Modeling Heteroscedasticity in the Presence of Serial Correlations in Discrete-Time Stochastic Series: A GARCH-in-Mean Approach

ABSTRACT

Background: In modeling heteroscedasticity of returns, it is often assumed that the series are uncorrelated. In practice, such series with small time periods between observations can be observed to contain significant serial correlations, hence the motivation for this research.
Aim: The aim of this research is to investigate the existence of serial correlations in the return series of Zenith bank Plc, which is targeted at identifying their effects on the parameter estimates of heteroscedastic models.
Material and Methods: The data were obtained from the Nigerian Stock Exchange spanning from January 3, 2006 to November 24, 2016 having 2690 observations. The joint Autoregressive Integrated Moving Average-Generalized Autoregressive Conditional Heteroscedaticity (ARIMA-GARCH-type) models such as Autoregressive Integrated Moving Average-Generalized Autoregressive Conditional Heteroscedasticity (ARIMA-GARCH), Autoregressive Integrated Moving Average-Exponential Generalized Autoregressive Conditional Heteroscedasticity (ARIMA-EGARCH) and the Autoregressive Integrated Moving Average-Glosten, Jagannathan and Runkle Generalized Autoregressive Conditional Heteroscedastic (ARIMA-GJRGARCH) under normal and student-t distributions were employed to model the conditional variance while the GARCH-in-Mean-GARCH-type model corresponding to the selected ARIMA-GARCH-type model was applied to appraise the possible existence of serial correlations.
Results: The findings of this study showed that heteroscedasticity exists and could adequately captured by ARIMA(2,1,1)-EGARCH(1,1) model under student-t distribution but failed to account for the presence of serial correlations in the series. Meanwhile, its counterpart, GARCH-in-Mean-EGARCH(1,1) model under student-t distribution sufficiently appraised the existence of serial correlations.
Conclusion: One remarkable implication is that, the estimates of the parameters of ARIMA-GARCH-type model are likely to be biased when the presence of serial correlations is ignored. Also, the application of GARCH-in-Mean-GARCH-type model possibly provides the feedback mechanism or interaction between the variance and mean equations.

Keywords: GARCH-type models, Heteroscedasticity, Time Series, Volatilty

1. INTRODUCTION

The existence of heteroscedasticity in financial series (returns) always leads to the violation of assumption of constant variance in linear time series. The linkage between the occurrence of heteroscedasticity in financial data and the violation of assumption of constant variance in linear time series has created a vast research area for professionals in Statistics, Economics and Finance. As required naturally, the assumption of constant variance assumes that the error term of the linear stationary model should be homogeneous. By implication, the constant error variance means that the conditional variance of the dependent variable is also constant. According to [1], the assumption of constant variance is required to ensure the accuracy of standard errors and asymptotic covariances amongst estimated parameters. It could be remarked that a major setback on linear stationary models when applying to financial data (returns) is their failure to account for changing variance. In other words, whenever the assumption of constant variance is violated, heteroscedasticity has occurred, implying that the conditional distribution of the dependent variable has different degrees of variability at different levels.

In the Statistical context, heteroscedasticity (i.e. non-constant variance) means the same thing as volatility in Finance and Economics, although they are generally used interhangeably by some authors. However, neglecting the presence of heteroscedasticity in linear models makes the ordinary least squares estimates of ARMA parameters inefficient. Although they are still consistent and asymptotically normally distributed, their variance-covariance matrix is no longer the usual one. As a result of this, the t-statistics become invalid and cannot be used to examine the significance of the individual explanatory variables in the model. Also, over-parameterization of an ARMA model and low statistical power are identified as part of the consequences for neglecting heteroscedasticity. Lastly, neglecting heteroscedasticity can lead to spurious nonlineality in the conditional mean and difficulty in computing the confidence interval for forecasts (see [2], [3], [4], [5]). Furthermore, details of heteroscedasticity modeling are documented in [6], [7], [8], [9], [10], [11], [12], [13].

Certainly, the motivation for this study is drawn from the fact that serial correlations (a relationship between a variable and its lagged-value over a period of time) tend to exist in most financial series though several analyses on such series are often based on the assumption that the series are uncorrelated. Moreover, these serial correlations are believed to be introduced by those in the time-varying heteroscedasticity process [14]. However, failure to account for these serial correlations when modeling heteroscedasticity would amount to obtaining a biased estimate of the true degree of persistence (see also [15]). To capture these high variations over time with regards to risk and volatility, [16] proposed the modification of standard (generalized autoregressive conditional heteroscedastic) GARCH-type model under the assumption that the variance coefficient in the mean equation measures the relative risk aversion. Also, according to [17], the increasing roles played by the risk and uncertainty in financial assets have led to the development of new time series techniques for measuring time variations. One of such techniques is the GARCH-in-Mean (GARCH-M) model. It allows the conditional variance of the series to influence the conditional mean. Also, the formulation of the GARCH-M model implies that there are serial correlations in the series and are being introduced by those in the heteroscedasticity process. This particular specification is useful and effective in modeling the risk-return relationship in financial series. The major advantage of GARCH-M model over the standard GARCH-type models is that any misspecification of variance function would not affect the consistency of the estimators of parameters of the mean. Meanwhile, prior studies of [18], [17], [19], [20], [21], [22], [23] and [24]) have applied GARCH-M technique to capture varying property of risk aversion and autocorrelation of return series as well as interaction between the mean and variance equations of GARCH-type models. Particularly, this study seeks to improve on the work of [25] that detected and modeled the asymmetric GARCH effects using GARCH-type models under the assumption that the return series is uncorrelated. This is captured by applying GARCH-M technique to ascertain the presence of serial correlation in the return series considered.

The study is further organized as follows: materials and methods are presented in section 2, discussion of results is handled in section 3 while section 4 concludes the study.

2. MATERIAL AND METHODS

2.1 Returns

The return series, R_t, can be obtained given that P_t is the price of a unit share at time t, and P_{t-1} is the share price at time t−1. Thus,

$$R_t = \nabla lnP_t = (1 - B)lnP_t = ln\,P_t - ln\,P_{t-1} \quad (1)$$

Here, R_t is regarded as a transformed series of the share price (P_t) meant to attain stationarity, that is, both mean and variance of the series are stable [25] while B is the backshift operator.

2.2 Autoregressive Integrated Moving Average (ARIMA) Model

[26] considered the extension of ARMA model to deal with homogenous non-stationary time series in which X_t, itself is non-stationary but its d^{th} difference is a stationary ARMA model. Denoting the d^{th} difference of X_t by

$$\varphi(B) = \phi(B)\nabla^d X_t = \theta(B)\varepsilon_t \quad (2)$$

where $\varphi(B)$ is the nonstationary autoregressive operator such that d of the roots of $\varphi(B) = 0$ are unity and the remainder lie outside the unit circle while$\phi(B)$ is a stationary autoregressive operator.

2.3 Standard GARCH-type Models

Conceptually, heteroscedastic models are hybridized of both mean and variance equations. The mean equation is represented by the ARIMA Model as shown in equation (3), $R_t = \mu_t + a_t$, (3)

where $\mu_t = \varphi_0 + \sum_{j=1}^{p} \varphi_j R_{t-j} + \sum_{i=1}^{q} \theta_i a_{t-i}$.

Also,

$$a_t = \sigma_t e_t, \qquad (4)$$

where e_t is a sequence of independent and identically distributed (i.i.d.) random variables with zero mean,i.e.E(e_t) = 0 and variance 1. In practice, e_t is often assumed to follow the standard normal or a standardized student-t distribution while a_t is the standardized residual term that follows autoregressive conditional heteroscedastic (ARCH(q)), generalized autoregressive conditional heteroscedastic (GARCH (q, p)), exponential generalized autoregressive conditional heteroscedastic (EGARCH(q,p)) and glosten, jagannathan and runkle generalized autoregressive conditional heteroscedastic (GJR-GARCH(q,p)) models in (5), (6), (7) and (8), respectively.

2.3.1 ARCH model

The first model that provides a systematic framework for modeling volatility is the ARCH model of [27]. Specifically, an ARCH (q) model assumes that,

$$\sigma_t^2 = \omega + \alpha_1 a_{t-1}^2 + \cdots + \alpha_q a_{t-q}^2, \qquad (5)$$

where $\omega > 0, and \alpha_1, .., \alpha_q \geq 0$ [28]. The coefficients α_i, for $i > 0$, must satisfy some regularity conditions to ensure that the unconditional variance of $a_t$is finite. From the structure of the model, it is seen that large squares of past shocks,$\{a_{t-i}^2\}_{i=1}^{q}$, imply a large conditional variance,σ_t^2, for the innovation, a_t. Consequently, a_t tends to assume a large value (in modulus). This means that, under the ARCH framework, large shocks tend to be followed by another large shock.

2.3.2 GARCH model

Although the ARCH model is simple, it often requires many parameters to adequately describe the volatility process of a share price return. As a functional alternative,[29] proposed a useful extension known as the generalized ARCH (GARCH) model. The GARCH (q, p) is defined as;

$$\sigma_t^2 = \omega + \sum_{i=1}^{q} \alpha_i a_{t-i}^{'2} + \sum_{j=1}^{p} \beta_j \sigma_{t-j}^2 \tag{6}$$

where $\omega > 0,\ \alpha_i \geq 0,\ \beta_j \geq 0,\ and \sum_{i=1}^{max(p,q)} (\alpha_i + \beta_i) < 1$ (Tsay, 2010).

Here, it is understood that $\alpha_i = 0,$ for $i > p$, and $\beta_i = 0$, for $i > q$. The later constraint on $\alpha_i + \beta_i$ implies that the unconditional variance of $a_t^{'}$ is finite, whereas its conditional variance,σ_t^2, evolves over time. In most cases, estimates of the GARCH (1,1) model on returns yield $\alpha_1 + \beta_1 \approx$ 1, and this results in an explosive process, that is, the volatility process is not mean-reverting. So, the conditional variance is nearly integrated (Integrated GARCH model) [14].

2.3.3 EGARCH model

The Exponential GARCH (EGARCH) model represents a major shift from ARCH and GARCH models [30]. Rather than model the variance directly, EGARCH models the natural logarithm of the variance, and so no parameter restrictions are required to ensure that the conditional variance is positive. The EGARCH (q, p) is defined as,

$$ln\sigma_t^2 = \omega + \sum_{k=1}^{r} \gamma_k a_{t-k} + \sum_{i=1}^{q} \alpha_i \left(|a_{t-i}| - \sqrt{2/\pi} \right) + \sum_{j=1}^{p} \beta_j \, ln\sigma_{t-j}^2 \tag{7}$$

Alternatively, EGARCH(q, p) model with respect to student-t distribution can be represented by

$$ln\sigma_t^2 = \omega + \sum_{k=1}^{r} \gamma_k a_{t-k} + \sum_{i=1}^{q} \alpha_i \left(|a_{t-i}| - \frac{2\sqrt{v-2}\,\Gamma(v+1)/2}{(v-1)\Gamma(v/2)\sqrt{\pi}} \right) + \sum_{j=1}^{p} \beta_j \, ln\sigma_{t-j}^2 \tag{8}$$

,

where γ_k is the asymmetric coefficient. In the original parameterization of Nelson (1991), p and r were assumed to be equal. The process is covariance stationary if and only if$\sum_{j=1}^{q} \beta_j < 1$. The γ_i parameter thus signifies the leverage effect of a_{t-i}. Again, we expect γ_i to be negative in real applications [14].

2.3.4 GJR-GARCH model

The GJR GARCH (q, p) model [31] is a variant, represented by

$$\sigma_t^2 = \omega + \sum_{i=1}^{q} \alpha_i a_{t-i}^2 + \sum_{i=1}^{p} \gamma_i I_{t-i} a_{t-i}^2 + \sum_{j=1}^{p} \beta_j \sigma_{t-j}^2 \tag{9}$$

Or written as

$$\sigma_t^2 = \omega + \sum_{i=1}^{q}(\alpha_i + \gamma_i I_{t-i})a_{t-i}^2 + \sum_{j=1}^{p}\beta_j \sigma_{t-j}^2 \quad (10)$$

where I_{t-1} is an indicator for negative a_{t-i}, that is,

$$I_{t-1} = \begin{cases} 0 & if \ a_{t-i} < 0, \\ 1 & if \ a_{t-i} \geq 0, \end{cases}$$

and α_i, γ_i, and β_j are nonnegative parameters satisfying conditions similar to those of GARCH models. Also the introduction of indicator parameter of leverage effect, I_{t-1} in the model accommodates the leverage effect, since it is supposed that the effect of a_{t-i}^2 on the conditional variance σ_t^2 is different accordingly to the sign ofa_{t-i}. From the model, it is obvious that a positive a_{t-i} contributes $\alpha_i a_{t-i}^2$ to σ_t^2, whereas a negative a_{t-i} has a larger impacts $(\alpha_i + \gamma_i)a_{t-i}^2$ with γ_i> 0 as established by (Tsay, 2010). The model uses zero as it threshold to separate the impacts of past shocks (see,[28], [14]).

2.4 GARCH-in-Mean Model

The mean equation (3) is modified to obtain GARCH-in-mean model in (11) such that the return series depends on its variance. The specification of GARCH-in-mean model implies that there are serial correlations in the return series (see [14]).

$$R_t = \mu_t + \tau\sigma_t^2 + a_t, \quad (11)$$

where the parameter τ is the variance functional coefficient. Thus the presence of variance functional coefficient σ_t^2, indicates that the return series has serial correlation, which implies that the return series is related to its variance.

3 RESULTS AND DISCUSSION.

3.1 Interpretation of Time Plot

The share price series of the Nigerian bank considered was found to be nonstationary given the random fluctuations away from the common mean (see Figure 1).

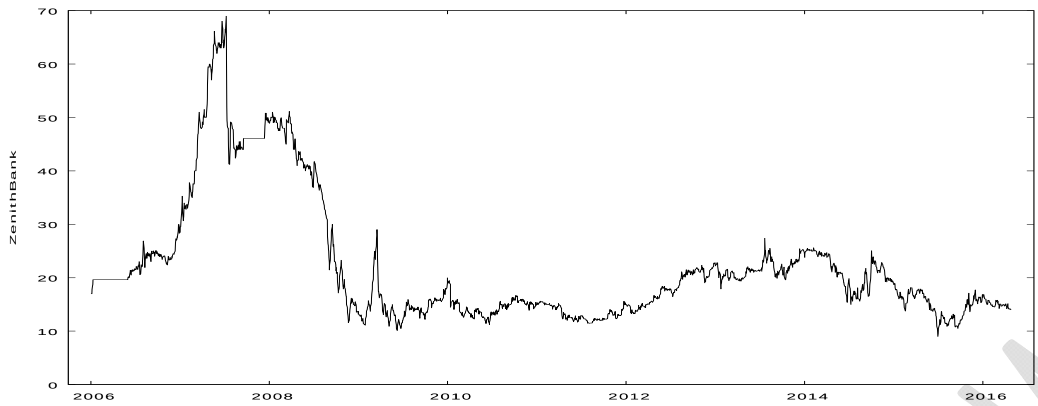

Figure 1: Share Price Series of Zenith Bank

Stationarity was achieved by transforming the share price series using equation (1) and the transformed series was found to cluster round the common mean and thus indicated the presence of heteroscedasticity (see Figure 2).

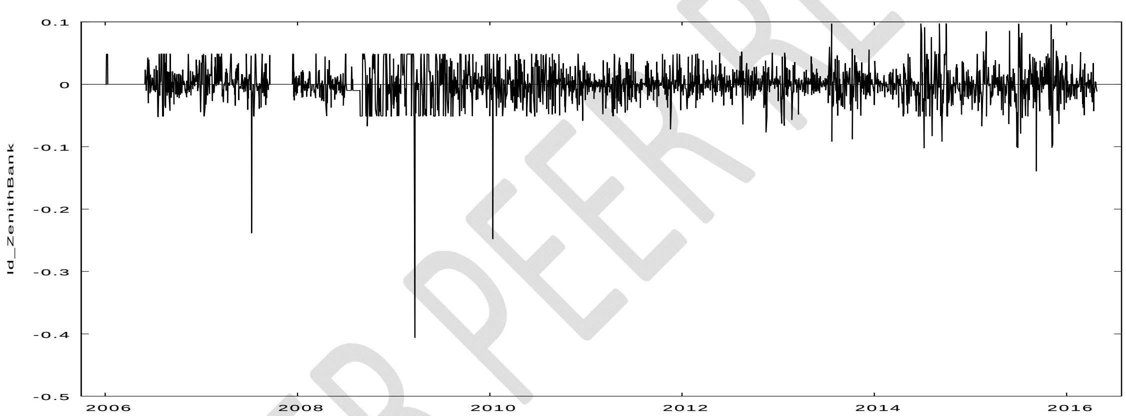

Figure 2: Return Series of Zenith Bank

3.2 Modeling Joint ARIMA-GARCH-type Processes of Return Series of Zenith Bank

Based on Box and Jenkins procedures, out of the several models identified tentatively, the following joint ARIMA-GARCH-type models with respect to both normal (norm) and student-t (std) distributions were considered (see Table 1).

Table 1: Output of ARIMA-GARCH-type Models of Return Series of Zenith Bank

						Information Criteria		
Model	**Parameter**	**Estimate**	**s.e**	**t-ratio**	**p-value**	**AIC**	**BIC**	**HQIC**
ARIMA(2,1,1)-GARCH (1,0)-std	μ	$-1.38e^{-4}$	$1.2e^{-4}$	-1.1518	0.2494	-6.4622	-6.4469	-6.4567
	φ_1	-1.0182	0.0094	-108.3242	0.0000			
	φ_2	-0.0828	0.0211	-3.9297	0.0001			
	θ_1	0.9268	0.0197	47.0444	0.0000			
	ω	$6.4e^{-5}$	$6.0e^{-6}$	10.7403	0.0000			

	α_1	0.9990	0.1339	7.4598	0.0000			
ARIMA(2,1,1)-GARCH (2,0)-norm	μ	$5.11e^{-4}$	$1.85e^{-4}$	2.7590	0.0058			
	φ_1	0.8695	0.0208	41.7691	0.0000			
	φ_2	0.1140	0.0202	5.6563	0.0000			
	θ_1	−0.9529	0.0022	−442.0869	0.0000			
	ω	$5.1e^{-5}$	$2.0e^{-6}$	20.9116	0.0000			
	α_1	0.4918	0.0463	10.6297	0.0000			
	α_2	0.2357	0.0314	7.5012	0.0000	−6.3503	−6.3350	−6.3448
ARIMA(2,1,1)-GARCH (2,0)-std	μ	$-2.48e^{-4}$	$2.3e^{-4}$	−1.0367	0.29987			
	φ_1	0.8644	0.0212	40.8256	0.0000			
	φ_2	0.1193	0.0211	5.6551	0.0000			
	θ_1	−0.9722	0.0011	−851.2935	0.0000			
	ω	$4.0e^{-5}$	$4.0e^{-6}$	11.2777	0.0000			
	α_1	0.6418	0.0754	8.5143	0.0000			
	α_2	0.3572	0.0553	6.4607	0.0000	−6.5041	−6.4866	−6.4978
ARIMA(2,1,1)-GARCH (1,1)-norm	μ	$7.4e^{-5}$	$1.4e^{-5}$	5.4230	0.0000			
	φ_1	0.1655	0.3888	0.4256	0.6704			
	φ_2	$5.9e^{-5}$	0.0386	0.0015	0.9988			
	θ_1	−0.2458	0.3883	−0.6330	0.52670			
	ω	$2.0e^{-6}$	0.0000	11.6607	0.0000			
	α_1	0.1753	0.0125	13.9806	0.0000			
	β_1	0.8237	0.0092	89.6875	0.0000	−6.4261	−6.4108	−6.4206
ARIMA(2,1,1)-GARCH(1,1)-std	μ	0.0000	0.0000	−0.0035	0.9972			
	φ_1	−0.1192	0.9558	−0.1247	0.9007			
	φ_2	0.0011	0.0860	0.0129	0.9897			
	θ_1	0.0139	0.9550	0.0145	0.9884			
	ω	0.0000	0.0000	0.0000	1.0000			
	α_1	0.2646	0.0101	26.3116	0.0000			
	β_1	0.7252	0.0055	131.5636	0.0000	−7.0699	−7.0524	−7.0635
ARIMA(2,1,1)-EGARCH(1,1)-norm	μ	$2.22e^{-4}$	$9.5e^{-5}$	2.33943	0.0193			
	φ_1	−0.0068	0.0139	-0.4929	0.6221			
	φ_2	−0.0061	0.0276	−0.21981	0.8260			
	θ_1	−0.0719	0.0226	−3.18238	0.0015			
	ω	−0.6502	0.0034	−191.7116	0.0000			
	α_1	−0.0040	0.0160	−0.2489	0.8034			
	β_1	0.9260	$1.9e^{-4}$	4871.3044	0.0000			
	γ_1	0.3794	0.0207	18.3618	0.0000	−6.4624	−6.4448	−6.4560
ARIMA(2,1,1)-**EGARCH(1,1)-std**	$\boldsymbol{\mu}$	**0.0000**	**0.0000**	**1.2065**	**0.2276**			
	$\boldsymbol{\varphi_1}$	**−0.3086**	**0.0106**	**−29.0801**	**0.0000**			
	$\boldsymbol{\varphi_2}$	**0.0495**	**0.0171**	**2.8972**	**0.0038**			
	$\boldsymbol{\theta_1}$	**0.2912**	**0.0105**	**27.7875**	**0.0000**			
	$\boldsymbol{\omega}$	**−0.0291**	$\mathbf{8.62e^{-4}}$	**−33.7888**	**0.0000**			
	$\boldsymbol{\alpha_1}$	**−0.6979**	$\mathbf{9.8e^{-5}}$	**−7150.4179**	**0.0000**			
	$\boldsymbol{\beta_1}$	**0.9996**	$\mathbf{6.3e^{-5}}$	**15825.7924**	**0.0000**			
	$\boldsymbol{\gamma_1}$	**0.6983**	$\mathbf{9.8e^{-5}}$	**7147.6333**	**0.0000**	**−7.0189**	**−6.9992**	**−7.0118**
ARIMA(2,1,1)-GJR-GARCH(1,0)-norm	μ	$-4.1e^{-4}$	$2.0e^{-6}$	−260.0268	0.0000			
	φ_1	1.7704	0.0049	363.9136	0.0000			
	φ_2	−1.2088	0.0019	−642.61109	0.6056			
	θ_1	0.7873	$6.41e^{-4}$	1228.7631	0.0000			
	ω	0.0000	$1.0e^{-6}$	0.0952	0.92418			
	α_1	0.961136	0.0029	328.0539	0.0000	1.6422	1.6576	1.6478

	β_1	0.8486	0.0064	131.9359	0.0000			
	γ_1	0.0754	0.0364	2.072042	0.0383			
ARIMA(2,1,1)-GJR-GARCH(1,0)-std	μ	$-4.32e^{-4}$	$2.57e^{-4}$	-1.677346	0.0935			
	φ_1	0.8733	0.0216	40.3732	0.0000			
	φ_2	0.1086	0.0217	5.0124	$1.0e^{-6}$			
	θ_1	-0.9684	0.0012	-810.3464	0.0000			
	ω	$6.3e^{-5}$	$6.0e^{-6}$	10.9327	0.0000			
	α_1	0.993393	0.1474	6.7378	0.0000			
	γ_1	0.0112	0.1536	0.0730	0.9418	-6.4675	-6.4499	-6.4611
ARIMA(2,1,1)-GJR-GARCH(2,0)-norm	μ	$-1.21e^{-4}$	$9.9e^{-5}$	-1.2223	0.2216			
	φ_1	0.8713	0.0233	37.3883	0.0000			
	φ_2	0.1115	0.0234	4.7763	0.0000			
	θ_1	-0.9526	0.0014	-659.3565	0.0000			
	ω	$5.0e^{-5}$	$2.0e^{-6}$	20.8695	0.0000			
	α_1	0.3549	0.0468	7.5802	0.0000			
	α_2	0.1918	0.0383	5.0032	$1.0e^{-6}$			
	γ_1	0.3147	0.0845	3.7230	0.0002			
	γ_2	0.0804	0.0561	1.4328	0.1519	-6.3556	-6.3359	-6.3485
ARIMA(2,1,1)-GJR-GARCH(2,0)-std	μ	$-3.46e^{-4}$	$2.52e^{-4}$	-1.3744	0.1693			
	φ_1	0.8722	0.0091	96.194	0.0000			
	φ_2	0.1181	0.0090	13.111	0.0000			
	θ_1	-0.9811	$9.5e^{-5}$	-1.0310	0.0000			
	ω	$4.0e^{-5}$	$4.0e^{-6}$	11.367	0.0000			
	α_1	0.6411	0.0911	7.0376	0.0000			
	α_2	0.2869	0.0614	4.6756	$3.0e^{-6}$			
	γ_1	-0.0047	0.1105	-0.0042	0.9663			
	γ_2	0.1467	0.0906	1.6205	0.1051	-6.5037	-6.4817	-6.4957
ARIMA(2,1,1)-GJR-GARCH(1,1)-norm	μ	$7.7e^{-5}$	$1.4e^{-5}$	5.6257	0.0000			
	φ_1	0.1732	0.1767	0.9802	0.3270			
	φ_2	$-1.13e^{-4}$	0.0342	-0.0033	0.9974			
	θ_1	-0.2540	0.1921	-1.3223	0.1861			
	ω	$2.0e^{-6}$	0.0000	4.1180	$3.8e^{-5}$			
	α_1	0.1775	0.0330	5.3738	0.0000			
	β_1	0.8243	0.0239	34.5612	0.0000			
	γ_1	-0.0056	0.0370	-0.1528	0.8785	-6.4254	-6.4079	-6.4191
ARIMA(2,1,1)-GJR-GARCH(1,1)-std	μ	0.0000	0.0000	0.306023	0.75959			
	φ_1	-0.0417	0.4469	-0.0933	0.9257			
	φ_2	-0.0029	0.0578	-0.0509	0.9594			
	θ_1	-0.0813	0.4390	-0.1851	0.8531			
	ω	0.0000	0.0000	0.0000	1.0000			
	α_1	0.2737	0.0186	14.7192	0.0000			
	β_1	0.7013	0.0067	104.9602	0.0000			
	γ_1	0.0367	0.0295	1.2459	0.2128	-7.0480	-7.0282	-7.0408

Comparing the values of the information criteria of the models as indicated in Table 1, it is shown that the information criteria for ARIMA(2,1,1)-GARCH(1,1)-std model is the smallest, followed by ARIMA(2,1,1)-GJR-GARCH(1,1)-std mode, although they are characterized by several non-significant parameters. However, ARIMA(2,1,1)-EGARCH(1,1)-std model, which is next to ARIMA(2,1,1)-GJR-

GARCH(1,1)-std model has all its parameters significant except the constant term of the mean equation, which assumes the value of zero. Hence, ARIMA(2,1,1)-EGARCH(1,1)-std model is selected as the appropriate heteroscedastic model for the return series of the Bank.

Table 2: Diagnostic Checking for ARIMA-GARCH-type Model of Return Series of Zenith Bank

Model	Standardized Residuals			Standardized Squared Residuals					
	Lag	Weighted LB	p-value	Lag	Weighted LB	p-value	Lag	Weighted ARCH-LM	p-value
ARIMA(2,1,1)-EGARCH(1,1)-std	1	0.0014	0.9697	1	0.0014	0.9704	3	0.0014	0.9704
	8	0.0066	1.0000	5	0.0041	1.0000	5	0.0033	0.9999
	14	0.0111	1.0000	9	0.0069	1.0000	7	0.0049	1.0000

The model was found to be adequate given that the p-values corresponding to weighted Ljung-Box Q statistics at lags 1, 8 and 14 on standardized residuals, weighted Ljung-Box Q statistics at lags 1, 5 and 9 on standardized squared residuals and weighted Lagrange Multiplier statistics at lags 3, 5 and 7 are all greater than 5% level of significance [see Table 2]. That is to say, the hypotheses of no autocorrelation and no remaining ARCH effect are not rejected.

3.4 Modeling GARCH-in-Mean-EGARCH Processes of the Return Series of Zenith Bank

Table 3: Output of GARCH-in-Mean-EGARCH Model of Return Series of Zenith Bank

Model	Parameter	Estimate	s.e	t-ratio	p-value	Information Criteria		
						AIC	BIC	HQIC
GARCH-in-Mean-EGARCH(1,1)-std	μ	0.0000	0.0000	0.15021	0.8806			
	φ_1	0.6845	0.0040	170.0493	0.0000			
	φ_2	0.0428	0.0038	11.3871	0.0000			
	θ_1	−0.7089	0.0041	−170.894	0.0000			
	τ	0.0428	0.0043	9.8773	0.0000			
	ω	-0.1279	0.0029	−43.9733	0.0000			
	α_1	-0.6616	$2.72e^{-4}$	−2436.7359	0.0000			
	β_1	0.9904	$3.8e^{-5}$	26149.3241	0.0000			
	γ_1	0.6632	$2.72e^{-4}$	2433.8259	0.0000	-6.9012	-6.8793	-6.8933

All the parameters of joint GARCH-in-Mean-EGARCH(1,1)-std model are significant at 5% level of significance except the constant term of the GARCH-in-Mean equation which assumes the value of zero. Of particular interest is the GARCH-in-Mean coefficient whose significance points to the presence of serial correlation in the return series(see Table 3).

Table 4: Diagnostic Checking for GARCH-in-Mean-EGARCH Models of Return Series of Zenith Bank

Model	Standardized Residuals			Standardized Squared Residuals					
GARCH-in-Mean-	Lag	Weighted LB	p-value	Lag	Weighted LB	p-value	Lag	Weighted ARCH-LM	p-value

EGARCH(1,1)-std	1	0.0008	0.978	1	0.0009	0.9757	3	0.0009	0.9757
	8	0.0059	1.0000	5	0.0028	1.0000	5	0.0022	0.9999
	14	0.0099	1.0000	9	0.0046	1.0000	7	0.0033	1.0000

The model was found to be adequate given that the p-values corresponding to weighted Ljung-Box Q statistics at lags 1, 8 and 14 on standardized residuals, weighted Ljung-Box Q statistics at lags 1, 5 and 9 on standardized squared residuals and weighted Lagrange Multiplier statistics at lags 3, 5 and 7 are all greater than 5% level of significance [see Table 4]. That is to say, the hypotheses of no autocorrelation and no remaining ARCH effect are not rejected.

3.5 Effects of Serial Correlation on Parameters of ARIMA-GARCH-type Model

Table 5: Biased Effects of Serial Correlations on the Parameters of ARIMA(2,1,1)-EGARCH(1,1)-std Model of Zenith Bank

Parameter	ARIMA(2,1,1)-EGARCH(1,1)-std Model fitted to Returns Series of Zenith Bank	GARCH-in-Mean-EGARCH(1,1)-std Model fitted to Returns Series of Zenith Bank	Biases introduced
Constant Term (μ)	0.0000	0.0000	0.0000
Autoregressive of order 1 Coefficient (φ_1)	0.6094	0.6845	-0.0751
Autoregressive of order 2 Coefficient (φ_2)	0.0852	0.0428	0.0424
Moving Average of order 1 Coefficient (θ_1)	-0.6012	-0.7089	0.1077
Garch-in-Mean Coefficient (τ)	0.0428	-	-
Constant Term (ω)	-0.0960	-0.1279	0.0319
ARCH Coefficient (α)	-0.8581	-0.6616	-0.1965
GARCH Coefficient(β)	0.9903	0.9904	-0.0001
Asymmetric Coefficient (γ)	0.8591	0.6632	0.1959

Substantial biases are being introduced into the parameters of the ARIMA(2,1,1)-EGARCH(1,1)-std model when the possible existence of serial correlation is ignored as indicated in Table 5. That is, in the presence of serial correlations, the Autoregressive of order 1, ARCH and GARCH parameters were reduced by 0.0751, 0.1965 and 0.0001, respectively while Autoregressive of order 2 Coefficient, Moving Average of order 1 Coefficient, Constant term of the variance equation and asymmetric parameters were hyped by 0.0424, 0.1077, 0.0319 and 0.1959, respectively. Hence, it can be deduced that the presence of serial correlations, the parameters of ARIMA-GARCH-type models are biased.

In brief, the findings of this study showed that serial correlations exist in the return series of the bank understudy. Thus building an ARIMA(2,1,1)-EGARCH(1,1)-std model without accounting for the existence of serial correlations results in biased parameters as indicated in Table 5.Consequently, the extent of bias associated with the existence of serial correlation was appraised by GARCH-in-Mean-EGARCH(1,1)-std model as shown in Table 3.

Although this study showed similarity to the work of (25) by confirming that EGARCH model is suitable to the return series of Zenith bank Plc, yet, it provides enough evidence of substantial improvement by modifying the mean equation of the model to account for the presence of serial correlations. In addition, the introduction of variance parameter in the mean equation creates a feedback mechanism between heteroscedasticity and returns.

By implication, the study revealed that the return is positively related to its variance, which implies that any high increase in conditional variance would likely lead to a high increase in the returns.

4 CONCLUSION

In summary, the findings of this very study revealed that the standard Joint ARIMA- GARCH-type model is not sufficient for capturing serial correlations and their application without considering the existence of serial correlations often results in biased parameters. Consequently, the GARCH-in-Mean-GARCH-type model provided the much needed modification that accounts for the existence of serial correlations in return series. Therefore, the formulation of GARCH-in-Mean equation by incorporating variance component ensures that the risk-return relationship is properly depicted. It is recommended that the similar formulation be undertaken by replacing the variance component with the standard deviation or probably the natural logarithm of the variance in future studies.

REFERENCES

[1] Rosopa PJ, Schaffer MM, Schroeder AN. Managing Heteroscedasticity in General Linear Models. *Psychological Methods* 2003;18(**3**): 335-351. https://doi.org/10.1037/a0032553.

[2] Deshon RP, Alexander RA. Alternative Procedures for Testing Regression Slope Homogeneity when Group Error Variances are unequal. *Psychological Methods* 1996; **1**:261-277. http://dx.doi.org/10.1037/1082-109X.1.3.261.

[3] Franses PH, van Dijk D. *Non-linear Time Series Models in Empirical Finance.* 2nd ed. New York,Cambridge University Press, pp 135-147; 2003.

[4] Fan J, Yao Q. *Nonlinear time series: Nonparametric and Parametric methods*. 2nd ed. New York: Springer, pp 143-171; 2003.

[5] Asteriou D, Hall SG. *Applied Econometrics.A Modern Approach.*3rd ed.New York: Palgrave Macmillan, pp 117-124; 2007.

[6] Moffat IU, Akpan E A. Selection of Heteroscedastic Models: A Time Series Forecasting Approach. *Applied Mathematics* 2019; **10**: 333-348. https://doi.org/10.4236/am.2019.105024.

[7] Akpan EA, Lasisi KE, Adamu A, Rann HB. Application of Iterative Approach in Modeling the Efficiency of ARIMA-GARCH Processes in the Presence of Outliers. *Applied Mathematics* 2019; **10**: 138-158. https://doi.org/10.4236/am.2019.103012.

[8] Xie H. Financial Volatility Modeling: The Feedback Asymmetric Conditional Autoregressive Range Model. *Journal of Forecasting* 2018; 38(**1**):11-28. https://doi.org/10.1002/for.2548.

[9] Siddiqui TA, Narula I. Stock Market Volatility and Anomalies in India: A Behavioural Approach. *Asia-Pacific Journal of Management Research and Innovation* 2017; 12(**3-4**): 194-202. https://doi.org/10.1177/2319510X17708368.

[10] Moffat IU, Akpan EA, Abasiekwere UA. A Time Series Evaluation of the Asymmetric Nature of Heteroscedasticity: An EGARCH Approach. *International Journal of Statistics and Applied Mathematics* 2017; 2(**6**):111-117.

[11] Akpan E A, Moffat I U, Ekpo NB. Arma- Arch Modeling of the Returns of First Bank of Nigeria. *European Scientific Journal* 2016; 12(**8**): 257-266. http://dx.doi.org/10.19044/esj.2016.v12n18p257.

[12] Huskaj B, Larsson K. An Empirical Study of the Dynamic of Implied Volatility Indices: International Evidence.*Quantitative Finance Letters* 2016; 4(**1**):77-85. https://doi.org/10.1080/21649502.2017.1292041.

[13] Adu G, Karimu A. Stock Return Distribution in the BRICS. *Review of Development Finance* 2015; 5(**2**): 98-109. https://doi.org/10.1016/j.rdf.2015.09.002.

[14] Tsay RS. *Analysis of Financial Time Series*. 3rd ed. New York: John Wiley & Sons Inc., pp 97-140; 2010. https://doi.org/10.1002/9780470644560.

[15] Zhao H, Huffer F, Niu X. Time-Varying Coefficient Models with ARMA-GARCH Structures for Longitudinal Data Analysis. *Journal of Applied Statistics* 2014; 42(**2**):309-326. https://doi.org/10.1080/02664763. 2014.949638.

[16] Engle RF, Lilien DM, Robin RP. Estimating Time Varying Risk Premia in the Term Structure: The Arch-M Model. *Econometrica* 1987; 55(**2**): 391-407. http://ideas.respec.org/a/ecmetrp/v55y1987i2p391-407.html.

[17] Mathur S, Chotia V, Rao NVM. Modeling the Impact of Global Financial Crisis on the Indian Stock Market through GARCH Models. *Asia-Pacific Journal of Management Research and Innovation* 2016; 12(**1**): 11-12. https://doi.org/0.1177/2319510/16650056.

[18] Dias GF. The Time-Varying GARCH-in-Mean Model. *Economics Letters*2002; **157**:129-132. https://doi.org/10.1016/j econlet. 2017.06.005.

[19] Conrad C, Karanasos M. On the Transmission of Memory in GARCH-in-Mean Models. *Journal of Time Series Analysis* 2015; 36(**5**):706-720. https://doi.org/10.1111/jtsa.12119.

[20] Islam MA. Applying Generalized Autoregressive Conditional Heteroscedasticity Models to Model Univariate Volatility. *Journal of Applied Science* 2014; 14(**7**):64-650. https://doi.org/10.3923/Jas.2014.641.650.

[21] Panait I, Slavescu EO. Using GARCH-in-Mean Model to investigate Volatility and persistence at Different Frequencies for Bucharest Stock Exchange during 1997-2012. *Theoretical and Applied Economics* 2012; **5(570**): 55-76.

[22] Zhang X, Wong H, Li Y, Lp W. An Alternative GARCH-in-Model: Structured and Estimation. *Communications inStatistics-Theory and Methods* 2011; 42(**10**): 1821-1839 https://doi.org/10.1080/03610926.2011.598999.

[23] Karanasos M. Prediction in ARMA Models with GARCH in Mean Effects. *Journal of Time Series Analysis*2001; 22(**5**): 555-576. http://doi.org/10.1111/1467.9892.00241.

[24] Hong EP. The Autocorrelation Structure for the GARCH-M Process. *Economics Letters* 1991; 37(**2**): 129-132. https://doi.org /10.1016/0165-1765(91)90120-A

[25] Akpan EA, Moffat IU. Detection and Modeling of Asymmetric GARCH Effects in a Discrete-Time Series. *International Journal of Statistics and Probability*2017; **6**: 111-119.https://doi.org/10.5539/ijsp.v6n6p111.

[26] Box GEP, JenkinsGM,Reinsel GC. *Time Series Analysis: Forecasting and Control.*3rd ed. Hoboken, NJ,John Wiley & Sons;2008.https://doi.org/10.1002/9781118619193.

[27] Engle RF. Autoregressive Conditional Heteroscedasticity with Estimates of the Variance of United Kingdom Inflations. *Econometrica* 1982; **50**: 987-1007. https://doi.org/10.2307/1912773.

[28] Francq C, Zakoian J. GARCH Models: Structure, Statistical Inference and Financial Applications . 1st ed.Chichester: John Wiley & Sons Ltd.19-220; 2010. https://doi.org/10.1002/9780470670057.

[29] Bollerslev T. Generalized Autoregressive Conditional Heteroscedasticity. *Econometrics* 1986; **31**, 307-327. https://doi.org/10.1016/0304-4076(86)90063-1.

[30] Nelson DB. Conditional Heteroscedasticity of Asset Returns. A New Approach.*Econometrica* 1991; **59**: 347-370. https://doi.org/10.2307/2938260.

[31] Glosten LR, Jagannathan R, Runkle D. On the Relation between the expected Values and the Volatility of the Nominal Excess Return on Stocks. *Journal of Finance* 1993; **48**: 1779-1801. https://doi.org/10.1111/j.1540-6261.1993.tb05128.x.

.

.