence, the electroweak theory of the $W^{\pm}$, Z particles is inconsistent with requirement C.6.

5.4. Quantum theories have the following general structure:

5.4.A Solutions of the theory's wave equations which are a set of linear partial differential equations describe adequately the relevant data.

5.4.B An appropriate Lagrangian density yields these quantum equations of motion. Such a Lagrangian density guarantees that the theory is consistent with general laws of physics.

Electromagnetic fields are compatible with the foregoing structure. Maxwell equations describe adequately these fields [20]. These equations can be derived from an appropriate Lagrangian density [21]. A massless spin=1 particle called photon is the particle form of electromagnetic fields.

The same is true for charged spin=1/2 massive particles. The Dirac equation describes adequately this kind of particles [14]. This equation can be derived from a Lagrangian density. Furthermore, the interaction between a charged Dirac particle and Maxwellian fields can also be derived from a Lagrangian density (see the first three terms of (3)). Details of these properties of electromagnetic fields and of Dirac particles are discussed in many textbooks.

The status of the $W^{\pm}$, Z electroweak particles is completely different. Textbooks that present the electroweak theory show the Lagrangian density of the $W^{\pm}$, Z but refrain from showing an explicit form of the corresponding partial differential equation of the wave function. A fortiori, no solution of this equation is presented. Indeed, as stated above, the $W^{\pm}$, Z equations of motion are *not* linear partial differential equations. Therefore, *they do not take the form of a wave equation* and they do not belong to the realm of quantum theories.

5.5. Wigner has analyzed the irreducible representations of the inhomogeneous Lorentz group [6, 22, 23, 24]. An important result of his work states that a massive quantum particle has a well defined mass and spin. Mass is a Lorentz scalar quantity. Therefore, Wigner's work implies that a massive particle can also carry another kind of scalar entity. In the Dirac equation, the charge takes the form of a Lorentz scalar. Therefore, this equation proves that Nature uses the scalar attribute of a massive particle for a description of a physical interaction.

This well-known evidence prompts the following question: Does Nature use spin, which is the second intrinsic property of a massive particle, as a basis for a physical interaction?

The dipole-dipole weak interaction theory provides a positive answer to this question. The last term of (3) depends on the spin operator $\sigma_{\mu\nu}$. It shows that the dipole-dipole weak interaction term is analogous to the electromagnetic interaction term: both have γ matrices that are coupled to an external field and both are free of derivatives. Therefore, it can be stated that the dipole-dipole weak interaction theory provides a positive answer to the forgoing question, and that in so doing it also closes a logical gap.

6 Conclusions

The structure of the Standard Model electroweak theory is compared with that of the dipole-dipole weak interaction theory. Several theoretical errors of the electroweak theory are pointed out. Furthermore, it is shown that the electroweak theory is extremely complicated and that its Lagrangian density contains more than 20 terms. By contrast, the dipole-dipole weak interaction theory together with QED contain four terms that represent the self energy of a Dirac particle, the self energy of electromagnetic fields, the charge-fields electromagnetic interaction and the dipole-dipole weak interaction. The first three of them are the ordinary QED terms.

The electroweak theory invokes the factor $(1 \pm \gamma^5)$ in order to account for the $\boldsymbol{V} - \boldsymbol{A}$ parity violating property of weak interactions. It means that the electroweak theory *assumes* parity violation. By contrast, the dipole-dipole weak interaction theory *proves* this property (5). Parity violation is a dramatic experimental property of weak interactions. The proof of this property is a successful experimental example of the physical merits of the dipole-dipole weak interaction theory.

It is also pointed out that in spite of the fact that the electroweak theory is about 50 years old, the wave equation of the $W^{\pm}$, Z particles is still not written explicitly in electroweak textbooks. By contrast, the wave equations of the photon (Maxwell equations) and the wave equation of a massive spin-1/2 particle (the Dirac equation) are discussed in every relevant textbook. The wave equation of the dipole-dipole weak interaction theory (4) is obtained in a straightforward manner.

It can be concluded that the dipole-dipole weak interaction theory is free of the theoretical discrepancies of the electroweak theory.

References

[1] Formaggio JA and Zeller GP. From eV to EeV: Neutrino cross sections across energy scales. Rev. Mod. Phys. 2012;84(3):1307-41.

[2] Patrignani C. et al. (Particle Data Group). Review of Particle Physics. Chin. Phys. C. 2016;40(10):100001-101808.

[3] Griffiths D. Introduction to Elementary Particles. 2nd ed. Wiley-VCH: Weinheim; 2008.

[4] Comay E. A Theory of Weak Interaction Dynamics. Open Access Library Journal, 2016:3(12):1-10.
https://www.scirp.org/journal/PaperInformation.aspx?paperID=72788

[5] Comay E. Further Aspects of Weak Interaction Dynamics. Open Access Library Journal. 2017;4(2):1-11.
https://www.scirp.org/journal/PaperInformation.aspx?PaperID=74373

[6] Weinberg S. The Quantum Theory of Fields. Vol. I. Cambridge University Press: Cambridge; 1995.

[7] Schiff LI. Quantum Mechanics. 2nd ed. McGraw-Hill: New York; 1955.

[8] Wikipedia. https://en.wikipedia.org/wiki/Correspondence_principle

[9] Messiah A. Quantum Mechanics. Vol. 1. North Holland: Amsterdam; 1967.

[10] Landau LD. Lifshitz EM. Quantum Mechanics. Pergamon: London; 1959.

[11] Dirac PAM. The Principles of Quantum Mechanics. 4th ed. Oxford University Press: London; 1958.

[12] Perkins DH. Introduction to High Energy Physics. Addison-Wesley: Menlo Park CA; 1987.

[13] Srednicki M. Quantum Field Theory. Cambridge University Press: Cambridge; 2007.

[14] Bjorken JD. Drell SD. Relativistic Quantum Mechanics. McGraw-Hill: New York; 1964.

[15] Bjorken JD. Drell SD. Relativistic Quantum Fields. McGraw-Hill: New York; 1965.

[16] Peskin ME, Schroeder DV. An Introduction to Quantum Field Theory. Addison-Wesley: Reading Mass; 1995.

[17] Thomson M. Modern Particle Physics. Cambridge University Press: Cambridge; 2013.

[18] Sterman G. An Introduction to Quantum Field Theory. Cambridge University Press: Cambridge; 1993.

[19] https://en.wikipedia.org/wiki/Electroweak_interaction#After_electroweak_symmetry_breaking

[20] Jackson JD. Classical Electrodynamics. 2nd ed. John Wiley: New York; 1975.

[21] Landau LD. Lifshitz EM. The Classical Theory of Fields. Elsevier: Amsterdam; 2005.

[22] Schweber SS. An Introduction to Relativistic Quantum Field Theory. Harper & Row: New York; 1964. (See pp. 44-53.)

[23] Wigner E. On Unitary Representations of the Inhomogeneous Lorentz Group. Ann. Math. 1939;40(1):149-204.

[24] Sternberg S. Group Theory and Physics. Cambridge University Press: Cambridge; 1994. (See pp. 143-150.)