Data Article

MODELLING THE MEAN WAITING TIMES FOR QUEUES IN SELECTED BANKS IN ELDORET TOWN-KENYA

Abstract

The mathematical study of waiting lines is mainly concerned with queue performance measures where several applications have been drawn in past studies. Among the vast uses and applications of the theory of queuing system in banking halls, is the main focus of this study where the theory has been used to solve the problem of long queues as witnessed in banks leads to resource waste. The study aims to model the waiting times for queues in selected banks within Eldoret town, Kenya. The latter component was put under D/D/1 framework and therein its mean derived while the stochastic component was put under the M/M/c framework. Harmonization of the moments of the deterministic and the stochastic components was done to come up with the mean of the overall bank queue traffic delay. The simulation was performed using MATLAB for traffic intensities ranging from 0.1 to 1.9. The results reveal that both deterministic and the stochastic delay components are compatible in modelling waiting time. The models also are applicable to real-time bank queue data whereupon simulation, both models depict fairly equal waiting times for server utilisation factors below 1 and an infinitely increasing delay at rho greater than 1. In conclusion, the models that estimate waiting time were developed and applied on real bank queue data. The models need to be implemented by the banks in their systems so that customers are in a position to know the expected waiting time to be served as soon as they get the ticket from the ticket dispenser.

Keywords: D/D/1, M/M/c, Utilization factor, Simulation.

1.0: Introduction

Waiting is one of the most unpleasant experiences in life. Queuing theory deals with delays and queues which are essentials in determining the levels of service in banking halls (Agbola & Salawu, 2008, Kimber, R. and Hollis, 1979). They also evaluate the adequacy of service channels and the economic losses that come about as a result of long waiting lines. Quantifying these delays accurately and appropriately in banks, critically use for planning design and analysis of teller services. Tellers referred to herein are the personnel in the bank and will be represented as servers or service channels (Agbola & Odunukwe, 2013; Bakari, 2014; Beckmann, 1956). In modern banking, queuing has been automated such that customers arrive and pick ticket numbers from a ticket dispensing machine (Tarko et al., 1993b; Teply et al., 1995). Electronic quality management systems were implemented for purposes of instilling order and eliminating or easing/reducing congestion in banks. Bishop et al. (2018) stated that the gains expected from this survey are to help review the efficiency of the models used by banks in such geographical locations in sub-Saharan countries as well as estimate the average waiting time and length of the queue(s).

Models incorporate both the deterministic and stochastic components of queue performance are very appealing in modelling bank queues since they are applied in a wide range of traffic intensities as well as to various types of teller services (Darroch, 1964; Erlang,1909; Gazis, 1974; Kendal,1953). They simplify theoretical models with delay terms that are numerically inconsequential. Of the various queueing models, D/D/1 and M/M /c were used in this study. The D/D/1 model assumed that the arrivals and departures were uniform and one service channel (teller) existed (Okagbue et al., 2017; Janos & Eger, 2010). This model is quite intuitive and easily solvable. Using this form of queueing with an arrival rate, denoted by λ and a service rate, indoicated by μ, certain useful values regarding the consequences of queues were computed (Lindley, 1952; Little, 1961). The M/M /c model used implied that the customers arrived at an intersection in a Poisson process with rate λ and were treated in the order of arrival with inter-arrival times following exponential distribution with parameter μ. The service times were treated as independent identically distributed with an arbitrary distribution. Similarly, several service channels (tellers) were considered in this model (Liping and Bruce, 1999; McNeil, 1968). The study aims to model the waiting times for queues in selected banks within Eldoret town, Kenya.

2.0: Modelling Waiting Times

The Mean of Deterministic Delay Model

To analyse the mean, it is assumed that customer entry and exit are uniformly distributed with rates λ and μ respectively.

To obtain the mean waiting time for the D/D/1 model, we note the following notations.

c_y– Cycle time (min).

g_e –Effective service time.

g_0 – Time necessary for the queue to dissipate.

r – Effective waiting time on the queue before service.

$D(t)$ - Cumulative departures.

λ – Arrival rate.

$A(t)$ – Cumulative arrivals.

ρ - Utilization factor

W_{t_1} – Deterministic queue delay component.

π_w – Probability of waiting on the queue.

P_0 – Steady state probability of having no customers in the system.

Such that the duration of C_y in the bank is given by

$$C_y = r + g_e \qquad 1$$

$$W_{t_1} = \frac{\lambda r^2}{2\left(1 - \frac{g_e}{C_y}\rho\right)} \qquad 2$$

Finally the expected deterministic delay in the bank queue is obtained by dividing W_{t_1} by the total number of customers in a cycle that is λC_y to yield

$$E(W_{t_1}) = \frac{C_y\left(1 - \frac{g_e}{C_y}\right)^2}{2\left(1 - \frac{g_e}{C_y}\rho\right)} \qquad 3$$

as the mean of the deterministic component, W_{t_1}.

Mean of Stochastic Delay Component

To obtain the mean of the stochastic delay component we also note the following notations,

We begin with the expected waiting time while on service is given by

$$W_s = {}^{1}/_{\mu} \qquad 4$$

Then proceed to the waiting time on the queue which is obtained as follows

$$E(t) = \int_0^{\infty} t.\pi_w c\mu(1-\rho)e^{-c\mu(1-\rho)t}\ dt \qquad 5$$

$$= \frac{\pi_w c\mu(1-\rho)}{[c\mu(1-\rho)^2]}\int_0^{\infty} ye^{-y}\,dy$$

$$\text{Thus}\quad E(t) = \frac{\pi_w}{c\mu(1-\rho)} = W_q$$

$$\therefore E(W_{t_2}) = {}^{1}/_{\mu} + \frac{\pi_w}{c\mu(1-\rho)}$$

Mean of the overall delay model

To obtain the mean of the overall delay model we sum up the expected waiting times for both stochastic and deterministic delay model.

$$E(W_t) = \frac{C_y\left(1 - \frac{g_e}{C_y}\right)^2}{2\left(1 - \frac{g_e}{C_y}\rho\right)} + {}^{1}/_{\mu} + \frac{\pi_w}{c\mu(1-\rho)} \qquad 9$$

3.0: Results

The overall traffic delay model was applied to real bank queue data collected at the various banks in Eldoret town between 1st August and 5th August 2016. The intermediate results from the data are given and simulation on the developed models using MATLAB software for traffic intensities ranging from 0.1 to 1.9.

Computation of Parameters

The average effective deterministic service time is

$$g_e = \frac{1}{5}\left(\frac{440}{6} + \frac{437}{6} + \frac{430}{6} + \frac{426}{6} + \frac{413}{6}\right)$$

$$= 68.23 \text{ sec}$$

The average arrival rate is

$$\lambda = \frac{\text{Total arrivals}}{\text{Total number of hours observed}}$$

$$= \frac{2146}{30}$$

$$= 71.5333 \text{ Customers per hour}$$

The average service rate is

$$\mu = \frac{\text{Total Departures}}{\text{Total number of hours observed}}$$

$$= \frac{2092}{30}$$

$$= 69.7333 \text{ Customers per hour}$$

The utilisation factor (probability that a server is busy) is

$$\rho = \frac{\text{Average arrival rate}}{\text{number of servers} * \text{Average service rate}}$$

$$= \frac{71.5333}{3 * 69.7333}$$

$$= 0.3419$$

The probability that a server is idle is

$$P_0 = \left\{1 + \frac{\left(\lambda/\mu\right)^1}{1!} + \frac{\left(\lambda/\mu\right)^2}{2!} + \cdots + \frac{\left(\lambda/\mu\right)^{c-1}}{(c-1)!} + \frac{\left(\lambda/\mu\right)^c}{c!}\left[1 + \left(\lambda/c\mu\right) + \left(\lambda/c\mu\right)^2 + \cdots\right]\right\}^{-1}$$

$$= \left\{1 + 1.0258 + \frac{(1.0258)^2}{2!} + \frac{(1.0258)^3}{3!\,(1 - 0.3419)}\right\}^{-1}$$

$$= (2.8253)^{-1}$$

$$= 0.3539$$

97 For two servers (c=2)

98 The utilization factor (probability that a sever is busy) is

$$\rho = \frac{\text{Average arrival rate}}{\text{number of servers} * \text{Average service rate}}$$

$$= \frac{71.5333}{2 * 69.7333}$$

$$= 0.5129$$

99 The probability that a server is idle is

$$P_0 = \left\{1 + \frac{\left(\lambda/\mu\right)^1}{1!} + \frac{\left(\lambda/\mu\right)^2}{2!} + \cdots + \frac{\left(\lambda/\mu\right)^{c-1}}{(c-1)!} + \frac{\left(\lambda/\mu\right)^c}{c!}\left[1 + \left(\lambda/c\mu\right) + \left(\lambda/c\mu\right)^2 + \cdots\right]\right\}^{-1}$$

$$= \left\{1 + 1.0258 + \frac{(1.0258)^2}{2!\,(1 - 0.5129)}\right\}^{-1}$$

$$= (1 + 1.0258 + 1.0801)^{-1}$$

$$= (3.1059)^{-1}$$

$$= 0.3219$$

4.0 Discussion and conclusion

4.0.1 Discussion

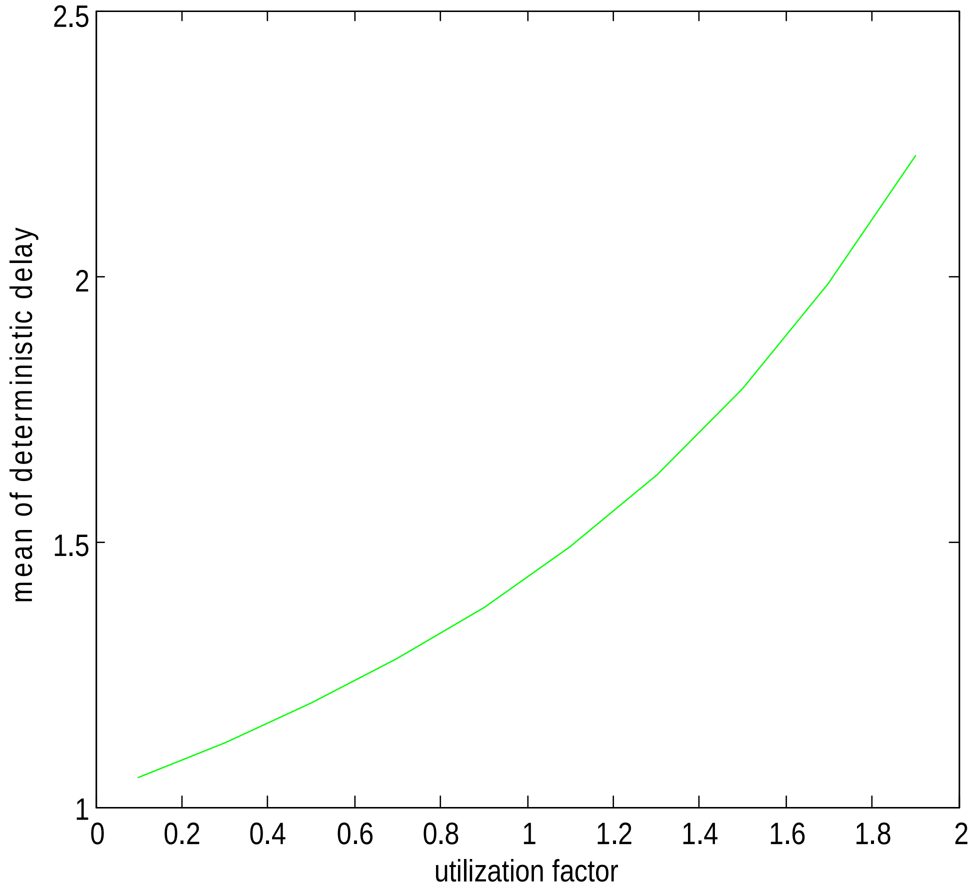

Figure 1 Diagram representing simulation of deterministic component $\mathbf{E}\left[\mathbf{W}_{t_1}\right]$ verses $\boldsymbol{\rho}$

From figure 1 , it is revelaed that the deterministic delay model analysed a continuous delay but does not accommodate the aspect of randomness when the arrival flows are close to capacity$\rho < 1$. The model reveals a steady increase in mean delay with a more increase in waiting when the flows approach capacity $\rho > 1$ which consequently implies infinite delays, in the long run, queuing of customers.

Simulation of $\mathbf{E(W_{t_2})}$

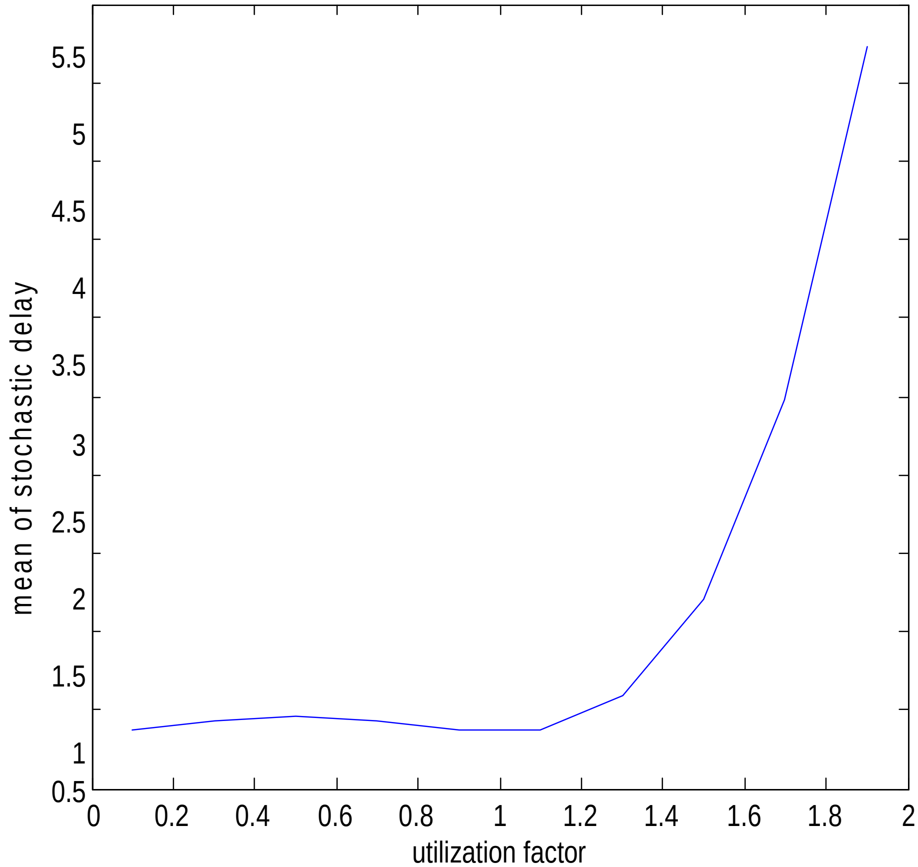

Figure 2 Diagram representing the simulation of stochastic component $\mathbf{E[W_{t_2}]}$ **verses $\boldsymbol{\rho}$** with two servers

From figure 2, the stochastic delay model with two servers is also applicable to under saturated conditions $\rho < 1$ and estimates delays tending to infinity when the arrival flow approaches capacity $\rho > 1$. However, comparing the delay with the three server model, it implies an increased delay which is quite natural due to decreased service channels (Wayne, 2003; Wenny and Whitney, 2004).

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Simulation of $\mathbf{E(W_t)}$

We split $E(W_t)$ into EW_{t_1} and EW_{t_2} as described in figure 7 by MATLAB software when service times and inter-arrival times follow exponential distributions with parameters $\frac{1}{\mu}$ and $\frac{1}{\lambda}$ respectively.

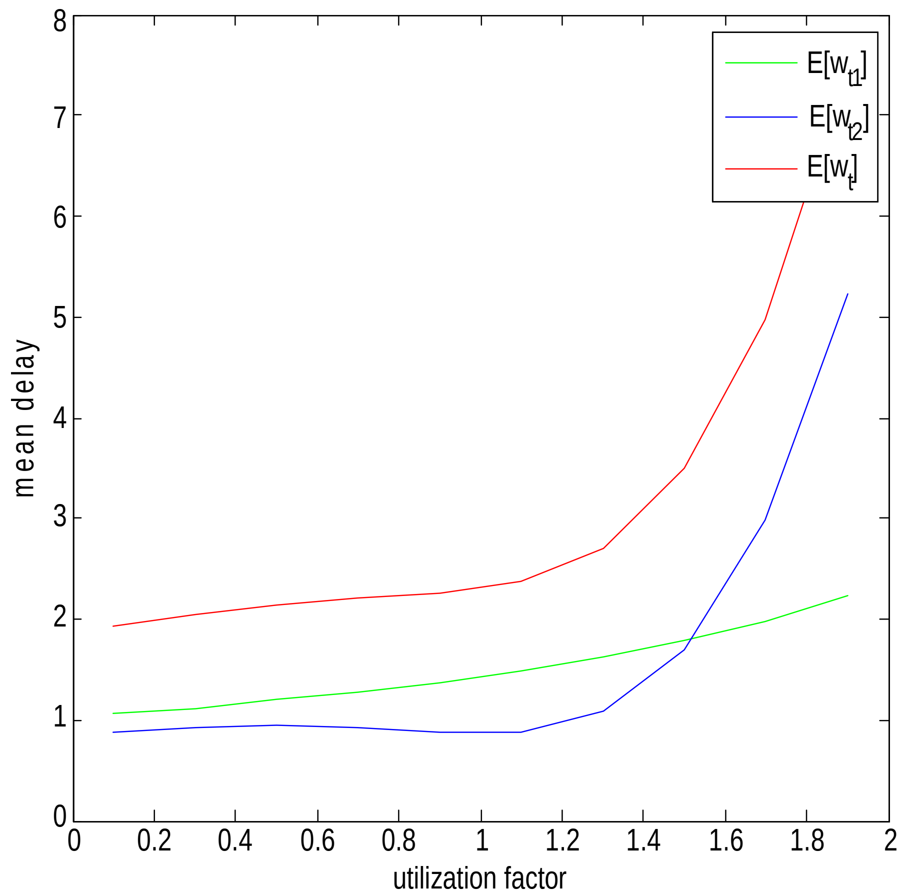

Figure 3 Diagram representing the simulation of overall model
$\mathbf{E[W_t]\ E[W_{t_1}]\ E[W_{t_2}]}$ **verses $\boldsymbol{\rho}$** with two servers

From figure 3 it is revealed that the stochastic delay model is only applicable to under saturated conditions $\rho < 1$ and estimates infinite delay when the arrival flow approaches capacity. However, when arrival flows exceed capacity, oversaturated queues exist and continuous delays occur. The deterministic delay model also depicts that it estimates a continuous delay which is definitely higher than that of a three sever queue but it does not completely deal with the effect of randomness when the arrival flows are close to capacity (Toshiba et al., 2013).

The figure shows that both components of the overall delay model are compatible when the utilisation factor is equal to 1.0. Therefore the overall delay model is used to bridge the gap between the two models. It is important to also note that ultimately the overall model also indicates of an increased waiting time which is explained by the reduced number of servers and also provides a more realistic point of view for the results in the estimation of delays in the bank queue delays for the oversaturated as well as the under saturated conditions is predicted without having any discontinuity (Yusuf, 2013; Zukerman, 2012).

4.0.2: Conclusion

The uniform and random properties of queues in banks was considered for estimating deterministic and stochastic delay components of bank queue delays successfully modelled waiting times in selected banks in Eldoret town. From the mean waiting time models of stochastic and deterministic delays, the models are conveniently applicable to real-time bank queue data. To validate the mean waiting time models, the model was applied to real bank queue data collected from the various selected banks namely; Kenya Commercial bank, Equity Bank, National Bank, Barclays Bank and Cooperative Bank for data between Monday 1st to Friday 5th August 2016 respectively and simulation was performed for utilization factors ranging from 0.1 to 1.9 using MATLAB software simulink functions. The simulation results show that when a queue system is not at equilibrium, it indicates continuous delays past the equilibrium point i.e. $\rho > 1$.

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