Effect of Variable Viscosity on Natural Convection Flow of Heat Generating/Absorbing Fluid in a Vertical Channel: An Approximate Solution

Abstract

The present paper was aimed at investigating the effects of variable viscosity on natural convection flow and heat generating/absorbing fluid in a channel formed by two vertical parallel plates. Equations of continuity, momentum and energy are solved using the Homotopy perturbation method (HPM). The temperature and velocity profiles are presented graphically for various values of physical parameters. During the course of investigation, it is found that as the heat generation increases, fluid temperature and velocity increase while it decreases with the increase in heat absorption. Also, velocity decreases with the increase of the viscosity of the fluid.

Keywords: variable viscosity; natural convection; heat generation/absorption; Homotopy perturbation

Nomenclature

g - acceleration due to gravity $[ms^{-2}]$
h - width of the channel [m]
S - dimensionless heat generation/absorption parameter
Q_0 - heat generation/absorption coefficient $[Kgm^{-1}s^{-3}K^{-1}]$
T^* - dimensional fluid temperature [K]
T_w^* - channel wall temperature [K]
T_0^* - temperature of the ambience [K]
T - dimensionless fluid temperature
u^* - dimensional velocity $[ms^{-1}]$
u - dimensionless velocity
U - dimensional velocity of the moving plate $[ms^{-1}]$
y^* - co-ordinate perpendicular to the plate [m]
y - dimensionless co-ordinate perpendicular to the plate
Gr - Grashof number
cp - specific heat at constant pressure $[m^2s^{-2}K^{-1}]$
ρ - density of the fluid $[Kgm^{-3}]$
α - thermal diffusivity $[m^2s^{-1}]$
p - embedding parameter
β - coefficient of thermal expansion $[K^{-1}]$
μ - coefficient of viscosity

ν - kinematic viscosity $[m^2 s^{-1}]$

1 Introduction

The study of natural convection flow in a vertical channel has received a great deal of attention due to its applications such as, engineering field, geophysics, oceanography and environmental problems. A detailed review of natural convection flow and heat transfer can be found. Abdou [1] studied the effect of radiation with temperature dependent viscosity and thermal conductivity on unsteady a stretching sheet through porous media. He concluded his work that velocity and temperature across the boundary layer increase with increasing viscosity variation parameter. Santana and Hazarika [2] examined the effects of variable viscosity and thermal conductivity on magnetohydrodynamics free convection and mass transfer flow over an inclined vertical surface in a porous medium with heat generation. They concluded that an increasing values of viscosity retard the velocity but enhance the temperature. Adel et al. [3] worked on the similarity solution for steady magnetohydrodynamics Falkne-skan heat and mass transfer flow over a wedge in porous media considering thermal-diffusion and diffusion-thermo effects with variable viscosity and thermal conductivity. They discovered that the velocity of the fluid is found to increase with increase of the temperature dependent fluid viscosity. Makungu et al. [4] studied the effects of variable viscosity of nanofluid flow over a permeable wedge embedded in saturated porous medium with chemical reaction and thermal radiation. Hazarika and Gopal [5] analyzed the effects of variable viscosity and thermal conductivity on magnetohydrodynamics flow past a vertical plate. They observed that, the velocity profile decreases with the increase of variable viscosity is not so prominent in case of temperature profile. Mohamed [6] examined dissipation and variable viscosity on steady magnetohydrodynamics free convective flow over a stretching sheet in presence of thermal radiation and chemical reaction. He discovered that, the velocity decreases with an increase in viscosity parameter. Phukan and Hazarika [7] studied the effects of variable viscosity and thermal conductivity on magnetohydrodynamics free convective flow of micropolar fluid past a stretching plate through porous medium with radiation, heat generation and Joule heating. They reported that velocity decreases with the increase of the viscosity parameter. In another article, Noghrehabadi et al. [8] examined the effects of variable viscosity and thermal conductivity on natural convection of nanofluids past a vertical plate in a porous media. The outcomes showed that, an increase of variable viscosity parameter increases the velocity profiles whereas decreases the concentration profiles. Moreover, variation of viscosity parameter does not show the significant effect on the temperature profiles. All the above mentioned studies did not consider the effect of heat generation/absorption.

The study of heat generation/absorption in moving fluids is important in several physical problems dealing with chemical reactions and those concerned with dissociating fluids. Possible heat generation effects may alter the temperature distribution and therefore, the particle deposition rate. Chamkha and Camille [9] solved hydro magnetic flow with heat and mass transfer over a flat plate in the presence of heat generation or absorption and thermophoresis. Natural convection with heat generation along a uniformly heat vertical wavy surface have been demonstrated by Molla et al. [10]. Veena et al. [11] worked on heat transfer characteristics in the laminar boundary layer flow of a viscoelastic fluid over a linearly stretching continuous surface with variable wall temperature subjected to suction or blowing. Jha and Ajibade [12] considered the case of unsteady free convective Couette flow of heat generating/absorbing fluid. Their outcomes showed that, the skin friction increased as the external heating/cooling increases, likewise an increase in heat absorption increases the rate of heat transfer on the moving plate and decreases the rate of heat transfer on the stationary plate.

The objective of this study is to investigates the effect of variable viscosity and heat generation/absorption fluid on natural convection flow in a vertical channel. The equation governing the flow have some non linear terms in them so that obtaining closed form solution is a daunting task. Such problems can therefore be approached by numerical schemes or some approximate solution methods. One of the efficient methods is the perturbation method. However, solutions obtained by perturbation method are restricted to small perturbation parameters, therefore to overcome this restriction, another method called Homotopy perturbation method was introduced. He [13] was first studied to solve linear, non-linear and couple problems in partial or ordinary form. He [14] studied a coupling method of a Homotopy technique and a perturbation technique for non-linear problems. In another article, He [15] studied a new non linear analytical technique using Homotopy perturbation methods. Da-Hua [16] studied Homotopy perturbation method for nonlinear oscillators.

2 Mathematical analysis

We considers a steady natural convection flow of an incompressible viscous fluid in a vertical channel of width h. The flow is assumed to be in the x^* - direction which is taken vertically along one of the plates while y^* - axis is taken normal to it. The second is placed h distance away from the first. The temperature of the fluid and one of the channel plates are kept at T_0 while the temperature of the plate $y^* = 0$ is raised or fell to T_w and thereafter maintained constant. Also, the plate $y^* = 0$ moves in its own plane impulsively at a uniform velocity $u^* = U$ while the other plate remains at rest. The flow configuration and coordinates system is shown in figure 1.

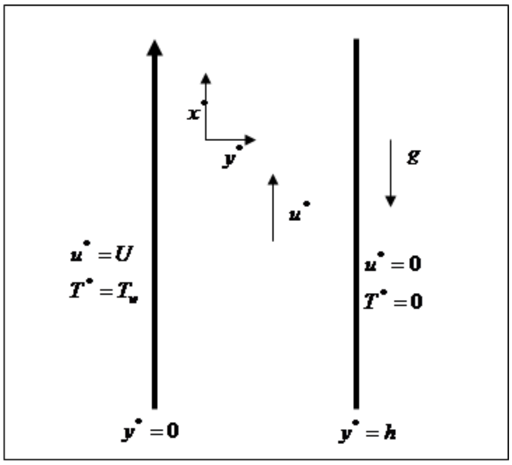

Figure 1: Schematic diagram of the problem

Under the usual assumption of Boussinesq's approximation, the governing equations of the energy and momentum as follows:

$$\frac{du^*}{dx^*} = 0, \tag{1}$$

$$\frac{1}{\rho}\frac{d}{dy^*}\left(\mu^*\frac{du^*}{dy^*}\right) + g\beta(T^* - T_0) = 0, \tag{2}$$

$$\alpha\frac{d^2T^*}{dy^{*2}} - \frac{Q_0}{\rho cp}(T^* - T_0) = 0. \tag{3}$$

The viscosity of the working fluid is assumed to vary linearly with temperature as follows

$$\mu^* = \mu_0(1 - \lambda^*(T^* - T_0))$$

while the boundary conditions are:

$$\begin{aligned} u^* = U, \quad T^* = T_w \quad at \quad y^* = 0, \\ u^* = 0, \quad T^* = T_0 \quad at \quad y^* = h. \end{aligned} \tag{4}$$

Due to the nature of the quantities that are given in different dimensions, we introduce some dimensionless quantities that can transform the governing equations and their conditions into dimensionless form. The dimensionless quantities used in equations (1) - (3) and the boundary condition (4) are:

$$\begin{aligned} y = \frac{y^*}{h}, \quad u = \frac{u^*}{U}, \quad T = \frac{T^* - T_0}{T_w - T_0}, \quad S = \frac{Q_0 h^2}{k}, \\ Gr = \frac{g\beta h^2(T_w - T_0)}{vU}, \quad \lambda = \lambda^*(T_w - T_0). \end{aligned} \tag{5}$$

By using the dimensionless quantities, the governing equations and the boundary conditions are transformed into non-dimensional form as

$$\frac{du}{dx} = 0, \tag{6}$$

$$(1-\lambda T)\frac{d^2u}{dy^2} - \lambda\frac{du}{dy}\cdot\frac{dT}{dy} + Gr(1-\lambda T)T = 0, \tag{7}$$

$$\frac{d^2T}{dy^2} - ST = 0. \tag{8}$$

And the boundary conditions are:

$$\begin{aligned} u = 1, \quad T = 1 \quad at \quad y = 0, \\ u = 0, \quad T = 0 \quad at \quad y = 1. \end{aligned} \tag{9}$$

2.1 Homotopy perturbation method

In order to illustrate the basic ideas of the Homotopy Perturbation Method (HPM), we consider the following nonlinear differential equation

$$A(u) - f(r) = 0, \qquad r \in \Omega, \tag{10}$$

with the boundary conditions of

$$B(u, \frac{\partial u}{\partial n}) = 0 \qquad r \in \Gamma, \tag{11}$$

where A is a general differential operator, B is a boundary operator, $f(r)$ is known analytical function and Γ is the boundary of the domain Ω, respectively. Generally speaking, the operator A can be divided into two parts which are L and N, where L is linear part and N is nonlinear part. Therefore (10) can be written as:

$$L(u) + N(u) - f(r) = 0, \qquad r \in \Omega, \tag{12}$$

By the homotopy techniques, we construct a homotopy as follows
$v(r,p) : \Omega \times [0,1] \to R$ which satisfies:

$$H(v,p) = (1-p)[L(v) - L(u_0)] + p[A(v) - f(r)] = 0, \tag{13}$$

in equation (13), $p \in [0,1]$ is an embedding parameter, while u_0 is an initial approximation of equation (10), which satisfies the boundary conditions. Clearly from eqn (13), we have

$$H(v,0) = L(v) - L(u_0) = 0, \tag{14}$$

$$H(v,1) = A(v) - f(r) = 0. \tag{15}$$

We can assume that the solution of equation (13) can be written as a power series in p:

$$v = v_0 + pv_1 + p^2 v_2 + ..., \tag{16}$$

setting $p = 1$ gives the approximate solution of eqn (10) as

$$u = \lim_{p \to 1} v = v_0 + v_1 + v_2 + ... \tag{17}$$

Applying the Homotopy perturbation technique to solve the governing equations in the present problem, we construct a convex Homotopy on eqs. (7) and (8) to get

$$H(u,p) = (1-p)\left[\frac{d^2u}{dy^2}\right] + p\left[\frac{d^2u}{dy^2} + \lambda T\frac{d^2u}{dy^2} + \lambda\frac{du}{dy}\cdot\frac{dT}{dy} - GrT + \lambda GrT^2\right] = 0, \tag{18}$$

$$H(T,p) = (1-p)\left[\frac{d^2T}{dy^2}\right] + p\left[\frac{d^2T}{dy^2} - ST\right] = 0, \tag{19}$$

using infinite series (18) and (19) to define u and T as follows

$$\begin{aligned} u &= u_0 + pu_1 + p^2u_2 + ..., \\ T &= T_0 + pT_1 + p^2T_2 + ... \end{aligned} \tag{20}$$

Substituting eqn. (20) into eqns. (18) and (19), we have

$$\begin{aligned} \frac{d^2u_0}{dy^2} + p\frac{d^2u_1}{dy^2} + p^2\frac{d^2u_2}{dy^2} + p^3\frac{d^2u_3}{dy^2} + ... = {} & p\lambda T_0\frac{d^2u_0}{dy^2} + p^2\left[\lambda T_0\frac{d^2u_1}{dy^2} + \lambda T_1\frac{d^2u_0}{dy^2}\right] \\ & + p^3\left[\lambda T_0\frac{d^2u_2}{dy^2} + \lambda T_2\frac{d^2u_0}{dy^2} + \lambda T_1\frac{d^2u_1}{dy^2}\right] + ... \\ & + p\lambda\frac{du_0}{dy}\cdot\frac{dT_0}{dy} + p^2\left[\frac{du_0}{dy}\cdot\frac{dT_1}{dy} + \lambda\frac{du_1}{dy}\cdot\frac{dT_0}{dy}\right] \\ & + p^3\left[\lambda\frac{du_0}{dy}\cdot\frac{dT_2}{dy} + \lambda\frac{du_2}{dy}\cdot\frac{dT_0}{dy} + \lambda\frac{du_1}{dy}\cdot\frac{dT_1}{dy}\right] + ... \\ & - pGrT_0 - p^2GrT_1 - p^3GrT_2 - ... \\ & + p\lambda GrT_0^2 + p^2\left[2\lambda GrT_0T_1\right] \\ & + p^3\left[2\lambda GrT_0T_2 + \lambda GrT_1^2\right] + ... \end{aligned} \tag{21}$$

$$\frac{d^2T_0}{dy^2} + p\frac{d^2T_1}{dy^2} + p^2\frac{d^2T_2}{dy^2} + p^3\frac{d^2T_3}{dy^2} + ... = pST_0 + p^2ST_1 + p^3ST_2 + ... \tag{22}$$

By comparing the coefficient of p^0, p^1, p^2 and p^3 of eqns. (21) and (22), we have

$$P^0 : \frac{d^2u_0}{dy^2} = 0, \tag{23}$$

$$P^0 : \frac{d^2T_0}{dy^2} = 0, \tag{24}$$

$$P^1 : \frac{d^2u_1}{dy^2} = \lambda T_0\frac{d^2u_0}{dy} + \lambda\frac{du_0}{dy}\cdot\frac{dT_0}{dy} - GrT_0 + \lambda GrT_0{}^2, \tag{25}$$

$$P^1 : \frac{d^2T_1}{dy^2} = ST_0, \tag{26}$$

$$P^2 : \frac{d^2u_2}{dy^2} = \lambda T_0\frac{d^2u_1}{dy^2} + \lambda T_1\frac{d^2u_0}{dy^2} + \lambda\frac{du_0}{dy}\cdot\frac{dT_1}{dy} + \lambda\frac{du_1}{dy}\cdot\frac{dT_0}{dy} - GrT_1 + 2\lambda GrT_0T_1, \tag{27}$$

$$P^2 : \frac{d^2T_2}{dy^2} = ST_1, \tag{28}$$

$$P^3 : \frac{d^2u_3}{dy^2} = \lambda T_0\frac{d^2u_2}{dy^2} + \lambda T_2\frac{d^2u_0}{dy^2} + \lambda T_1\frac{d^2u_1}{dy^2} + \lambda\frac{du_0}{dy}\cdot\frac{dT_2}{dy} + \lambda\frac{du_2}{dy}\cdot\frac{dT_0}{dy} + \lambda\frac{du_1}{dy}\cdot\frac{dT_1}{dy} - GrT_2 + 2\lambda GrT_0T_2 + \lambda GrT_1^2, \tag{29}$$

$$P^3 : \frac{d^2T_3}{dy^2} = ST_2. \tag{30}$$

$$\vdots$$

The boundary conditions are transformed also as

$$\begin{aligned} T_0(0) = 1, \quad & T_1(0) = T_2(0) = T_3(0) = ... = 0, \\ & T_0(1) = T_1(1) = T_2(1) = ... = 0, \\ u_0(0) = 1, \quad & u_1(0) = u_2(0) = u_3(0) = ... = 0, \\ & u_0(1) = u_1(1) = u_2(1) = ... = 0. \end{aligned} \tag{31}$$

Since the zeroth order of the Homotopy gives a linear ordinary differential equations, it is easily solvable without making recourse to initial guess. Therefore solving eqs. (23) and (24) and applying the boundary conditions $T_0(0) = 1$ and $T_0(1) = 0$, $u_0(0) = 1$ and $u_0(1) = 0$, we obtain eqs. (32) and (33) as

$$u_0 = A_1y + A_2, \tag{32}$$

$$T_0 = B_1y + B_2. \tag{33}$$

Solving eqs. (25) and (26) and applying the boundary conditions $T_1(0) = 0$ and $T_1(1) = 0$, $u_1(0) = 0$ and $u_1(1) = 0$, we obtain eqs. (34) and (35) as

$$u_1 = \frac{\lambda y^2}{2} + \lambda Gr\left[\frac{y^2}{2} - \frac{y^3}{3} + \frac{y^4}{12}\right] - Gr\left[\frac{y^2}{2} - \frac{y^3}{6}\right] + A_3y + A_4, \tag{34}$$

$$T_1 = S\left[\frac{y^2}{2} - \frac{y^3}{6}\right] + B_3y + B_4. \tag{35}$$

Solving eqs. (27) and (28) and applying the boundary condition $T_2(0) = 0$ and $T_2(1) = 0$, $u_2(0) = 0$ and $u_2(1) = 0$, we obtain eqs. (36) and (37) as

$$\begin{aligned} u_2 = {} & \lambda^2\left[\frac{y^2}{2}-\frac{y^3}{6}\right]+\lambda^2 Gr\left[\frac{y^2}{2}-\frac{y^3}{2}+\frac{y^4}{4}-\frac{y^5}{20}\right]-\lambda Gr\left[\frac{y^2}{2}-\frac{y^3}{3}+\frac{y^4}{12}\right] \\ & -\lambda S\left[\frac{y^3}{6}-\frac{y^4}{24}\right]+\frac{\lambda S y^2}{6}-\frac{\lambda^2 y^3}{6}-\lambda^2 Gr\left[\frac{y^3}{6}-\frac{y^4}{24}+\frac{y^5}{60}\right]-\frac{\lambda Gr y^2}{6} \\ & +\lambda Gr\left[\frac{y^3}{6}-\frac{y^4}{24}\right]+\frac{\lambda^2 y^2}{4}+\frac{\lambda^2 Gr y^2}{8}-GrS\left[\frac{y^4}{24}-\frac{y^5}{120}\right]+\frac{GrSy^3}{18} \\ & +2\lambda GrS\left[\frac{y^4}{24}-\frac{y^5}{30}+\frac{y^6}{180}\right]-\frac{2\lambda GrS}{3}\left[\frac{y^3}{6}-\frac{y^4}{12}\right]+A_5y+A_6, \end{aligned} \tag{36}$$

$$T_2 = S^2\left[\frac{y^4}{24}-\frac{y^5}{120}\right]-\frac{S^2y^3}{18}+B_5y+B_6. \tag{37}$$

Solving eqs. (29) and (30) and applying the boundary condition $T_2(0) = 0$ and $T_2(1) = 0$, $u_2(0) = 0$ and $u_2(1) = 0$, we obtain eqs. (38) and (39) as

$$\begin{aligned} u3 = {} & \lambda^3\left[\frac{y^2}{2}-\frac{y^3}{3}+\frac{y^4}{12}\right]+\lambda^3 Gr\left[\frac{y^2}{2}-\frac{2y^3}{3}+\frac{y^4}{2}-\frac{y^5}{5}+\frac{y^6}{30}\right]-\lambda^3\left[\frac{y^3}{6}-\frac{y^4}{12}\right] \\ & -\lambda^2 Gr\left[\frac{y^2}{2}-\frac{y^3}{2}+\frac{y^4}{4}-\frac{y^5}{20}\right]+\frac{\lambda^2 S}{3}\left[\frac{y^2}{2}-\frac{y^3}{6}\right]-\lambda^2 S\left[\frac{y^3}{6}-\frac{y^4}{8}+\frac{y^5}{40}\right] \\ & -\lambda^3 Gr\left[\frac{y^3}{6}-\frac{y^4}{6}+\frac{y^5}{15}-\frac{y^6}{90}\right]+\lambda^2 Gr\left[\frac{y^3}{6}-\frac{y^4}{8}+\frac{y^5}{40}\right]+\frac{\lambda^3}{2}\left[\frac{y^2}{2}-\frac{y^3}{6}\right] \\ & -\frac{\lambda^2 Gr}{3}\left[\frac{y^2}{2}-\frac{y^3}{6}\right]+\frac{\lambda^3 Gr}{4}\left[\frac{y^2}{2}-\frac{y^3}{6}\right]+\frac{\lambda S^2 y^2}{72}+\frac{\lambda GrS}{3}\left[\frac{y^3}{6}-\frac{y^4}{12}\right] \\ & -\lambda GrS\left[\frac{y^4}{24}-\frac{y^5}{30}+\frac{y^6}{180}\right]+2\lambda^2 GrS\left[\frac{y^4}{24}-\frac{7y^5}{120}+\frac{y^6}{36}-\frac{y^7}{252}\right]-\frac{\lambda S^2y^2}{90} \\ & -\frac{2\lambda^2 GrS}{3}\left[\frac{y^3}{6}-\frac{y^4}{6}+\frac{y^5}{20}\right]+\lambda^2 S\left[\frac{y^4}{24}-\frac{y^5}{120}\right]-\lambda GrS\left[\frac{y^4}{24}-\frac{y^5}{30}+\frac{y^6}{180}\right] \\ & +\lambda^2 GrS\left[\frac{y^4}{24}-\frac{7y^5}{120}+\frac{y^6}{36}-\frac{y^7}{252}\right]-\frac{\lambda^2 Sy^3}{18}-\frac{\lambda^2 GrS}{3}+\left[\frac{y^3}{6}-\frac{y^4}{6}+\frac{y^5}{20}\right] \\ & +\frac{\lambda GrS}{3}\left[\frac{y^3}{6}-\frac{y^4}{12}\right]-\lambda S^2\left[\frac{y^5}{120}-\frac{y^6}{720}\right]-\lambda^3\left[\frac{y^3}{6}-\frac{y^4}{24}\right]+\frac{\lambda^2 Gry^3}{18}-\frac{\lambda^3y^3}{12} \\ & -\lambda^3 Gr\left[\frac{y^3}{6}-\frac{y^4}{8}+\frac{y^5}{20}-\frac{y^6}{120}\right]+\lambda^2 S\left[\frac{y^4}{24}-\frac{y^5}{120}\right]+\lambda^2 Gr\left[\frac{y^3}{6}-\frac{y^4}{12}+\frac{y^5}{60}\right] \\ & +\lambda^3 Gr+\left[\frac{y^4}{24}-\frac{y^5}{60}+\frac{y^6}{360}\right]-\lambda^2 Gr\left[\frac{y^4}{24}-\frac{y^5}{120}\right]+\lambda GrS\left[\frac{y^5}{120}-\frac{y^6}{720}\right] \\ & +\frac{\lambda^3y^4}{24}-\frac{\lambda^3 Gry^3}{24}-\frac{\lambda^2 Sy^3}{18}-2\lambda^2 GrS\left[\frac{y^5}{120}-\frac{y^6}{180}+\frac{y^7}{1260}\right]-\frac{\lambda GrSy^4}{72} \\ & +\frac{2\lambda^2 GrS}{3}\left[\frac{y^4}{24}-\frac{y^5}{60}\right]+\frac{5\lambda^3y^2}{24}+\frac{9\lambda^3 Gry^2}{80}-\frac{7\lambda^2 Gry^2}{48}+\frac{\lambda^2 Sy^2}{48}+\frac{\lambda GrSy^2}{90} \\ & -\frac{\lambda^2 GrSy^2}{72}+\lambda^2 S\left[\frac{y^4}{12}-\frac{y^5}{40}\right]-\frac{\lambda^2 Sy^3}{18}+\lambda^2 GrS\left[\frac{y^4}{12}-\frac{3y^5}{40}+\frac{y^6}{36}-\frac{y^7}{252}\right] \end{aligned} \tag{38}$$

$$
-\frac{\lambda^2 GrS}{3}\left[\frac{y^3}{6}-\frac{y^4}{12}+\frac{y^5}{60}\right]-\lambda GrS\left[\frac{y^4}{12}-\frac{y^5}{20}+\frac{y^6}{120}\right]-\frac{\lambda^2 S}{2}\left[\frac{y^3}{6}-\frac{y^4}{24}\right]
+\frac{\lambda GrS}{3}\left[\frac{y^3}{3}-\frac{y^4}{12}\right]-\frac{\lambda GrSy^2}{18}+\frac{\lambda^2 Sy^2}{12}-\frac{\lambda^2 GrS}{4}\left[\frac{y^3}{6}-\frac{y^4}{24}\right]+\frac{\lambda^2 GrS}{24}
-GrS^2\left[\frac{y^6}{720}-\frac{y^7}{5040}\right]+\frac{GrS^2y^3}{360}-\frac{GrS^2y^3}{270}-\frac{\lambda GrS^2}{9}\left[\frac{y^5}{20}-\frac{y^6}{30}\right]-\frac{\lambda^2 Sy^3}{18}
+\lambda GrS^2\left[\frac{y^6}{120}-\frac{y^7}{252}+\frac{y^8}{2016}\right]+2\lambda GrS^2\left[\frac{y^6}{720}-\frac{y^7}{840}+\frac{y^8}{6720}\right]+\frac{\lambda GrS^2y^4}{108}
-\frac{2\lambda GrS^2}{3}\left[\frac{y^5}{40}-\frac{y^6}{180}\right]+A_7y+A_8,
$$

$$
T_3 = S^3\left[\frac{y^6}{720}-\frac{y^7}{5040}\right]-\frac{S^3y^5}{360}+\frac{S^3y^3}{270}+B_7y+B_8. \quad (39)
$$

Eqns (32) - (39) gives the expressions for the velocity and temperature as

$$
u = u_0 + u_1 + u_2 + u_3 + ..., \quad (40)
$$

$$
T = T_0 + T_1 + T_2 + T_3 + ... \quad (41)
$$

where,

$$
A_1 = B_1 = -1, \quad A_2 = B_2 = 1, \quad A_3 = \frac{Gr}{3}-\frac{\lambda}{2}-\frac{\lambda Gr}{4}, \quad B_3 = -\frac{S}{3},
$$
$$
A_4 = B_4 = A_6 = B_6 = A_8 = B_8 = 0,
$$
$$
A_5 = -\frac{5\lambda^2}{12}-\frac{9\lambda^2 Gr}{40}+\frac{7\lambda Gr}{24}-\frac{\lambda S}{24}-\frac{GrS}{45}+\frac{\lambda GrS}{36}, \quad B_5 = \frac{S^2}{45},
$$
$$
A_7 = -\frac{3\lambda^3}{8}-\frac{151\lambda^3 Gr}{720}+\frac{193\lambda^2 Gr}{720}-\frac{2\lambda^2 S}{45}-\frac{13\lambda GrS}{720}+\frac{1673\lambda^2 GrS}{70560}
+\frac{\lambda S^2}{240}+\frac{2GrS^2}{945}-\frac{\lambda GrS^2}{270}, \quad B_7 = -\frac{2S^3}{945}.
$$

To obtain the skin friction and rate of heat transfer at the surfaces of the channel boundaries, the expressions for velocity and temperature are differentiated with respect to y, that is $\tau = (1-\lambda T)\frac{du}{dy}|_{y=0,y=1}$ and $Nu = \frac{dT}{dy}|_{y=0,y=1}$ so that,

$$
\frac{du}{dy}|_{y=0} = -1 + A_3 + A_5,
$$

$$
\tau_0 = (1-\lambda T)\frac{du}{dy}|_{y=0}, \quad (42)
$$

$$
\frac{du}{dy}|_{y=1} = -1+\lambda-\frac{Gr}{2}+A_3+\frac{\lambda^2}{2}+\frac{\lambda^2 Gr}{12}+\frac{GrS}{24}-\frac{2\lambda GrS}{45}+A_5,
$$

$$
\tau_1 = (1-\lambda T)\frac{du}{dy}|_{y=1}, \quad (43)
$$

$$\frac{dT}{dy}|_{y=0} = -1 + B_3 + B_5, \tag{44}$$

$$\frac{dT}{dy}|_{y=1} = -1 + \frac{S}{2} + B_3 - \frac{S^2}{24} + B_5, \tag{45}$$

To obtain the mass flux Q, we have

$$Q = \frac{1}{2} + \frac{\lambda}{6} - \frac{\lambda Gr}{45} - \frac{Gr}{8} + \frac{A_3}{2} + \frac{\lambda^2}{6} + \frac{4\lambda^2 Gr}{45} + \frac{GrS}{144} - \frac{\lambda GrS}{840} + \frac{\lambda S}{45} + \frac{A_5}{2}, \tag{46}$$

and mean temperature θ_m, we have

$$\theta_m = \frac{\int_0^1 uT(y)dy}{\int_0^1 u(y)dy}, \tag{47}$$

3 Results and discussion

The present work analyses the effects of variable viscosity on natural convection flow of heat generating/absorbing fluid in a vertical channel using Homotopy perturbation method. The velocity and temperature fields are presented graphically in figures 2-5 for various values of Grashof number (Gr), heat generation/absorption parameter (S) and variable viscosity (λ). For the purpose of this discussion, the parameters of interest are carefully selected between $1 \leq Gr \leq 10$, $-2 \leq S \leq 2$ and $-1 \leq \lambda \leq 1$.
Figures 2 and 3 display temperature and velocity profiles for different values of heat generation/absorption parameter (S). It should be noted that positive values of S signifies heat absorption while negative values of S signifies heat generation. It is seen from the figures that as the heat generation (S ¡ 0) increases, fluid temperature and velocity increase while, fluid temperature and velocity decreases with increase in heat absorption (S ¿ 0). Increasing the heat generation parameter causes the fluid temperature to increase and it strengthens the convection current within the channel which in turn increases the fluid velocity. In addition, fluid temperature drop as a result of increasing the heat absorption parameter and the thermal boundary layer becomes thinner thereby reduces the velocity distribution of the fluid as shown in figure 3.
Figure 4 shows the influence of thermal buoyancy parameter (Gr) on the fluid velocity for fixed values of heat generation/absorption parameter (S) and variable viscosity parameter (λ). It is clear from this figure, the velocity profile increases with increases in the values of thermal buoyancy. Increasing the buoyancy parameter is made possible by decreasing the fluid viscosity which lead to thickening of the momentum boundary layer and hence an increase in velocity with growing Gr.
Figure 5 depict the effect of viscosity parameter (λ) on the velocity profile for fixed values of heat generation/absorption parameter (S) and Grashof number (Gr). It is seen from the figure that velocity decreases with the increase of the viscosity parameter and hence reduced resistance to flow.
The skin friction on both plate is simulated and presented in Table 1. From table 1 it is evident to shows that growing buoyancy parameter, heat generation as well as viscosity have tendency to increase the skin friction on both plates. However,

heat absorption contributes a decrease in the skin friction and this due to velocity decrease caused by increasing heat absorption which consequently leads to decrease in the skin friction on both plates.
Table 2 reveals the numerical values of rate of heat transfer on both plates. A general view of this table indicates that growing buoyancy parameter, heat generation as well as viscosity leads to a significant changes in the rate of heat transfer, this can be attributed to decrease on the heated plate while the opposite trend is observed on the cold plate. Furthermore, heat absorption leads to increase in the heat transfer on the heated plate.
Table 3 presents the mass flux Q within the channels. It is clearly seen that the mass flux increase with the increase in heat generation and decreases with increasing heat absorption. The table further shows that growing buoyancy parameter and viscosity leads to increase the mass flux.
Table 4 shows the numerical values of mean temperature θ_m. It is observed that with the increase in heat generation, mean temperature decreases and the reverse trend is observed in heat absorption.
To validate this problem, we compare our results obtained for temperature as well as velocity are in good agreement with those of Jha and Ajibade [12] as shown in table 5 which shows that the Homotopy perturbation method is an efficient tool for solving coupled and nonlinear system of differential equations.

4 Conclusion

In this paper we have studied the effect of variable viscosity and heat generating/absorbing fluid in a vertical channel, the work concluded that as the heat generation increases, fluid temperature and velocity increase while fluid temperature and velocity decreases with increase in heat absorption and also the velocity profile increases with increase in thermal buoyancy parameter. In addition, velocity decreases with the increase of the viscosity parameter.

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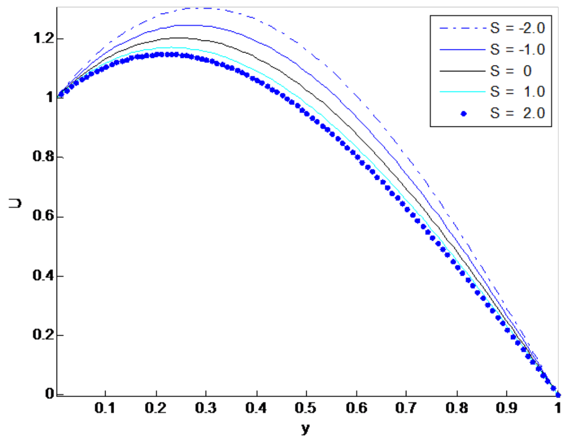

Figure 2: Velocity profile for different values of S $(Gr = 8.0, \lambda = -0.2)$

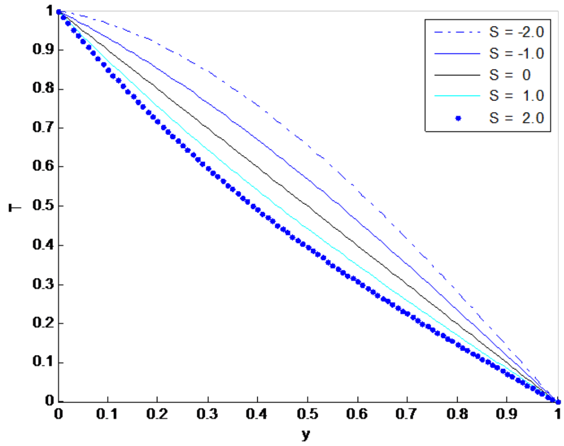

Figure 3: Temperature profile for different values of S $(Gr = 8.0, \lambda = -0.2)$

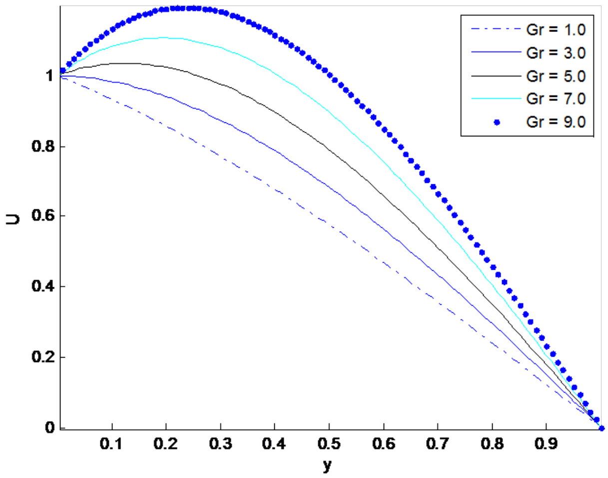

Figure 4: velocity profile for different values of Gr $(S = 2.0, \lambda = -0.2)$

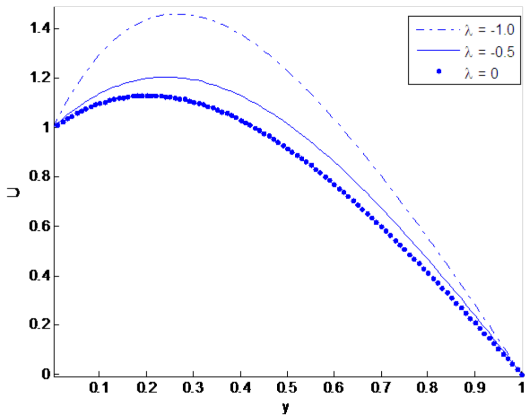

Figure 5: Velocity profile for different values of λ $(S = 2.0, Gr = 8.0)$

Table 1: Estimated numerical values of skin friction τ_0 and τ_1

	$Gr = 5.0, \lambda = -0.3$		$Gr = 8.0, \lambda = -0.1$	
S	τ_0	τ_1	τ_0	τ_1
-1	0.98240	2.23681	2.04270	2.60022
-0.5	0.89122	2.16944	1.93499	2.51369
0.5	0.70886	2.03472	1.71958	2.34064
1	0.61768	1.96736	1.61187	2.25411

Table 2: Estimated numerical values of rate of heat transfer Nu_0 and Nu_1

	$Gr = 5.0, \lambda = -0.3$		$Gr = 8.0, \lambda = -0.1$	
S	Nu_0	Nu_1	Nu_0	Nu_1
-1	0.64444	1.18611	0.64313	1.93121
-0.5	0.82778	1.08819	0.81146	1.96662
0.5	1.16111	0.92153	1.16010	1.99301
1	1.31111	0.85278	1.29000	2.03011

Table 3: Estimated numerical values of mass flux Q

	$Gr = 5.0, \lambda = -0.3$	$Gr = 8.0, \lambda = -0.1$
S	Q	Q
-1	0.64444	0.64513
-0.5	0.82778	0.83146
0.5	1.16111	1.18210
1	1.31111	1.41000

Table 4: Estimated numerical values of mean temperature θ_m

	$Gr = 5.0, \lambda = -0.3$	$Gr = 8.0, \lambda = -0.1$
S	θ_m	θ_m
-1	0.49416	0.40148
-0.5	0.55081	0.50380
0.5	0.67386	0.72479
1	0.74085	0.84436

Table 5: Comparison of numerical values between the present problem and of Jha and Ajibade (12)

	Jha and Ajibade (12) $Gr = 8.0, y = 0.5$		Present work $Gr = 8.0, \lambda = 0, y = 0.5$	
S	*Temperature*	*Velocity*	*Temperature*	*Velocity*
-1	0.56974696	1.05797571	0.56967230	1.05737847
-0.5	0.53296476	1.02743612	0.53289252	1.02736545
0.5	0.47029886	0.97521826	0.47029486	0.97528212
1	0.44340944	0.95272446	0.44348524	0.95321181