Original Research Article

Calculation of Relative Uncertainty when Measuring Physical Constants: CODATA Technique vs Information Method

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ABSTRACT

Aims: To analyze the results of measurements of the Boltzmann, gravitational and Planck constants using a theoretically sound information approach in comparison with the CODATA technique.
Place and Duration of Study: Beer-Sheba, between January 2019 and May 2019.
Methodology: Using the concepts of information theory, the amount of information contained in the measurement model of a physical constant is calculated. This allows us to find the value of the comparative uncertainty proposed by Brillouin, and the achievable value of the relative uncertainty, taking into account the basic SI values used on each test bench when measuring physical constants.
Results: An unsolved question was to find the amount of information contained in the model of the measurement of a physical constant, which can be converted to the value of the achievable absolute uncertainty. This value now has an exact analytical formula. It is notoriously difficult to study the consistency of the measurement results of physical constants, but the proposed mathematical tool, developed using the concepts of information theory, allow us to simplify the analysis completely.
Conclusion: The information method leads to an intuitive and logically justified calculation of the relative uncertainty, which is compatible with the current practice of CODATA. This allows you to identify the threshold discrepancy between the model and the object under study. Proof of this is the calculation of the achievable value of the relative uncertainty when measuring the Boltzmann, gravitational and Planck constants. The proposed information-oriented method for calculating the relative uncertainty in measuring physical constants represents a new tool when formulating a modernized SI.

Keywords: Boltzmann, gravitational and Planck constants, CODATA, Information theory, Least squares correction, Relative uncertainty

1. INTRODUCTION

The Committee’s decision [1], seemingly so far from the consciousness and understanding of the majority of the 7.5 billion population of the Earth, opened a new era not only in measurement theory and metrology but also in all areas of human life. Over the last decade, thanks to huge investments, unique test benches, advanced mathematical methods, super-powerful computers and accumulated knowledge, the modification of the International system of units (SI) has become possible. It lies in the fact that four new definitions and four fixed numerical values of the basic constants have been established [1]. In this case, the uncertainty, necessarily associated with the data used, is discarded, and the value is assumed to be exact by agreement.

The process of fixing the value of the constant is carried out using the method of least squares correction (LSA). There is no published evidence that the latest adjustments can be considered equivalent, therefore, CODATA can only be trusted for the correctness of their work [2].

The LSA method is aimed at checking the consistency of the results, and for this, the initial experimental values are “corrected,” that is, changed to optimize the final dispersion of the set. However, in these cases, the initial values are adjusted. This shortcoming is not available to the scientific community because the adjustments are not presented in CODATA publications.

The purpose of this article is to point out some features inherent in the approved CODATA method for calculating the value of the constants and their relative uncertainty. A positive discussion of this becomes important in view of the implementation of the revision of the International System of Measurement Units (SI) in 2019. The existing statistical features of the CODATA method, along with the mandatory discussion and formulation of an expert opinion, may still raise doubts about the complexity and possible subjectivity of the tools used. The application of the LSA method and its influence on the decisions made by CODATA causes some skepticism. Don't forget the joke: statistics is one form of lies.

Unlike the accepted CODATA procedure for processing the results of experiments using LSA, we propose a new procedure for finding the recommended value of relative uncertainty, which simulates the results using the comparative uncertainty proposed by Brillouin and takes into account the basic SI values implemented on each test bench when measuring physical constants.

2. PROVISIONS AND SOME FORMULAS RELATED TO THE INFORMATION METHOD

Background premises and evidence are presented in previously published works [3-6]. Below, in a condensed form, the data necessary for subsequent reasoning and analysis of the results of measurements of physical constants are given.

The total number of the dimensionless criteria $\boldsymbol{\mu}_{\mathrm{SI}}$ in SI equals

$$\boldsymbol{\mu}_{\mathrm{SI}} = 38,265 \qquad (1)$$

SI is a set of dimensional quantities, base and derived, used for descriptions of different classes of phenomena (*CoP*), which depend on seven base quantities: meter, the length $\boldsymbol{L}$; kilogram, the mass $\boldsymbol{M}$; second, the time $\boldsymbol{T}$; kelvin, the thermodynamic temperature $\boldsymbol{\theta}$; ampere, the electrical current $\boldsymbol{I}$; mole, the amount of substance $\boldsymbol{F}$; candela, the luminous intensity $\boldsymbol{J}$ [7]. For example, when measuring the gravitational constant by electromechanical methods, the basis {the length $\boldsymbol{L}$, weight $\boldsymbol{M}$, time $\boldsymbol{T}$, electrical current $\boldsymbol{I}$} is used, i.e., $CoP_{\mathrm{SI}} \equiv \boldsymbol{LMTI}$.

The dimensionless measurement absolute uncertainty $\boldsymbol{\Delta u}$ of the dimensionless quantity $\boldsymbol{u}$ with a changed interval $\boldsymbol{S}$ can be calculated

$$\boldsymbol{\Delta u} = \boldsymbol{S}\cdot(\boldsymbol{z}' - \boldsymbol{\beta}')/\boldsymbol{\mu}_{\mathrm{SI}} + (\boldsymbol{z}'' - \boldsymbol{\beta}'')/(\boldsymbol{z}' - \boldsymbol{\beta}'), \qquad (2)$$

where $\boldsymbol{\beta}'$ is the number of the base quantities of the chosen *CoP*, $\boldsymbol{z}'$ is the total number of the dimensional quantities of the chosen *CoP*, $\boldsymbol{z}''$ is a given number of the dimensional physical quantities recorded in the model, $\boldsymbol{\beta}''$ is the number of the base quantities recorded in the model, $\boldsymbol{\varepsilon}$ is the comparative uncertainty suggested by Brillouin [8], $\boldsymbol{\varepsilon} = \boldsymbol{\Delta u}/\boldsymbol{S}$.

Equation (2) sets an ultimate limit on the accuracy of measuring a physical constant, which cannot be overcome by any measuring instruments, perfect mathematical methods, and using unique materials or software. This limit already exists before performing any calculations or implementing algorithms on a computer. Its value depends on the class of the phenomenon and the number of quantities taken into account.

The information-oriented method can be applied for measurements of *any dimensional or dimensionless physical constant* because the relative and comparative uncertainties of the dimensional quantity $\boldsymbol{U}$ and the dimensionless quantity $\boldsymbol{u}$ are equal:

$$\begin{gathered}\boldsymbol{\Delta U}/\boldsymbol{S^*} = (\boldsymbol{\Delta U}/\boldsymbol{a})/(\boldsymbol{S^*}/\boldsymbol{a}) = (\boldsymbol{\Delta u}/\boldsymbol{S}),\\ (\boldsymbol{r}/\boldsymbol{R}) = (\boldsymbol{\Delta U}/\boldsymbol{U})/(\boldsymbol{\Delta u}/\boldsymbol{u}) = (\boldsymbol{\Delta U}/\boldsymbol{U})\cdot(\boldsymbol{a}/\boldsymbol{\Delta U})\cdot(\boldsymbol{U}/\boldsymbol{a}) = 1,\end{gathered} \qquad (3)$$

where $\boldsymbol{\Delta u}$ is the total absolute uncertainty in determining the dimensionless quantity $\boldsymbol{u}$; $\boldsymbol{S^*}$ and $\boldsymbol{\Delta U}$ are dimensional quantities (respectively, the range of variations and the total absolute uncertainty in determining the dimensional quantity $\boldsymbol{U}$); $\boldsymbol{a}$ is the dimensional scale parameter with the same dimension as that of $\boldsymbol{U}$ and $\boldsymbol{S^*}$; $\boldsymbol{r}$ is the relative uncertainty of the dimensional quantity $\boldsymbol{U}$; and $\boldsymbol{R}$ is the relative uncertainty of the dimensionless quantity $\boldsymbol{u}$.

Taking into account (2), one can verify conditions for calculating the minimum comparative uncertainty for a particular *CoP*:

$$(\boldsymbol{z}' - \boldsymbol{\beta}')^2/\boldsymbol{\mu}_{\mathrm{SI}} = (\boldsymbol{z}'' - \boldsymbol{\beta}''). \qquad (4)$$

According to (4), it is possible to check (Table 1) the optimal number of quantities in the model and the achievable comparative uncertainties recommended in the framework of the information method, as well as the *CoP* commonly used when measuring the Boltzmann, gravitational and Planck constants:

Table 1. Comparative uncertainties and recommended number of dimensionless criteria

$\boldsymbol{CoP_{\mathrm{SI}}}$	Comparative uncertainty	Recommended number of criteria
LMT	0.0048	0.2 < 1
LMTF	0.0146	≅2
LMTI	0.0245	≅6
LMTθF	0.1331	≅169
LMTθI	0.2220	≅471

It should be noted that the comparative uncertainty and the recommended number of values in the model are different and depend to the choice of *CoP*. From the data in Table 1, it can be seen that ***LMT** and **LMTF** are not recommended for use in measurements of physical constants* because there are very few criteria that can be used in the model. This causes a situation where an increase in the number of variables/criteria taken into account leads to an increase in experimental comparative uncertainty that can be achieved, which is much more than the recommended. Consequently, the discrepancy between the model and the really emerging process of measuring a physical constant increases.

An objective assessment of the achieved accuracy of measuring a physical constant, within the framework of the information approach, is confirmed using two metrics, denoted as *IARU* (information approach with relative uncertainty) and *IACU* (information approach with comparative uncertainty). In *IARU*, the interval of change of the physical constant ***S*** is calculated as the difference between the maximum and minimum values of the physical constant measured by various research groups in recent years. This is due to the need to consider the appearance of each experimental result in a given range as an independent event. In this case, knowing the comparative uncertainty inherent in the chosen class of phenomena, the recommended relative uncertainty is calculated. Its value, in turn, is compared with the relative uncertainty of each published study.

For *IACU*, ***S*** is calculated in accordance with the technical limitations of measurement devices [8]. In this case, the standard uncertainty calculated in the experiment when measuring a physical constant is taken as the possible interval for the placement of its true value. The experimental absolute uncertainty is calculated by multiplying the value of the fundamental physical constant and its relative uncertainty achieved in each experiment. The achieved experimental comparative uncertainty of each published study is calculated by dividing the experimental absolute uncertainty by the standard uncertainty. Then, the experimentally calculated comparative uncertainty is compared with the selected comparative uncertainty (Table 1), which is inherent in the model describing the measurement of the fundamental constant.

3. ANALYZING RESULTS OF MEASURING THE BOLTZMANN, GRAVITATIONAL AND PLANCK CONSTANTS

A detailed analysis of the measurement of the Boltzmann, gravitational and Planck constants from the positions of *IARU* and *IACU* is presented in [4-6]. Methods and results with data on the values of physical constants, relative measurement uncertainties and standard uncertainties, published in scientific journals during 2000–2018 and confirmed by CODATA, were taken into account. Below, we present a summary of these studies (Tables 1, 2, 3), taking into account the application of *IARU*.

From the data presented in Tables 2–4, you can simply draw the following obvious conclusions.

Table 2. Summarized data of the Boltzmann constant, *k*

Variable	AGT	DCGT	JNT	DBT
CoP	***LMTθF***	***LMTθI***	***LMTθI***	***LMTθF***
Comparative uncertainty according to CoP_{SI}	0.1331	0.2220	0.2220	0.1331
Possible observed range S_k of ***k*** placing, $m^2 \cdot kg/(s^2 \cdot K)$	$2.4 \cdot 10^{-29}$	$2.7 \cdot 10^{-29}$	$9.2 \cdot 10^{-29}$	$2.2 \cdot 10^{-27}$
Relative uncertainty according to CoP_{SI}, r_k	$2.3 \cdot 10^{-7}$	$4.3 \cdot 10^{-7}$	$1.4 \cdot 10^{-6}$	$2.1 \cdot 10^{-5}$
Achieved experimental lowest relative uncertainty, r_{kexp}	$6.0 \cdot 10^{-7}$	$\mathbf{3.7 \cdot 10^{-7}}$	$2.7 \cdot 10^{-6}$	$2.4 \cdot 10^{-5}$
Ratio of r_{kexp}/r_k	2.6	0.9	1.9	1.1

Impressive advances in measuring physical constants have been achieved using DCGT for ***k***. This is because of the significant difference in the magnitude of the comparative uncertainties between CoP_{SI} ≡ ***LMTΘF*** (AGT – 0.1331) and CoP_{SI} ≡ ***LMTΘI*** (DCGT – 0.2220). The only concern is that the experimental relative uncertainty is less than the relative uncertainty theoretically calculated (Table 2), which contradicts the information method. Therefore, a researcher using DCGT needs to recheck everything, if possible, and within the framework of the information approach—necessarily, potential sources of uncertainty;

Table 3. Summarized data of the Planck constant, *h*

Variable	KB	XRCD
CoP	***LMTI***	***LMTF***
Comparative uncertainty according to CoP_{SI}	0.0245	0.0146
Possible observed range S_h of h placing, $m^2 \cdot kg/s$	$1.2\cdot10^{-40}$	$4.6\cdot10^{-41}$
Relative uncertainty according to CoP_{SI} (*IARU*), r_k	$4.5\cdot10^{-9}$	$1.0\cdot10^{-9}$
Achieved experimental lowest relative uncertainty, r_{kexp}	$1.3\cdot10^{-8}$	$9.1\cdot10^{-9}$
Ratio of r_{kexp}/r_k	3.0	9.1

Table 4. Summarized data of the gravitational constant, *G*

Variable	Mechanistic methods	Electromechanical methods
CoP	***LMT***	***LMTI***
Comparative uncertainty according to CoP_{SI}	0.0048	0.0245
Possible observed range S_G of G placing, $m^3/(kg \cdot s^2)$	$2.1\cdot10^{-14}$	$1.7\cdot10^{-14}$
Relative uncertainty according to CoP_{SI} (*IARU*), r_G	$1.5\cdot10^{-6}$	$6.3\cdot10^{-6}$
Achieved experimental lowest relative uncertainty, r_{Gexp}	$1.9\cdot10^{-5}$	$1.2\cdot10^{-5}$
Ratio of r_{Gexp}/r_G	12.7	1.9

- KB for *h*. This is because there is a twofold difference between the comparative uncertainties for CoP_{SI} ≡ ***LMTF*** (XRCD – 0.0146) and CoP_{SI} ≡ ***LMTI*** (KB – 0.0245) and almost equal placement interval of *h*;
- Electromechanical methods for *G*. This is due to the huge difference in comparative uncertainties between CoP_{SI} ≡ ***LMT*** (ε_{LMT} = 0.0048) and CoP_{SI} ≡ ***LMTI*** (ε_{LMTI} = 0.0245) and the closeness of the achieved lowest experimental value of relative uncertainty ($1.2\cdot10^{-5}$) to the recommended one ($6.3\cdot10^{-6}$). That is why further and detailed research of the current electromechanical methods should be continued.

Within the framework of the information method, several methods seem limited for future improvement:
- DBT (CoP_{SI} ≡ ***LMTΘF***) for ***k*** in terms of the possibility of achieving higher accuracy. This is because the values of relative uncertainty, theoretically calculated and achieved in the experiment, are very close ($2.1\cdot10^{-5}$ and $2.4\cdot10^{-5}$);
- AGT (CoP_{SI} ≡ ***LMTΘF***) for ***k***. Given the fact that the interval of the possible placement of ***k*** for the AGT method ($2.4\cdot10^{-29}$ m² kg/(s² K)) is the smallest compared with other methods, it is difficult to expect any achievements in increasing its accuracy;

- Mechanistic methods (CoP_{SI} ≡ ***LMT***) for *G*. There are two reasons to stick to that point of view. The latest results for the relative uncertainty of the gravitational constant are very different from the relative uncertainty calculated by the *IARU* method (Table 4). Second, and, perhaps more importantly, in this case, the use of even one or several variables leads to an increase in the attainable experimental uncertainty, which is much more than the theoretically recommended value of the comparative uncertainty (Table 1).

To compare all the methods used, Table 5 was compiled. As shown in Table 5, despite the huge differences between the methods in order of magnitude of relative values according to CoP_{SI}, relative uncertainty according to CoP_{SI} (*IARU*) r_{SI}, and experimental minimum relative uncertainty r_{exp}, the ratio r_{exp}/r_{SI} varies in a rather small interval (0.9–3.0) compared with models V (mechanistic methods, gravitational constant) and VIII (XRCD, Planck constant). Consistency is a basic requirement for a new SI, but you may ask why V and VIII stand out?

Table 5. Comparison data of measuring the Boltzmann, gravitational and Planck constants

Fundamental constant	Boltzmann constant				Gravitational constant		Planck constant	
Variable/ Method	AGT	DCGT	JNT	DBT	Mechanistic methods	Electro-mechanical methods	KB	XRCD
	I	II	III	IV	V	VI	VII	VIII
CoP	***LMTθF***	***LMTθI***	***LMTθI***	***LMTθF***	***LMT***	***LMTI***	***LMTI***	***LMTF***
Comparative uncertainty according to CoP_{SI}	0.1331	0.2220	0.2220	0.1331	0.0048	0.0245	0.0245	0.0146
Relative uncertainty according to CoP_{SI} (*IARU*), r_{SI}	$2.3\cdot10^{-7}$	$4.3\cdot10^{-7}$	$1.4\cdot10^{-6}$	$2.1\cdot10^{-5}$	$1.5\cdot10^{-6}$	$6.3\cdot10^{-6}$	$4.5\cdot10^{-9}$	$1.0\cdot10^{-9}$
Achieved experimental lowest relative uncertainty, r_{exp}	$6.0\cdot10^{-7}$	$3.7\cdot10^{-7}$	$2.7\cdot10^{-6}$	$2.4\cdot10^{-5}$	$1.9\cdot10^{-5}$	$1.2\cdot10^{-5}$	$1.3\cdot10^{-8}$	$9.1\cdot10^{-9}$
Ratio of r_{exp}/r_{SI}	**2.6**	**0.9**	**1.9**	**1.1**	***12.7***	**1.9**	**3.0**	***9.1***

We wonder if it arises straight from the application of the LSA method, or is it due to any further culling of the data—including by CODATA [9]? In fact, this degree of consistency can exist simply due to the application of the LSA method and as a result of reducing the uncertainty of the measurement data. Perhaps the situation will change for the better if the new method of processing the results of measurements of physical constants is used in the CODATA technique [10].

However, there is another reason to explain this situation in the context of an information-oriented approach.

Already in the process of formulating the method of measuring the physical constant, there is an unremovable uncertainty, called comparative uncertainty, due to the number of variables and the qualitative set of base quantities in the model. It is not constant and changes depending on the number of recorded base quantities. In addition, according to the calculations formulated within the framework of the presented approach, the use of ***LMT*** and ***LMTF*** *is not recommended* because the achievement of the theoretical value of comparative uncertainty in practice is impossible. This is because when using these *CoP*, numerous potential effects are not taken into account, and the recommended number of selected criteria is less than two. That is why, within the framework of the information-oriented method in contrast to the method adopted in CODATA, *it is inappropriate to establish only one value* *of relative uncertainty when measuring physical constants by various methods*. This is explained by the fact that for models inherent in different *CoP*, there are different values of comparative uncertainties and a different number of quantities, which is recommended to choose.

4. CONCLUDING REMARKS

An unsolved question was to find the amount of information contained in the model of the measurement of a physical constant, which can be converted to the value of the achievable absolute uncertainty. This value now has an exact analytical formula. It is notoriously difficult to study the consistency of the measurement results of physical constants, but the proposed mathematical tool, developed using the concepts of information theory, allowed us to simplify the analysis completely.

It is obvious from the analyzed data that an information approach is a universal tool for verifying accuracy and recommended values of relative uncertainties. The information-oriented method does not depend on the subjective judgment of the expert and is free from any inaccuracies, weighting factors inherent in statistical methods and accessible to all (meaning no hierarchy). It is very easy to use; it is available even to students, user understandable and it does not require complex calculations and is performed in a short time. It is not unimportant that this method is theoretically justified and conceptually correct.

The approach implements a simple and reliable way of formulating a model with the optimal number of quantities taken into account. Thus, the duration of the experiments and their cost could be significantly reduced.

From the point of view of the author, the information method leads to a theoretically proven, intuitive, and logically sound calculation of relative uncertainty, which is compatible with modern CODATA practice. This allows you to identify the threshold discrepancy between the model and the object under study. Proof of this is the calculation of the achievable value of the relative uncertainty when measuring the Boltzmann, gravitational and Planck constants.

The author does not want to look like a person who automatically criticizes the CODATA methodology. Of course, recent years have been marked by great achievements in measuring fundamental constants with reduced uncertainty, which led to outstanding results. However, one should keep in mind the possible "enthusiasm" of CODATA scientists in search of the threshold value of uncertainty. Therefore, the information approach can serve as a theoretically justified tool for confirming certain values of relative uncertainties.

Based on the foregoing, it seems correct to assume that the proposed information-oriented method for calculating the relative uncertainty in measuring physical constants represents a new tool when formulating a modernized SI.

In the end, the author expresses the hope that the proposed method, along with the current version of SI, can be labeled as "for all times, for all people."

COMPETING INTERESTS

The author has declared that no competing interests exist.

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