Data Article

MODELLING THE MEAN WAITING TIMES FOR QUEUES IN SELECTED BANKS IN ELDORET TOWN-KENYA

Abstract

The mathematical study of waiting lines is mainly concerned with queue performance measures where several applications have been drawn in past studies. Among the vast uses and applications of the theory is the queuing system in banking halls which sets in here as the main focus of this study where the theory has been used to solve the problem of long queues as witnessed in banks which leads to resource waste. The main aim of this study was to model waiting times for queues in selected banks within Eldoret town-Kenya. The latter component was put under D/D/1 framework and therein its mean derived while the stochastic component was put under the M/M/c framework. Harmonization of the moments of the deterministic and the stochastic components was done to come up with the mean of the overall bank queue traffic delay. Simulation was performed using MATLAB for traffic intensities ranging from 0.1 to 1.9. The results reveal that both deterministic and the stochastic delay components are compatible in modelling waiting time. The models also are applicable to real time bank queue data where upon simulation, both models depict fairly equal waiting times for server utilization factors below 1 and an infinitely increasing delay at rho greater than 1. In conclusion, the models that estimate waiting time were developed and applied on real bank queue data. The models need be implemented by the banks in their systems so that customers are in a position to know the expected waiting time to be served as soon as they get the ticket from the ticket dispenser.

Key words: D/D/1, M/M/c, Utilization factor, Simulation.

1.0: Introduction

Waiting is one of the most unpleasant experiences of life. Queuing theory deals with delays and queues which are essentials in determining the levels of service in banking halls. They also evaluate the adequacy of service channels and the economic losses that come about as a result of long waiting lines. Quantifying these delays accurately and appropriately in banks is critical for planning design and analysis of teller services. Tellers referred herein are the personnel in the bank and will be represented as servers or service channels. In modern banking, queuing has been automated such that customers arrive and pick ticket numbers from a ticket dispensing machine. Electronic quality management systems were implemented for purposes of instilling order and eliminating or easing/reducing congestion in banks.

Models that incorporate both deterministic and stochastic components of queue performance are very appealing in modelling bank queues since they are applied in a wide range of traffic intensities as well as to various types of teller services. They simplify theoretical models with delay terms that are numerically inconsequential. Of the various queueing models, D/D/1 and M/M /c were used in this study. The D/D/1 model assumed that the arrivals and departures were uniform and one service channel (teller) existed. This model is quite intuitive and easily solvable. Using this form of queueing with an arrival rate, denoted by λ and a service rate, denoted by λ, certain useful values regarding the consequences of queues were computed. The M/M /c model used implied that the customers arrived at an intersection in a Poisson process with rate λ and were treated in the order of arrival with inter arrival times following exponential distribution with parameter μ. The service times were treated as independent identically distributed with an arbitrary distribution. Similarly, several service channels (tellers) were considered in this model.

2.0: Modelling Waiting Times

The Mean of Deterministic Delay Model

To compute the mean, it is assumed that customer arrivals and departures are uniformly distributed with rates λ and μ respectively.

To obtain the mean waiting time for the D/D/1 model, we note the following notations.

c_y – Cycle time (min).

g_e –Effective service time.

g_0 – Time necessary for the queue to dissipate.

r – Effective waiting time on the queue before service.

$D(t)$ - Cumulative departures.

λ – Arrival rate.

$A(t)$ – Cumulative arrivals.

ρ - Utilization factor

W_{t_1} – Deterministic queue delay component.

π_w – Probability of waiting on the queue.

P_0 – Steady state probability of having no customers in the system.

Such that the duration of C_y in the bank is given by

$$C_y = r + g \qquad (1)$$

$$W_{t_1} = \frac{\lambda r^2}{2\left(1 - \frac{g_e}{C_y}\rho\right)} \qquad 2$$

Finally the expected deterministic delay in the bank queue is obtained by dividing W_{t_1} by the total number of customers in the in a cycle, that is λC_y to yield

$$E(W_{t_1}) = \frac{C_y\left(1 - \frac{g_e}{C_y}\right)^2}{2\left(1 - \frac{g_e}{C_y}\rho\right)} \qquad 3$$

as the mean of the deterministic component, W_{t_1}.

Mean of Stochastic Delay Component

To obtain the mean of the stochastic delay component we also note the following notations,

We begin with the expected waiting time while on service is given by

$$W_s = {}^1/_\mu \qquad 4$$

Then proceed to the waiting time on the queue which is obtained as follows

$$E(t) = \int_0^\infty t.\pi_w c\mu(1-\rho)e^{-c\mu(1-\rho)t} \, dt \qquad 5$$

$$= \frac{\pi_w c\mu(1-\rho)}{[c\mu(1-\rho)^2]}\int_0^\infty ye^{-y}\, dy \qquad 6$$

$$\text{Thus } \; E(t) = \frac{\pi_w}{c\mu(1-\rho)} = W_q \qquad 7$$

$$\therefore E(W_{t_2}) = {}^1/_\mu + \frac{\pi_w}{c\mu(1-\rho)} \qquad 8$$

Mean of the overall delay model

To obtain the mean of the overall delay model we sum up the expected waiting times for both stochastic and deterministic delay model.

$$E(W_t) = \frac{C_y\left(1 - \frac{g_e}{C_y}\right)^2}{2\left(1 - \frac{g_e}{C_y}\rho\right)} + {}^1/_\mu + \frac{\pi_w}{c\mu(1-\rho)} \qquad 9$$

3.0: Results

The developed overall traffic delay model was applied on real bank queue data collected at the various banks in Eldoret town between 1st August and 5th August 2016. The intermediate results from the data are given and simulation on the developed models using MATLAB software is performed for traffic intensities ranging from 0.1 to 1.9.

Computation of Parameters

The average effective deterministic service time is

$$g_e = \frac{1}{5}\left(\frac{420}{6} + \frac{417}{6} + \frac{410}{6} + \frac{406}{6} + \frac{394}{6}\right)$$

$$= 68.23 \text{ sec}$$

The average arrival rate is

$$\lambda = \frac{\text{Total arrivals}}{\text{Total number of hours observed}}$$

$$= \frac{2146}{30}$$

$$= 71.5333 \text{ Customers per hour}$$

The average service rate is

$$\mu = \frac{\text{Total Departures}}{\text{Total number of hours observed}}$$

$$= \frac{2092}{30}$$

$$= 69.7333 \text{ Customers per hour}$$

The utilization factor (probability that a sever is busy) is

$$\rho = \frac{\text{Average arrival rate}}{\text{number of servers} * \text{Average service rate}}$$

$$= \frac{71.5333}{3 * 69.7333}$$

$$= 0.3419$$

The probability that a server is idle is

$$P_0 = \left\{1 + \frac{\left(\lambda/\mu\right)^1}{1!} + \frac{\left(\lambda/\mu\right)^2}{2!} + \cdots + \frac{\left(\lambda/\mu\right)^{c-1}}{(c-1)!} + \frac{\left(\lambda/\mu\right)^c}{c!}\left[1 + \left(\lambda/c\mu\right) + \left(\lambda/c\mu\right)^2 + \cdots\right]\right\}^{-1}$$

$$= \left\{1 + 1.0258 + \frac{(1.0258)^2}{2!} + \frac{(1.0258)^3}{3!\,(1 - 0.3419)}\right\}^{-1}$$

$$= (2.8253)^{-1}$$

$$= 0.3539$$

For two servers (c=2)

The utilization factor (probability that a sever is busy) is

$$\rho = \frac{\text{Average arrival rate}}{\text{number of servers} * \text{Average service rate}}$$

$$= \frac{71.5333}{2 * 69.7333}$$

$$= 0.5129$$

The probability that a server is idle is

$$P_0 = \left\{1 + \frac{\left(\lambda/\mu\right)^1}{1!} + \frac{\left(\lambda/\mu\right)^2}{2!} + \cdots + \frac{\left(\lambda/\mu\right)^{c-1}}{(c-1)!} + \frac{\left(\lambda/\mu\right)^c}{c!}\left[1 + \left(\lambda/c\mu\right) + \left(\lambda/c\mu\right)^2 + \cdots\right]\right\}^{-1}$$

$$= \left\{1 + 1.0258 + \frac{(1.0258)^2}{2!\,(1 - 0.5129)}\right\}^{-1}$$

$$= (1 + 1.0258 + 1.0801)^{-1}$$

$$= (3.1059)^{-1}$$

$$= 0.3219$$

4.0 Discussion and conclusion

4.0.1 Discussion

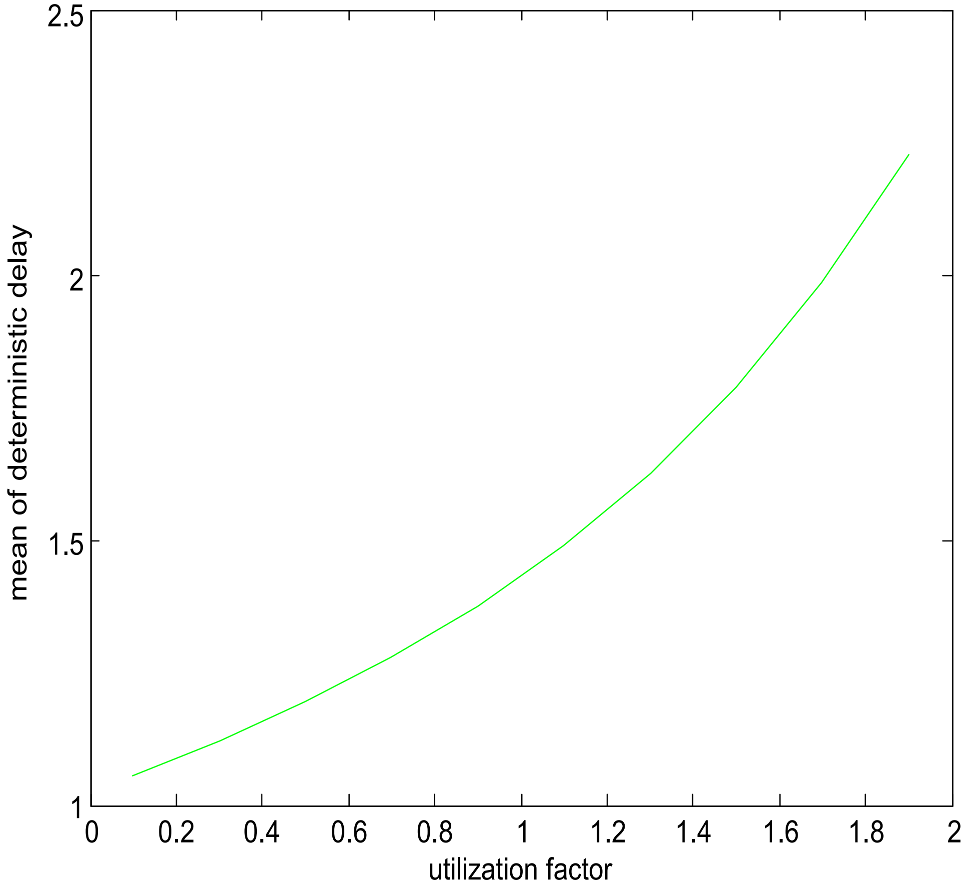

Figure 1 Diagram representing simulation of deterministic component $\mathbf{E}\left[\mathbf{W}_{t_1}\right]$ verses $\boldsymbol{\rho}$

From figure 1 , it is clear that the deterministic delay model estimates a continuous delay but does not accommodate the aspect of randomness when the arrival flows are close to capacity$\rho < 1$. The model reveals a steady increase in mean delay with a more increase in waiting when the flows approach capacity $\rho > 1$ which consequently implies infinite delays in the long run queuing of customers.

Simulation of $\mathbf{E}(\mathbf{W}_{t_2})$

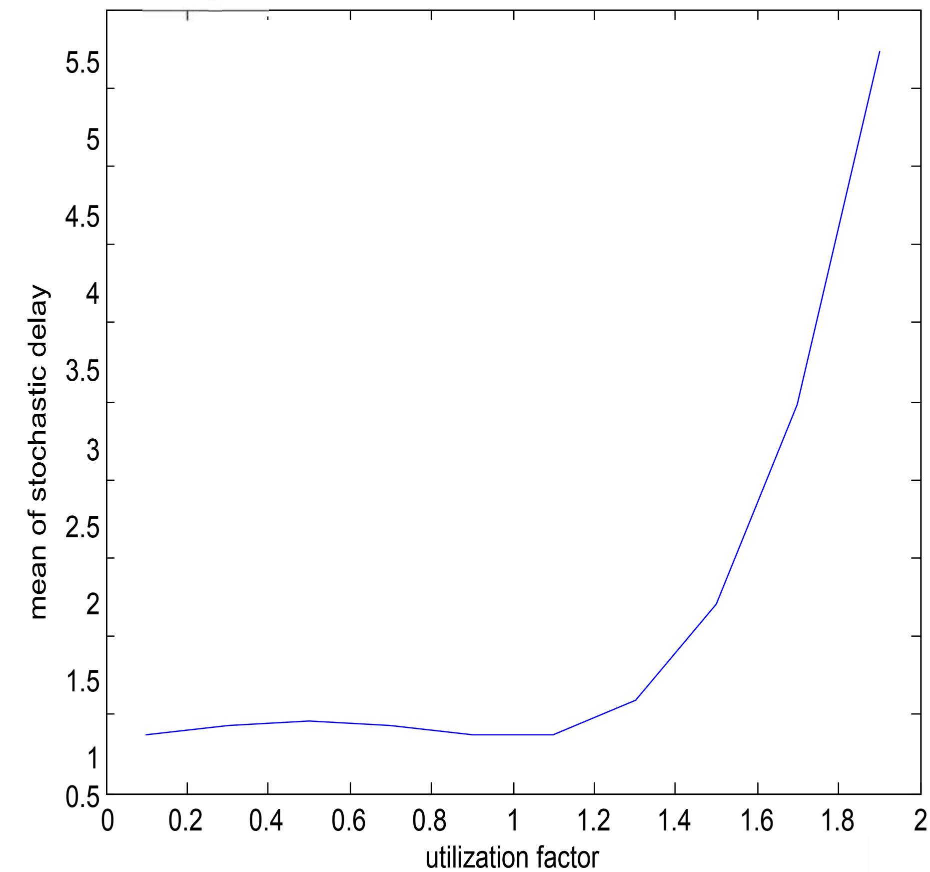

Figure 2 Diagram representing simulation of stochastic component $\mathbf{E}[\mathbf{W}_{t_2}]$ verses ρ vρ with two servers

From figure 2, the stochastic delay model with two servers is also applicable to under saturated conditions $\rho < 1$ and estimates delays tending to infinity when the arrival flow approaches capacity $\rho > 1$. However, comparing the delay with the three server model, it implies an increased delay which is quite natural due to decreased service channels.

Simulation of $\mathbf{E}(\mathbf{W}_t)$

We split $E(W_t)$ into EW_{t_1} and EW_{t_2} as described in figure 7 by MATLLAB software when service times and inter arrival times follow exponential distributions with parameters $\frac{1}{\mu}$ and $\frac{1}{\lambda}$ respectively.

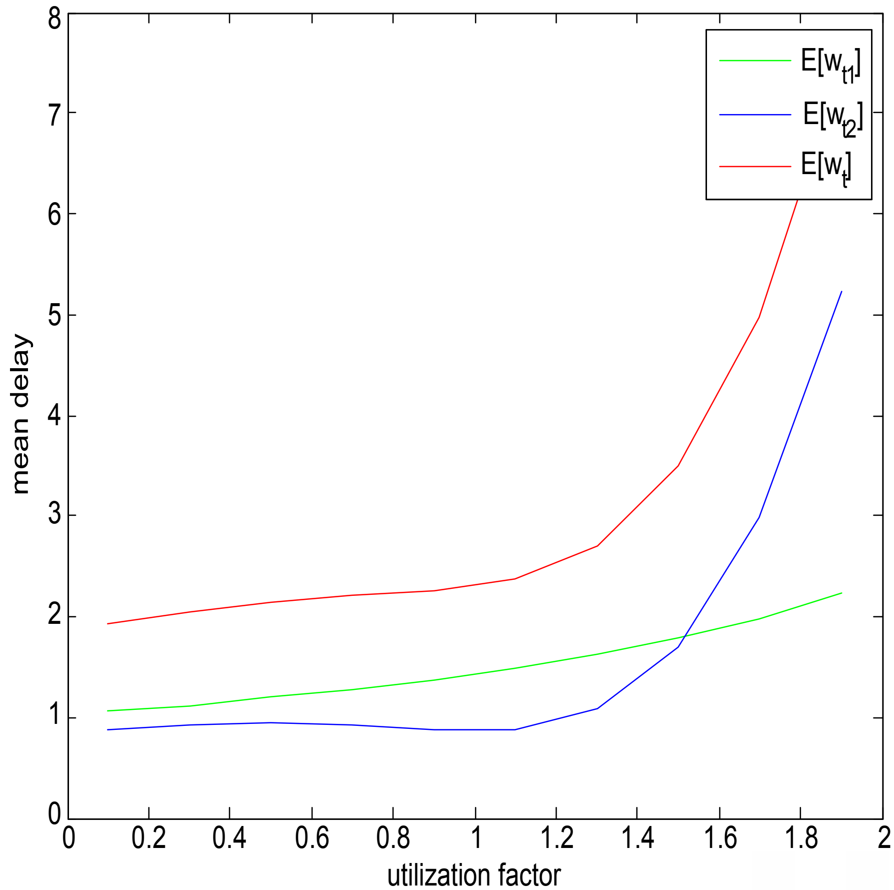

Figure 3 Diagram representing simulation of overall model $\mathbf{E}[\mathbf{W}_{\mathbf{t}}]$ $\mathbf{E}[\mathbf{W}_{\mathbf{t}_1}]$ $\mathbf{E}[\mathbf{W}_{\mathbf{t}_2}]$ verses ρ $\boldsymbol{\rho}$ with two servers

From figure 3 it is clear to note that the stochastic delay model is only applicable to under saturated conditions $\rho < 1$ and estimates infinite delay when the arrival flow approaches capacity. However, when arrival flows exceed capacity, oversaturated queues exist and continuous delays occur. The deterministic delay model also depicts that it estimates a continuous delay which is definitely higher than that of a three sever queue but it does not completely deal with the effect of randomness when the arrival flows are close to capacity.

The figure shows that both components of the overall delay model are compatible when the utilization factor is equal to 1.0. Therefore the overall delay model is used to bridge the gap between the two models. It is important to also note that ultimately the overall model also indicates of an increased waiting time which is explained by the reduced number of servers and also provides a more realistic point of view for the results in the estimation of delays in the bank queue delays for the oversaturated as well as the under saturated conditions is predicted without having any discontinuity.

4.0.2: Conclusion

Considering the uniform and random properties of queues in banks, the models for estimating deterministic and stochastic delay components of bank queue delays successfully modelled waiting times in selected banks in Eldoret town.

From the mean waiting time models of stochastic and deterministic delays, the models are conveniently applicable on real time bank queue data.

To validate the mean waiting time models, the model was applied to real bank queue data collected from the various selected banks namely; Kenya Commercial bank, Equity Bank, National Bank, Barclays Bank and Cooperative Bank for data between Monday 1st to Friday 5th August 2016 respectively and simulation was performed for utilization factors ranging from 0.1 to 1.9 using MATLAB software simulink functions. The simulation results show that when a queue system is not at equilibrium, it indicates continuous delays past the equilibrium point i.e. $\rho > 1$.

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