

# Original Research Article

## An ARMA Model for Short-Term Prediction of Hepatitis B Virus Seropositivity among Blood Donors in Lafia-Nigeria

### ABSTRACT

In this paper, we attempt to search for an optimal Autoregressive Moving Average (ARMA) model that best forecast hepatitis B virus infection among blood donors in Lafia-Nigeria. The study uses monthly data in Lafia-Nigeria for the period of 11 years 6 months from January 2007 to June 2018. The data was obtained as secondary data from General Hospital Lafia and Dalhatu Araf Specialist Hospital, Lafia. The time series and stationarity properties of the data are explored using time plots and Dickey-Fuller Generalized Least Squares unit root test. The results indicate that the series is integrated of order zero,  $I(0)$ . An ARMA (p,q) model in line with Box-Jenkins procedure was employed to model the time series data. The result shows that ARMA (1,1) was the best candidate to model and forecast hepatitis B virus infection among blood donors in Lafia- Nigeria. Critical analysis of the model shows that the HBV infection is chronic among blood donors in the study area. The estimated ARMA (1,1) model was then used to forecast future values of hepatitis B infection among blood donors in Lafia-Nigeria from July 2018 to June 2019. The forecast shows a stable level of infection for the forecasted period. The study provided some policy recommendations.

*Keywords: Hepatitis B virus, Blood donors, ARMA Model, Forecasting, Nigeria.*

### 1. INTRODUCTION

Hepatitis B is a highly contagious liver disease caused by infection with the hepatitis B virus (HBV). The hepatitis B virus is known as a blood-borne virus because it is transmitted from one person to another. The virus is spread when blood, semen, saliva, vaginal fluids (including menstrual blood) and other bodily fluids from an infected person enter the body of an uninfected person. Possible methods of transmission include: transfer from mother to baby during birth, being pricked with a contaminated needle, close contact with a person with HBV, sex (oral, vaginal, and anal), using an infected toothbrush or razor. Symptoms may not occur for a few days or longer after contracting the virus. However, one is still contagious, even without symptoms. Symptoms of hepatitis B may not be apparent for months or years. However, common symptoms include: dark urine, joint pain, loss of appetite, fever, abdominal discomfort, weakness, yellowing of the whites of the eyes (sclera) and skin (jaundice).

The complications of HBV without early treatment include: liver scarring (cirrhosis), liver failure, kidney cancer, kidney failure and liver cancer. Another possible complication is hepatitis D infection. It is only people with HBV that can contract hepatitis D. A combined infection can cause serious liver problems.

Hepatitis B virus infects liver cells (hepatocytes) and can cause both acute and chronic disease. Acute hepatitis lasts for less than 6 months while chronic hepatitis lasts for more than 6 months [1]. Acute infection does not usually require treatment. Most people overcome an acute infection on their own. Chronic infection requires antiviral medications for treatment which help in fighting the virus and may also reduce the risk of future liver complications [2]. Persons with chronic hepatitis B virus (CHB) are at risk of developing serious sequelae, such as cirrhosis and hepatocellular carcinoma [3].

Hepatitis B is one of the prevalent diseases in the world and a major cause of morbidity and mortality [4]. According to [5] an approximate population of 2 billion people worldwide has been infected with the hepatitis B virus (HBV). Despite the availability of highly effective vaccine against hepatitis B virus there are still over 350 million chronic carriers worldwide, of whom possibly one million die annually

38 from cirrhosis and/or hepatocellular carcinoma [6]. HBV infection accounts for 0.6 to 1.2 million global  
 39 deaths annually [7, 8].

40 In Nigeria, HBV is reported to be the most common cause of liver disease. Several authors have  
 41 reported on the prevalence of HBV among sub-populations in Nigeria with varying estimates  
 42 depending on population studied and methods used. However, there is no reliable nationwide survey  
 43 of HBV exposure in the average risk population and in subgroups most likely to benefit from early  
 44 detection, surveillance, and treatment. Vaccination against the hepatitis B virus (HBV) is lower in  
 45 Nigeria than any other West African nation of the many Sub-Saharan African countries.

46 Due to the severe health impact of hepatitis B infection, there is a growing need for methods that will  
 47 allow forecasting and early warning with timely case detection in areas of unstable transmission, So  
 48 that effective control and preventive measures can be implemented. This study contributes and  
 49 extends the existing literature by modeling and providing short-term forecasts on hepatitis B virus  
 50 infection among blood donors in Lafia-Nigeria using time series techniques and more recent data.

## 51 2. MATERIAL AND METHODS

### 52 2.1 Data and Source

53 The data use for this study comprises serologically confirmed cases of hepatitis B virus infection  
 54 among blood donors in Lafia town, Nassarawa state in Nigeria from January 2007 to June 2018. The  
 55 data consists of 138 monthly observations of persons believed to be residents of Lafia town. The data  
 56 was obtained as secondary data from the two tertiary health institutions in Lafia town. The General  
 57 Hospital Lafia and Dalhatu Araf Specialist Hospital, Lafia, Nassarawa state-Nigeria.

### 58 2.2 Some Basic Concepts

59 Let  $\{HBV_t\}$  be a stochastic time series process. We define  $HBV_t$  as a sequence of hepatitis B virus  
 60 infection indexed by time. We shall be using  $HBV_t$  to refer to a series throughout our study.

#### 61 2.2.1 Autocorrelation Function (ACF)

62 We define the Autocorrelation function (ACF) of a stationary series  $\{HBV_t\}$  as:

$$63 \rho_k = \frac{Cov(HBV_t, HBV_{t-k})}{Var(HBV_t)} = \frac{Cov(HBV_t, HBV_{t-k})}{\sqrt{Var(HBV_t) \cdot (HBV_{t-k})}} \quad (1)$$

64 where  $\rho_0 = 1$  and  $-1 \leq \rho_k \leq 1$  otherwise. The sample autocorrelation function can be estimated by:

$$65 \hat{\rho}_k = \frac{\frac{1}{T-k} \sum_{t=k+1}^T (HBV_t - \overline{HBV_t})(HBV_{t-k} - \overline{HBV_{t-k}})}{\frac{1}{T-k} \sum_{t=k+1}^T (HBV_{t-k} - \overline{HBV_{t-k}})^2} \quad (2)$$

66 which is the OLS estimator in  $HBV_t = c + \rho_k HBV_{t-k} + e_t$ . The 95% confidence bounds are given by  
 $\pm 1.96/\sqrt{T}$ , where T is the number of observations.

#### 67 2.2.2 Partial Autocorrelation Function (ACF)

68 The partial Autocorrelation Function (PACF) is the correlation between  $HBV_t$  and  $HBV_{t-k}$  after the  
 69 data has been corrected for intermediate lags  $HBV_{t-1}, \dots, HBV_{t-k+1}$ . The PACF can be estimated as  
 70 the OLS estimator  $\hat{\beta}_k$  in the regression

$$71 HBV_t = \sigma + \beta_1 HBV_{t-1} + \beta_2 HBV_{t-2} + \dots + \beta_k HBV_{t-k} + e_t \quad (3)$$

72 where the intermediate lags are included. Under the assumption of white noise,  $\beta_1 = \beta_2 = \dots = 0$ , it  
 73 holds that  $Var(\hat{\beta}_k) = T^{-1}$ .

#### 74 2.2.3 Stationarity of Order M

75 A stochastic time series process  $\{HBV_t\}$  is stationary of order M if for any admissible set  $\{t_1, t_2, \dots, t_m\}$   
 76 and for any  $k$ , the joint moments of  $\{HBV_{t_1}, HBV_{t_2}, \dots, HBV_{t_m}\}$  up to order M exists, and are equal to the

77 joint moments of  $\{HBV_{t1+k}, HBV_{t2+k}, \dots, HBV_{tm+k}\}$  up to order M. That is  
 78  $E\{(HBV_{t1})^\alpha (HBV_{t2})^\beta \dots (HBV_{tm})^\gamma\} = E\{(HBV_{t1+k})^\alpha (HBV_{t2+k})^\beta \dots (HBV_{tm+k})^\gamma\}$  for all  $\alpha, \beta, \dots, \gamma$  such that  
 79  $\alpha + \beta + \dots + \gamma \leq M$ .

## 80 2.2.4 Weakly or Covariance Stationary

81 A stochastic time series process  $\{HBV_t\}$  is said to be weakly or covariance stationary if its mean and  
 82 variance are constant over time and its covariance function depends only on the time lag. A  
 83 covariance stationary series satisfies the following conditions:

84 (i)  $E(HBV_t) = \mu$ , where  $\mu$  is a constant (ii)  $E(HBV_t - \mu)^2 = \text{Var}(HBV_t) = \sigma^2$ , where  $\sigma^2$  is a constant and  
 85 (iii)  $E(HBV_t, HBV_s) = E(HBV_t, HBV_{t+k})$  is a function of  $s - t = k$  only where  $k$  is the lag.

## 86 2.3 Model Specification

### 87 2.3.1 Autoregressive Model

88 An autoregressive model of order one, AR (1) is specified as:

$$89 HBV_t = \phi_0 + \phi_1 HBV_{t-1} + \varepsilon_t \quad (4)$$

90 where  $HBV_t$  is hepatitis B virus infection response variable at time  $t$ ,  $\varepsilon_t$  is a purely random process  
 91 with mean zero and variance  $\sigma^2$ ,  $\phi_0$  is a constant and  $\phi_1$  is an autoregressive parameter and the  
 92 subscript 1 is the order of the autoregressive parameters which increase with increases in  $HBV_t$ . The  
 93 values of  $\phi$  which would make the process to be stationary are such that the roots of the polynomial  
 94 equation  $\phi[L] = 0$  lie outside the unit circle in the complex plane. L is the lag operator such that  
 95  $L_1 HBV_t = HBV_{t-1}$  and  $\phi L = 1 - \phi L$ .

### 96 2.3.2 Moving Average Model

97 Suppose that  $\{\varepsilon_t\}$  is a white noise process with mean zero and variance  $\sigma^2$ , then the process  $HBV_t$  is  
 98 said to be a moving average model of order one, MA (1) if

$$99 HBV_t = \varepsilon_t + \beta_1 \varepsilon_{t-1} \quad (5)$$

100 Where  $\beta_1$  is the moving average parameter. The subscript on  $\beta_1$  is called the order of moving average  
 101 parameter.

### 102 2.3.3 Autoregressive Moving Average Model

103 A stochastic process resulting from the combination of autoregressive and moving average models is  
 104 called an Autoregressive Moving Average (ARMA) model. An ARMA model of order one, ARMA (1,1)  
 105 is specified as:

$$106 HBV_t = \phi_0 + \phi_1 HBV_{t-1} + \varepsilon_t - \beta_1 \varepsilon_{t-1} \quad (6)$$

107 To obtain stationarity for this model the equation  $\phi[L] = 0$  has its roots outside the unit circle and the  
 108 root of  $\beta[L] = 0$  must lie outside the unit circle for the process to be invertible. Equation (6) is the  
 109 theoretical model which serves as a basic framework of our analysis.

## 110 3.4 Model Order Selection

111 We used the following information criteria for model order selection in conjunction with log likelihood  
 112 function: Akaike Information Criterion (AIC) due to [9] and Schwarz information Criterion (SIC) due to  
 113 [10].

$$114 AIC(P) = -2 \ln(L) + 2P \quad (7)$$

$$115 SIC(P) = -2 \ln(L) + P \ln(T) \quad (8)$$

116 where  $P$  is the number of free parameters to be estimated in the model,  $T$  is the number of  
 117 observations and  $L$  is the maximum likelihood function .

## 118 2.5 Some Statistical Tests

119 **2.5.1 Dickey-Fuller Generalized Least Squares (DF GLS) Unit Root Test**

120 If  $HBV_t$  is the series under investigation, the DF GLS test is based on testing

121  $H_0: \psi = 0$  (The series contains unit root) against

122  $H_1: \psi < 0$  (The series is stationary) in the following regression

123 
$$\Delta HBV_t^d = \psi_0 HBV_{t-1}^d + \psi_1 \Delta HBV_{t-1}^d + \dots + \psi_{p-1} \Delta HBV_{t-p+1}^d + u_t \quad (9)$$

124 where  $HBV_t^d$  is the detrended series. Detrending depends on whether a constant or a constant and  
125 trend are included in the model. We reject  $H_0$  if the DF-GLS test statistic is less than the critical value  
126 of the test at the designated test sizes. Elliot *et al.* [11] show that de-trending in this way produces a  
127 test that has good power properties.

128 **2.5.2 Portmanteau Test**

129 A Portmanteau test is a test used for investigating the presence of autocorrelation in time series. The  
130 test checks the following pairs of hypotheses:

131  $H_0: \rho_{k,1} = \rho_{k,2} = \dots = \rho_{k,T} = 0$  (all lags correlations are zero) against

132  $H_1: \rho_{k,1} \neq \rho_{k,2} \neq \dots \neq \rho_{k,T} \neq 0$  (there is at least one lag with non-zero correlation). The test statistic is  
133 given by:

$$Q^{(LB)} = T(T+2) \sum_{k=1}^h \frac{\hat{\rho}_k^2}{T-k}, \quad (10)$$

where  $\hat{\rho}_k^2 = \frac{T}{T-k} \left( T \sum_{t=k+1}^T (\hat{\varepsilon}_t^2 - \bar{\varepsilon})(\hat{\varepsilon}_{t-k}^2 - \bar{\varepsilon}) / \sum_{t=1}^T (\hat{\varepsilon}_t^2 - \bar{\varepsilon})^2 \right)$ , for  $\bar{\varepsilon} = T^{-1} \sum_{t=1}^T \varepsilon_t^2$

134 denotes the autocorrelation estimate of squared standardized residuals at  $k$  lags.  $T$  is the sample  
135 size,  $Q$  is the sample autocorrelation at lag  $k$ . We reject  $H_0$  if p-value is less than  $\alpha = 0.05$  level of  
136 significance [12].

137 **2.6 Forecast and Forecast Evaluation**

138 Suppose the sample we wish to forecast is  $j = T+1, T+2, \dots, T+h$ , and denote the actual and  
139 forecasted value in period  $t$  as  $HBV_t$  and  $\widehat{HBV}_t$ , respectively. The reported forecast error statistics are  
140 computed as follows:

$$\text{Root Mean Square Error (RMSE)} = \sqrt{\sum_{t=T+1}^{T+h} (\widehat{HBV}_t - HBV_t)^2 / h}$$

$$\text{Mean Absolute Error (MAE)} = \sum_{t=T+1}^{T+h} |\widehat{HBV}_t - HBV_t| / h$$

$$\text{Mean Absolute Percentage Error (MAPE)} = 100 \times \sum_{t=T+1}^{T+h} \left| \frac{\widehat{HBV}_t - HBV_t}{HBV_t} \right| / h$$

$$\text{Theil Inequality Coefficient (TIC)} = \frac{\sqrt{\sum_{t=T+1}^{T+h} (\widehat{HBV}_t - HBV_t)^2 / h}}{\sqrt{\sum_{t=T+1}^{T+h} \widehat{HBV}_t^2 / h} + \sqrt{\sum_{t=T+1}^{T+h} HBV_t^2 / h}}$$

$$\text{Bias Proportion (BP)} = ((\sum_{t=T+1}^{T+h} \widehat{HBV}_t / h) - \overline{HBV_t})^2 / \sum_{t=T+1}^{T+h} (\widehat{HBV}_t - HBV_t)^2 / h$$

$$\text{Variance Proportion (VP)} = (S_{\widehat{HBV}} - S_{HBV})^2 / \sum_{t=T+1}^{T+h} (\widehat{HBV}_t - HBV_t)^2 / h$$

$$\text{Covariance Proportion (CP)} = 2(1-r) S_{\widehat{HBV}} S_{HBV} / \sum_{t=T+1}^{T+h} (\widehat{HBV}_t - HBV_t)^2 / h$$

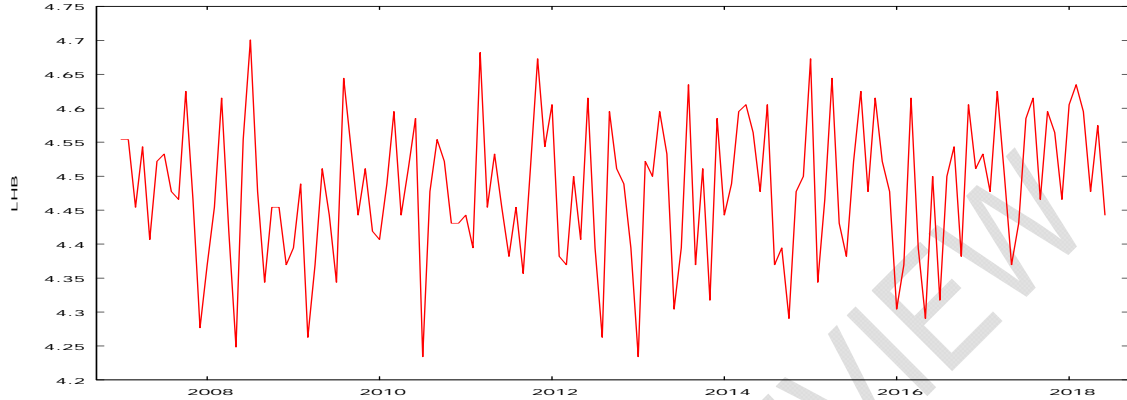
141 where  $h$  is the number of steps ahead that we want to predict, and  $T$  is the total sample size. For  
142 additional discussion of forecast evaluation see [13].

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144 **3.0 RESULTS AND DISCUSSION**

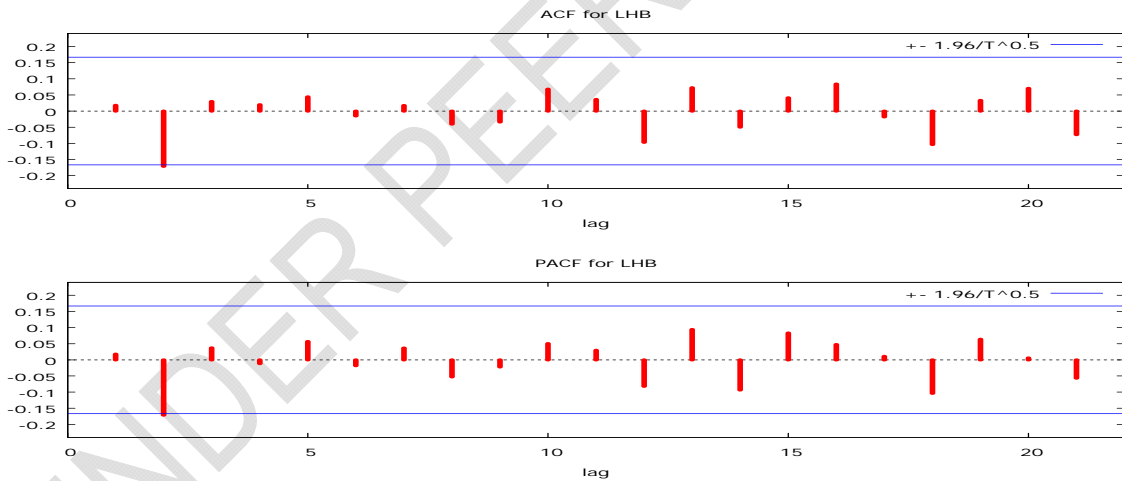
145 **3.1 Graphical Examination of the Series**

146 The data generating process of the series are first examined using time plot after transforming the  
 147 original series into natural logarithms. The result of time plot of the series is presented in Figure 1.



148 **Figure 1: Time Plot of Hepatitis-B Infection in Lafia (Natural Log)**

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 150 The time plot of the transformed series reported in Figure 1 indicates a stable and smooth trend which  
 151 suggests that the mean and variance of the series are constant over time (homoskedastic). This  
 152 means that the natural log of the series in level is weakly stationary. Although, we will further  
 153 investigate this by considering the autoregressive function (ACF) and partial autoregressive function  
 154 (PACF) of the series reported in Figure 2.  
 155



156 **Figure 2: ACF and PACF Plot of Hepatitis-B Infection in Lafia (Natural Log)**

157 The plots of ACF and PACF of the series reported in Figure 2 suggest that the series is stationary in  
 158 level since all the lags are inside the confidence bounds. This is an indication that the residual of the  
 159 series are purely random process. This also shows that the series is independent of time (i.e., the  
 160 infection in the current month does not depend on the infection of the previous month and vice versa).  
 161 We also consider the Q-statistics for autocorrelation of the series. The result is presented in Table 1.  
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 164 **Table 1: Autocorrelation Function and Ljung-Box Q-statistics**

Lag	ACF	Std. Error	Ljung-Box Q-statistic	P-value
1	-0.0260	0.090	0.084	0.773
5	0.0374	0.089	6.270	0.281
7	-0.0433	0.088	6.711	0.460
8	-0.0343	0.087	6.865	0.551
10	0.0604	0.087	7.548	0.673
11	0.0132	0.086	7.571	0.751

12	-0.1322	0.086	9.943	0.621
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166 The p-values of the Q-statistics of the series reported in Table 1 are highly statistically insignificant.  
 167 This is one of the properties of a dynamically stable and stationary series whose residuals are purely  
 168 random process. The Q-statistics of the ACF thus help to confirm that the series is stationary in level.

### 169 3.2 Dickey Fuller (DF) GLS Unit Root Test

170 To further confirm the stationarity of the series in level as shown by the result of time plot, ACF and  
 171 PACF plots as well as Q-statistic test and to know the order of integration of the series, we conduct  
 172 unit root test in level of the series using Dickey-Fuller Generalized Least Squares unit root test  
 173 procedure. The result of the DF-GLS unit root test in level is reported in Table 2.

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175 Table 2: Elliot-Rothenberg & Stock DF-GLS Unit Root Test Results

Option	DF-GLS Statistic	Test	DF-GLS Test Critical Values		
			1%	5%	10%
Intercept only	-8.8306*		-2.5845	-1.9435	-1.6149
Intercept & Trend	-10.3205*		-3.5572	-3.0110	-2.7210

176 **Note:** \* denotes the significant of the DF-GLS test statistic at 1% significance level.

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178 The unit root test result reported in Table 2, shows that the series is weakly stationary in level since  
 179 the DF-GLS test statistics both with intercept only and with intercept and trend are all less (or more  
 180 negative) than the critical values of the test at the conventional test sizes. This shows that the series  
 181 is integrated of order zero, I(0). That is, stationary in level. Having obtained the order of integration of  
 182 the series, we proceed with other analysis using the stationary series.

### 183 3.3 Selection of Model Order

184 The spikes of ACF and PACF in Figure 3 both decayed quickly to zero. This suggest a mixed ARMA  
 185 model for the series while the DF-GLS unit root test shows the order of integration of the series to be  
 186 zero, I(0). We need to marry these two basic ideas to search for an optimal ARMA (p,d,q) model using  
 187 information criteria, log likelihood and R<sup>2</sup> statistic bearing in mind that d = 0. The result is reported in  
 188 Table 3.

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190 Table 3: Model Order Selection Using Information Criteria and Log Likelihood

S/n	Model	AIC	BIC	LogL	R <sup>2</sup>
1	ARMA (0,1)	-1.6222	-1.5758	99.3339	0.0012
2	ARMA (0,2)	-1.6668	-1.5971	103.0081	0.0606
3	ARMA (0,3)	-1.6692	-1.6043	104.9034	0.0871
4	ARMA (1,0)	-1.6181	-1.5714	98.2779	0.0007
5	ARMA (2,0)	-1.6479	-1.5775	100.2268	0.0497
6	ARMA (3,0)	-1.6601	-1.5794	101.9015	0.0696
7	<b>ARMA (1,1)**</b>	<b>-1.7980</b>	<b>-1.6797</b>	<b>113.1629</b>	<b>0.6480</b>
8	ARMA (1,2)	-1.6509	-1.5575	102.2314	0.0649
9	ARMA (1,3)	-1.6405	-1.5718	101.8241	0.0894
10	ARMA (1,4)	-1.6307	-1.5701	103.1862	0.1290
11	ARMA (2,1)	-1.6358	-1.5419	100.5134	0.0534
12	ARMA (2,2)	-1.6815	-1.5641	104.2075	0.1169
13	ARMA (2,3)	-1.6706	-1.5297	104.5628	0.1170
14	ARMA (2,4)	-1.6519	-1.5215	103.4832	0.1392
15	ARMA (3,1)	-1.6447	-1.5276	101.2161	0.0867
16	ARMA (3,2)	-1.7852	-1.6436	110.4354	0.2199
17	ARMA (3,3)	-1.6430	-1.4778	103.1172	0.1192
18	ARMA (3,4)	-1.6516	-1.4872	104.4437	0.1152
19	ARMA (4,1)	-1.6320	-1.4896	100.6567	0.0956
20	ARMA (4,2)	-1.6491	-1.4829	102.6471	0.1262

21	ARMA (4,3)	-1.7533	-1.5634	109.6932	0.2261
22	ARMA (4,4)	-1.7348	-1.5211	109.6162	0.2251

Note: \*\* denotes ARMA model selected by the criteria

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The result of Table 3 indicates that ARMA (1,1) model has the least information criteria, largest log likelihood and highest  $R^2$ . Based on Box-Jenkins procedure, this seems to describe our time series data more adequately. We therefore select ARMA (1,1) as the best candidate to model and forecast hepatitis B virus infection among volunteer blood donors in Lafia, Nassarawa state-Nigeria . The parameter estimates of ARMA (1,1) are presented in Table 4.

Table 4: OLS Parameter Estimates of ARMA (1,1) Model

Variable	Coefficient	Std. Error	t-Statistic	P-value	
C	4.469523	0.004667	957.5877	0.0000	
AR(1)	0.724005	0.146412	4.944999	0.0000	
MA(1)	0.274561	0.101923	-8.580618	0.0000	
R-squared	0.647981	Akaike info criterion		-1.797973	
Adjusted $R^2$	0.431567	Schwarz criterion		-1.679735	
Log likelihood	113.1629	Hannan-Quinn criterion		-1.621347	
F-statistic	7.923161	Prob(F-statistic)		0.000035	
Inverted AR Roots	0.72	Inverted MA Roots	0.87	Durbin W.	1.86016

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From the result of Table 4, the estimated ARMA (1,1) model is represented in equation (11):

$$HBV_t = 4.469523 + 0.724005HBV_{t-1} + \varepsilon_t + 0.274561\varepsilon_{t-1} \quad (11)$$

The result of equation (11) shows that the intercept (C) is positively related with hepatitis B infection and statistically significant. This implies that the predicted value of hepatitis B infection will be 4.469523 units in log form (i.e., approximately 87 persons) if all other explanatory variables are kept constant. The AR and MA slope coefficients of the model are all statistically significant at marginal significant levels. The estimated model have also satisfied the stationarity condition because  $\alpha_1 + \beta_1 = 0.724005 + 0.274561 = 0.998566 < 1$ . This shows that the estimated ARMA (1,1) is stationary

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The coefficient of determination ( $R^2$ ) of the regression model is 0.647981 indicating that about 64.80% of the total variations in hepatitis B infection has been explained by independent variables while the remaining 35.20% unexplained variations is being accounted for by the error term or by factors not included in the model. The F-statistic is a goodness of fit test which measures the overall fitness of the regression parameters.  $F=7.923161$  with a p-value of 0.000035 indicates that the regression model is a good fit. The value of Durbin Watson statistic is 1.86016 which is greater than  $R^2$  and  $R^2$  adjusted indicating that the model is not spurious.

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### 3.4 Model Validation and Diagnostic Checks

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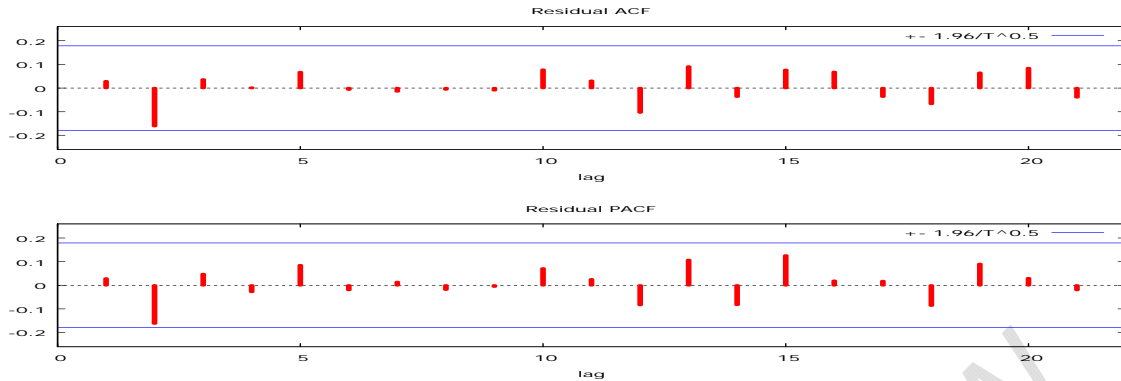
We now validate our model by carrying out residual diagnostic check on the estimated ARMA (1,1) model.

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#### 3.4.1 ACF and PACF Plots of Residual

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We examine the adequacy and goodness of fit of the model by means of plotting the ACF and PACF of residuals. If all the sample autocorrelation coefficients of the residuals are within the 95% confidence bounds, then the residuals are white noise indicating that the model is a good fit. The ACF and PACF plots are presented in Figure 3.



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224 **Figure 3: ACF and PACF Plot of Residual**  
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226 Figure 3 shows that all the sample autocorrelation coefficients of the residuals are within the  
227 confidence bounds indicating that the residuals are white noise and the fitted model is stable and  
228 stationary.

229 We also conduct Ljung-Box Q-statistic test of serial correlation (autocorrelation) for residuals of the  
230 fitted model. The result of the test is presented in Table 5.

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232 **Table 5: Ljung-Box Q-statistic Test for Serial Correlation of Residuals**

Lag	Q-statistics	P-value
1	0.1054	0.745
2	3.3254	0.190
3	3.4907	0.322
4	4.4911	0.479
5	4.0721	0.539
6	4.0775	0.666
7	4.1036	0.768
8	4.1080	0.847
9	4.1204	0.903
10	4.9234	0.896
11	5.0548	0.928
12	6.5083	0.888

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234 From the result of Table 5, the null Hypothesis of no serial correlation in the residuals of the fitted  
235 model at all lags is accepted since the p-values of the Q-statistics are all greater than 0.05. This  
236 shows that the estimated model is stationary and dynamically stable.

### 237 **3.4.2 Stability and Invertibility Analysis**

238 Another evidence to show that the estimated model is dynamically stable is that the inverse roots of  
239 AR/MA polynomials are all within a unit circle as reported in Figure 5.



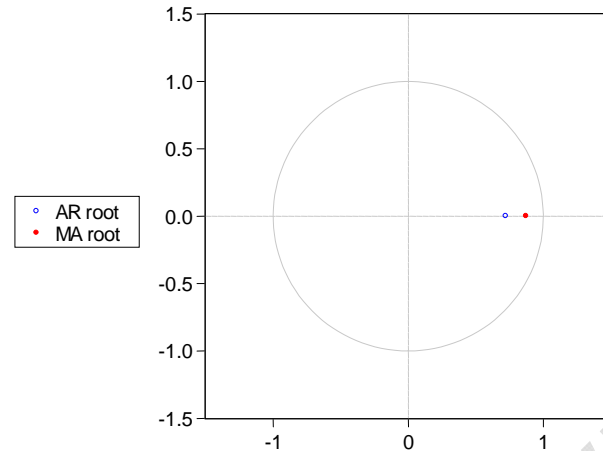


Figure 5: Inverse Roots of AR/MA Polynomials

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243 From the root of AR and MA polynomials of the fitted model presented in Table 4, AR root =  
244 0.72 and MA root = 0.87 and we estimate that  $\tan\theta = y/x = 0.72/0.87 = 0.8276$  and  $\theta = 39.61^\circ$ . Thus,  
245 the life cycle of hepatitis B virus infection among blood donor in the study area is  
246  $360^\circ/39.61^\circ = 9.09 \approx 9$  months and we say that hepatitis B virus infection among blood donors in  
247 Lafia-Nigeria has a life cycle of 9 months which could be describe as chronic, a disease condition in  
248 which if not properly treated will lead to severe liver complications and high risk of developing serious  
249 sequelae, such as cirrhosis and hepatocellular carcinoma.

### 250 3.4.3 Forecast Evaluation

251 Having validated our model, we now seek an appropriate forecast mode that best forecast future  
252 relevant series. Here we consider in-sample and out-of-sample forecasts using seven accuracy  
253 measures. The forecast mode with the least accuracy measures stands as the best to predict hepatitis  
254 B virus infection among blood donors in Lafia-Nigeria. The result of forecast comparison is presented  
255 in Table 6.

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Table 6: Forecast Comparison Using Accuracy Measures

Forecast Mode	RMSE	MAE	MAPE	TIC	BP	VP	CP
In-Sample	0.1057	0.0855	1.9157	0.0118	0.0000	0.8899	0.1101
Out-of-Sample*	0.1034	0.0824	1.8469	0.0116	0.0000	0.6372	0.3628

258 **Note:** \* denotes the forecast mode selected by accuracy measures  
259

260 The accuracy measures automatically select out-of-sample forecast mode for our model. This is  
261 because the out-of-sample forecast has the least accuracy measures except for covariance proportion  
262 (CP).

### 263 3.4.4 Short-Term Forecast of Hepatitis B Infection in Lafia-Nigeria

264 Using the out-of-sample forecast approach for the series, the estimated ARMA (1,1) model is use to  
265 forecast future values of hepatitis B virus infection among volunteer blood donors in Lafia-Nigeria for  
266 the period of 1 year (12 months) starting from July 2018 to June 2019. The result of the forecast is  
267 presented in Table 7.

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Table 7: Forecast of Hepatitis-B Infection in Lafia from July 2018-June 2019

Year: Month	Forecast (in natural log)	Actual Forecast (no of persons)	Std. Error (in log)	95% Interval (no of persons)
2018:07	4.45626	86	0.102313	[71, 106]
2018:08	4.45809	86	0.103097	[71, 106]
2018:09	4.45970	86	0.103695	[71, 106]
2018:10	4.46110	87	0.104151	[71, 106]
2018:11	4.46233	87	0.104500	[71, 106]
2018:12	4.46341	87	0.104767	[71, 106]
2019:01	4.46435	87	0.104971	[71, 107]
2019:02	4.46518	87	0.105128	[71, 107]
2019:03	4.46591	87	0.105247	[71, 107]
2019:04	4.46654	87	0.105339	[71, 107]
2019:05	4.46710	87	0.105410	[71, 107]
2019:06	4.46758	87	0.105464	[71, 107]

272 **Note:** For 95% confidence intervals,  $Z_{0.025} = 1.96$

273

274 The forecast value for the month of July 2018 is 86 persons with a 95% confidence interval of [71,  
275 106] persons. By this we are 95% confident that the outcome for the next period will fall within this  
276 interval. Comparing with the monthly infection in June 2018 (85 persons), we predict that in July 2018  
277 the hepatitis B virus infection will slightly increase from the current month. The interval [71, 106]  
278 persons imply that the monthly increase may lie between 71 and 106 persons (i.e. it may increase at  
279 least by 1 person or at most by 20 persons) in July 2018. The forecasts for the following months show  
280 a stable level in the virus infection in Lafia-Nigeria. The confidence intervals of the forecast suggest a  
281 stable level of infection during the forecasted period of July 2018 to June 2019. This implies that  
282 hepatitis B virus infection among volunteer blood donors in Lafia-Nigeria will remain stable within the  
283 years 2018 and 2019. This could possibly be as a result of better and improved control and preventive  
284 measures, enhanced awareness and campaign strategies, medical care and treatment facilities  
285 provided by the state government and other NGOs and international donors in the region.

#### 286 4 CONCLUSION AND RECOMMENDATIONS

287 In this paper, attempt has been made to search for an optimal Autoregressive Moving Average  
288 (ARMA) model that best forecast hepatitis B virus infection among blood donors in Lafia-Nigeria. The  
289 study uses monthly data in Lafia-Nigeria for the period of 11 years 6 months from January 2007 to  
290 June 2018. The data comprises of 138 consecutive observations and was obtained as secondary  
291 data from General Hospital Lafia and Dalhatu Araf Specialist Hospital, Lafia. The time series and  
292 stationarity properties of the data are explored using time plot, ACF and PACF plots and Dickey-Fuller  
293 Generalized Least Squares unit root test. The results indicate that the series is stationary in level and  
294 hence integrated of order zero, I(0). An ARMA (p,q) model in line with Box-Jenkins procedure were  
295 employed to model the time series data. The result shows that ARMA (1,1) was the best candidate to  
296 model and forecast hepatitis B virus infection among blood donors in Lafia- Nigeria. The analysis of  
297 the model shows that hepatitis B infection is chronic among blood donors in Lafia-Nigeria. Persons  
298 with chronic HBV infection are at high risk of developing serious sequelae, such as cirrhosis and  
299 hepatocellular carcinoma. The estimated ARMA (1,1) model was then used to forecast future values  
300 of hepatitis B infection among blood donors in Lafia-Nigeria from July 2018 to June 2019. The  
301 forecast shows a stable level of infection for the forecasted period.

302 Based on the findings of this study the following recommendations/suggestions are hereby presented:

- 303 i. To further reduce the spread of HBV, government in collaboration with public health  
304 authorities need to educate the community and health care providers about HBV transmission  
305 routes based on known HBV epidemiology in Lafia and its neighbouring communities.
- 306 ii. Hepatitis B vaccine programme should be initiated with a target of reducing the infection rate  
307 from its current state.
- 308 iii. Future research should be carried out with focus on factors associated with hyper-endemic  
309 levels of HBV infection in the community.

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