Review Paper

Modeling Nonlinear Partial Differential Equations and Construction of Solitary Waves Solutions in an Inductive Electrical Line

Abstract: In this paper, we apply Kirchhoff laws to the networks of a nonlinear inductive electrical line to model new partial differential equations with higher-order of nonlinearity which govern the dynamics of solitary waves on the given line, the construction of solitary wave solutions of these equations by effective methods has permitted us to realize that solitary wave of type Kink and type Pulse can easily propagate in the line when certain conditions we have presented are respected.

Keywords: inductive electrical line, modeling, construction, soliton solution, solitary wave, Nonlinear Partial Differential Equation, Kink, Pulse.

1. Introduction

Solitary waves, have evolved from the level of a simple water wave to the displacement of solitons in optical fibers [1]. From a solitary wave which is defined as a wave capable of displacing on longer distances without changing its shape and its velocity, we have borne in mind the fact that if one of such signals is used in engineering of information through an inductive electrical line, it will resist best on different dissipation factors. In this effect, we have decided to render two definitions of nonlinear magnetic flux linkage of inductors constituting networks of an inductive electrical transmission line. Then, we have applied them to model new nonlinear partial differential equations, which govern the dynamics of solitary waves in the said line. In order to construct exact solitary wave solutions of every nonlinear partial differential equation obtained, we rely first on methods presented in [2-15]. Furthermore, we have decided to adopt the new Bogning-Djeumem Tchaho-Kofane method [16-21] reason being that it facilitates the construction of a solitary wave solution by identification of the basic hyperbolic function coefficients of nonlinear partial differential equations in a direct and effective manner. Having solved the equations, we have come up with solitary wave solutions of type Kink and type Pulse. The work presented in this paper is partitioned as follows: In the part 2, we present a general modeling of a nonlinear inductive electrical line; In part 3, we construct solitary wave solutions of type Kink; In part 4, we construct solitary wave solutions of type Pulse and we present at the end the conclusion in part 5.

2. General modeling of a nonlinear inductive electrical line

Let us consider an electrical line constituting a good number of identical networks shown in figure 1 where G is the conductance of the resistor and R the resistance of another resistor, connected in a series branch with an inductor whose the magnetic flux linkage $\phi(i_n)$ changes in nonlinear manner in terms of the current i_n flowing through that inductor. By applying Kirchhoff's laws to the circuit shown in figure 1, we obtain the following equations

$$u_n - u_{n+1} = Ri_n + \frac{\partial \phi_n}{\partial t} \quad , \tag{1}$$

$$i_n = i_{n+1} + Gu_{n+1} \quad . \tag{2}$$

Where n is a positive integer that numbers each network of the line, i_n and i_{n+1} indicate respectively the current that flows through the inductor network order n and the inductor network order $n+1$, u_n and u_{n+1} indicate respectively the voltage across resistors with conductance G of the network order n and the network order $n+1$. ϕ_n Indicates the nonlinear magnetic flux linkage of the inductor network order n. considering equation (1), equation (2) become

$$i_n = i_{n+1} + Gu_n - RGu_n - G\frac{\partial \phi_n}{\partial t} - RGi_n \quad . \tag{3}$$

The substitution of $Gu_n = i_{n-1} - i_n$ of equation (2) obtained during the previous order in equation (3), one obtains the differential equation below

$$i_{n+1} - 2i_n + i_{n-1} = G\frac{\partial \phi_n}{\partial t} + RGi_n \quad . \tag{4}$$

To obtain the continuum model, the left hand side of equation (4) has to be approximated to a spatial partial derivative with respect to $x = nh$ which represents the distance measured from the beginning of the line. h represents the distance that separates two consecutive nodes and which is equivalent to the spatial sampling derivatives period. We obtain as such spatial partial derivatives using Taylor expansion of i_{n+1} and i_{n-1} closely to i_n by considering the terms till fourth order in the following manner

$$i_{n+1} = i_n + \frac{h}{1!}\frac{\partial i_n}{\partial x} + \frac{h^2}{2!}\frac{\partial^2 i_n}{\partial x^2} + \frac{h^3}{3!}\frac{\partial^3 i_n}{\partial x^3} + \frac{h^4}{4!}\frac{\partial^4 i_n}{\partial x^4} \quad , \tag{5}$$

$$i_{n-1} = i_n - \frac{h}{1!}\frac{\partial i_n}{\partial x} + \frac{h^2}{2!}\frac{\partial^2 i_n}{\partial x^2} - \frac{h^3}{3!}\frac{\partial^3 i_n}{\partial x^3} + \frac{h^4}{4!}\frac{\partial^4 i_n}{\partial x^4} \quad , \tag{6}$$

and

$$i_{n+1} - 2i_n + i_{n-1} = h^2\frac{\partial^2 i_n}{\partial x^2} + \frac{h^4}{12}\frac{\partial^4 i_n}{\partial x^4} \quad . \tag{7}$$

Equation (7) and (4) permits us to derive the result as follows

$$-h^2\frac{\partial^2 i_n}{\partial x^2} - \frac{h^4}{12}\frac{\partial^4 i_n}{\partial x^4} + G\frac{\partial \phi_n}{\partial t} + RGi_n = 0 \quad . \tag{8}$$

Finally, we obtain the continuum model of the nonlinear inductive electrical line presented in figure1 by the nonlinear partial differential equation below

$$-h^2\frac{\partial^2 i(x,t)}{\partial x^2}-\frac{h^4}{12}\frac{\partial^4 i(x,t)}{\partial x^4}+G\frac{\partial\phi(i(x,t))}{\partial t}+RGi(x,t)=0\ . \qquad (9)$$

Let's find out the solitary wave solutions of equation (9).

3. Construction of solitary wave solution of type Kink of partial differential equation (9).

We define the nonlinear magnetic flux linkage of inductors on the analytical shape as follows:

$$\phi(i(x,t))=B_1 i^4(x,t)+B_2 i^2(x,t)+B_3\ln(i^2(x,t)-B_0^2)\,. \qquad (10)$$

With $|i(x,t)|>|B_0|$. B_1 ; B_2 and B_3 are non-nil real numbers which will be chosen conveniently. By substituting the flux $\phi(i(x,t))$ of (10) in equation (9) we obtain the nonlinear partial differential equation written as

$$\begin{aligned}&\frac{B_0^2h^4}{12}\frac{\partial^4 i(x,t)}{\partial x^4}-\frac{h^4}{12}i^2(x,t)\frac{\partial^4 i(x,t)}{\partial x^4}+B_0^2h^2\frac{\partial^2 i(x,t)}{\partial x^2}-h^2i^2(x,t)\frac{\partial^2 i(x,t)}{\partial x^2}\\&+(2GB_3-2B_0^2GB_2)i(x,t)\frac{\partial i(x,t)}{\partial t}+(2GB_2-4B_0^2GB_1)i^3(x,t)\frac{\partial i(x,t)}{\partial t}\\&+4GB_1i^5(x,t)\frac{\partial i(x,t)}{\partial t}-B_0^2RGi(x,t)+RGi^3(x,t)=0.\end{aligned} \qquad (11)$$

Considering : $m_1=\frac{B_0^2h^4}{12}$, $m_2=-\frac{h^4}{12}$, $m_3=B_0^2h^2$, $m_4=-h^2$, $m_5=2GB_3-2B_0^2GB_2$, $m_6=2GB_2-4B_0^2GB_1$, $m_7=4GB_1$, $m_8=-B_0^2RG$, $m_9=RG$, equation (11) takes the following shape

$$\begin{aligned}&m_1\frac{\partial^4 i(x,t)}{\partial x^4}+m_2i^2(x,t)\frac{\partial^4 i(x,t)}{\partial x^4}+m_3\frac{\partial^2 i(x,t)}{\partial x^2}+m_4i^2(x,t)\frac{\partial^2 i(x,t)}{\partial x^2}+m_5i(x,t)\frac{\partial i(x,t)}{\partial t}\\&+m_6i^3(x,t)\frac{\partial i(x,t)}{\partial t}+m_7i^5(x,t)\frac{\partial i(x,t)}{\partial t}+m_8i(x,t)+m_9i^3(x,t)=0.\end{aligned} \qquad (12)$$

Let us use Bogning-Djeumen Tchaho-Kofane method [16-21] to come out with the solution of equation (12) under the analytical shape below

$$i(x,t)=a\tanh(kx-vt) \qquad (13)$$

Where a , k and v are non-nil real numbers to be determined. Replacing $i(x,t)$ given by (13) in equation (12) we yield the following equation

$$
\begin{aligned}
&(-24m_2a^3k^4 - m_7a^6v)\frac{\sinh(kx-vt)}{\cosh^7(kx-vt)} \\
&+(2m_7a^6v + 24m_1ak^4 + m_6a^4v + 32m_2a^3k^4 + 2m_4a^3k^2)\frac{\sinh(kx-vt)}{\cosh^5(kx-vt)} + \\
&(-m_7a^6v - 8m_1ak^4 - 2m_3ak^2 - 8m_2a^3k^4 - m_9a^3 - m_5a^2v - m_6a^4v - 2m_4a^3k^2)\frac{\sinh(kx-vt)}{\cosh^3(kx-vt)} + \\
&(m_8a + m_9a^3)\frac{\sinh(kx-vt)}{\cosh(kx-vt)} = 0.
\end{aligned} \tag{14}
$$

Equation (14) is valid if and only if each of its basic hyperbolic function coefficients is nil. This permits us to obtain the following set of fours equations

$$
\begin{cases}
-24m_2a^3k^4 - m_7a^6v = 0, \\
2m_7a^6v + 24m_1ak^4 + m_6a^4v + 32m_2a^3k^4 + 2m_4a^3k^2 = 0, \\
-m_7a^6v - 8m_1ak^4 - 2m_3ak^2 - 8m_2a^3k^4 - m_9a^3 - m_5a^2v - m_6a^4v - 2m_4a^3k^2 = 0, \\
m_8a + m_9a^3 = 0.
\end{cases} \tag{15}
$$

Solving the set of equation (15) has permitted us to obtain the following results:

$$
a = \pm\sqrt{\frac{-m_8}{m_9}}, \qquad k = \pm\frac{1}{2}\sqrt{\frac{m_4m_8m_7}{2m_2m_8m_7 + 3m_1m_9m_7 - 3m_6m_2m_9}}, \qquad m_8m_9 < 0,
$$

$$
v = \frac{3m_2m_4^2n_7\left(-m_8m_9\right)^{\frac{3}{2}}}{2m_8\left(2m_2m_8m_7 + 3m_1m_9m_7 - 3m_6m_2m_9\right)^2},
$$

$$
\left(
\begin{aligned}
&m_8m_1^2m_9^2m_7^2 - 2m_8m_1m_9^2 - m_7m_6m_2 + m_8m_6^2m_2^2m_9^2 + \frac{4}{3}m_2m_8^2m_7^2m_1m_9 \\
&-\frac{1}{6}m_8m_9m_7^2m_1m_3m_4 - \frac{4}{3}m_2^2m_8^2m_7m_6m_9 - \frac{1}{6}m_8m_5m_2m_4^2m_7m_9 + \frac{1}{6}m_8m_9m_7m_2m_4m_3m_6 \\
&+\frac{4}{9}m_2^2m_8^3m_7^2 + \frac{1}{9}m_4^2m_8^2m_7^2m_1 - \frac{1}{9}m_8^2m_7^2m_3m_4m_2 = 0
\end{aligned}
\right). \tag{16}
$$

Replacing m_1 , m_2 , m_3 , m_4 , m_5 , m_6 , m_7 , m_8 and m_9 by their different expressions in (16), we obtain the solution of the nonlinear partial differential equation (11) which models the dynamic of solitary waves of type Kink in the inductive line as follow

$$
a = B_0\ ,\ k = \pm B_0\left(\frac{-RGB_1}{h^4B_3}\right)^{\frac{1}{4}},\ v = \frac{-RB_0}{2B_3}\ ,\ B_2 = -\frac{4}{3}B_1B_0^2 - 2\sqrt{\frac{-B_1B_3}{RG}}\ ;\ B_1B_3 < 0
$$

$$
i(x,t) = B_0\tanh\left(\pm B_0\left(\frac{-RGB_1}{h^4B_3}\right)^{\frac{1}{4}}x + \frac{RB_0}{2B_3}t\right). \tag{17}
$$

4. Construction of solitary wave solution of type Pulse relative to nonlinear partial differential equation (9)

We define the nonlinear magnetic flux linkage of inductors with analytical shape as given:

$$\phi(i(x,t)) = B_1 i(x,t)\sqrt{1-\left(\frac{i(x,t)}{B_0}\right)^2} + B_2 i^3(x,t)\sqrt{1-\left(\frac{i(x,t)}{B_0}\right)^2} + B_3 \arctan\left(\sqrt{\frac{i^2(x,t)}{B_0^2 - i^2(x,t)}}\right) . \quad (18)$$

With $|B_0| > |i(x,t)|$. B_1 ; B_2 and B_3 are non-nil real numbers whose conditions of choice will be established. A substituting of $\phi(i(x,t))$ of (18) in differential equation (9) permits us to obtain the nonlinear partial differential equation bellow

$$\begin{aligned} & B_0\sqrt{B_0^2 - i^2(x,t)}\left(-h^2\frac{\partial^2 i(x,t)}{\partial x^2} - \frac{h^4}{12}\frac{\partial^4 i(x,t)}{\partial x^4} + RGi(x,t)\right) \\ & + G\left(-4B_2 i^4(x,t) + \left(3B_0^2 B_2 - 2B_1\right) i^2(x,t) + B_0^2 B_1 + B_0 B_3\right)\frac{\partial i(x,t)}{\partial t} = 0. \end{aligned} \quad (19)$$

Finding out the result of equation (19) on the analytical shape:

$$i(x,t) = a\,\text{sech}\left(kx - vt\right) \quad (20)$$

Where a , k and v are non-nil real numbers to be determined. Substituting $i(x,t)$ of (20) in differential equation (19), we obtain the equation as follows

$$\begin{aligned} & \left(3a^3 vGB_0^2 B_2 - 2a^3 vGB_1\right)\frac{\sinh(kx-vt)}{\cosh^4(kx-vt)} + \left(avGB_0 B_3 + avGB_0^2 B_1\right)\frac{\sinh(kx-vt)}{\cosh^2(kx-vt)} \\ & -4a^5 vGB_2 \frac{\sinh(kx-vt)}{\cosh^6(kx-vt)} + \left(B_0 RGa - \frac{1}{12}B_0 h^4 ak^4 - B_0 h^2 ak^2\right)\frac{\sqrt{B_0^2 - \frac{a^2}{\cosh^2(kx-vt)}}}{\cosh(kx-vt)} \\ & + \left(2B_0 h^2 ak^2 + \frac{5}{3}B_0 h^4 ak^4\right)\frac{\sqrt{B_0^2 - \frac{a^2}{\cosh^2(kx-vt)}}}{\cosh^3(kx-vt)} - 2B_0 h^4 ak^4 \frac{\sqrt{B_0^2 - \frac{a^2}{\cosh^2(kx-vt)}}}{\cosh^5(kx-vt)} = 0. \end{aligned} \quad (21)$$

We realize that to be able to transform the hyperbolic functions of (21) to the basic hyperbolic functions as recommended by the new Bogning-Djeumen Tchaho-Kofane [16-21] we must consider $B_0 = a$ such that

$$\sqrt{B_0^2 - \frac{a^2}{\cosh^2(kx-vt)}} = a\tanh\left(kx - vt\right) \quad (22)$$

The right-hand side of (22) has enables us to rearrange (21) as

$$\left(3a^3vGB_0^2B_2+2B_0a^2h^2k^2+\frac{5}{3}B_0a^2h^4k^4-2a^3vGB_1\right)\frac{\sinh(kx-vt)}{\cosh^4(kx-vt)}$$
$$+\left(B_0a^2RG+avGB_0B_3-B_0a^2h^2k^2-\frac{1}{12}B_0a^2h^4k^4+avGB_0^2B_1\right)\frac{\sinh(kx-vt)}{\cosh^2(kx-vt)} \quad (23)$$
$$+\left(-2B_0a^2h^4k^4-4a^5vGB_2\right)\frac{\sinh(kx-vt)}{\cosh^6(kx-vt)}=0.$$

Equation (23) is valid if each coefficient of its basic hyperbolic function is equal to zero. This enables us to obtain the set of three equations as follows

$$\begin{cases} 3a^3vGB_0^2B_2+2B_0a^2h^2k^2+\frac{5}{3}B_0a^2h^4k^4-2a^3vGB_1=0, \\ B_0a^2RG+avGB_0B_3-B_0a^2h^2k^2-\frac{1}{12}B_0a^2h^4k^4+avGB_0^2B_1=0, \\ -2B_0a^2h^4k^4-4a^5vGB_2=0. \end{cases} \quad (24)$$

The result of the set of nonlinear equation (24) enables us to realize that solitary waves of type Pulse are easily displaced in the nonlinear inductive line with analytical shape given below:

$$a=B_0\ ,\ v=\frac{-RB_0}{B_3}\ ,\ k=\pm\left(\frac{2B_0^3B_2RG}{h^4B_3}\right)^{\frac{1}{4}}\ ,\ B_3=\frac{RG\left(B_2^2B_0^4+12B_2B_0^2B_1+36B_1^2\right)}{72B_0B_2}\ ,$$

$$i(x,t)=B_0\,\mathrm{sech}\left(\pm\left(\frac{2B_0^3B_2RG}{h^4B_3}\right)^{\frac{1}{4}}x+\frac{RB_0}{B_3}t\right)\ . \quad (25$$

5. Conclusion

At the end of this work, where we have modeled and constructed solitary wave solution by two different nonlinear partial derivative equations of an inductive electrical line; it is therefore important to point out that the results obtained will first of all enable us in the domain of physics and telecommunication of engineering, the manufacturing of new transmission lines as inductive electrical lines whose magnetic flux linkage of inductors varies one in a nonlinear shape defined in (10) and varies for the other in a nonlinear shape defined in (18). In addition, these results will permit us to ameliorate the quality of signals that will be displaced in those new lines. In fact, those signals are solitary waves of type Pulse obtained in (25) and type Kink obtained in (17) which by their definitions, displace on a very long distance maintaining their shape; their speed and resist best on different dissipation factors. Finally, in a typical mathematics domain, the results obtained has permitted us to define in (11) and (19) two new nonlinear partial derivative equations which have respectively for exact solutions solitary wave (17) and (25). This augments the field of mathematical knowledge. In order to inquire ideas concerning the stability of obtained solitary wave, it seems for us to study later their modulational instability before carrying out the practical survey where we will experiment the applicability and the perfection of these new inductive electrical lines.

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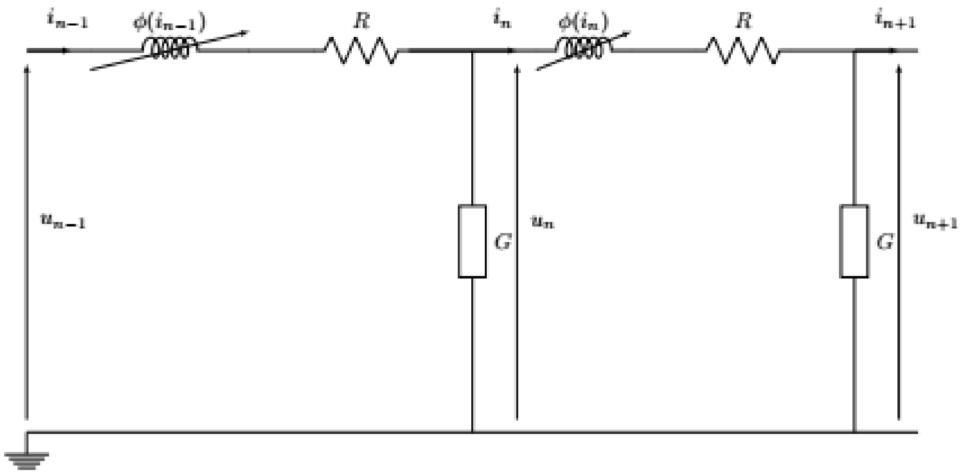
Figure 1: presentation of a nonlinear inductive electrical line.


i_{n-1}
$\phi(i_{n-1})$
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i_n
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