Original Research Article

Statistical Measure of Second Order Response Surface Rotatability using an infinite class of supplementary Difference Sets.

ABSTRACT

Rotatability is a desirable feature of a response surface experimental design. In case a design is non rotatable or exhibit surface of constant prediction variances that are nearly spherical then an attempt is made to make the design rotatable. In this paper, a measure of rotatability of five level second order rotatable designs using an infinite class of supplementary difference sets is suggested. The variance function of a second-order response design and an infinite class of supplementary difference sets is used in coming up with the design

Keywords: Response surface Methodology; Rotatable designs; second order designs; five leve; Supplementary difference sets.

1. INTRODUCTION

The property of rotatability as a desirable quality of an experimental design was first advanced by [1] and requires that the variances of the estimated response are constant on circles or spheres about the center of the design.

Koukouvinos et al. [4] gave a general construction method for five level second order rotatable designs. . Further research was done by Mutiso, Kerich and Ng'eno [5&6] where they constructed five level rotatable designs using an infinite class of supplementary difference sets. Park et al. [9&10] developed a measure of rotatability that is invariant under rotation and Ng'eno [7] developed a measure of modified rotatability using an infinite class of supplementary difference sets by fixing c=5. This article presents a measure of Box- Hunter [1] rotatability by fixing c = 3. The measure will be investigated using an infinite class of supplementary difference sets.

The following symmetry conditions (Moments conditions) needs to be satisfied for a design to form a Second order rotatable arrangement [1]

i) $\sum_{u=i}^{N}\prod_{i=1}^{v}x_{iu}^{\alpha_i}=0$ if any α_i is odd for $\sum\alpha_i\leq 4$

ii) $\sum_{u=1}^{N}x_{iu}^{2}=Constant=\ N\lambda_2$ for i = 1,2,…,v

iii) $\sum_{u=1}^{N}x_{iu}^{4}=Constant=cN\lambda_4$ for i= 1,2,…,v **(1.1)**

iv) $\sum_{u=1}^{N} x_{iu}^2 x_{ju}^2 = Constant = N\lambda_4$

v) $\frac{\sum_{u=1}^{N} x_{iu}^4}{\sum_{u=1}^{N} X_{iu}^2 x_{ju}^2} = c$ for i≠j

where c, λ_2 and λ_4 are constants.

Using these symmetry conditions the variance and the covariance of the estimates are obtained and are shown below

i) V(b_0) = $\frac{(c+v-1)\,\lambda_4\delta^2}{N[(c+v-1)\,\lambda_4 - v\lambda_2^2]}$

ii) V(b_i) = $\frac{\delta^2}{N\lambda_2}$

iii) V(b_{ij})= $\frac{\delta^2}{N\lambda_4}$

iv) V(b_{ii}) = $\frac{\delta^2}{N(c-1)\lambda_4}\left[\frac{(c+v-2)\lambda_4 - \lambda_2^2(v-1)}{(c+v-1)\lambda_4 - v\lambda_2^2}\right]$ **(1.2)**

v) Cov(b_0, b_{ii}) = $\frac{-\lambda_2\delta^2}{N[(c+v-1)\lambda_4 - v\lambda_2^2]}$

vi) Cov(b_{ii}, b_{ij}) = $\frac{\delta^2}{N\lambda_4(c-1)}\left[\frac{\lambda_2^2 - \lambda_4}{(c+v-1)\lambda_4 - v\lambda_2^2}\right]$

and all other covariances are zero.

An inspection of the variances shows that a necessary condition for the existence of a non-singular second order design is $(c + v - 1)\lambda_4 - v\lambda_2^2 > 0$ which leads to the following non-singularity condition first developed by Box and Hunter [1]

$$\frac{\lambda_4}{\lambda_2^2} > \frac{v}{c+v-1} \qquad \textbf{(1.3)}$$

Harder and Park [3] studied estimates in response at two different points in the factor space. They introduced the analogous form of Box Hunter rotatability and termed it as slope rotatability. Harder and Park [6] and Park [8] stated that the necessary and sufficient condition for slope rotatability is 4V(b_{ii}) = $V(b_{ij})$. The condition was simplified by Victorbabu and Narasimham [12] and they developed equation (1.4) below which is the necessary and sufficient condition for a 2nd order design to be slope rotatable

$$\lambda_4[v(5-c) - (c-3)^2] + \lambda_2^2[v(c-5)+4] = 0 \qquad \textbf{(1.4)}$$

2. METHODOLOGY

Seberry Wallis [11] defined supplementary difference sets and stated that the parameters of e- [v; $k_1, k_2, \ldots, k_e$,λ] SDS satisfies λ (v-1) = ek(k-1). Koukouvinos et al [4] came up with the following useful relationships which we will utilize in this study.

i) $\sum_{u=1}^{N} x_{1u} = \sum_{u=1}^{N} x_{2u} = \sum_{u=1}^{N} x_{3u} = \sum_{u=1}^{N} x_{4u} = 0$

ii) $\sum_{u=1}^{N} x_{iu}^2 = 2^{t(m)}(e-1) + 2b^2 = N\lambda_2$

iii) $\sum_{u=1}^{N} x_{iu}^4 = 2^{t(m)}(e-1) + 2b^4 = cN\lambda_4$

iv) $\sum_{u=1}^{N} x_{iu}^2 x_{ju}^2 = 2^{t(m)}(e-2) = N\lambda_4$

Mutiso et el. [6] stated in a theorem that the Supplementary difference sets with parameters e-[v:2:1] gives a five level second order rotatable design in $b^4 = \frac{2^{t(m)}[2e-5]}{2}$ and $n_0 = \frac{-[-2v+4][2^{t(m)}(e-1)+2b^2]^2}{2v[2^{t(m)}(e-2)]}$ $-m2^{t(m)} - 2m$

Using (iii) and (iv) above we obtain

$$b^4 = \frac{2^{t(m)}[c(e-2) - (e-1)]}{2} \qquad \textbf{(2.1)}$$

We use equation (1.4) to get the number of centre points (n_0) to be added to the design i.e

$$\lambda_4[v(5-c) - (c-3)^2] + \lambda_2^2[v(c-5)+4] = 0 \text{ where } \lambda_2 = \frac{2^{t(m)}(e-1)+2b^2}{N} \; ; \; \lambda_4 = \frac{2^{t(m)}(e-2)}{N}$$

on simplification we obtain

$$\text{N} = -\frac{[v(c-5)+4][2^{t(m)}(e-1)+2b^2]^2}{[v(5-3)-(c-3)^2]2^{t(m)}(e-2)} \qquad \textbf{(2.2)}$$

therefore $n_0 = N - m2^{t(m)} - 2m$

$$\therefore n_0 = \frac{-[v(c-5)+4][2^{t(m)}(e-1)+2b^2]^2}{[v(5-c)-(c-3)^2]2^{t(m)}(e-2)} - m2^{t(m)} - 2m$$

Basing on Box –Hunter rotatability criteria [1] we fix c to be 3 in 2.1 and 2.2 and therefore we have

$$b^4 = \frac{2^{t(m)}[2e-5]}{2} \qquad \textbf{(2.3)}$$

and therefore,

$$n_0 = \frac{-[-2v+4][2^{t(m)}(e-1)+2b^2]^2}{2v[2^{t(m)}(e-2)]} - m2^{t(m)} - 2m \qquad \textbf{(2.4)}$$

A design whose moments do not conform to the moment conditions of rotatability is said to be non-rotatable. If circumstances are such that exact rotatability is unattainable, it is still a good idea to make the design nearly rotatable [9&10]

The article utilizes the measure developed by Park et al [10] to access the degree of rotatability. If the region of interest is 0≤ $\rho \leq 1$, then the rotatability measure is expresses as

$$P_v[D] = \frac{1}{1+R_v[D]} \qquad \textbf{(2.5)}$$

Where

$R_v[D] = [\frac{N}{\delta^2}]^2 \left\{\frac{6v\left[v(b_{ij}) + 2Cov(b_{ii},b_{ij}) - 2V(b_{ii})\right]^2(v-1)}{(v+2)^2(v+4)(v+6)(v+8)g^8}\right\}$ and g is a scaling factor.

on simplification the numerator portion of $v(b_{ij}) + 2Cov(b_{ii}, b_{ij}) - 2V(b_{ii})$ become $\frac{(c-3)\delta^2}{(c-1)N\lambda_4}$ thus $R_v[D]$ becomes, $R_v[D] = [\frac{N}{\delta^2}]^2 \left\{\frac{6v\left[(c-3)\delta^2\right]^2(v-1)}{((c-1)N\lambda_4)^2 \; (v+2)^2(v+4)(v+6)(v+8)g^8}\right\}$ **(2.6)**

For second order rotatable design, we have c=3. Substituting the value of c=3 in the above equation we get $R_v[D] = 0$. Hence $P_v[D]$ takes the value of 1 if and only if a design is rotatable and it's smaller than one for non rotatable design.

To compare the design, Park et al [10] considered the scaling of the design. In this article, the design is scaled in such a way that all the point's lie inside or on the unit sphere [2].

If we have a set of points $\boldsymbol{x_i'}$ = $(x_{1u}, x_{2u}, \dots, x_{vu})$ i=1,2,…,N then the scaled point, gx_i should satisfy

$$0 \le g\sqrt{(x_{1u})^2 + (x_{2u})^2 + \dots + (x_{vu})^2} \le 1 \text{ i=1,2,…,N} \quad \textbf{(2.7)}$$

One advantage of this is that, when we add center point, the remaining points do not have to be rescaled.

To construct the measure we will use equation (2.5) developed by park et al [10] and replace v with m because in supplementary difference sets we have $m = \frac{v-1}{2}$ factors. We therefore obtain the following expressions $P_m[D] = \frac{1}{1+R_m[D]}$

Where $$R_m[D] = [\frac{N}{\delta^2}]^2 \left\{\frac{6m[(c-3)\delta^2]^2(m-1)}{(c-1)N\lambda_4)^2\ (m+2)^2(m+4)(m+6)(m+8)g^8}\right\} \quad \textbf{(2.8)}$$

$$c = \frac{\sum_{u=1}^{N} x_{iu}^4}{\sum_{u=1}^{N} x_{iu}^2 x_{ju}^2} = \frac{2^{t(m)}(e-1) + 2b^4}{2^{t(m)}(e-2)},\ b^4 = \frac{2^{t(m)}[2e-5]}{2},\ \lambda_4 = \frac{2^{t(m)}(e-2)}{N}$$

$$g = \begin{cases} \frac{1}{b}\ if\ b \le \sqrt{2^{t(m)-1} + m} \\ \frac{1}{\sqrt{\sqrt{2^{t(m)-1}+m}}}\ if\ b > 2^{t(m)-1} + m \end{cases}$$

On simplifying 2.8 we have $$R_m[D] = \frac{6m(m-1)(c-3)^2}{(m+2)^2(m+4)(m+6)(m+8)\lambda_4{}^2(c-1))^2\ g^8} \quad \textbf{(2.9)}$$

2. RESULTS, DISCUSION AND CONCLUSION

For a second order rotatable design, we have c=3. Substituting the value of c=3 in (4.6), we get $R_m[D] = 0$. Hence $P_m[D]$ takes the value of 1 if the design is rotatable and less than 1 if it is not rotatable.

Illustration

Let us consider 3-[7:2:1] SDS. In this case we have, $v = 7, m = e = \frac{v-1}{2} = 3$, and $\lambda_4 = \frac{2^{t(m)}(e-2)}{N} = \frac{2^2(3-2)}{21} = \frac{4}{21} = 0.1905$

Using (2.9) we have $$R_m[D] = \frac{6m(m-1)(c-3)^2}{(m+2)^2(m+4)(m+6)(m+8)\lambda_4{}^2(c-1))^2\ g^8} = 0.05727\ \frac{(c-3)^2}{(c-1))^2\ g^8}$$

For rotatable design, c=3 meaning that $R_m[D]$= 0 and therefore $P_m[D] = \frac{1}{1+R_m[D]} = 1$. Suppose c= 3.92, We use (2.8) to get the value of g. In this case g=$\frac{1}{b}$ =0.7143

$$R_m[D] = 0.05727\ \frac{(c-3)^2}{(c-1))^2\ g^8} = 0.05727\frac{(3.92-3)^2}{(3.92-1))^2\ g^8} = 8.3886 \times 10^{-2}$$

Therefore $P_m[D] = \frac{1}{1+R_m[D]} \quad = \frac{1}{1+8.3886 \times 10^{-2}} = 0.9226.$

The above value is less than one but very close to one meaning that the design in nearly rotatable.

Suppose c=13.71, $R_m[D]$= 0.05727$\frac{(13.71-3)^2}{(13.71-1))^2 \; g^8}$ = 22.3328

Therefore $P_m[D] = \frac{1}{1+R_m[D]} = \frac{1}{1+22.3328} = 4.285 \times 10^{-2}$

The above value is less than one but not close to one meaning that the design in not rotatable.

The expressions for calculating $R_m[D]$ for different class of supplementary difference sets are given in the following equations. The results of the measure for different class of SDS are as shown in tables 1 to 10 in the appendix.

3-[7:2:1] SDS - $R_m[D]$ = 5.7258$\times 10^{-2} \; \frac{(c-3)^2}{(c-1))^2 \; g^8}$

4-[9:2:1] SDS- $R_m[D]$ = 7.9991 $\times 10^{-3} \; \frac{(c-3)^2}{(c-1))^2 \; g^8}$

5-[11:2:1] SDS– Taking half replicate of factorial part- $R_m[D]$ = 1.1111 $\times 10^{-2} \; \frac{(c-3)^2}{(c-1))^2 \; g^8}$

5-[11:2:1] SDS - $R_m[D]$ = 8.4266 $\times 10^{-3} \; \frac{(c-3)^2}{(c-1))^2 \; g^8}$

6-[13:2:1] SDS – Taking $\frac{1}{4}$replicate of factorial part - $R_m[D]$= 7.5599 $\times 10^{-3} \; \frac{(c-3)^2}{(c-1))^2 \; g^8}$

7-[15:2:1] SDS – Taking $\frac{1}{4}$replicate of factorial part- $R_m[D]$= 4.3792 $\times 10^{-3} \; \frac{(c-3)^2}{(c-1))^2 \; g^8}$

8-[17:2:1] SDS – Taking $\frac{1}{8}$replicate of factorial part- $R_m[D]$ = 3.3859 $\times 10^{-3} \; \frac{(c-3)^2}{(c-1))^2 \; g^8}$

9-[19:2:1] SDS – Taking $\frac{1}{16}$ replicate of factorial part- $R_m[D]$= 2.6592$\times 10^{-3} \; \frac{(c-3)^2}{(c-1))^2 \; g^8}$

10-[21:2:1] SDS – Taking $\frac{1}{32}$ replicate of factorial part - $R_m[D]$= 2.1586$\times 10^{-3} \; \frac{(c-3)^2}{(c-1))^2 \; g^8}$

11-[23:2:1] SDS – Taking $\frac{1}{64}$ replicate of factorial part – $R_m[D]$= 1.7802$\times 10^{-3} \; \frac{(c-3)^2}{(c-1))^2 \; g^8}$

COMPETİNG İNTERESTS

Authors have declared that no competing interests exist.”.

REFERENCES

[1] Box, G.E.P., Hunter, J.S.(1957). Multifactor experimental designs for exploring response surfaces. Annals of Mathematical Statistics. 28 195-241.

[2] Draper, N.R., Pukelsheim, F. (1990). Another look at rotatability. Technometrics. 32 195-202.

[3] Hader, R.J., Park, S.H. (1978). Slope rotatable central composite designs. Technometrics. 20 413-417.

[4] Koukiuvinos, C., Mylona, K., Skountzou, A.,Goos, P. (2013). A general construction Method for five level second order rotatable designs. Communication in statistics- simulation and computation. 42 1961-1969.

[5] Mutiso, J.M., Kerich, G.K., Ng'eno, H.M. (2016a). Construction of five level modified second order rotatable designs using supplementary difference sets. Far East Journal of Theoretical Statistics. 52: 333-343.

[6] Mutiso, J.M., Kerich, G.K., Ng'eno, H.M. (2016b). Construction of five level second order rotatable designs using supplementary difference sets. Advances and Application in statistics. 49:21-30

[7] Ng'eno, M.N. (2018). Measure of rotatability of modified second-order rotatabled design using supplementary difference sets. Statistics Theory and Related field.

[8] Park, S.H. (1987). A class of multifactor designs for estimating the slope of response surfaces. Technometrics. 29 449-453.

[9] Park, S.H., Kim, H.J. (1992). A measure of slope rotatability for second order response surface experimental designs. Journal of Applied Statistics. 19(3) 391-404.

[10] Park, S.H., Lim, J.H., Baba, Y. (1993). A measure of rotatability for second order response surface designs. Ann. Inst. Statist. math. 45 655-664

[11] Seberry W.J. (1973). Some remarks on supplementary difference sets. Calloquia Mathematica Societalis Janos Bolyai Hungary. 10 1503-1526.

[12] Victorbabu, B.Re., Narasimham, V.L. (1991). Construction of second order slope rotatable designs through balanced incomplete block designs. Communication in Statistics- Theory and Methods. 20(8) 2467-2478.

APPENDIX

Table 1 – Measure of rotatability for five level SORD for 3-[7:2:1] SDS

v=7, m=3, N = 21 λ_4= 0.1905

b	c	g	$R_m[D]$	$P_m[D]$
1.00	2.50	1.0000	6.3621×10^{-3}	0.9937
1.19	3.00	0.8403	0.0000	1.0000
1.20	3.04	0.8333	9.4686×10^{-5}	0.9999
1.40	3.92	0.7143	8.3886×10^{-2}	0.9226
1.60	5.28	0.6250	0.6988	0.5889
1.80	7.25	0.5556	2.9158	0.2554
2.00	10.00	0.5000	8.8672	0.1013
2.20	13.71	0.4545	22.3283	4.287×10^{-2}
2.40	18.58	0.4472	28.1139	3.4235×10^{-2}
2.60	24.85	0.4472	30.0435	3.2213×10^{-2}
2.80	32.73	0.4472	31.4249	3.0834×10^{-2}
3.00	42.50	0.4472	32.4282	2.9915×10^{-2}
3.20	54.43	0.4472	33.1656	2.9269×10^{-2}
3.40	68.81	0.4472	33.7148	2.8806×10^{-2}
3.60	85.98	0.4472	34.1302	2.8466×10^{-2}
3.80	106.26	0.4472	34.4478	2.8210×10^{-2}
4.00	130.00	0.4472	34.6939	2.8016×10^{-2}
4.20	157.58	0.4472	34.8866	2.7866×10^{-2}
4.40	189.40	0.4472	35.0393	2.7748×10^{-2}
Rotatable b = 1.19				

Table 2 - Measure of rotatability for five level SORD for 4-[9:2:1] SDS

m= 4, v=9, N = 47, λ_4= 0.5106

b	c	g	$R_m[D]$	$P_m[D]$
1.00	1.63	1.0000	3.7788×10^{-2}	0.9636
1.20	1.76	0.8333	9.1496×10^{-2}	0.9161
1.40	1.98	0.7143	0.1277	0.8867
1.60	2.32	0.6250	9.1080×10^{-2}	0.9165
1.80	2.81	0.5556	9.6972×10^{-3}	0.9904
1.86	3.00	0.5376	0.0000	1.0000
2.00	3.50	0.5000	8.1827×10^{-2}	0.9244
2.20	4.43	0.4545	0.7628	0.5673
2.40	5.65	0.4167	2.8549	0.2594
2.60	7.21	0.3846	7.6719	0.1153
2.80	9.18	0.3571	17.2483	5.4799×10^{-2}
3.00	11.63	0.3536	21.5503	4.4345×10^{-2}
3.20	14.61	0.3536	23.7928	4.0334×10^{-2}
3.40	18.20	0.3536	25.5346	3.7687×10^{-2}
3.60	22.49	0.3536	26.8937	3.5851×10^{-2}
3.80	27.56	0.3536	27.9576	3.4533×10^{-2}
4.00	35.50	0.3536	29.0153	3.3316×10^{-2}
4.20	40.39	0.3536	29.4604	3.2829×10^{-2}
4.40	48.35	0.3536	29.9925	3.2266×10^{-2}
Rotatable b = 1.86				

Table 3 - Measure of rotatability for five level SORD for 5-[11:2:1] SDS

m= 5($\frac{1}{2}$ replicate), v=11, N=58, λ_4= 0.4138

b	c	g	$R_m[D]$	$P_m[D]$
1.00	1.42	1.0000	0.1573	0.8641
1.20	1.51	0.8333	0.4079	0.7102
1.40	1.65	0.7143	0.7073	0.5857
1.60	1.88	0.6250	0.7731	0.5639
1.80	2.21	0.5556	0.5216	0.6571
2.00	2.67	0.5000	0.1110	0.9000
2.12	3.00	0.4717	0.0000	1.0000
2.20	3.29	0.4545	9.7877×10^{-2}	0.91088
2.40	4.09	0.4167	1.5212	0.3966
2.60	5.14	0.3846	6.2023	0.1388
2.80	6.16	0.3571	15.7609	5.9663×10^{-2}
3.00	8.08	0.3333	37.5668	2.5929×10^{-2}
3.20	10.07	0.3333	44.3371	2.2057×2
3.40	12.47	0.3333	49.7412	1.9708×2
3.60	15.33	0.3333	54.0227	1.8174×10^{-2}
3.80	18.71	0.3333	57.4194	1.7118×10^{-2}
4.00	22.67	0.3333	60.1221	1.6361×2
4.20	27.26	0.3333	62.2781	1.5803×10^{-2}
4.40	32.57	0.3333	64.0171	1.5380×10^{-2}
Rotatable b = 2.12				

Table 4 - Measure of rotatability for five level SORD for 5-[11:2:1] SDS

m= 5, v=11, N=101, λ_4= 0.4752

b	c	g	$R_m[D]$	$P_m[D]$
1.00	1.38	1.0000	0.1531	0.8672
1.20	1.42	0.8333	0.5129	0.6609
1.40	1.49	0.7143	1.1808	0.4583
1.60	1.61	0.6250	1.8792	0.3473
1.80	1.77	0.5556	2.3680	0.2969
2.00	2.00	0.5000	2.1572	0.3167
2.20	2.31	0.4545	1.2839	0.4378
2.40	2.72	0.4167	0.2457	0.8028
2.50	2.96	0.4000	5.3552×10^{-2}	0.9947
2.52	3	0.3968	0.0000	1.0000
2.60	3.24	0.3846	0.2021	0.8319
2.80	3.89	0.3571	3.0221	0.2486
3.00	4.71	0.3333	11.7548	7.8402×10^{-2}
3.20	5.70	0.3125	30.5762	3.1669×10^{-2}
3.40	6.90	0.2941	65.7838	1.4974×10^{-2}
3.60	8.33	0.2778	125.6152	7.8980×10^{-3}
3.80	10.02	0.2778	143.89873	6.9010×10^{-3}
4.00	12.00	0.2778	159.0360	6.2490×10^{-3}
4.20	14.30	0.2778	173.4942	5.7970×10^{-3}
4.40	16.95	0.2778	181.7284	5.4730×10^{-3}
Rotatable b = 2.52				

Table 5 - Measure of rotatability for five level SORD for 6-[13:2:1] SDS

m= 6($\frac{1}{4}$ replicate), v=13, N=68, λ_4= 0.4706

b	c	g	$R_m[D]$	$P_m[D]$
1.00	1.31	1.0000	0.2247	0.8166
1.20	1.38	0.8333	0.5909	0.6286
1.40	1.49	0.7142	1.0604	0.4853
1.60	1.66	0.6250	1.3383	0.4277
1.80	1.91	0.5556	1.1944	0.4557
2.00	2.25	0.50	0.6967	0.5894
2.20	2.71	0.4545	0.1194	0.8933
2.30	3.00	0.4348	0.0000	1.0000
2.40	3.32	0.4167	0.1582	0.8634
2.60	4.11	0.3846	2.0115	0.3321
2.80	5.09	0.3571	7.4646	0.1181
3.00	6.31	0.3333	19.2869	4.9292×10^{-2}
3.20	7.80	0.3162	37.6919	2.5845×10^{-2}
3.40	9.60	0.3162	44.5528	2.1953×10^{-2}
3.60	11.75	0.3162	50.1168	1.9563×10^{-2}
3.80	14.28	0.3162	54.5766	1.7993×10^{-2}
4.00	17.25	0.3162	58.1711	1.6900×10^{-2}
4.20	20.70	0.3162	61.0659	1.6112×10^{-2}
4.40	24.68	0.3162	63.4073	1.5526×10^{-2}
Rotatable b = 2.30				

Table 6 - Measure of rotatability for five level SORD for 7-[15:2:1] SDS

m= 7 ($\frac{1}{4}$ replicate), v=15, N=139, λ_4= 0.5755

b	c	g	$R_m[D]$	$P_m[D]$
1.00	1.23	1.0000	0.2593	0.7941
1.20	1.25	0.8333	0.9229	0.5200
1.40	1.30	0.7143	2.0773	0.3252
1.60	1.36	0.6250	3.9033	0.2039
1.80	1.46	0.5556	5.4054	0.1561
2.00	1.60	0.5000	6.1037	0.1408
2.20	1.79	0.4545	5.6421	0.1506
2.40	2.01	0.4177	4.5405	0.1805
2.60	2.34	0.3846	2.2192	0.3106
2.80	2.74	0.3571	0.3698	0.7301
2.90	2.97	0.3448	5.0840×10^{-3}	0.9949
2.91	3.00	0.3436	0.0000	1.0000
3.00	3.23	0.3333	0.3059	0.7658
3.20	3.82	0.3125	4.0712	0.1972
3.40	4.54	0.2941	14.8072	6.3263×10^{-2}
3.60	5.40	0.2778	36.7330	2.6502×10^{-2}
3.80	6.41	0.2632	75.5483	1.3064×10^{-2}
4.00	7.60	0.2582	107.6900	9.2005×10^{-3}
4.20	8.98	0.2582	124.4928	7.9686×10^{-3}
4.40	10.57	0.2582	138.7124	7.1576×10^{-3}
Rotatable b = 2.91				

Table 7 - Measure of rotatability for five level SORD for 8-[17:2:1] SDS

m= 8 ($\frac{1}{8}$ replicate), v=17, N=158, λ_4= 0.6076

b	c	g	$R_m[D]$	$P_m[D]$
1.00	1.19	1.0000	0.3073	0.7649
1.20	1.21	0.8333	1.0581	0.4859
1.40	1.25	0.7143	2.4508	0.2900
1.60	1.30	0.6250	4.6697	0.1764
1.80	1.39	0.5556	6.3548	0.1360
2.00	1.50	0.5000	7.8011	0.1136
2.20	1.65	0.4545	8.0213	0.1108
2.40	1.86	0.4167	6.5448	0.1325
2.60	2.12	0.3846	4.3665	0.1863
2.80	2.45	0.3571	1.8422	0.3518
3.00	2.85	0.3333	0.1462	0.8725
3.06	3.00	0.3268	0.0000	1.0000
3.20	3.35	0.3125	0.8258	0.5477
3.40	3.95	0.2941	6.2736	0.1375
3.60	4.67	0.2778	19.7659	4.8156×10^{-2}
3.80	5.51	0.2632	45.5388	2.1487×10^{-2}
4.00	6.50	0.2500	89.8597	1.1006×10^{-2}
4.20	7.65	0.2500	108.4967	9.1328×10^{-3}
4.40	8.98	0.2500	124.6094	7.9612×10^{-3}
Rotatable b = 3.06				

Table 8 - Measure of rotatability for five level SORD for 9-[19:2:1] SDS

m= 9 ($\frac{1}{16}$ replicate), v=19, N=176, λ_4= 0.6364

b	c	g	$R_m[D]$	$P_m[D]$
1.00	1.16	1.0000	0.3517	0.7398
1.20	1.18	0.8333	1.1693	0.4609
1.40	1.21	0.7143	2.8508	0.2597
1.60	1.26	0.6250	5.1152	0.1635
1.80	1.33	0.5556	7.4999	0.1176
2.00	1.43	0.5000	9.0751	9.9254×10^{-2}
2.20	1.56	0.4545	9.6567	9.3837×10^{-2}
2.40	1.74	0.4167	8.4809	0.1055
2.60	1.96	0.3846	6.5193	0.1329
2.80	2.24	0.3571	3.7776	0.2093
3.00	2.59	0.3333	1.1610	0.4627
3.10	2.79	0.3226	0.3120	0.7622
3.19	3.00	0.3135	0.0000	1.0000
3.20	3.02	0.3125	2.8662×10^{-3}	0.9971
3.40	3.53	0.2941	2.0849	0.3242
3.60	4.14	0.2778	9.8820	9.1894×10^{-2}
3.80	4.87	0.2632	26.9603	3.5765×10^{-2}
4.00	5.71	0.2500	57.6936	1.7037×10^{-2}
4.20	6.70	0.2425	93.6938	1.0560×10^{-3}
4.40	7.84	0.2425	111.3340	8.9019×10^{-3}
Rotatable b = 3.19				

Table 1 - Measure of rotatability for five level SORD for 10-[21:2:1] SDS

m= 10 ($\frac{1}{32}$ replicate),   v=21, N=195, λ_4= 0.6564

b	c	g	$R_m[D]$	$P_m[D]$
1.00	1.14	1.0000	0.3810	0.7241
1.20	1.16	0.8333	1.2279	0.4488
1.40	1.19	0.7143	2.8905	0.2570
1.60	1.23	0.6250	5.4906	0.1541
1.80	1.29	0.5556	8.2655	0.1079
2.00	1.38	0.5000	10.0432	9.0553×10^{-2}
2.20	1.49	0.4545	11.2580	8.1579×10^{-2}
2.40	1.64	0.4167	10.7226	8.5330×10^{-2}
2.60	1.84	0.3846	8.5991	0.1042
2.80	2.09	0.3571	5.6896	0.1495
3.00	2.39	0.3333	2.7297	0.2681
3.20	2.76	0.3125	0.4413	0.6938
3.30	2.98	0.3030	3.0998×10^{-3}	0.9967
3.31	3.00	0.3021	0.0000	1.0000
3.40	3.21	0.2941	0.3482	0.7417
3.60	3.75	0.2778	4.5266	0.1809
3.80	4.38	0.2632	15.6246	6.0152×10^{-2}
4.00	5.13	0.2500	37.6280	2.5888×10^{-2}
4.20	5.99	0.2357	81.3648	1.2141×10^{-2}
4.40	6.98	0.2357	100.3828	9.8636×10^{-3}
Rotatable b = 3.31				

Table 10 - Measure of rotatability for five level SORD for 11-[23:2:1] SDS

m= 11 ($\frac{1}{64}$ replicate),   v=23, N=214, λ_4= 0.6729

b	c	g	$R_m[D]$	$P_m[D]$
1.00	1.13	1.0000	0.3684	0.7308
1.20	1.14	0.8333	1.3515	0.4253
1.40	1.16	0.7143	3.4739	0.2235
1.60	1.20	0.6250	6.1932	0.1390
1.80	1.26	0.5556	8.7806	0.1022
2.00	1.33	0.5000	11.6712	7.8920×10^{-2}
2.20	1.44	0.4545	12.2897	7.5247×10^{-2}
2.40	1.57	0.4167	12.3255	7.5045×10^{-2}
2.60	1.75	0.3846	10.3298	8.8264×10^{-2}
2.80	1.96	0.3571	7.9009	0.1123
3.00	2.24	0.3333	4.3911	0.1855
3.20	2.57	0.3125	1.4683	0.4051
3.40	2.97	0.2941	7.3760×10^{-3}	0.9927
3.41	3.00	0.2933	0.0000	1.0000
3.60	3.44	0.2778	1.6321	0.3799
3.80	4.01	0.2632	8.7035	0.1031
4.00	4.67	0.2500	24.1574	3.9750×10^{-2}
4.20	5.43	0.2381	51.8560	1.8919×10^{-2}
4.40	6.32	0.2294	90.4012	1.0941×10^{-2}
Rotatable b = 3.41				