

A CHINESE REMAINDER THEOREM BASED ENHANCEMENTS OF LEMPEL-ZIV-WELCH AND HUFFMAN CODING IMAGE COMPRESSION

ABSTRACT

Data size minimization is the focus of data compression procedures by altering representation and reducing redundancy of information to a more effective kind. In general, lossless approach is favoured by a number of compression methods for the purpose of maintaining the content integrity of the file afterwards. The benefits of compression include saving storage space, speed up of data transmission and high quality of data. This paper observes the effectiveness of Chinese Remainder Theorem (CRT) enhancement in the implementation of Lempel-Ziv-Welch (LZW) and Huffman coding algorithms for the purpose of compressing large size images. Ten images of Yale database was used for testing. The outcomes revealed that CRT-LZW compression saved more space and speedy compression (or redundancy removal) of original images to CRT-Huffman coding. Similarly, CRT-Huffman coding (28.13) offered better quality Peak-Signal-to-Noise-Ratio (PSNR) for the reconstructed images when compared to CRT-LZW (3.54) and (25.59) obtained in other investigated paper.

Keywords: LZW, Huffman coding, CRT, compression time, size reduction, image, compression.

1. INTRODUCTION

Data compression is the method of decreasing the size of information to be transmitted or stored by the process of eliminating redundancy in information without the loss or the ability to reconstruct the original data. There are several file formats that can be effectively compressed including text, image, video, and audio [1]. Image compression addresses the difficulty involved in decreasing the volume of data vital in denoting an image with no major loss of information. In recent times, Chinese Remainder Theorem (CRT) has developed in fields of coding theory, phase unwrapping, frequency estimation and distributed data storage. This is certainly due to the fact; CRT is profound for isolating errors in residue caused by noise. The traditional CRT reconstructs a single integer for error-free co-prime and residues [2]. Images have limited applications in real life situations such as medical, scientific, prepress and artistic applications, due to enormous sizes for broadcast or storage given low bandwidth [3]. In dealing with memory capacity insufficiency, compression schemes have been deployed; thereby offering the prospect of broadcasting images/video under scarce bandwidth. The classical image compression scheme converts a spatial domain representation to frequency domain [3].

In classical JPEG codec, images are encoded independently. The surge in cloud data storage has thrown up issues of content duplication and large redundancy, which must be considered. Inter-coding is one of such solution for traditional video encoding of consecutive frames from previous frames. Another method is the inter-prediction tools of video codecs to encode comparable images as pseudo-video arrangements [4].

Again, lossy compression algorithms such as Huffman coding gives relatively good quality as well as compression rates with images but blocky look of reconstructed images [5]. The reverse is the case of LZW in which compressed image data quality is retained at the expense of little size decreases [6]. In this paper, CRT enhancement is proposed for independent implementation of Huffman coding and LZW algorithm for image compression procedures.

2. RELATED STUDIES

The goal of image compression in variety of application is to decrease the quantity of bytes in a graphic file but retaining the quality. In the study, [3] considered several approaches for compressing images especially in medicine. The forms of compression for image involve spatial to frequency

48 domains. The main concept is that pictures are composed of neighbouring pixels; though related not
49 without repetitive data. However, colour images require treatment of distinct colour segments.

50 A novel prediction scheme for performing cloud-based compression was initiated by [4]. This
51 approach utilizes the semi-local geometric and photometric prediction scheme to compensate in a
52 region-wise style the distortion between two images rather than inter-coding schemes (such as high
53 efficiency video coding). This is most useful for highly correlated image content applications such as
54 traditional video coding, cloud gaming streaming, photo albums compression. This minimizes the
55 redundancy arising from similar content already available in cloud. Nevertheless, cloud multiple
56 frames exploitations, and determining scalability of cloud-based image compression system.

57 Lossless compression schemes of LZW and Huffman coding were combined by [6]. The target was to
58 enable medical imagery suitable for storage, quality retention and broadcast. Huffman coding offered
59 massive size decreases in a speedy manner, but, poor quality of compressed image. Conversely,
60 LZW algorithm produced finest quality with little size decreases. The combined compression schemes
61 gave rise to high compression ratio and high PSNR values.

62 A fresh algorithm was proposed by [7] for encoding, decoding or regenerating the replica of the
63 encoded data. The first step uses the forward difference scheme on Huffman. Then, the values are
64 regenerated into fixed length code representation with twos complement for further new probabilities
65 computation along Huffman's algorithm. There are improvements in compression factors for the new
66 algorithm against the traditional Huffman encoding. Residue Number System is introduced in data
67 encryption and decryption with Shannon Fano compression scheme by [16]. The outcomes revealed
68 significant improvements in the security, speed and memory needed for existing information
69 communication networks.

70 Typically, multimedia files (such as image data) are composed of redundancy and irrelevance limiting
71 their usage on widespread basis. Since the advent of internetworks and communication infrastructure,
72 there are possibilities of broadcasting or storing digital images seamlessly [4]. The sizes of the
73 multimedia data make them inefficient for broadcast or storage purpose [6]. Majority of lossless
74 compression methods are founded on probability or dictionary and entropy because they make use of
75 the availability of the identical string or character within data in order to realize compression [6].
76 Researchers are focusing attention on removing redundancy and irrelevance in image data, which
77 gave rise to the concept of data compression schemes such as Huffman coding and Lempel-Ziv-
78 Welch algorithm [6]. In general, the performance of compression schemes is estimated with standard
79 metrics such as effectiveness (compression ratio) and efficiency (speedup or throughput) [7]. In this
80 paper, these compression algorithms are highlighted in certain details.

81 One common entropy encoding algorithm deployed for lossless image compression is Huffman
82 coding [8]. The encoding strategy commences with calculation of each symbol probability in the
83 image. Thereafter, these symbols probabilities are placed in a descending magnitude as to create leaf
84 nodes of a tree. By individually coding the symbols, the Huffman code is built combining the
85 lowermost probable symbols. These entire steps are continued until only two probabilities of two
86 compound symbols are present. Eventually, a code tree is produced and the labelling of the code tree
87 generates the Huffman code [9].

88 The Huffman codes for the symbols are realised by analysing the branch digits in succession from the
89 root node to the respective terminal node or leaf using symbols 0 and 1. Huffman coding is the most
90 deployed method for redundancy or relevance minimization [8]. The operational principle of Huffman
91 code is based on these observations:

- 92 a) The more frequently occurring symbols are assigned shorter code words than less frequent
93 occurring ones;
- 94 b) The two symbols occurring least frequently is assigned the similar length.

95 On the average, code length is determined as the average of the product of symbol probability and
96 amount of encoding bits [10] [11]. The Huffman code efficiency is calculated as the ratio of entropy to
97 the average length. The target of Huffman encoding creates the optimal code for a collection of
98 symbols and probabilities whenever, the symbols are coded currently within the same time frame [8].
99

100 LZW algorithm is a popular lossless data compression scheme initiated by Abraham Lempel, Jacob
101 Ziv, and Terry Welch. In 1984, as an improvement over the traditional LZ78 algorithm released 1978
102 by Lempel and Ziv, which is easy to deploy with the prospect of offering significantly high throughput

103 in hardware applications. According [12], the algorithm encodes sequences of 8-bit data as fixed-
 104 length 12-bit codes. The codes from 0 to 255 depict 1-character sequences composed of the
 105 matching 8-bit character, and the codes 256 through 4095 are created in a dictionary for sequences
 106 contained in the data during the process of encoding [1] [12] [13].

107

108 2.1 The Concept of Chinese Remainder Theorem (CRT)

109

110 The basic operation of Chinese Remainder Theorem (CRT) is to generate a single integer through its
 111 residue modulo within moduli set [2]. CRT is an alternative to the Mixed Radix Conversion (MRC) in
 112 which large modulo M derivations are unnecessary. MRC accepts a low complexity of $O(n)$ unlike the
 113 CRT having computation complexity of order $O(n^3)$. In CRT, arithmetic operations for modulo M are to
 114 be manually executed. CRT residue converters are much more complex. In contrast, the MRC
 115 procedure requires arithmetic operations for modulo m_i only, thereby simplifying all operations. In
 116 MRC method, a number x is expressed in mixed-radix system. Suppose for moduli set
 117 (m_1, m_2, \dots, m_n) , RNS representation of a number x is given by (x_1, x_2, \dots, x_n) . The number x can be
 118 expressed in mixed-radix form as:

119

$$|x|_{m_1} = a_1$$

120

$$X = a_n \prod_{i=1}^{n-1} m_i + \dots a_3 m_1 m_2 + a_2 m_1 + a_1 \quad (1)$$

121 where, the a_i 's are the mixed-radix coefficients. These a_i 's are determined sequentially, starting with
 122 a_1 , in the following manner:

123 Equation (1) is first taken in modulo m_1 . Since, all terms except the last are multiples of m_1 to give

124

$$|x|_{m_1} = a_1$$

125 Hereafter, a_1 is just the first residue digit. To obtain a_2 , first subtract a_1 from x . The quantity $x - a_1$ is
 126 divided by m_1 , and doing modulo operation with respect to m_2 , we have

127

$$\left| \frac{x - a_1}{m_1} \right|_{m_2} = a_2$$

128 Similarly, for a_3 , $(a_2 m_1 + a_1)$ is subtracted from x . By dividing the quantity $(x - a_2 m_1 - a_1)$ by $m_1 m_2$ and
 129 performing modulo operation with respect to m_3 , we get

130

$$\left| \frac{x - a_2 m_1 - a_1}{m_1 m_2} \right|_{m_3} = a_3$$

131 In this way, by successive subtraction and division in residue notation, all the mixed-radix digits may
 132 be obtained.

133 Conversely, an RNS number $(x_1, x_2, x_3, \dots, x_k)$ for the moduli set $(m_1, m_2, m_3, \dots, m_k)$ whose the
 134 decimal equivalent is given by:

135

$$a_1 = x_1 \quad (3)$$

136

$$a_2 = |(x_2 - a_1) m_1^{-1}|_{m_2} \quad (4)$$

137

$$a_3 = |((x_3 - a_1) m_1^{-1}|_{m_3} - a_2 |m_2^{-1}|_{m_3})|_{m_3} \quad (5)$$

138 Therefore, a general expression is given by:

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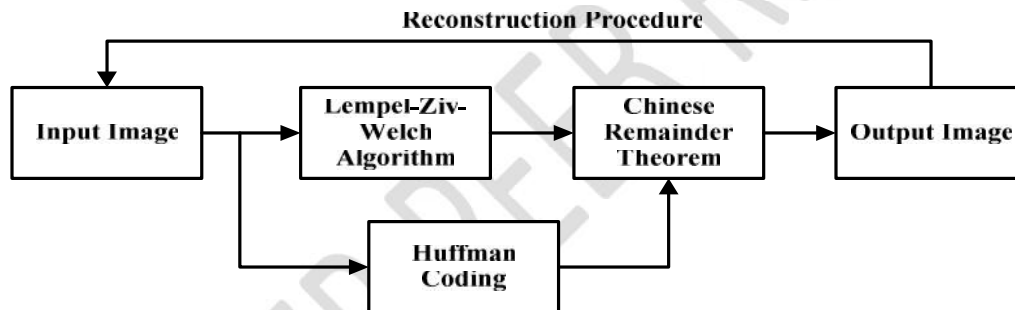
$$a_n = \left| \left(\left(\left((x_n - a_1) \right|_{m_1}^{-1} \right|_{m_1} - a_2 \right) \right|_{m_2}^{-1} \right|_{m_2} - \dots - a_{n-1} \right) \right|_{m_{n-1}}^{-1} \Big|_{m_{n-1}} \Big|_{m_n} \quad (6)$$

140 The mixed radix digit (MRD) of a_i , $0 \leq a_i < m_i$, any positive number in the interval by $[0, \prod_{i=1}^k m_i - 1)$
 141 can be uniquely represented. The major advantage of MRC, as can be seen from Equation (5) above
 142 is that the calculation of a_i , $i = 1, k$ can be done only using arithmetic mod- m_i contrasting CRT, which
 143 entails arithmetic mod-M, M being the system dynamic range, a rather large constant. It can be noted
 144 that equations (5) and (6) are directly utilized, only if the moduli set $\{m_1 m_2 m_3 \dots m_k\}$ are relatively prime
 145 and that Euclidean algorithm is the common way to verify this, i.e., if $\text{gcd}(m_i, m_j) = 1$, for $i \neq j$.

146 The residue independence, carry-free operation and parallelism attributes of the RNS have been
 147 intensively used in variety of areas, such as digital signal processing (DSP), digital filtering, digital
 148 communications, cryptography, error detection and correction [14] [15]. The addition, subtraction and
 149 multiplication are dominant. And, division, comparison, overflow and sign detection are negligible.
 150 One key field of RNS-based applications is finite impulse response (FIR) filters. Likewise, digital
 151 image processing benefits from the RNS's features such as enhancing digital image processing
 152 applications [15].

153 3. METHODOLOGY

154 In this image compression process, the implementation process was coded from scratch using
 155 MATLAB R2015a programming language. The paper studied the performances of traditional
 156 compression schemes of LZW and Huffman coding with CRT. The purpose of the employing CRT is
 157 to enhance their individual effectiveness using image media lossless compression technique. The
 158 arrangement of the planned enhancements of compression approaches is illustrated in Figure 1.



159

160 **Fig. 1. The layout for the enhancement of compression schemes.**

161 The input image is used to acquire the various formats of images for the complete data compression
 162 processes. These input images are composed of diverse degree of redundancy which is expected to
 163 be removed or minimized during planned compression processes. The data compression phase
 164 encompasses three distinct operations; firstly, the input image is received at Lempel-Ziv-Welch
 165 Algorithm block to commence the data compression. Similarly, the Huffman coding performs
 166 preliminary compression operation on the input images. Secondly, the complete compression
 167 of original image is achieved with CRT using the outcomes of LZW and Huffman coding schemes.
 168 Finally, the output image is realized from the last compression process of CRT computation, which
 169 enhanced the traditional image compressed format when compared to input images.

170 This paper considered four metrics in evaluating the effectiveness of the planned image compression
 171 schemes including:

- 172 1) *Compression Ratio* (CR) is expressed as the amount of uncompressed data size divided by
 173 compressed data size. This provides the relative size of compressed image data.
- 174 2) *Compression Time* (CT) calculates the time taken to compress bits in data in a second.
- 175 3) *Peak Signal-to-Noise Ratio* (PSNR) is used to estimate the amount of noise in the signals of
 176 compressed data relative to original data.
- 177 4) *Imperceptibility* calculates the rate of bits distributions of image data after complete
 178 compression. This infers on the appearance of compressed image data.

179 **The operational algorithm of proposed LZW-CRT image compression scheme is presented**
180 **below**

- 181 **Step 1-** Extract first byte from input STRING.
- 182 **Step 2-** Extract the next byte from input CHARACTER.
- 183 **Step 3-** Lookup in table for the STRING and CHARACTER stored up.
- 184 **Step 4-** Generate code for the STRING and update the lookup table.
- 185 **Step 5-** Output STRING same as CHARACTER.
- 186 **Step 6-** STRING = STRING and CHARACTER.
- 187 **Step 7-** Apply CRT on the resulting STRING.
- 188 **Step 8-** The moduli set is chosen to obtain the best redundancy in data.
- 189 **Step 9-** The compressed image data is attained as final encoded values.
- 190 **Step 10-** The reconstructed image is obtained by applying decoding of LZW and CRT.
- 191 **Step 11-** Output is reconstructed image data.

192 Similarly, the steps for performing CRT-Huffman coding image compression scheme is presented in
193 algorithm 2 below;

- 194
- 195 **Step 1-** INPUT original image
- 196 **Step 2-** Run Huffman coding functions
- 197 **Step 3-** Extract symbols of the pixels from input IMAGE.
- 198 **Step 4-** Create the probability of pixel symbols and organize in decreasing magnitude and smaller
199 probabilities are combined.
- 200 **Step 5-** Concatenate the Huffman codeword ready for CRT
- 201 **Step 6-** Generate code for the STRING and update the lookup table.
- 202 **Step 7-** Apply CRT on the resulting STRING.
- 203 **Step 8-** The moduli set is chosen to obtain the best redundancy in data.
- 204 **Step 9-** The compressed image data is attained as final encoded values.
- 205 **Step 10-** The reconstructed image is obtained by applying decoding of LZW and CRT
- 206 **Step 11-** Output is reconstructed image data.

208 4. PRESENTATION OF RESULTS

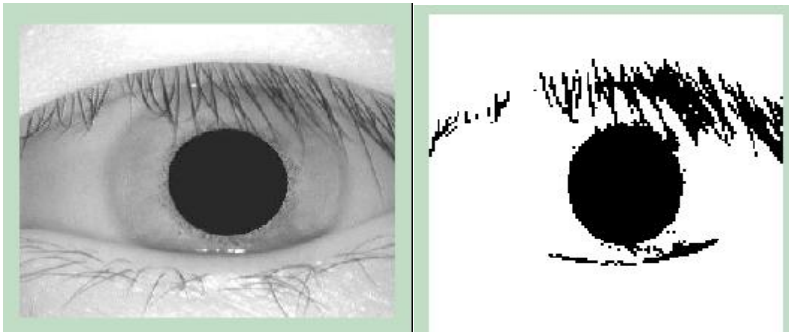
209 The paper utilized 10 different image samples from Yale database for the purpose of validating the
210 proposed concepts of CRT-LZW and CRT-Huffman coding compression. The outcomes of
211 compression procedures on the sampled images using CRT and LZW are shown in Table 1.

212 **Table 1. CRT-LZW based image compression**

Image sample	Size before compression	Size after compression	Compression time	PSNR	Compression ratio
1	12282	3524	10.10	3.11	0.38
2	12906	3571	8.77	3.17	0.40
3	12353	3522	12.28	3.21	0.39
4	12762	3410	10.18	3.22	0.36
5	12872	3606	9.99	3.31	0.38
6	12357	3548	8.61	3.13	0.38
7	12150	4266	9.08	4.09	0.41
8	12243	3889	10.45	3.99	0.40
9	12530	3882	9.76	4.09	0.39
10	12319	3935	10.27	4.05	0.40
Total	124774	37153	99.49	35.37	3.89
Average	12477.4	3715.3	9.95	3.54	0.39

213

214 From Table 1, there is a significant decrease in the compressed images when compared to the
215 original images at 12477.4 kb and 3715.3 kb on the average (that is 3:1) respectively. Consequently,
216 3 kb is used to represent 1 kb in the original image after performing the compression processes on
217 the sampled images. In the same vain, the average values for reduced image size, compression time,
218 PSNR and CR were 29.78%, 9.95 s, 3.54 and 0.39 respectively. The outlook of the compressed
219 image is poor for Human Visual System (HVS) as depicted in Figure 2.



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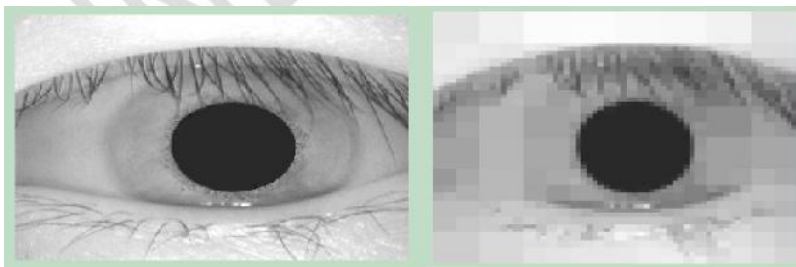
Fig. 2. Original image against CRT-LZW compressed image

223 In Figure 2, the image on the left hand side is the original image without compression operations. The
224 image on the right hand side is realized after performing compression on the CRT and LZW. The
225 compressed image looks washed out due to uneven distribution of bits composition to the HVS when
226 matched with the reconstructed image. The outcomes of applying CRT to Huffman coding based
227 compression using the sampled images are obtainable in Table 4.

228 **Table 2. CRT-Huffman coding based image compression**

Image sample	Size before compression	Size after compression	Compression time	PSNR	Compression ratio (%)
1	12282	1853	18.40	28.29	4.13
2	12906	1819	18.35	28.34	4.14
3	12353	1745	16.86	27.73	4.28
4	12762	1766	18.56	28.27	4.15
5	12872	1854	21.05	27.85	4.02
6	12357	1783	18.02	28.29	4.28
7	12150	1593	14.91	28.22	4.92
8	12243	1638	31.48	28.07	4.59
9	12530	1614	17.53	28.10	4.63
10	12319	1672	16.37	28.09	4.65
Total	124774	17337	191.53	281.25	43.79
Average	12477.4	1733.7	19.15	28.13	4.38

229 In Table 2, the data compression procedure of CRT and Huffman coding revealed substantial
230 improvements in terms of the resultant image sizes, PSNR and compression ratio. On the average,
231 CRT-Huffman based compressed scored 14.00%, 19.15 s, 28.13, and 4.38 for image size saved, time
232 of compression, PSNR and CR. Again, the outlook of reconstructed image when compared to the
233 original images is depicted in Figure 3.



234
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Fig. 3. Original image against CRT-Huffman compressed image

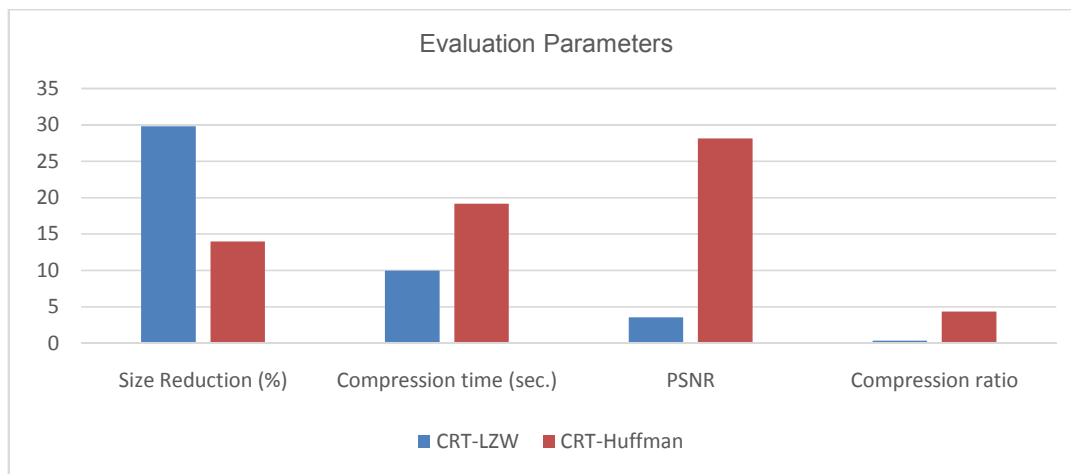
236 In Figure 3, the first image is the original sample image before applying the proposed compression
237 algorithm. The second image is the output of compression procedure with CRT-Huffman coding. The
238 two images showed large similarities, that is, the original and the reconstructed images, because of
239 even distribution of bits compositions to the HVS.

240 The paper compared the performances of compression procedures of CRT-LZW and CRT-Huffman
241 coding as shown in Table 3.

242 **Table 3. Comparisons of CRT-LZW and CRT-Huffman coding based image compression**

Evaluation Parameters	CRT-LZW	CRT-Huffman
Size Reduction (%)	29.78	14.00
Compression time (sec.)	9.95	19.15
PSNR	3.54	28.13
Compression ratio	0.39	4.38
Compressed Imperceptibility	Low	High

243



244

245 **Fig. 4. Graphical view for parameters evaluation of LZW and Huffman coding**

246 In Table 3 and figure 4, the introduction of CRT for LZW and Huffman coding based image
247 compression showed significant performances with LZW saving more space and speedy compression
248 (or redundancy removal) of original images. Conversely, Huffman coding offered better quality for the
249 reconstructed images as against LZW [5].

250 5. CONCLUSION

251 The fundamental principle of data compression procedures ensure minimization of data redundancy
252 (or resized data), better data compression time, improved or retention of data quality and high
253 compression ratio. The conventional compression algorithms, such as LZW and Huffman coding,
254 have shortcomings which limited their widespread implementations especially in image processing.
255 One parameter for measuring the suitability compression procedure in image is the bits distribution
256 because it reveals the similarity or otherwise of compressed image and original images to the Human
257 Visual System (HVS). This paper implemented CRT in LZW and Huffman coding in order to improve
258 their individual performances. The outcomes revealed that more space saving (or redundancy
259 removal) and faster compression time were offered by CRT-LZW. But, CRT-Huffman coding (28.13)
260 provided better quality (PSNR) for reconstructed images against CRT-LZW (3.54) and 25.59 in the
261 study [5]. However, there is need for further implementation of these concepts in other media files
262 such as text, videos and audio.

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