Review Paper

(Kink; Kink; kink; Kink) and (Pulse; pulse; Pulse; pulse) Solutions of a Set of four Equations Modeled in a Nonlinear Hybrid Electrical Line with crosslink capacitor

Abstract: The physics system that helps us in the study of this paper is a nonlinear hybrid electrical line with crosslink capacitor. Meaning it is composed of two different nonlinear hybrid parts Linked by capacitors with identical constant capacitance. We apply Kirchhoff laws to the circuits of the line to obtain new set of four nonlinear partial differential equations which describe the simultaneous dynamics of four solitary waves. Furthermore, we apply efficient mathematical methods based on the identification of coefficients of basic hyperbolic functions to construct exact solutions of those set of four nonlinear partial differential equations. The obtained results have enabled us to discover that, one of the two nonlinear hybrid electrical line with crosslink capacitor that we have modeled accepts the simultaneous displacement of a set of four solitary waves of type (Pulse; Pulse; Pulse; Pulse), while the other accepts the simultaneous displacement of a set of four solitary waves of type (Kink; Kink; Kink; Kink) when certain conditions we have established are respected. We ameliorate the quality of the signals by changing the sinusoidal waves that are displacing in hybrid electrical lines with crosslink capacitor to solitary waves which are displacing in the new nonlinear hybrid electrical lines; we therefore facilitate the choice of the type of line relative to the type of signal that we want to transmit.

Keywords: Hybrid electrical line, crosslink capacitor, construction, solitons solution, solitary wave, Nonlinear Partial Differential Equation, Kink, Pulse

1. Introduction

The signal that is displaced in electrical lines where the parameters of its components are constant is a sinusoidal wave whose amplitude decreases exponentially and loses a lot of energy contrary to solitary wave signal which conserves its velocity, its shape and does not loses energy during its displacement. If solitons could be displaced in electrical lines, they will resist better on dissipation factors; for this reason we have decided to search on what means we could modify the component parameters of a hybrid electrical line with crosslink capacitor so that it accepts the displacement of solitary waves. We therefore define analytically the nonlinear flux linkage of inductors and the nonlinear charge of capacitors constituting the two parts linked by capacitors in the line. The use of these definitions and the application of Kirchhoff laws to the circuit of nonlinear hybrid electrical line with crosslink capacitor has enabled us to model a set of four nonlinear partial differential equations which describe the dynamics of solitary waves in the line. To construct exact solitary wave solution of each set of four nonlinear partial differential equations, we have used the mathematical methods presented in [1-14] and particularly the Bogning-Djeumen Tchaho-Kofane method [15-20]. For one of the set of four nonlinear partial differential equations we have obtained a solution which is a set of four solitary waves of type (Pulse; Pulse; Pulse; Pulse) and for the other we have obtained a solution which is a set of four solitary waves of type (Kink; Kink; Kink; Kink). Our work is developed in the following order: in section two, we model a nonlinear hybrid electrical line with crosslink capacitor; in section three we find the solitary wave solution of type (Kink; Kink; Kink; Kink); in section four we find the solitary wave solution of type (Pulse; Pulse; Pulse; Pulse). We conclude our work in section 5.

2. General modeling of nonlinear hybrid electrical line with crosslink capacitor.

Let us consider a nonlinear hybrid electrical line shown in figure 1. The line is constituted by a good number of identical networks numbered by the positive integer n. The network number n is constituted by a capacitor with capacitance C_0 which link the two nonlinear hybrid parts; two capacitors in which each of the charge q_1^n and q_2^n changes respectively in nonlinear manner in terms of the voltage u_1^n and u_2^n across each capacitor; two inductors in which each of the magnetic flux ϕ_1^n and ϕ_2^n changes respectively in nonlinear manner in terms of the current i_1^n and i_2^n that flow through each inductor. Applying Kirchhoff's laws to the circuit shown in figure 1, we obtain the following equations:

$$u_1^n - u_1^{n-1} = -\frac{\partial \phi_1^n}{\partial t} \tag{1}$$

$$u_2^n - u_2^{n-1} = -\frac{\partial \phi_2^n}{\partial t} \tag{2}$$

$$i_1^n - i_1^{n+1} = C_0 \frac{\partial \left(u_1^n - u_2^n\right)}{\partial t} + \frac{\partial q_1^n}{\partial t} \tag{3}$$

$$i_2^n - i_2^{n+1} = -C_0 \frac{\partial \left(u_1^n - u_2^n\right)}{\partial t} + \frac{\partial q_2^n}{\partial t} \tag{4}$$

To obtain the continuum model, the left hand side of each equation (1); (2) ; (3) et (4) has to be approximated to a spatial partial derivative with respect to $x = nh$ which represents the distance measured from the beginning of the line. h represent the distance that separates two consecutive nodes and which is equivalent to the spatial sampling derivatives period. Using respectively Taylor expansion of u_1^{n-1} ; u_2^{n-1} ; i_1^{n+1} and i_2^{n+1} closely to u_1^n ; u_2^n ; i_1^n and i_2^n by considering the terms till fourth order we obtain the set of four partial differential equations in the following manner:

$$\begin{cases} \frac{h^4}{24}\frac{\partial^4 u_1^n}{\partial x^4}-\frac{h^3}{6}\frac{\partial^3 u_1^n}{\partial x^3}+\frac{h^2}{2}\frac{\partial^2 u_1^n}{\partial x^2}-h\frac{\partial u_1^n}{\partial x}-\frac{\partial \phi_1^n}{\partial t}=0 \\ \frac{h^4}{24}\frac{\partial^4 u_2^n}{\partial x^4}-\frac{h^3}{6}\frac{\partial^3 u_2^n}{\partial x^3}+\frac{h^2}{2}\frac{\partial^2 u_2^n}{\partial x^2}-h\frac{\partial u_2^n}{\partial x}-\frac{\partial \phi_2^n}{\partial t}=0 \\ \frac{h^4}{24}\frac{\partial^4 i_1^n}{\partial x^4}+\frac{h^3}{6}\frac{\partial^3 i_1^n}{\partial x^3}+\frac{h^2}{2}\frac{\partial^2 i_1^n}{\partial x^2}+h\frac{\partial i_1^n}{\partial x}+C_0\frac{\partial\left(u_1^n-u_2^n\right)}{\partial t}+\frac{\partial q_1^n}{\partial t}=0 \\ \frac{h^4}{24}\frac{\partial^4 i_2^n}{\partial x^4}+\frac{h^3}{6}\frac{\partial^3 i_2^n}{\partial x^3}+\frac{h^2}{2}\frac{\partial^2 i_2^n}{\partial x^2}+h\frac{\partial i_2^n}{\partial x}-C_0\frac{\partial\left(u_1^n-u_2^n\right)}{\partial t}+\frac{\partial q_2^n}{\partial t}=0 \end{cases} \tag{5}$$

Finally, we obtain the continuum model of the nonlinear hybrid electrical line with crosslink capacitor presented in figure1 by the set of four nonlinear partial differential equations below:

$$\begin{cases} \frac{h^4}{24}\frac{\partial^4 u_1(x,t)}{\partial x^4}-\frac{h^3}{6}\frac{\partial^3 u_1(x,t)}{\partial x^3}+\frac{h^2}{2}\frac{\partial^2 u_1(x,t)}{\partial x^2} \\ -h\frac{\partial u_1(x,t)}{\partial x}-\frac{\partial \phi_1\left(i_1(x,t)\right)}{\partial t}=0 \\ \frac{h^4}{24}\frac{\partial^4 u_2(x,t)}{\partial x^4}-\frac{h^3}{6}\frac{\partial^3 u_2(x,t)}{\partial x^3}+\frac{h^2}{2}\frac{\partial^2 u_2(x,t)}{\partial x^2} \\ -h\frac{\partial u_2(x,t)}{\partial x}-\frac{\partial \phi_2\left(i_2(x,t)\right)}{\partial t}=0 \\ \frac{h^4}{24}\frac{\partial^4 i_1(x,t)}{\partial x^4}+\frac{h^3}{6}\frac{\partial^3 i_1(x,t)}{\partial x^3}+\frac{h^2}{2}\frac{\partial^2 i_1(x,t)}{\partial x^2}+h\frac{\partial i_1(x,t)}{\partial x} \\ +C_0\frac{\partial\left(u_1(x,t)-u_2(x,t)\right)}{\partial t}+\frac{\partial q_1\left(u_1(x,t)\right)}{\partial t}=0 \\ \frac{h^4}{24}\frac{\partial^4 i_2(x,t)}{\partial x^4}+\frac{h^3}{6}\frac{\partial^3 i_2(x,t)}{\partial x^3}+\frac{h^2}{2}\frac{\partial^2 i_2(x,t)}{\partial x^2}+h\frac{\partial i_2(x,t)}{\partial x} \\ -C_0\frac{\partial\left(u_1(x,t)-u_2(x,t)\right)}{\partial t}+\frac{\partial q_2\left(u_2(x,t)\right)}{\partial t}=0 \end{cases} \tag{6}$$

3. Construction of a set of four solitary wave solutions of type (Kink ; Kink; Kink ; Kink) relative to general differential equation (6)

We define each of nonlinear charges $q_1\left(u_1(x,t)\right)$, $q_2\left(u_2(x,t)\right)$ of the capacitors and each of nonlinear magnetic flux linkage $\phi_1\left(i_1(x,t)\right)$, $\phi_2\left(i_2(x,t)\right)$ of the inductors under the analytical shape given below:

$$\begin{cases} \phi_1\left(i_1\left(x,t\right)\right) = E_1 i_1\left(x,t\right) + E_2 i_1^2\left(x,t\right) + E_3 i_1^3\left(x,t\right) + E_4 i_1^4\left(x,t\right) \\ \phi_2\left(i_2\left(x,t\right)\right) = F_1 i_2\left(x,t\right) + F_2 i_2^2\left(x,t\right) + F_3 i_2^3\left(x,t\right) + F_4 i_2^4\left(x,t\right) \\ q_1\left(u_1\left(x,t\right)\right) = A_1 u_1\left(x,t\right) + A_2 u_1^2\left(x,t\right) + A_3 u_1^3\left(x,t\right) + A_4 u_1^4\left(x,t\right) \\ q_2\left(u_2\left(x,t\right)\right) = B_1 u_2\left(x,t\right) + B_2 u_2^2\left(x,t\right) + B_3 u_2^3\left(x,t\right) + B_4 u_2^4\left(x,t\right) \end{cases} \quad (7)$$

With E_1 ; E_2 ; E_3 ; E_4 ; F_1 ; F_2 ; F_3 ; F_4 ; A_1 ; A_2 ; A_3 ; A_4 ; B_1 ; B_2 ; B_3 and B_4 are non-nil real numbers which will be chosen conveniently. By substituting each of the nonlinear charge $q_1\left(u_1\left(x,t\right)\right)$, $q_2\left(u_2\left(x,t\right)\right)$ and each of the nonlinear magnetic flux $\phi_1\left(i_1\left(x,t\right)\right)$, $\phi_2\left(i_2\left(x,t\right)\right)$ of (7) in (6) we obtain the set of four nonlinear partial differential equation written as:

$$\begin{cases} \frac{h^4}{24}\frac{\partial^4 u_1(x,t)}{\partial x^4} - \frac{h^3}{6}\frac{\partial^3 u_1(x,t)}{\partial x^3} + \frac{h^2}{2}\frac{\partial^2 u_1(x,t)}{\partial x^2} - h\frac{\partial u_1(x,t)}{\partial x} \\ \quad + \left(-E_1 - 2E_2 i_1(x,t) - 3E_3 i_1^2(x,t) - 4E_4 i_1^3(x,t)\right)\frac{\partial i_1(x,t)}{\partial t} = 0 \\ \frac{h^4}{24}\frac{\partial^4 u_2(x,t)}{\partial x^4} - \frac{h^3}{6}\frac{\partial^3 u_2(x,t)}{\partial x^3} + \frac{h^2}{2}\frac{\partial^2 u_2(x,t)}{\partial x^2} - h\frac{\partial u_2(x,t)}{\partial x} \\ \quad + \left(-F_1 - 2F_2 i_2(x,t) - 3F_3 i_2^2(x,t) - 4F_4 i_2^3(x,t)\right)\frac{\partial i_2(x,t)}{\partial t} = 0 \\ \frac{h^4}{24}\frac{\partial^4 i_1(x,t)}{\partial x^4} + \frac{h^3}{6}\frac{\partial^3 i_1(x,t)}{\partial x^3} + \frac{h^2}{2}\frac{\partial^2 i_1(x,t)}{\partial x^2} + h\frac{\partial i_1(x,t)}{\partial x} - C_0\frac{\partial u_2(x,t)}{\partial t} \\ + \left(C_0 + A_1 + 2A_2 u_1(x,t) + 3A_3 u_1^2(x,t) + 4A_4 u_1^3(x,t)\right)\frac{\partial u_1(x,t)}{\partial t} = 0 \\ \frac{h^4}{24}\frac{\partial^4 i_2(x,t)}{\partial x^4} + \frac{h^3}{6}\frac{\partial^3 i_2(x,t)}{\partial x^3} + \frac{h^2}{2}\frac{\partial^2 i_2(x,t)}{\partial x^2} + h\frac{\partial i_2(x,t)}{\partial x} - C_0\frac{\partial u_1(x,t)}{\partial t} \\ + \left(C_0 + B_1 + 2B_2 u_2(x,t) + 3B_3 u_2^2(x,t) + 4B_4 u_2^3(x,t)\right)\frac{\partial u_2(x,t)}{\partial t} = 0 \end{cases} \quad (8)$$

Let us use Bogning-Djeumen Tchaho-Kofane method [15-20] to come out with the solution of (8) under the analytical shape below:

$$\begin{pmatrix} u_1\left(x,t\right) = a\tanh\left(kx - vt\right) \\ u_2\left(x,t\right) = b\tanh\left(kx - vt\right) \\ i_1\left(x,t\right) = e\tanh\left(kx - vt\right) \\ i_2\left(x,t\right) = f\tanh\left(kx - vt\right) \end{pmatrix} \quad (9)$$

Where a; b; e; f; k and v are non-nil real numbers to be determined in terms of modeled line parameters. Replacing $u_1\left(x,t\right)$; $u_1\left(x,t\right)$; $i_1\left(x,t\right)$ et $i_2\left(x,t\right)$ given by (9) in (8) we yield the following set of four equations which are written in a simplified form:

$$
\begin{cases}
\left(3E_3e^3v-hak-\dfrac{2}{3}h^3ak^3+E_1ev\right)\dfrac{1}{\cosh^2(kx-vt)} \\
+\left(2E_2e^2v-\dfrac{1}{3}h^4ak^4-h^2ak^2+4E_4e^4v\right)\dfrac{\sinh(kx-vt)}{\cosh^3(kx-vt)} \\
+\left(h^4ak^4-4E_4e^4v\right)\dfrac{\sinh(kx-vt)}{\cosh^5(kx-vt)}+\left(-3E_3e^3v+h^3ak^3\right)\dfrac{1}{\cosh^4(kx-vt)}=0 \\
\left(3F_3f^3v-hbk-\dfrac{2}{3}h^3bk^3+F_1fv\right)\dfrac{1}{\cosh^2(kx-vt)} \\
+\left(2F_2f^2v-\dfrac{1}{3}h^4bk^4-h^2bk^2+4F_4f^4v\right)\dfrac{\sinh(kx-vt)}{\cosh^3(kx-vt)} \\
+\left(h^4bk^4-4F_4f^4v\right)\dfrac{\sinh(kx-vt)}{\cosh^5(kx-vt)}+\left(-3F_3f^3v+h^3bk^3\right)\dfrac{1}{\cosh^4(kx-vt)}=0 \\
\quad\left(-3A_3a^3v+hek+\dfrac{2}{3}h^3ek^3-A_1av-C_0av+C_0bv\right)\dfrac{1}{\cosh^2(kx-vt)} \\
\quad+\left(3A_3a^3v-h^3ek^3\right)\dfrac{1}{\cosh^4(kx-vt)}+\left(h^4ek^4+4A_4a^4v\right)\dfrac{\sinh(kx-vt)}{\cosh^5(kx-vt)} \\
\quad+\left(-2A_2a^2v-\dfrac{1}{3}h^4ek^4-h^2ek^2-4A_4a^4v\right)\dfrac{\sinh(kx-vt)}{\cosh^3(kx-vt)}=0 \\
\quad\left(-3B_3b^3v+hfk+\dfrac{2}{3}h^3fk^3-B_1bv-C_0bv+C_0av\right)\dfrac{1}{\cosh^2(kx-vt)} \\
\quad+\left(3B_3b^3v-h^3fk^3\right)\dfrac{1}{\cosh^4(kx-vt)}\left(h^4fk^4+4B_4b^4v\right)\dfrac{\sinh(kx-vt)}{\cosh^5(kx-vt)} \\
\quad+\left(-2B_2b^2v-\dfrac{1}{3}h^4fk^4-h^2fk^2-4B_4b^4v\right)\dfrac{\sinh(kx-vt)}{\cosh^3(kx-vt)}+=0
\end{cases}
\tag{10}
$$

The set of equations (10) is valid if and only if each of its basic hyperbolic function coefficients is nil. This permits us to obtain the following set of sixteen equations:

$$
\begin{cases}
3E_3e^3v - hak - \frac{2}{3}h^3ak^3 + E_1ev = 0 \\
2E_2e^2v - \frac{1}{3}h^4ak^4 - h^2ak^2 + 4E_4e^4v = 0 \\
h^4ak^4 - 4E_4e^4v = 0 \\
-3E_3e^3v + h^3ak^3 = 0 \\
3F_3f^3v - hbk - \frac{2}{3}h^3bk^3 + F_1fv = 0 \\
2F_2f^2v - \frac{1}{3}h^4bk^4 - h^2bk^2 + 4F_4f^4v = 0 \\
h^4bk^4 - 4F_4f^4v = 0 \\
-3F_3f^3v + h^3bk^3 = 0 \\
-3A_3a^3v + hek + \frac{2}{3}h^3ek^3 - A_1av - C_0av + C_0bv = 0 \\
-2A_2a^2v - \frac{1}{3}h^4ek^4 - h^2ek^2 - 4A_4a^4v = 0 \\
3A_3a^3v - h^3ek^3 = 0 \\
h^4ek^4 + 4A_4a^4v = 0 \\
-3B_3b^3v + hfk + \frac{2}{3}h^3fk^3 - B_1bv - C_0bv + C_0av = 0 \\
-2B_2b^2v - \frac{1}{3}h^4fk^4 - h^2fk^2 - 4B_4b^4v = 0 \\
3B_3b^3v - h^3fk^3 = 0 \\
h^4fk^4 + 4B_4b^4v = 0
\end{cases}
\qquad (11)
$$

Haven solved the set of equation (11), it has permitted us to present in (12) the solution with conditions of the set of four nonlinear partial differential equations obtained in (8) which model the dynamic of a set of four solitary wave of type (Kink; Kink; Kink; Kink):

$$
a = \frac{\sqrt{-48A_2A_4 + 54A_3^2}}{8A_4} \;;\; e = \frac{\left(A_3E_3^2\right)^{\frac{1}{4}}\sqrt{-48A_2A_4 + 54A_3^2}}{8E_3A_4} \;;\; f = \frac{B_3^4A_4^2\left(A_3E_3^3\right)^{\frac{1}{4}}\sqrt{-48A_2A_4 + 54A_3^2}}{8E_3A_3^4B_4^3} \;;
$$

$$
b = \frac{E_3}{B_3\left(A_3E_3^3\right)^{\frac{1}{4}}}\left(\frac{B_3^6\left(A_3E_3^3\right)^{\frac{3}{4}}\left(-48A_2A_4 + 54A_3^2\right)^{\frac{3}{2}}}{512A_3^3E_3^3B_4^3}\right)^{\frac{1}{3}} \;;\; v = \frac{-8A_4^2\left(A_3E_3^3\right)^{\frac{1}{4}}\sqrt{-48A_2A_4 + 54A_3^2}}{81E_3A_3^4} \;;
$$

$$k=\frac{1}{h\left(A_3E_3^3\right)^{\frac{1}{4}}}\left(\frac{-\left(A_3E_3^3\right)^{\frac{3}{4}}\left(-48A_2A_4+54A_3^2\right)^{\frac{3}{2}}}{216A_3^3}\right)^{\frac{1}{3}}\ ;\ E_4=\frac{h^4ak^4}{4e^4v}\ ;\ F_3=\frac{h^3bk^3}{4f^3v}\ ;\ F_4=\frac{h^4bk^4}{4f^4v}\ ;$$

$$A_1=\frac{-h^3ek^3a^3+3heka^3-3C_0va^4+3C_0va^3b}{3va^4}\ ;\ B_1=\frac{-h^3fk^3b^3+3hfkb^3-3C_0vb^4+3C_0vb^3a}{3vb^4}\ ;$$

$$E_1=\frac{-hak\left(-3e^3+h^2k^2e^3\right)}{3e^4v}\ ;\qquad E_2=\frac{-hak\left(-\frac{3}{2}hke^2+h^3k^3e^2\right)}{3e^4v}\ ;\qquad F_1=\frac{-hbk\left(-3f^3+h^2k^2f^3\right)}{3f^4v}\ ;$$

$$F_2=\frac{-hbk\left(-\frac{3}{2}hkf^2+h^3k^3f^2\right)}{3f^4v}\ ;\ A_3<0\ ;\ E_3<0\ ;\ 54A_3^2>48A_2A_4\ ;$$

$$\left(\begin{array}{c}
u_1(x,t)=\frac{\sqrt{-48A_2A_4+54A_3^2}}{8A_4}\tanh\left(\begin{array}{l}\frac{1}{h\left(A_3E_3^3\right)^{\frac{1}{4}}}\left(\frac{-\left(A_3E_3^3\right)^{\frac{3}{4}}\left(-48A_2A_4+54A_3^2\right)^{\frac{3}{2}}}{216A_3^3}\right)^{\frac{1}{3}}x\\+\frac{8A_4^2\left(A_3E_3^3\right)^{\frac{1}{4}}\sqrt{-48A_2A_4+54A_3^2}}{81E_3A_3^4}t\end{array}\right)\\
u_2(x,t)=\frac{E_3}{B_3\left(A_3E_3^3\right)^{\frac{1}{4}}}\left(\frac{B_3^6\left(A_3E_3^3\right)^{\frac{3}{4}}\left(-48A_2A_4+54A_3^2\right)^{\frac{3}{2}}}{512A_3^3E_3^3B_4^3}\right)^{\frac{1}{3}}\tanh\left(\begin{array}{l}\frac{1}{h\left(A_3E_3^3\right)^{\frac{1}{4}}}\left(\frac{-\left(A_3E_3^3\right)^{\frac{3}{4}}\left(-48A_2A_4+54A_3^2\right)^{\frac{3}{2}}}{216A_3^3}\right)^{\frac{1}{3}}x\\+\frac{8A_4^2\left(A_3E_3^3\right)^{\frac{1}{4}}\sqrt{-48A_2A_4+54A_3^2}}{81E_3A_3^4}t\end{array}\right)\\
i_1(x,t)=\frac{\left(A_3E_3^2\right)^{\frac{1}{4}}\sqrt{-48A_2A_4+54A_3^2}}{8E_3A_4}\tanh\left(\begin{array}{l}\frac{1}{h\left(A_3E_3^3\right)^{\frac{1}{4}}}\left(\frac{-\left(A_3E_3^3\right)^{\frac{3}{4}}\left(-48A_2A_4+54A_3^2\right)^{\frac{3}{2}}}{216A_3^3}\right)^{\frac{1}{3}}x\\+\frac{8A_4^2\left(A_3E_3^3\right)^{\frac{1}{4}}\sqrt{-48A_2A_4+54A_3^2}}{81E_3A_3^4}t\end{array}\right)\\
i_2(x,t)=\frac{B_3^4A_4^2\left(A_3E_3^3\right)^{\frac{1}{4}}\sqrt{-48A_2A_4+54A_3^2}}{8E_3A_3^4B_4^3}\tanh\left(\begin{array}{l}\frac{1}{h\left(A_3E_3^3\right)^{\frac{1}{4}}}\left(\frac{-\left(A_3E_3^3\right)^{\frac{3}{4}}\left(-48A_2A_4+54A_3^2\right)^{\frac{3}{2}}}{216A_3^3}\right)^{\frac{1}{3}}x\\+\frac{8A_4^2\left(A_3E_3^3\right)^{\frac{1}{4}}\sqrt{-48A_2A_4+54A_3^2}}{81E_3A_3^4}t\end{array}\right)
\end{array}\right)\qquad(12)$$

4. Construction of a set of four solitary wave solutions of type (Pulse; Pulse; Pulse; Pulse) relative to general differential equation (6)

We define each of nonlinear charges $q_1\left(u_1\left(x,t\right)\right)$, $q_2\left(u_2\left(x,t\right)\right)$ of the capacitors and each of nonlinear magnetic flux linkage $\phi_1\left(i_1\left(x,t\right)\right)$, $\phi_2\left(i_2\left(x,t\right)\right)$ of the inductors under the analytical shape given below:

$$\begin{cases} \phi_1\left(i_1\left(x,t\right)\right)=E_1 i_1\left(x,t\right)+E_2 i_1^3\left(x,t\right)+\left(E_3 i_1\left(x,t\right)+E_4 i_1^3\left(x,t\right)\right)\sqrt{1-\dfrac{i_1^2\left(x,t\right)}{E_0^2}} \\ \phi_2\left(i_2\left(x,t\right)\right)=F_1 i_2\left(x,t\right)+F_2 i_2^3\left(x,t\right)+\left(F_3 i_2\left(x,t\right)+F_4 i_2^3\left(x,t\right)\right)\sqrt{1-\dfrac{i_2^2\left(x,t\right)}{F_0^2}} \\ q_1\left(u_1\left(x,t\right)\right)=A_1 u_1\left(x,t\right)+A_2 u_1^3\left(x,t\right)+\left(A_3 u_1\left(x,t\right)+A_4 u_1^3\left(x,t\right)\right)\sqrt{1-\dfrac{u_1^2\left(x,t\right)}{A_0^2}} \\ q_2\left(u_2\left(x,t\right)\right)=B_1 u_2\left(x,t\right)+B_2 u_2^3\left(x,t\right)+\left(B_3 u_2\left(x,t\right)+B_4 u_2^3\left(x,t\right)\right)\sqrt{1-\dfrac{u_2^2\left(x,t\right)}{B_0^2}} \end{cases} \tag{13}$$

With $\left|E_0\right|>\left|i_1\left(x,t\right)\right|$; $\left|F_0\right|>\left|i_2\left(x,t\right)\right|$; $\left|A_0\right|>\left|u_1\left(x,t\right)\right|$; $\left|B_0\right|>\left|u_2\left(x,t\right)\right|$. E_1 ; E_2 ; E_3 ; E_4 ; F_1 ; F_2 ; F_3 ; F_4 ; A_1 ; A_2 ; A_3 ; A_4 ; B_1 ; B_2 ; B_3 and B_4 are non-nil real numbers which will be chosen conveniently. By substituting each of the nonlinear charge $q_1\left(u_1\left(x,t\right)\right)$, $q_2\left(u_2\left(x,t\right)\right)$ and each of the nonlinear magnetic flux $\phi_1\left(i_1\left(x,t\right)\right)$, $\phi_2\left(i_2\left(x,t\right)\right)$ of (13) in (6) we obtain the set of four nonlinear partial differential equation written as:

$$
\left\{
\begin{array}{l}
\frac{h^4}{24}\frac{\partial^4 u_1(x,t)}{\partial x^4}-\frac{h^3}{6}\frac{\partial^3 u_1(x,t)}{\partial x^3}+\frac{h^2}{2}\frac{\partial^2 u_1(x,t)}{\partial x^2}-h\frac{\partial u_1(x,t)}{\partial x} \\
+\left(-E_1-3E_2 i_1^2(x,t)-\left(E_3+3E_4 i_1^2(x,t)\right)\sqrt{1-\frac{i_1^2(x,t)}{E_0^2}}+\frac{E_3 i_1^2(x,t)+E_4 i_1^4(x,t)}{E_0^2\sqrt{1-\frac{i_1^2(x,t)}{E_0^2}}}\right)\frac{\partial i_1(x,t)}{\partial t}=0 \\
\frac{h^4}{24}\frac{\partial^4 u_2(x,t)}{\partial x^4}-\frac{h^3}{6}\frac{\partial^3 u_2(x,t)}{\partial x^3}+\frac{h^2}{2}\frac{\partial^2 u_2(x,t)}{\partial x^2}-h\frac{\partial u_2(x,t)}{\partial x} \\
+\left(-F_1-3F_2 i_2^2(x,t)-\left(F_3+3F_4 i_2^2(x,t)\right)\sqrt{1-\frac{i_2^2(x,t)}{F_0^2}}+\frac{F_3 i_2^2(x,t)+F_4 i_2^4(x,t)}{F_0^2\sqrt{1-\frac{i_2^2(x,t)}{F_0^2}}}\right)\frac{\partial i_2(x,t)}{\partial t}=0 \\
\frac{h^4}{24}\frac{\partial^4 i_1(x,t)}{\partial x^4}+\frac{h^3}{6}\frac{\partial^3 i_1(x,t)}{\partial x^3}+\frac{h^2}{2}\frac{\partial^2 i_1(x,t)}{\partial x^2}+h\frac{\partial i_1(x,t)}{\partial x}-C_0\frac{\partial u_2(x,t)}{\partial t} \\
+\left(C_0+A_1+2A_2 u_1^2(x,t)+\left(A_3+3A_4 u_1^2(x,t)\right)\sqrt{1-\frac{u_1^2(x,t)}{A_0^2}}-\frac{A_3 u_1^2(x,t)+A_4 u_1^4(x,t)}{A_0^2\sqrt{1-\frac{u_1^2(x,t)}{A_0^2}}}\right)\frac{\partial u_1(x,t)}{\partial t}=0 \\
\frac{h^4}{24}\frac{\partial^4 i_2(x,t)}{\partial x^4}+\frac{h^3}{6}\frac{\partial^3 i_2(x,t)}{\partial x^3}+\frac{h^2}{2}\frac{\partial^2 i_2(x,t)}{\partial x^2}+h\frac{\partial i_2(x,t)}{\partial x}-C_0\frac{\partial u_1(x,t)}{\partial t} \\
+\left(C_0+B_1+2B_2 u_2^2(x,t)+\left(B_3+3B_4 u_2^2(x,t)\right)\sqrt{1-\frac{u_2^2(x,t)}{B_0^2}}-\frac{B_3 u_2^2(x,t)+B_4 u_2^4(x,t)}{B_0^2\sqrt{1-\frac{u_2^2(x,t)}{B_0^2}}}\right)\frac{\partial u_2(x,t)}{\partial t}=0
\end{array}
\right. \tag{14}
$$

Let us use Bogning-Djeumen Tchaho-Kofane method [15-20] to come out with the solution of (14) under the analytical shape below:

$$
\begin{pmatrix}
u_1(x,t)=a\,\mathrm{sech}(kx-vt) \\
u_2(x,t)=b\,\mathrm{sech}(kx-vt) \\
i_1(x,t)=e\,\mathrm{sech}(kx-vt) \\
i_2(x,t)=f\,\mathrm{sech}(kx-vt)
\end{pmatrix} \tag{15}
$$

Where a; b; e; f; k and v are non-nil real numbers to be determined in terms of modeled line parameters. Replacing $u_1(x,t)$; $u_1(x,t)$; $i_1(x,t)$ et $i_2(x,t)$ given by (15) in (14) we yield the following set of four equations which are written in a simplified form $a = A_0$; $b = B_0$; $e = E_0$ et $f = F_0$:

$$\begin{cases} \left(-20h^4A_0k^4 + 48E_0vE_3 - 72E_0^3vE_4 - 24h^2A_0k^2\right)\sinh(kx-vt)\cosh^2(kx-vt) \\ +\left(h^4A_0k^4 + 12h^2A_0k^2 - 24E_0vE_3\right)\sinh(kx-vt)\cosh^4(kx-vt) + \left(96E_0^3vE_4 + 24h^4A_0k^4\right)\sinh(kx-vt) \\ +\left(24h^3A_0k^3 + 72E_0^3vE_2\right)\cosh(kx-vt) + \left(-72E_0^3vE_2 - 24hA_0k + 24E_0vE_1 - 28h^3A_0k^3\right)\cosh^3(kx-vt) \\ +\left(24hA_0k - 24E_0vE_1 + 4h^3A_0k^3\right)\cosh^5(kx-vt) = 0 \\ \left(-20h^4B_0k^4 + 48F_0vF_3 - 72F_0^3vF_4 - 24h^2B_0k^2\right)\sinh(kx-vt)\cosh^2(kx-vt) \\ +\left(h^4B_0k^4 + 12h^2B_0k^2 - 24F_0vF_3\right)\sinh(kx-vt)\cosh^4(kx-vt) + \left(96F_0^3vF_4 + 24h^4B_0k^4\right)\sinh(kx-vt) \\ +\left(24h^3B_0k^3 + 72F_0^3vF_2\right)\cosh(kx-vt) + \left(-72F_0^3vF_2 - 24hB_0k + 24F_0vF_1 - 28h^3B_0k^3\right)\cosh^3(kx-vt) \\ +\left(24hB_0k - 24F_0vF_1 + 4h^3B_0k^3\right)\cosh^5(kx-vt) = 0 \\ \left(-20h^4E_0k^4 - 48A_0vA_3 + 72A_0^3vA_4 - 24h^2E_0k^2\right)\sinh(kx-vt)\cosh^2(kx-vt) \\ +\left(-24h^3E_0k^3 - 72A_0^3vA_2\right)\cosh(kx-vt) + \left(-96A_0^3vA_4 + 24h^4E_0k^4\right)\sinh(kx-vt) \\ +\left(h^4E_0k^4 + 12h^2E_0k^2 + 24A_0vA_3\right)\sinh(kx-vt)\cosh^4(kx-vt) \\ +\left(72A_0^3vA_2 + 24hE_0k - 24A_0vA_1 + 28h^3E_0k^3 - 24A_0vC_0 + 24B_0vC_0\right)\cosh^3(kx-vt) \\ +\left(-24hE_0k + 24A_0vA_1 - 4h^3E_0k^3 + 24A_0vC_0 - 24B_0vC_0\right)\cosh^5(kx-vt) = 0 \\ \left(-20h^4F_0k^4 - 48B_0vB_3 + 72B_0^3vB_4 - 24h^2F_0k^2\right)\sinh(kx-vt)\cosh^2(kx-vt) \\ +\left(-24h^3F_0k^3 - 72B_0^3vB_2\right)\cosh(kx-vt) + \left(-96B_0^3vB_4 + 24h^4F_0k^4\right)\sinh(kx-vt) \\ +\left(h^4F_0k^4 + 12h^2F_0k^2 + 24B_0vB_3\right)\sinh(kx-vt)\cosh^4(kx-vt) \\ +\left(72B_0^3vB_2 + 24hF_0k - 24B_0vB_1 + 28h^3F_0k^3 - 24B_0vC_0 + 24A_0vC_0\right)\cosh^3(kx-vt) \\ +\left(-24hF_0k + 24B_0vB_1 - 4h^3F_0k^3 + 24B_0vC_0 - 24A_0vC_0\right)\cosh^5(kx-vt) = 0 \end{cases} \quad (16)$$

The set of equations (16) is valid if and only if each of its basic hyperbolic function coefficients is nil. This permits us to obtain the following set of twenty four equations:

$$
\left\{
\begin{array}{c}
-20h^4A_0k^4+48E_0vE_3-72E_0^3vE_4-24h^2A_0k^2=0\\
h^4B_0k^4+12h^2B_0k^2-24F_0vF_3=0\\
96E_0^3vE_4+24h^4A_0k^4=0\\
24h^3A_0k^3+72E_0^3vE_2=0\\
-72E_0^3vE_2-24hA_0k+24E_0vE_1-28h^3A_0k^3=0\\
24hA_0k-24E_0vE_1+4h^3A_0k^3=0\\
-20h^4B_0k^4+48F_0vF_3-72F_0^3vF_4-24h^2B_0k^2=0\\
h^4B_0k^4+12h^2B_0k^2-24F_0vF_3=0\\
96F_0^3vF_4+24h^4B_0k^4=0\\
24h^3B_0k^3+72F_0^3vF_2=0\\
-72F_0^3vF_2-24hB_0k+24F_0vF_1-28h^3B_0k^3=0\\
24hB_0k-24F_0vF_1+4h^3B_0k^3=0\\
-20h^4E_0k^4-48A_0vA_3+72A_0^3vA_4-24h^2E_0k^2=0\\
-24h^3E_0k^3-72A_0^3vA_2=0\\
-96A_0^3vA_4+24h^4E_0k^4=0\\
h^4E_0k^4+12h^2E_0k^2+24A_0vA_3=0\\
72A_0^3vA_2+24hE_0k-24A_0vA_1+28h^3E_0k^3-24A_0vC_0+24B_0vC_0=0\\
-24hE_0k+24A_0vA_1-4h^3E_0k^3+24A_0vC_0-24B_0vC_0=0\\
-20h^4F_0k^4-48B_0vB_3+72B_0^3vB_4-24h^2F_0k^2=0\\
-24h^3F_0k^3-72B_0^3vB_2=0\\
-96B_0^3vB_4+24h^4F_0k^4=0\\
h^4F_0k^4+12h^2F_0k^2+24B_0vB_3=0\\
72B_0^3vB_2+24hF_0k-24B_0vB_1+28h^3F_0k^3-24B_0vC_0+24A_0vC_0=0\\
-24hF_0k+24B_0vB_1-4h^3F_0k^3+24B_0vC_0-24A_0vC_0=0
\end{array}
\right. \qquad (17)
$$

Haven solved the set of equation (17), it has permitted us to present in (18) the solution with conditions of the set of four nonlinear partial differential equations obtained in (14) which model the dynamic of a set of four solitary wave of type (pulse ; pulse ; pulse ; pulse):

$$
a=A_0\ ;\ b=B_0\ ;\ e=E_0\ ;\ f=F_0\ ;\ k=\frac{A_0}{E_0h}\left(\frac{-64A_4^3E_0^3}{27A_2^3A_0^3}\right)^{\frac{1}{3}}\ ;\ v=\frac{64A_4^3E_0}{81A_2^4A_0^3}\ ;\ A_2A_4<0\ ;\ B_2=-\frac{h^3fk^3}{3B_0^2v}\ ;
$$

$$
B_4=\frac{h^4fk^4}{4B_0^3v}\ ;\qquad E_2=-\frac{h^3A_0k^3}{3e^2v}\ ;\qquad E_4=-\frac{h^4A_0k^4}{4e^3v}\ ;\qquad F_2=-\frac{h^3B_0k^3}{3f^2v}\ ;\qquad F_4=-\frac{h^4B_0k^4}{4f^3v}\ ;
$$

$$
A_1=-C_0+\frac{C_0B_0}{A_0}+\frac{h^3ek^3}{6A_0v}+\frac{hek}{A_0v}\ ;\qquad A_3=-\frac{h^4ek^4}{24A_0v}-\frac{h^2ek^2}{2A_0v}\ ;\qquad B_1=-C_0+\frac{C_0A_0}{B_0}+\frac{h^3fk^3}{6B_0v}+\frac{hfk}{B_0v}\ ;
$$

$$B_3 = -\frac{h^4 fk^4}{24B_0 v} - \frac{h^2 fk^2}{2B_0 v} \ ; \quad E_1 = \frac{hA_0 k}{ev} + \frac{h^3 A_0 k^3}{6ev} \ ; \quad E_3 = \frac{h^2 A_0 k^2}{2ev} + \frac{h^4 A_0 k^4}{24ev} \ ; \quad F_1 = \frac{hB_0 k}{fv} + \frac{h^3 B_0 k^3}{6fv} \ ;$$

$$F_3 = \frac{h^2 B_0 k^2}{2fv} + \frac{h^4 B_0 k^4}{24fv} \ ; \quad \begin{pmatrix} u_1(x,t) = A_0 \operatorname{sech}\left(\frac{A_0}{E_0 h}\left(\frac{-64A_4^3 E_0^3}{27A_2^3 A_0^3}\right)^{\frac{1}{3}} x - \frac{64A_4^3 E_0}{81A_2^4 A_0^3} t \right) \\ u_2(x,t) = B_0 \operatorname{sech}\left(\frac{A_0}{E_0 h}\left(\frac{-64A_4^3 E_0^3}{27A_2^3 A_0^3}\right)^{\frac{1}{3}} x - \frac{64A_4^3 E_0}{81A_2^4 A_0^3} t \right) \\ i_1(x,t) = E_0 \operatorname{sech}\left(\frac{A_0}{E_0 h}\left(\frac{-64A_4^3 E_0^3}{27A_2^3 A_0^3}\right)^{\frac{1}{3}} x - \frac{64A_4^3 E_0}{81A_2^4 A_0^3} t \right) \\ i_2(x,t) = F_0 \operatorname{sech}\left(\frac{A_0}{E_0 h}\left(\frac{-64A_4^3 E_0^3}{27A_2^3 A_0^3}\right)^{\frac{1}{3}} x - \frac{64A_4^3 E_0}{81A_2^4 A_0^3} t \right) \end{pmatrix} \tag{18}$$

5. Conclusion

The choice of nonlinear hybrid electrical line with crosslink capacitor for our study is due to the fact that it permits the simultaneous displacement of four signals contrary to a non-coupled hybrid electrical line which permits the simultaneous displacement of two signals; let us recall that the more we will multiply the crosslink capacitor in the line, the more we will multiply the simultaneous displacement of signals in the line. In mathematical domain, the nonlinear hybrid electrical line with crosslink capacitor presented in figure 1 has permitted us in the one hand to discover in (8) a set of four nonlinear partial differential equations which have for exact solution a set of four solitary waves given in (12) and on the other hand to discover in (14) another set of four nonlinear partial differential equations which have for exact solution another set of four solitary waves given in (18). In the domain of physics in general and particularly in the domain of telecommunication, the set of four solitary waves obtained in (12) will permit the manufacturing of a new hybrid electrical line with crosslink capacitor where the flux linkage of its inductors and the charge of its capacitors vary in nonlinear manner defined in (7). In the same light, the set of four solitary waves obtained in (18) will permit the manufacturing of another hybrid electrical line with crosslink capacitor where the flux linkage of its inductors and the charge of its capacitors vary in nonlinear manner defined in (13). The set of four solitary waves obtained in (12) and in (18) prove that the quality of signals which are being displaced in the nonlinear hybrid electrical line with crosslink capacitor was ameliorated as compared to sinusoidal signals which are being displaced in hybrid electrical line with crosslink capacitor.

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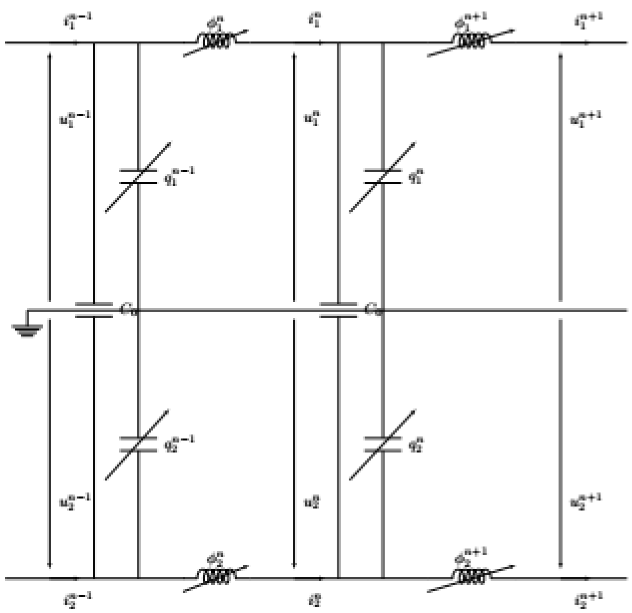

Figure 1: presentation of a nonlinear hybrid electrical line with crosslink capacitor.