Original Research Article

Solutions to the equations governing convective flow of two viscous immiscible dusty and pure fluids in a vertical corrugated wall and a parallel flat wall.

ABSTRACT

This paper presents the solutions to the equations governing convective flow and heat transfer of two viscous immiscible dusty and pure fluids confined between a vertical corrugated wall and a parallel flat wall. The nonlinear partial differential equations governing the flow have been reduced to nonlinear ordinary differential equations using the regular perturbation method. The transformed nonlinear ordinary differential equations have been solved numerically using the linear approximation theorem. The effects of the governing parameters on the velocity and temperature fields for the two fluids and the dust particles have been obtained and graphically represented using Matlab.

Keywords: *Immiscible fluids, Dusty fluid, Perturbation method, corrugated wall.*

INTRODUCTION

Corrugated surfaces are, for example, utilized in compact heat exchangers and in industrial processes to enhance heat transfer efficiency. Dusty fluids are applicable in areas such as petroleum extraction, purification of crude oil and nuclear waste treatment. Yao and Moulic [1] studied natural convection along a wavy surface with uniform heat flux. Sastry et al [2] analyzed Couette flow of two immiscible fluids between two permeable beds. Umavathi et al. [3] analyzed the problem of unsteady mixed convective heat transfer of two immiscible fluids confined between long vertical wavy wall and parallel flat wall. Umavathi et al [4, 5] studied unsteady flow and heat transfer of three immiscible fluids. Vajravelu and Sastri [6] investigated free convective heat transfer in a viscous incompressible fluid between a vertical wavy wall and a parallel flat wall. Verma and Bhatt [7] considered the steady flow of two immiscible incompressible fluids with suction at the stationary plate. Wang et al [8, 9, 10] studied free and forced convective flow in wavy channels. Yao [11] studied natural convection along a vertical complex wavy surface.

Most recently, Siddiqa et al [12] analyzed flow of a dusty fluid in two phase natural convection. Attia et al [13] used a porous medium in a circular pipe to study unsteady dusty Bingham fluid flow. Abba et al [14] also used two parallel plates with heat transfer to investigate Couette flow of two immiscible dusty fluids.

All the above cited references except Abba et al [14] investigated on dusty fluids and pure immiscible fluids through different channels but none studied flow and heat transfer of two viscous immiscible dusty and pure fluids between a corrugated wall and a parallel flat wall.

Thus, the objective of the present work is to study convective flow of two viscous immiscible dusty and pure fluids between a vertical corrugated wall and a parallel flat wall.

The flow is taken to be steady, two dimensional and the fluid is liquid and not gas, incompressible and electrically non-conducting. The governing nonlinear equations for the dusty and pure fluids are solved numerically by Perturbation Method with linear approximation theorem.

MATHEMATICAL FORMULATION

A two dimensional steady laminar flow of two electrically non-conducting immiscible dusty and pure fluids in a vertical channel with one wavy wall and another flat wall is considered as shown in figure 1. The X-axis which represented by $Y=-w^{(2)}+\eta\cos(\lambda X)$ is taken parallel to the flat wall, while the Y- axis represented by $Y=w^{(1)}$ is taken to be perpendicular. The wavy and flat walls are maintained at constant temperatures $\tilde{T}_2$ and $\tilde{T}_1$ respectively. Region I is occupied by a fluid of density $\rho^{(1)}$, viscosity $\mu^{(1)}$, thermal conductivity $k^{(1)}$, thermal expansion coefficient $\beta^{(1)}$, specific heat at constant pressure $c_p^{(1)}$ and Region II is occupied by the fluid of density $\rho^{(2)}$, viscosity $\mu^{(2)}$, thermal conductivity $k^{(2)}$, thermal expansion coefficient $\beta^{(2)}$, specific heat at constant pressure $c_p^{(2)}$.

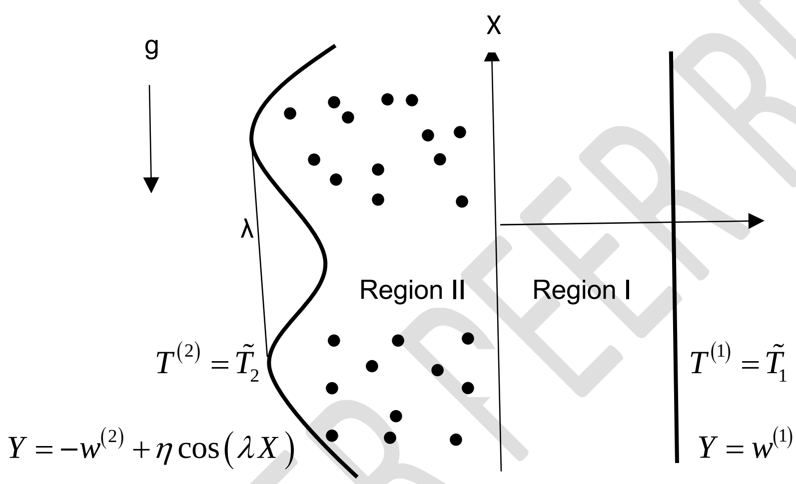

Figure 1: Physical configuration

The following assumptions are considered in this study. The fluid in region-II is dusty while the fluid in region-I is considered to be a pure fluid. Except the density in the buoyancy term in the momentum equation, all fluid properties are assumed constant. The transport properties of both fluids are assumed to be constant and the fluid rises in the channel driven by buoyancy forces. The dust particles in region II are assumed to be electrically non-conducting, spherical in shape, solid, same radius and mass (uniform in size), un-deformable, and uniformly distributed throughout the flow. This means that, by conduction through their spherical surface, the dust particles gain heat energy from the fluid. The number density N of the particles is constant throughout the flow and volume fraction of the dust particles is neglected and the temperature between the particles is uniform throughout the motion. The concentration of particles is very small that it is not interfering with the continuity and the net effect of the dust on the fluid particles is equivalent to $FN\left(u^{(2)}-u^{p}\right)$ per unit volume. Where F is stoke's law (drag force) where $F=6\pi\mu rv$ and r is average radius of the dust particles, μ is coefficient of fluid viscosity (dynamic viscosity, v is flow velocity relative to the object and N is density number of particles per unit volume of the fluid.

GOVERNING EQUATIONS

Region I (Pure fluid)

$$\frac{\partial u^{(1)}}{\partial x^{(1)}}+\frac{\partial v^{(1)}}{\partial y^{(1)}}=0$$ Continuity

(1)

$$\rho^{(1)}\left(u^{(1)}\frac{\partial u^{(1)}}{\partial x^{(1)}}+v^{(1)}\frac{\partial u^{(1)}}{\partial y^{(1)}}\right)=-\frac{\partial p^{(1)}}{\partial x^{(1)}}+\mu^{(1)}\nabla^2u^{(1)}+\rho^{(1)}g\beta^{(1)}\left(T^{(1)}-T_s\right)$$ X-Momentum (2)

$$\rho^{(1)}\left(u^{(1)}\frac{\partial v^{(1)}}{\partial x^{(1)}}+v^{(1)}\frac{\partial v^{(1)}}{\partial y^{(1)}}\right)=-\frac{\partial p^{(1)}}{\partial y^{(1)}}+\mu^{(1)}\nabla^2v^{(1)}$$ Y-Momentum (3)

$$\rho^{(1)}c_p^{(1)}\left(u^{(1)}\frac{\partial T^{(1)}}{\partial x^{(1)}}+v^{(1)}\frac{\partial T^{(1)}}{\partial y^{(1)}}\right)=k^{(1)}\nabla^2T^{(1)}$$ Energy

(4)

Region II (Dusty fluid)

$$\frac{\partial u^{(2)}}{\partial x^{(2)}}+\frac{\partial v^{(2)}}{\partial y^{(2)}}=0$$ Continuity

(5)

$$\rho^{(2)}\left(u^{(2)}\frac{\partial u^{(2)}}{\partial x^{(2)}}+v^{(2)}\frac{\partial u^{(2)}}{\partial y^{(2)}}\right)=-\frac{\partial p^{(2)}}{\partial x^{(2)}}+\mu^{(2)}\nabla^2u^{(2)}+\rho^{(2)}g\beta^{(2)}\left(T^{(2)}-T_s\right)-FN\left(u^{(2)}-u^p\right)$$ X-Momentum

(6)

$$\rho^{(2)}\left(u^{(2)}\frac{\partial v^{(2)}}{\partial x^{(2)}}+v^{(2)}\frac{\partial v^{(2)}}{\partial y^{(2)}}\right)=-\frac{\partial p^{(2)}}{\partial x^{(2)}}+\mu^{(2)}\nabla^2v^{(2)}-FN\left(v^{(2)}-v^p\right)$$ Y-Momentum (7)

The equation of motion of the dust particles by taking Newton’s second law in the X direction is given by

$$m_p\left(u^p\frac{\partial u^p}{\partial x}+\frac{\partial u^p}{\partial y}\right)=FN\left(u^{(2)}-u^p\right)$$

(8)

m_p is average mass of dust particles.

$$\rho^{(2)}c_p^{(2)}\left(u^{(2)}\frac{\partial T^{(2)}}{\partial x^{(2)}}+v^{(2)}\frac{\partial T^{(2)}}{\partial y^{(2)}}\right)=k^{(2)}\nabla^2T^{(2)}+\frac{\rho^p c_s}{\gamma_t}\left(T^p-T^{(2)}\right)$$ Energy equation of the fluid (9)

$$u^{(2)}\frac{\partial T^p}{\partial x^{(2)}}+v^{(2)}\frac{\partial T^p}{\partial y^{(2)}}=\frac{-1}{\gamma_t}\left(T^p-T^{(2)}\right)$$ Energy equation of the particles

(10)

For both the velocity and temperature, the relevant boundary and interface conditions used to solve Eqns. (1) to (10) are

$$u^{(2)}=v^{(2)}=u^{P}=v^{P}=0 \text{ at } Y=-w^{(2)}+\eta\cos(\lambda X),\ u^{(1)}=v^{(1)}=0 \text{ at } Y=w^{(1)},\ u^{(1)}=u^{(2)}=u^{p}\ ,$$

$$v^{(1)}=v^{(2)}=v^{p} \text{ at } Y=0,\ \mu^{(1)}\left(\frac{\partial u}{\partial y}+\frac{\partial v}{\partial x}\right)^{(1)}=\mu^{(2)}\left(\frac{\partial u}{\partial y}+\frac{\partial v}{\partial x}\right)^{(2)}=\mu^{(2)}\left(\frac{\partial u}{\partial y}+\frac{\partial v}{\partial x}\right)^{p} \text{ at } Y=0$$

$$\frac{\partial p^{(1)}}{\partial x^{(1)}}=\frac{\partial p^{(2)}}{\partial x^{(2)}} \text{ at } Y=0\ ,\quad T^{(2)}=\tilde{T}_2 \text{ at } Y=-w^{(2)}+\eta\cos(\lambda X),\ T^{(1)}=\tilde{T}_1 \text{ at } Y=w^{(1)}$$

$$T^{(1)}=T^{(2)}=T^{p}\ ;\ k^{(1)}\left(\frac{\partial T}{\partial y}+\frac{\partial T}{\partial x}\right)^{(1)}=k^{(2)}\left(\frac{\partial T}{\partial y}+\frac{\partial T}{\partial x}\right)^{(2)} \text{ at } Y=0$$

The non-dimensional flow variables are:

$$\hat{x}^{(1)}=\frac{x^{(1)}}{w^{(1)}}\ ,\ \hat{y}^{(1)}=\frac{y^{(1)}}{w^{(1)}}\ ,\ \hat{x}^{(2)}=\frac{x^{(2)}}{w^{(2)}}\ ,\ \hat{y}^{(2)}=\frac{y^{(2)}}{w^{(2)}}\ ,\ \hat{u}^{(1)}=\frac{w^{(1)}}{v^{(1)}}u^{(1)},\ \hat{v}^{(1)}=\frac{w^{(1)}}{v^{(1)}}v^{(1)}\ ,$$

$$\hat{u}^{(2)}=\frac{w^{(2)}}{v^{(2)}}u^{(2)},\ \hat{v}^{(2)}=\frac{w^{(2)}}{v^{(2)}}v^{(2)}\ ,\ Gr=\frac{{w^{(1)}}^{3}g\beta^{(1)}\Delta T}{{v^{(1)}}^{2}},\ \Delta T=T_2-T_s\ ,\ \Pr=\frac{{c_p}^{(1)}\mu^{(1)}}{k^{(1)}}\ ,\ \hat{\lambda}=\lambda w^{(2)}$$

$$\hat{p}^{(1)}=\frac{p^{(1)}}{\rho^{(1)}\left(v^{(1)}/w^{(1)}\right)^{2}},\ \hat{p}^{(2)}=\frac{p^{(2)}}{\rho^{(2)}\left(v^{(2)}/w^{(2)}\right)^{2}},\ \theta^{(1)}=\frac{T^{(1)}-T_s}{T_2-T_s},\theta^{(2)}=\frac{T^{(2)}-T_s}{T_2-T_s}\ ,\theta^{p}=\frac{T^{p}-T_s}{T_2-T_s},T_0=\frac{T_1-T_s}{T_2-T_s}$$

$$\beta_0=\frac{\beta^{(2)}}{\beta^{(1)}},\ w_0=\frac{w^{(2)}}{w^{(1)}},\ \mu_0=\frac{\mu^{(1)}}{\mu^{(2)}},\ \rho_0=\frac{\rho^{(2)}}{\rho^{(1)}},\ k_0=\frac{k^{(2)}}{k^{(1)}},\ c_p=\frac{c_p^{(1)}}{c_p^{(2)}},\ v^{(1)}=\frac{\mu^{(1)}}{\rho^{(1)}}\ ,\ v^{(2)}=\frac{\mu^{(2)}}{\rho^{(2)}}$$

The non-dimensional variables are substituted in to Eqns. (3.1) to (3.10) and dropping the (caps) for simplicity, the equations obtained are as follows

Region I (Pure fluid)

$$\frac{\partial u^{(1)}}{\partial x}+\frac{\partial v^{(1)}}{\partial y}=0 \tag{11}$$

$$u^{(1)}\frac{\partial u^{(1)}}{\partial x}+v^{(1)}\frac{\partial u^{(1)}}{\partial y}=-\frac{\partial p^{(1)}}{\partial x}+\frac{\partial^2 u^{(1)}}{\partial x^2}+\frac{\partial^2 u^{(1)}}{\partial y^2}+Gr\theta^{(1)} \tag{12}$$

$$u^{(1)}\frac{\partial v^{(1)}}{\partial x}+v^{(1)}\frac{\partial v^{(1)}}{\partial y}=-\frac{\partial p^{(1)}}{\partial y}+\frac{\partial^2 v^{(1)}}{\partial x^2}+\frac{\partial^2 v^{(1)}}{\partial y^2} \tag{13}$$

$$u^{(1)}\frac{\partial\theta^{(1)}}{\partial x}+v^{(1)}\frac{\partial\theta^{(1)}}{\partial y}=\frac{1}{\Pr}\left(\frac{\partial^2\theta^{(1)}}{\partial x^2}+\frac{\partial^2\theta^{(1)}}{\partial y^2}\right) \qquad (14)$$

Region II (Dusty fluid)

$$\frac{\partial u^{(2)}}{\partial x}+\frac{\partial v^{(2)}}{\partial y}=0 \qquad (15)$$

$$u^{(2)}\frac{\partial u^{(2)}}{\partial x}+v^{(2)}\frac{\partial u^{(2)}}{\partial y}=-\frac{\partial p^{(2)}}{\partial x}+\frac{\partial^2 u^{(2)}}{\partial x^2}+\frac{\partial^2 u^{(2)}}{\partial y^2}+Gr\,\beta_0 w_0^3\mu_0^2\rho_0^2\theta^{(2)}-R_0\left(u^{(2)}-u^p\right) \qquad (16)$$

$$u^{(2)}\frac{\partial v^{(2)}}{\partial x}+v^{(2)}\frac{\partial v^{(2)}}{\partial y}=-\frac{\partial p^{(2)}}{\partial y}+\frac{\partial^2 v^{(2)}}{\partial x^2}+\frac{\partial^2 v^{(2)}}{\partial y^2}-R_0\left(v^{(2)}-v^p\right) \qquad (17)$$

$$u^p\frac{\partial u^p}{\partial x}+v^p\frac{\partial u^p}{\partial y}=\frac{1}{G_0}\left(u^{(2)}-u^p\right) \qquad (18)$$

$$u^{(2)}\frac{\partial\theta^{(2)}}{\partial x}+v^{(2)}\frac{\partial\theta^{(2)}}{\partial y}=\frac{k_0\mu_0 c_p}{\Pr}\left(\frac{\partial^2\theta^{(2)}}{\partial x^2}+\frac{\partial^2\theta^{(2)}}{\partial y^2}\right)+\frac{2R_0}{3\Pr}\left(\theta^p-\theta^{(2)}\right) \qquad (19)$$

$$u^p\frac{\partial\theta^p}{\partial x}+v^p\frac{\partial\theta^p}{\partial y}=-L\left(\theta^{(2)}-\theta^p\right) \qquad (20)$$

The boundary and interface conditions are non - dimensionalized as follows

$$u^{(2)}=v^{(2)}=u^p=v^p \text{ at } y=-1+\eta\cos(\lambda x),\quad u^{(1)}=v^{(1)}=0 \text{ at } y=1$$

$$u^{(1)}=\frac{u^{(2)}}{\mu_0 w_0\rho_0}=\frac{u^p}{\mu_0 w_0\rho_0},\quad v^{(1)}=\frac{v^{(2)}}{\mu_0 w_0\rho_0}=\frac{v^p}{\mu_0 w_0\rho_0} \text{ at } y=0$$

$$\left(\frac{\partial u}{\partial y}+\frac{\partial v}{\partial x}\right)^{(1)}=\frac{1}{w_0^2\mu_0^2\rho_0}\left(\frac{\partial u}{\partial y}+\frac{\partial v}{\partial x}\right)^{(2)}=\frac{1}{w_0^2\mu_0^2\rho_0}\left(\frac{\partial u}{\partial y}+\frac{\partial v}{\partial x}\right)^{p} \text{ at } y=0$$

$$\frac{\partial p^{(1)}}{\partial x}=\frac{1}{\rho_0\mu_0^2 w_0^2}\frac{\partial p^{(2)}}{\partial x} \text{ at } y=0,\quad \theta^{(2)}=1 \text{ at } y=-1+\eta\cos(\lambda x),\quad \theta^{(1)}=T \text{ at } y=1$$

$$\theta^{(1)}=\theta^{(2)}=\theta^p,\quad \left(\frac{\partial\theta}{\partial y}+\frac{\partial\theta}{\partial x}\right)^{(1)}=\frac{k_0}{w_0}\left(\frac{\partial\theta}{\partial y}+\frac{\partial\theta}{\partial x}\right)^{(2)}=\frac{k_0}{w_0}\left(\frac{\partial\theta}{\partial y}+\frac{\partial\theta}{\partial x}\right)^{p} \text{ at } y=0$$

SOLVING OF THE EQUATIONS

Perturbation techniques can be used to obtain approximate solutions since analytical solutions are difficult because of their nonlinear form. By introducing a small parameter τ and assuming that the solutions consists of a Zeroth order (no perturbation, hence no new information) and a first order (meaning full perturbation) the velocity, pressure and temperature can be written as

Region I (Pure fluid)

$$u^{(1)}(x,y) = u_0^{(1)}(y) + \tau u_1^{(1)}(x,y) + ...$$
(21)

$$v^{(1)}(x,y) = \tau v_1^{(1)}(x,y) + ...$$
(22)

$$p^{(1)}(x,y) = \tau p_1^{(1)}(x,y) + ...$$
(23)

$$\theta^{(1)}(x,y) = \theta_0^{(1)}(y) + \tau\theta_1^{(1)}(x,y) + ...$$
(24)

Region II (Dusty fluid)

$$u^{(2)}(x,y) = u_0^{(2)}(y) + \tau u_1^{(2)}(x,y) + ...$$
(25)

$$u^{p}(x,y) = u_0^{p}(y) + \tau u_1^{p}(x,y) + ...$$
(26)

$$v^{(2)}(x,y) = \tau v_1^{(2)}(x,y) + ...$$
(27)

$$v^{p}(x,y) = \tau v_1^{p}(x,y) + ...$$
(28)

$$p^{(2)}(x,y) = \tau p_1^{(2)}(x,y) + ...$$
(29)

$$\theta^{(2)}(x,y) = \theta_0^{(2)}(y) + \tau\theta_1^{(2)}(x,y) + ...$$
(30)

$$\theta^{p}(x,y) = \theta_0^{p}(y) + \tau\theta_1^{p}(x,y) + ...$$
(31)

RESULTS AND DISCUSSION

After substituting Eqns. (21)-(31) in to Eqns. (11)-(20), the obtained ordinary differential equations are solved numerically using Perturbation method and the linear approximation theorem. In each graph, Grashof number, viscosity ratio, width ratio and conductivity ratio are fixed at 6, 3, 3, and 3 respectively except the temperature ratio, T_0 and the parameter in question. The temperature ratio is increasing from -2 to 2 in all the graphs.

From Fig. 4.1(a), it is observed that, as the Grashof number and the temperature ratio increases, the zeroth order velocity of the dust particles and that of the fluid increases in both regions for $T_0 = 2$ and $T_0 = 0$. For $T_0 = -2$, the velocity decreases from the start of region II (wavy wall) up to near the middle of region I (flat wall) and again starts to increase as it approaches the end of region I. Physically, increase in Grashof number is an increase in the buoyancy force because, in the momentum equation, the Grashof number acts a driving mechanism of the buoyancy force which supports the motion. From Fig. 4.1(b), as the Grashof number increases, the first order velocity diminishes sharply in both regions. For $T_0 = -2$, the velocity increases from the start of region II (wavy wall) up to near the middle of region I (flat wall) and again starts to decrease as it approaches the end of region I. From Figure 4.2 (a), it is observed that, as the width ratio and temperature ratio increases, the zeroth order velocity increases in both regions. For a larger width, physically this means an increase in velocity. From Figure 4.2 (b), it is observed that, as the width ratio and temperature ratio increases, the first order velocity decreases in both regions.

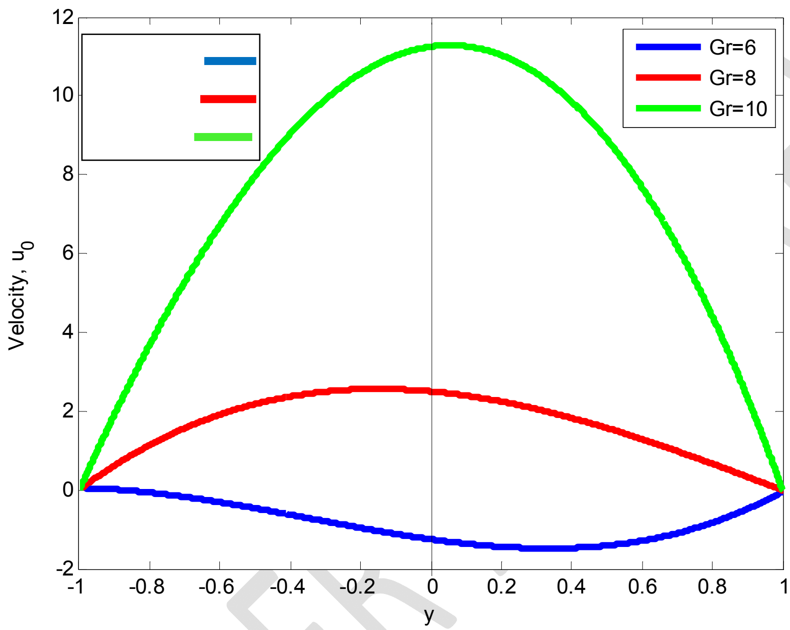

Figure 4.1 (a): Effect of Grashof number, Gr on the velocity profiles.

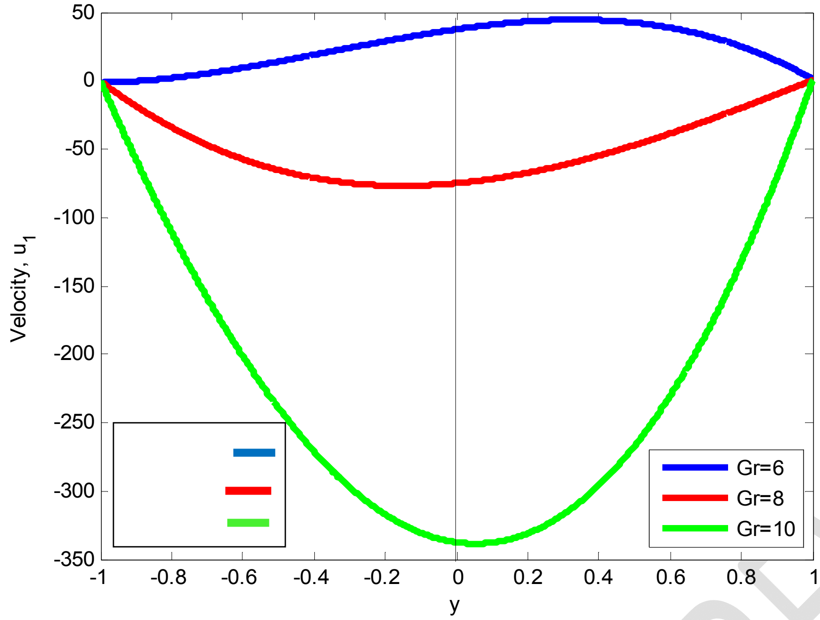

Figure 4.1 (b): Effect of Grashof number, Gr **on the velocity profiles**.

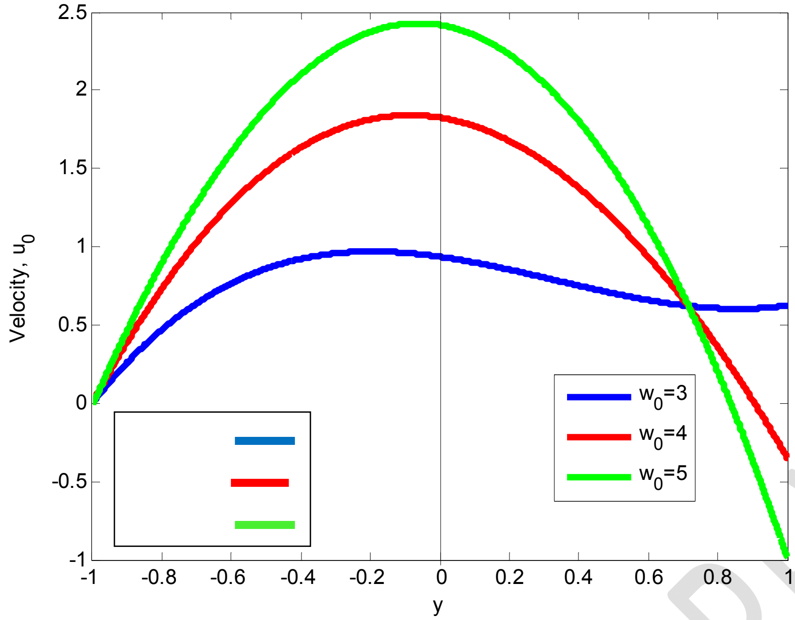

Figure 4.2(a): Effect of width ratio, w_0 on the velocity profiles.

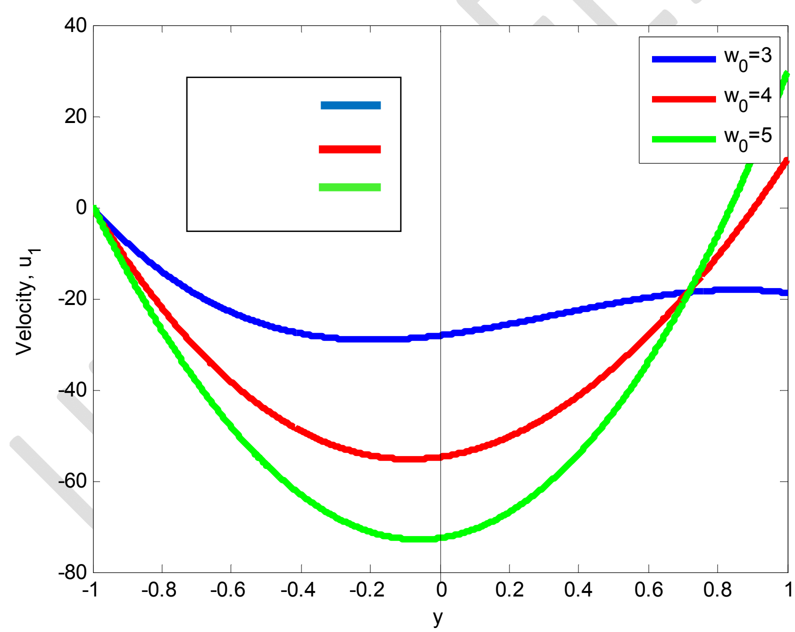

Figure 4.2(b): Effect of width ratio, w_0 on the velocity profiles.

CONCLUSIONS

As the Grashof number and temperature ratio increases, zeroth order velocity of the dust particles and that of the fluid increases in both regions as the first order velocity decreases in both regions. As the width ratio and temperature ratio increases, the zeroth order temperature increases in both regions as the first order velocity decreases in both regions.

COMPETING INTERESTS

Authors have declared that no existing competing interests.

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