1 **Modeling Heteroscedasticity in the Presence of Serial Correlations in Discrete-**
2 **Time Stochastic Series: A GARCH-in-Mean Approach**

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5 **ABSTRACT**

6 **Background:** In modeling heteroscedasticity of returns, it is often assumed that the series are
7 uncorrelated. In practice, such series with small time periods between observations can be observed to
8 contain significant serial correlations, hence the motivation for this research.
9 **Aim:** The aim of this research is to investigate the existence of serial correlations in the return series of
10 Zenith Bank Plc, which is targeted at identifying their effects on the parameter estimates of
11 heteroscedastic models.
12 **Material and Methods:** The data were obtained from the Nigerian Stock Exchange spanning from
13 January 3, 2006 to November 24, 2016 having 2690 observations. The joint Autoregressive Integrated
14 Moving Average-Generalized Autoregressive Conditional Heteroscedaticity (ARIMA-GARCH-type)
15 models such as Autoregressive Integrated Moving Average-Generalized Autoregressive Conditional
16 Heteroscedasticity (ARIMA-GARCH), Autoregressive Integrated Moving Average-Exponential
17 Generalized Autoregressive Conditional Heteroscedasticity (ARIMA-EGARCH) and the Autoregressive
18 Integrated Moving Average-Glosten, Jagannathan and Runkle Generalized Autoregressive Conditional
19 Heteroscedastic (ARIMA-GJRGARCH) under normal and student-t distributions were employed to model
20 the conditional variance while the GARCH-in-Mean-GARCH-type model corresponding to the selected
21 ARIMA-GARCH-type model was applied to appraise the possible existence of serial correlations.
22 **Results:** The findings of this study showed that heteroscedasticity exists and appeared to be adequately
23 captured by ARIMA(2,1,1)-EGARCH(1,1) model under student-t distribution but failed to account for the
24 presence of serial correlations in the series. Meanwhile, its counterpart, GARCH-in-Mean-EGARCH(1,1)
25 model under student-t distribution sufficiently appraised the existence of serial correlations.
26 **Conclusion:** One remarkable implication is that, the estimates of the parameters of ARIMA-GARCH-type
27 model are likely to be biased when the presence of serial correlations is ignored. Also, the application of
28 GARCH-in-Mean-GARCH-type model possibly provides the feedback mechanism or interaction between
29 the variance and mean equations.
30 **Keywords:** GARCH-type models, Heteroscedasticity, Time Series, Volatility

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35 **1. INTRODUCTION**

36 The existence of heteroscedasticity in financial series (returns) always leads to the violation of

37 assumption of constant variance in linear time series. The linkage between the occurrence of

38 heteroscedasticity in financial data and the violation of assumption of constant variance in linear time

39 series has created a vast research area for professionals in Statistics, Economics and Finance. As

40 required naturally, the assumption of constant variance assumes that the error term of the linear

41 stationary model should be homogeneous. By implication, the constant error variance means that the

42 conditional variance of the dependent variable is also constant. According to [1], the assumption of

43 constant variance is required to ensure the accuracy of standard errors and asymptotic covariances

44 amongst estimated parameters. It could be remarked that a major setback on linear stationary models

45 when applying to financial data (returns) is their failure to account for changing variance. In other words,

46 whenever the assumption of constant variance is violated, heteroscedasticity has occurred, implying that

47 the conditional distribution of the dependent variable has different degrees of variability at different levels.

48   In the Statistical context, heteroscedasticity (i.e. non-constant variance) means the same thing as

49 volatility in Finance and Economics, although they are generally used interchangeably by some authors.

50 However, neglecting the presence of heteroscedasticity in linear models makes the ordinary least

51 squares estimates of ARIMA parameters inefficient. Although they are still consistent and asymptotically

52 normally distributed, their variance-covariance matrix is no longer the usual one. As a result of this, the t-

53 statistics become invalid and cannot be used to examine the significance of the individual explanatory

54 variables in the model. Also, over-parameterization of an ARIMA model and low statistical power are

55 identified as part of the consequences for neglecting heteroscedasticity. Lastly, neglecting

56 heteroscedasticity can lead to spurious nonlineality in the conditional mean and difficulty in computing the

57 confidence interval for forecasts (see [2], [3], [4], [5]). Furthermore, details of heteroscedasticity modeling

58 are documented in [6], [7], [8], [9], [10], [11], [12], [13].

59   Certainly, the motivation for this study is drawn from the fact that serial correlations (a relationship

60 between a variable and its lagged-value over a period of time) tend to exist in most financial series though

61 several analyses on such series are often based on the assumption that the series are uncorrelated.

62 Moreover, these serial correlations are believed to be introduced by those in the time-varying

63 heteroscedasticity process [14]. However, failure to account for these serial correlations when modeling

64 heteroscedasticity would amount to obtaining a biased estimate of the true degree of persistence (see

65 also [15]). To capture these high variations over time with regards to risk and volatility, [16] proposed the

66 modification of standard (generalized autoregressive conditional heteroscedastic) GARCH-type model

67 under the assumption that the variance coefficient in the mean equation measures the relative risk

68 aversion. Also, according to [17], the increasing roles played by the risk and uncertainty in financial

69 assets have led to the development of new time series techniques for measuring time variations. One of

70 such techniques is the GARCH-in-Mean (GARCH-M) model. It allows the conditional variance of the

71 series to influence the conditional mean. Also, the formulation of the GARCH-M model implies that there

72 are serial correlations in the series and are being introduced by those in the heteroscedasticity process.

73 This particular specification is useful and effective in modeling the risk-return relationship in financial

74 series. The major advantage of GARCH-M model over the standard GARCH-type models is that any

75 misspecification of variance function would not affect the consistency of the estimators of parameters of

76 the mean. Meanwhile, prior studies of [18], [17], [19], [20], [21], [22], [23] and [24]) have applied GARCH-

77 M technique to capture varying property of risk aversion and autocorrelation of return series as well as

78 interaction between the mean and variance equations of GARCH-type models. Particularly, this study

79 seeks to improve on the work of [25] that detected and modeled the asymmetric GARCH effects using

80 GARCH-type models under the assumption that the return series is uncorrelated. This is captured by

81 applying GARCH-M technique to ascertain the presence of serial correlation in the return series

82 considered.

83   The study is further organized as follows: materials and methods are presented in section 2,

84 discussion of results is handled in section 3 while section 4 concludes the study.

85 **2. MATERIAL AND METHODS**
86 **2.1 Returns**

87 The return series, R_t, can be obtained given that P_t is the price of a unit share at time t, and P_{t-1} is the

88 share price at time t−1. Thus,

89 $R_t = \nabla ln P_t = (1 - B)ln P_t = ln P_t - ln P_{t-1}$ (1)

90 Here, R_t is regarded as a transformed series of the share price (P_t) meant to attain stationarity, that is,

91 both mean and variance of the series are stable [25] while B is the backshift operator.

92 **2.2   Autoregressive Integrated Moving Average (ARIMA) Model**

93 [26] considered the extension of ARMA model to deal with homogenous non-stationary time series in

94 which X_t, itself is non-stationary but its dth difference is a stationary ARMA model. Denoting the dth

95 difference of X_t by

96 $\varphi(B) = \phi(B)\nabla^d X_t = \theta(B)\varepsilon_t$ (2)

97   where $\varphi(B)$ is the nonstationary autoregressive operator such that d of the roots of $\varphi(B) = 0$ are unity

98   and the remainder lie outside the unit circle while $\phi(B)$ is a stationary autoregressive operator.

2.3   Standard GARCH-type Models

100   Conceptually, heteroscedastic models are hybridized of both mean and variance equations. The

101   mean equation is represented by the ARIMA Model as shown in equation (3),

102      $R_t = \mu_t + a_t,$ (3)

103   where $\mu_t = \varphi_0 + \sum_{j=1}^{p} \varphi_j R_{t-j} + \sum_{i=1}^{q} \theta_i\, a_{t-i}.$

104   Also,

105      $a_t = \sigma_t e_t,$ (4)

106   where e_t is a sequence of independent and identically distributed (i.i.d.) random variables with zero

107   mean,i.e.$E(e_t) = 0$ and variance 1. In practice, e_t is often assumed to follow the standard normal or a

108   standardized student-t distribution while a_t is the standardized residual term that follows autoregressive

109   conditional heteroscedastic (ARCH(q)), generalized autoregressive conditional heteroscedastic (GARCH

110   (q, p)), exponential generalized autoregressive conditional heteroscedastic (EGARCH(q,p)) and glosten,

111   jagannathan and runkle generalized autoregressive conditional heteroscedastic (GJR-GARCH(q,p))

112   models in (5), (6), (7) and (8), respectively.

2.3.1   ARCH model

114   The first model that provides a systematic framework for modeling volatility is the ARCH model of

115   [27]. Specifically, an ARCH (q) model assumes that,

116      $\sigma_t^2 = \omega + \alpha_1 a_{t-1}^2 + \cdots + \alpha_q a_{t-q}^2,$ (5)

117   where $\omega > 0, and \alpha_1,\dots,\alpha_q \geq 0$ [28]. The coefficients α_i, for $i > 0$, must satisfy some regularity conditions

118   to ensure that the unconditional variance of $a_t$is finite. From the structure of the model, it is seen that

119   large squares of past shocks,$\{a_{t-i}^2\}_{i=1}^{q}$, imply a large conditional variance,σ_t^2, for the innovation,a_t.

120   Consequently, a_t tends to assume a large value (in modulus). This means that, under the ARCH

121   framework, large shocks tend to be followed by another large shock.

2.3.2   GARCH model

123   Although the ARCH model is simple, it often requires many parameters to adequately describe

124   the volatility process of a share price return. As a functional alternative,[29] proposed a useful extension

125   known as the generalized ARCH (GARCH) model. The GARCH (q, p) is defined as;

126    $\sigma_t^2 = \omega + \sum_{i=1}^{q} \alpha_i a_{t-i}^{'2} + \sum_{j=1}^{p} \beta_j \sigma_{t-j}^2$ (6)

127   where $\omega > 0,\ \alpha_i \geq 0,\ \beta_j \geq 0,\ and\ \sum_{i=1}^{max(p,q)} (\alpha_i + \beta_i) < 1$ (Tsay, 2010).

128   Here, it is understood that $\alpha_i = 0,\ for\ i > p$, and $\beta_i = 0$, for $i > q$. The later constraint on $\alpha_i + \beta_i$ implies

129   that the unconditional variance of $a_t^{'}$ is finite, whereas its conditional variance,σ_t^2, evolves over time. In

130   most cases, estimates of the GARCH (1,1) model on returns yield $\alpha_1 + \beta_1 \approx 1$, and this results in an

131   explosive process, that is, the volatility process is not mean-reverting. So, the conditional variance is

132   nearly integrated (Integrated GARCH model) [14].

133   **2.3.3   EGARCH model**

134   The Exponential GARCH (EGARCH) model represents a major shift from ARCH and GARCH

135   models [30]. Rather than model the variance directly, EGARCH models the natural logarithm of the

136   variance, and so no parameter restrictions are required to ensure that the conditional variance is positive.

137   The EGARCH (q, p) is defined as,

138    $ln\sigma_t^2 = \omega + \sum_{k=1}^{r} \gamma_k a_{t-k} + \sum_{i=1}^{q} \alpha_i \left(|a_{t-i}| - \sqrt{2/\pi}\right) + \sum_{j=1}^{p} \beta_j ln\sigma_{t-j}^2$ (7)

139   Alternatively, EGARCH(q, p) model with respect to student-t distribution can be represented by

140    $ln\sigma_t^2 = \omega + \sum_{k=1}^{r} \gamma_k a_{t-k} + \sum_{i=1}^{q} \alpha_i \left(|a_{t-i}| - \frac{2\sqrt{v-2}\Gamma(v+1)/2}{(v-1)\Gamma(v/2)\sqrt{\pi}}\right) + \sum_{j=1}^{p} \beta_j ln\sigma_{t-j}^2$ (8)

141   ,

142   where γ_k is the asymmetric coefficient. In the original parameterization of Nelson (1991), p and r were

143   assumed to be equal. The process is covariance stationary if and only if $\sum_{j=1}^{q} \beta_j < 1$. The γ_i parameter

144   thus signifies the leverage effect of a_{t-i}. Again, we expect γ_i to be negative in real applications [14].

145   **2.3.4   GJR-GARCH model**

146   The GJR GARCH (q, p) model [31] is a variant, represented by

147    $\sigma_t^2 = \omega + \sum_{i=1}^{q} \alpha_i a_{t-i}^2 + \sum_{i=1}^{p} \gamma_i I_{t-i} a_{t-i}^2 + \sum_{j=1}^{p} \beta_j \sigma_{t-j}^2$ (9)

148   Or written as

149    $\sigma_t^2 = \omega + \sum_{i=1}^{q}(\alpha_i + \gamma_i I_{t-i})a_{t-i}^2 + \sum_{j=1}^{p}\beta_j\sigma_{t-j}^2$ (10)

150   where I_{t-1} is an indicator for negative a_{t-i}, that is,

151    $I_{t-1} = \begin{cases} 0 & if \ a_{t-i} < 0, \\ 1 & if \ a_{t-i} \geq 0, \end{cases}$

152   and α_i, γ_i, and β_j are nonnegative parameters satisfying conditions similar to those of GARCH models.

153   Also the introduction of indicator parameter of leverage effect, I_{t-1} in the model accommodates the

154   leverage effect, since it is supposed that the effect of a_{t-i}^2 on the conditional variance σ_t^2 is different

155   accordingly to the sign of a_{t-i}. From the model, it is obvious that a positive a_{t-i} contributes $\alpha_i a_{t-i}^2$ to σ_t^2,

156   whereas a negative a_{t-i} has a larger impacts $(\alpha_i + \gamma_i)a_{t-i}^2$ with $\gamma_i > 0$ as established by (Tsay, 2010). The

157   model uses zero as it threshold to separate the impacts of past shocks (see,[28], [14]).

158   **2.4   GARCH-in-Mean Model**

159   The mean equation (3) is modified to obtain GARCH-in-mean model in (11) such that the return

160   series depends on its variance. The specification of GARCH-in-mean model implies that there are serial

161   correlations in the return series (see [14]).

162    $R_t = \mu_t + \tau\sigma_t^2 + a_t,$ (11)

163   where the parameter τ is the variance functional coefficient. Thus the presence of variance functional

164   coefficient σ_t^2, indicates that the return series has serial correlation, which implies that the return series is

165   related to its variance.

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169   **3   RESULTS AND DISCUSSION**.

170   **3.1   Data**

171   Data collection was based on secondary source as documented in the records of Nigerian Stock

172   Exchange. The data on daily closing share prices of sampled bank (Zenith Bank) from January 3,

173   2006 to November 24, 2016 were obtained through contactcentre@nigerianstockexchange.com.

174   <mark>Since the data were obtained from a credible and secured source hence reliable. The data</mark>

175   <mark>analyses were implemented using Gretl 1.10.1 [32] and Rugarch 1.4-1 [33].</mark>

176   **<mark>3.2</mark>**   **Interpretation of Time Plot**

177   The share price series of the Nigerian bank considered was found to be nonstationary given the

178 random fluctuations away from the common mean (see Figure 1).

179

180 **Figure 1: Share Price Series of Zenith Bank**

181   Stationarity was achieved by transforming the share price series using equation (1) and the

182 transformed series was found to cluster round the common mean and thus indicated the presence of

183 heteroscedasticity (see Figure 2).

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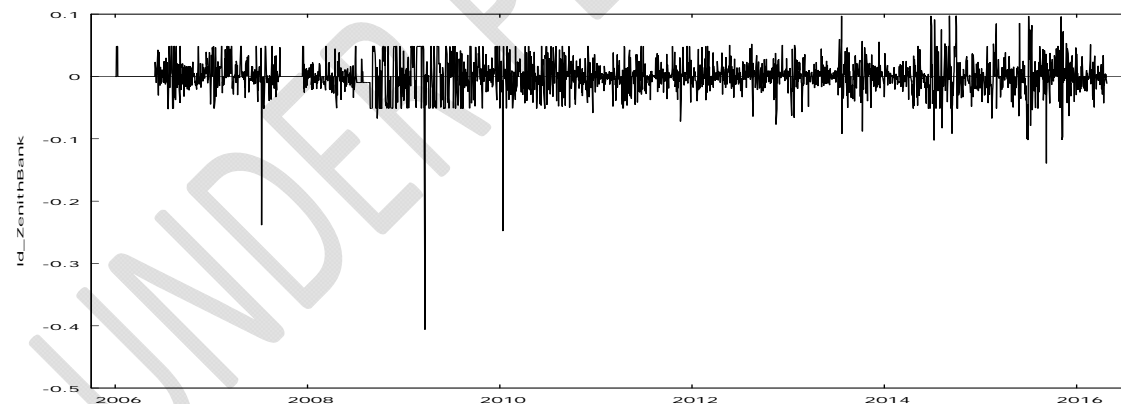185 **Figure 2: Return Series of Zenith Bank**

186 **<mark>3.3</mark>**   **Modeling Joint ARIMA-GARCH-type Processes of Return Series of Zenith Bank**

187   Based on Box and Jenkins procedures, out of the several models identified tentatively, the

188 following joint ARIMA-GARCH-type models with respect to both normal (norm) and student-t (std)

189 distributions were considered (see Table 1).

190

191 **Table 1: Output of ARIMA-GARCH-type Models of Return Series of Zenith Bank**

| Model | Parameter | Estimate | s.e | t-ratio | p-value | Information Criteria | | |
						AIC	BIC	HQIC
ARIMA(2,1,1)-GARCH (1,0)-std	μ	$-1.38e^{-4}$	$1.2e^{-4}$	-1.1518	0.2494			
	φ_1	-1.0182	0.0094	-108.3242	0.0000			
	φ_2	-0.0828	0.0211	-3.9297	0.0001			
	θ_1	0.9268	0.0197	47.0444	0.0000			
	ω	$6.4e^{-5}$	$6.0e^{-6}$	10.7403	0.0000			
	α_1	0.9990	0.1339	7.4598	0.0000	-6.4622	-6.4469	-6.4567
ARIMA(2,1,1)-GARCH (2,0)-norm	μ	$5.11e^{-4}$	$1.85e^{-4}$	2.7590	0.0058			
	φ_1	0.8695	0.0208	41.7691	0.0000			
	φ_2	0.1140	0.0202	5.6563	0.0000			
	θ_1	-0.9529	0.0022	-442.0869	0.0000			
	ω	$5.1e^{-5}$	$2.0e^{-6}$	20.9116	0.0000			
	α_1	0.4918	0.0463	10.6297	0.0000			
	α_2	0.2357	0.0314	7.5012	0.0000	-6.3503	-6.3350	-6.3448
ARIMA(2,1,1)-GARCH (2,0)-std	μ	$-2.48e^{-4}$	$2.3e^{-4}$	-1.0367	0.29987			
	φ_1	0.8644	0.0212	40.8256	0.0000			
	φ_2	0.1193	0.0211	5.6551	0.0000			
	θ_1	-0.9722	0.0011	-851.2935	0.0000			
	ω	$4.0e^{-5}$	$4.0e^{-6}$	11.2777	0.0000			
	α_1	0.6418	0.0754	8.5143	0.0000			
	α_2	0.3572	0.0553	6.4607	0.0000	-6.5041	-6.4866	-6.4978
ARIMA(2,1,1)-GARCH (1,1)-norm	μ	$7.4e^{-5}$	$1.4e^{-5}$	5.4230	0.0000			
	φ_1	0.1655	0.3888	0.4256	0.6704			
	φ_2	$5.9e^{-5}$	0.0386	0.0015	0.9988			
	θ_1	-0.2458	0.3883	-0.6330	0.52670			
	ω	$2.0e^{-6}$	0.0000	11.6607	0.0000			
	α_1	0.1753	0.0125	13.9806	0.0000			
	β_1	0.8237	0.0092	89.6875	0.0000	-6.4261	-6.4108	-6.4206
ARIMA(2,1,1)-GARCH(1,1)-std	μ	0.0000	0.0000	-0.0035	0.9972			
	φ_1	-0.1192	0.9558	-0.1247	0.9007			
	φ_2	0.0011	0.0860	0.0129	0.9897			
	θ_1	0.0139	0.9550	0.0145	0.9884			
	ω	0.0000	0.0000	0.0000	1.0000			
	α_1	0.2646	0.0101	26.3116	0.0000			
	β_1	0.7252	0.0055	131.5636	0.0000	-7.0699	-7.0524	-7.0635
ARIMA(2,1,1)-EGARCH(1,1)-norm	μ	$2.22e^{-4}$	$9.5e^{-5}$	2.33943	0.0193			
	φ_1	-0.0068	0.0139	-0.4929	0.6221			
	φ_2	-0.0061	0.0276	-0.21981	0.8260			
	θ_1	-0.0719	0.0226	-3.18238	0.0015			
	ω	-0.6502	0.0034	-191.7116	0.0000			
	α_1	-0.0040	0.0160	-0.2489	0.8034			
	β_1	0.9260	$1.9e^{-4}$	4871.3044	0.0000			
	γ_1	0.3794	0.0207	18.3618	0.0000	-6.4624	-6.4448	-6.4560
ARIMA(2,1,1)-**EGARCH(1,1)-std**	**μ**	**0.0000**	**0.0000**	**1.2065**	**0.2276**			
	φ_1	**-0.3086**	**0.0106**	**-29.0801**	**0.0000**			
	φ_2	**0.0495**	**0.0171**	**2.8972**	**0.0038**			
	θ_1	**0.2912**	**0.0105**	**27.7875**	**0.0000**			
	ω	**-0.0291**	**$8.62e^{-4}$**	**-33.7888**	**0.0000**	**-7.0189**	**-6.9992**	**-7.0118**

Model	Parameter	Estimate	Std. Error	t-value	p-value	AIC	BIC	HQ
	α_1	**−0.6979**	**9.8e^{-5}**	**−7150.4179**	**0.0000**			
	β_1	**0.9996**	**6.3e^{-5}**	**15825.7924**	**0.0000**			
	γ_1	**0.6983**	**9.8e^{-5}**	**7147.6333**	**0.0000**			
ARIMA(2,1,1)-GJR-GARCH(1,0)-norm	μ	−4.1e^{-4}	2.0e^{-6}	−260.0268	0.0000			
	φ_1	1.7704	0.0049	363.9136	0.0000			
	φ_2	−1.2088	0.0019	−642.61109	0.6056			
	θ_1	0.7873	6.41e^{-4}	1228.7631	0.0000			
	ω	0.0000	1.0e^{-6}	0.0952	0.92418			
	α_1	0.961136	0.0029	328.0539	0.0000			
	β_1	0.8486	0.0064	131.9359	0.0000			
	γ_1	0.0754	0.0364	2.072042	0.0383	1.6422	1.6576	1.6478
ARIMA(2,1,1)-GJR-GARCH(1,0)-std	μ	−4.32e^{-4}	2.57e^{-4}	−1.677346	0.0935			
	φ_1	0.8733	0.0216	40.3732	0.0000			
	φ_2	0.1086	0.0217	5.0124	1.0e^{-6}			
	θ_1	−0.9684	0.0012	−810.3464	0.0000			
	ω	6.3e^{-5}	6.0e^{-6}	10.9327	0.0000			
	α_1	0.993393	0.1474	6.7378	0.0000			
	γ_1	0.0112	0.1536	0.0730	0.9418	−6.4675	−6.4499	−6.4611
ARIMA(2,1,1)-GJR-GARCH(2,0)-norm	μ	−1.21e^{-4}	9.9e^{-5}	−1.2223	0.2216			
	φ_1	0.8713	0.0233	37.3883	0.0000			
	φ_2	0.1115	0.0234	4.7763	0.0000			
	θ_1	−0.9526	0.0014	−659.3565	0.0000			
	ω	5.0e^{-5}	2.0e^{-6}	20.8695	0.0000			
	α_1	0.3549	0.0468	7.5802	0.0000			
	α_2	0.1918	0.0383	5.0032	1.0e^{-6}			
	γ_1	0.3147	0.0845	3.7230	0.0002			
	γ_2	0.0804	0.0561	1.4328	0.1519	−6.3556	−6.3359	−6.3485
ARIMA(2,1,1)-GJR-GARCH(2,0)-std	μ	−3.46e^{-4}	2.52e^{-4}	−1.3744	0.1693			
	φ_1	0.8722	0.0091	96.194	0.0000			
	φ_2	0.1181	0.0090	13.111	0.0000			
	θ_1	−0.9811	9.5e^{-5}	−1.0310	0.0000			
	ω	4.0e^{-5}	4.0e^{-6}	11.367	0.0000			
	α_1	0.6411	0.0911	7.0376	0.0000			
	α_2	0.2869	0.0614	4.6756	3.0e^{-6}			
	γ_1	−0.0047	0.1105	−0.0042	0.9663			
	γ_2	0.1467	0.0906	1.6205	0.1051	−6.5037	−6.4817	−6.4957
ARIMA(2,1,1)-GJR-GARCH(1,1)-norm	μ	7.7e^{-5}	1.4e^{-5}	5.6257	0.0000			
	φ_1	0.1732	0.1767	0.9802	0.3270			
	φ_2	−1.13e^{-4}	0.0342	−0.0033	0.9974			
	θ_1	−0.2540	0.1921	−1.3223	0.1861			
	ω	2.0e^{-6}	0.0000	4.1180	3.8e^{-5}			
	α_1	0.1775	0.0330	5.3738	0.0000			
	β_1	0.8243	0.0239	34.5612	0.0000			
	γ_1	−0.0056	0.0370	−0.1528	0.8785	−6.4254	−6.4079	−6.4191
ARIMA(2,1,1)-GJR-GARCH(1,1)-std	μ	0.0000	0.0000	0.306023	0.75959			
	φ_1	−0.0417	0.4469	−0.0933	0.9257			
	φ_2	−0.0029	0.0578	−0.0509	0.9594			
	θ_1	−0.0813	0.4390	−0.1851	0.8531			
	ω	0.0000	0.0000	0.0000	1.0000			
	α_1	0.2737	0.0186	14.7192	0.0000			
	β_1	0.7013	0.0067	104.9602	0.0000			
	γ_1	0.0367	0.0295	1.2459	0.2128	−7.0480	−7.0282	−7.0408

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Comparing the values of the information criteria of the models as indicated in Table 1, it is shown that the information criteria for ARIMA(2,1,1)-GARCH(1,1)-std model is the smallest, followed by ARIMA(2,1,1)-GJR-GARCH(1,1)-std mode, although they are characterized by several non-significant parameters. However, ARIMA(2,1,1)-EGARCH(1,1)-std model, which is next to ARIMA(2,1,1)-GJR-GARCH(1,1)-std model has all its parameters significant except the constant term of the mean equation, which assumes the value of zero. Hence, ARIMA(2,1,1)-EGARCH(1,1)-std model is selected as the appropriate heteroscedastic model for the return series of the Bank.

Table 2: Diagnostic Checking for ARIMA-GARCH-type Model of Return Series of Zenith Bank

Model	Standardized Residuals			Standardized Squared Residuals					
	Lag	Weighted LB	p-value	Lag	Weighted LB	p-value	Lag	Weighted ARCH-LM	p-value
ARIMA(2,1,1)-EGARCH(1,1)-std	1	0.0014	0.9697	1	0.0014	0.9704	3	0.0014	0.9704
	8	0.0066	1.0000	5	0.0041	1.0000	5	0.0033	0.9999
	14	0.0111	1.0000	9	0.0069	1.0000	7	0.0049	1.0000

202

The model was found to be adequate given that the p-values corresponding to weighted Ljung-Box Q statistics at lags 1, 8 and 14 on standardized residuals, weighted Ljung-Box Q statistics at lags 1, 5 and 9 on standardized squared residuals and weighted Lagrange Multiplier statistics at lags 3, 5 and 7 are all greater than 5% level of significance [see Table 2]. That is to say, the hypotheses of no autocorrelation and no remaining ARCH effect are not rejected.

3.4 Modeling GARCH-in-Mean-EGARCH Processes of the Return Series of Zenith Bank

Table 3: Output of GARCH-in-Mean-EGARCH Model of Return Series of Zenith Bank

Model	Parameter	Estimate	s.e	t-ratio	p-value	Information Criteria		
						AIC	BIC	HQIC
GARCH-in-Mean-EGARCH(1,1)-std	μ	0.0000	0.0000	0.15021	0.8806			
	φ_1	0.6845	0.0040	170.0493	0.0000			
	φ_2	0.0428	0.0038	11.3871	0.0000			
	θ_1	-0.7089	0.0041	-170.894	0.0000			
	τ	0.0428	0.0043	9.8773	0.0000			
	ω	-0.1279	0.0029	-43.9733	0.0000			
	α_1	-0.6616	$2.72e^{-4}$	-2436.7359	0.0000			
	β_1	0.9904	$3.8e^{-5}$	26149.3241	0.0000			
	γ_1	0.6632	$2.72e^{-4}$	2433.8259	0.0000	-6.9012	-6.8793	-6.8933

211

All the parameters of joint GARCH-in-Mean-EGARCH(1,1)-std model are significant at 5% level of significance except the constant term of the GARCH-in-Mean equation which assumes the value of zero.

214 Of particular interest is the GARCH-in-Mean coefficient whose significance points to the presence of

215 serial correlation in the return series (see Table 3).

216 **Table 4: Diagnostic Checking for GARCH-in-Mean-EGARCH Models of Return**
217 **Series of Zenith Bank**

Model	Standardized Residuals			Standardized Squared Residuals					
	Lag	Weighted LB	p-value	Lag	Weighted LB	p-value	Lag	Weighted ARCH-LM	p-value
GARCH-in-Mean-EGARCH(1,1)-std	1	0.0008	0.978	1	0.0009	0.9757	3	0.0009	0.9757
	8	0.0059	1.0000	5	0.0028	1.0000	5	0.0022	0.9999
	14	0.0099	1.0000	9	0.0046	1.0000	7	0.0033	1.0000

218

219 The model was found to be adequate given that the p-values corresponding to weighted Ljung-

220 Box Q statistics at lags 1, 8 and 14 on standardized residuals, weighted Ljung-Box Q statistics at lags 1, 5

221 and 9 on standardized squared residuals and weighted Lagrange Multiplier statistics at lags 3, 5 and 7

222 are all greater than 5% level of significance [see Table 4]. That is to say, the hypotheses of no

223 autocorrelation and no remaining ARCH effect are not rejected.

224 **3.5 Effects of Serial Correlation on Parameters of ARIMA-GARCH-type Model**

225 **Table 5: Biased Effects of Serial Correlations on the Parameters of**
226 **ARIMA(2,1,1)-EGARCH(1,1)-std Model of Zenith Bank**

Parameter	ARIMA(2,1,1)-EGARCH(1,1)-std Model fitted to Returns Series of Zenith Bank	GARCH-in-Mean-EGARCH(1,1)-std Model fitted to Returns Series of Zenith Bank	Biases introduced
Constant Term (μ)	0.0000	0.0000	0.0000
Autoregressive of order 1 Coefficient (φ_1)	0.6094	0.6845	-0.0751
Autoregressive of order 2 Coefficient (φ_2)	0.0852	0.0428	0.0424
Moving Average of order 1 Coefficient (θ_1)	-0.6012	-0.7089	0.1077
Garch-in-Mean Coefficient (τ)	0.0428	-	-
Constant Term (ω)	-0.0960	-0.1279	0.0319
ARCH Coefficient (α)	-0.8581	-0.6616	-0.1965
GARCH Coefficient(β)	0.9903	0.9904	-0.0001
Asymmetric Coefficient (γ)	0.8591	0.6632	0.1959

227

228 Substantial biases are being introduced into the parameters of the ARIMA(2,1,1)-EGARCH(1,1)-

229 std model when the possible existence of serial correlation is ignored as indicated in Table 5. That is, in

230 the presence of serial correlations, the Autoregressive of order 1, ARCH and GARCH parameters were

231 reduced by 0.0751, 0.1965 and 0.0001, respectively while Autoregressive of order 2 Coefficient, Moving

232 Average of order 1 Coefficient, Constant term of the variance equation and asymmetric parameters were

233   hyped by 0.0424, 0.1077, 0.0319 and 0.1959, respectively. Hence, it can be deduced that the presence of

234   serial correlations, the parameters of ARIMA-GARCH-type models are biased.

235   In brief, the findings of this study showed that serial correlations exist in the return series of the

236   bank understudy. Thus building an ARIMA(2,1,1)-EGARCH(1,1)-std model without accounting for the

237   existence of serial correlations results in biased parameters as indicated in Table 5.Consequently, the

238   extent of bias associated with the existence of serial correlation was appraised by GARCH-in-Mean-

239   EGARCH(1,1)-std model as shown in Table 3.

240   Although this study showed similarity to the work of (25) by confirming that EGARCH model is

241   suitable to the return series of Zenith bank Plc, yet, it provides enough evidence of substantial

242   improvement by modifying the mean equation of the model to account for the presence of serial

243   correlations. In addition, the introduction of variance parameter in the mean equation creates a feedback

244   mechanism between heteroscedasticity and returns.

245   By implication, the study revealed that the return is positively related to its variance, which implies

246   that any high increase in conditional variance would likely lead to a high increase in the returns.

247   **4   CONCLUSION**

248   In summary, the findings of this very study revealed that the standard Joint ARIMA- GARCH-type

249   model is not sufficient for capturing serial correlations and their application without considering the

250   existence of serial correlations often results in biased parameters. Consequently, the GARCH-in-Mean-

251   GARCH-type model provided the much needed modification that accounts for the existence of serial

252   correlations in return series. Therefore, the formulation of GARCH-in-Mean equation by incorporating

253   variance component ensures that the risk-return relationship is properly depicted. It is recommended that

254   the similar formulation be undertaken by replacing the variance component with the standard deviation or

255   probably the natural logarithm of the variance in future studies.

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