

A CHINESE REMAINDER THEOREM BASED ENHANCEMENTS OF LEMPEL-ZIV-WELCH AND HUFFMAN CODING IMAGE COMPRESSION

ABSTRACT

Data size minimization is the focus of data compression procedures by altering representation and reducing redundancy of information to a more effective kind. In general, lossless approach is favoured by a number of compression methods for the purpose of maintaining the content integrity of the file afterwards. The benefits of compression include saving storage space, speed up of data transmission and high quality of data. This paper observes the effectiveness of Chinese Remainder Theorem (CRT) enhancement in the implementation of Lempel-Ziv-Welch (LZW) and Huffman coding algorithms for the purpose of compressing large size images. Ten images of Yale database was used for testing. The outcomes revealed that CRT-LZW compression saved more space and speedy compression (or redundancy removal) of original images to CRT-Huffman coding by 29.78% to 14.00% respectively. In terms of compression time, CRT-LZW approach outperformed CRT-Huffman approach by 9.95 sec. to 19.15 sec. For compression ratio, CRT-LZW also outperformed CRT-Huffman coding by 0.39db to 4.38 db, which is connected to low quality and imperceptibility of the former. Similarly, CRT-Huffman coding (28.13db) offered better quality Peak-Signal-to-Noise-Ratio (PSNR) for the reconstructed images when compared to CRT-LZW (3.54db) and (25.59db) obtained in other investigated paper.

Keywords: LZW, Huffman coding, CRT, compression time, size reduction, image, compression.

1. INTRODUCTION

Data compression is the method of decreasing the size of information to be transmitted or stored by the process of eliminating redundancy in information without the loss or the ability to reconstruct the original data. There are several file formats that can be effectively compressed including text, image, video, and audio [1]. Image compression addresses the difficulty involved in decreasing the volume of data vital in denoting an image with no major loss of information. In recent times, Chinese Remainder Theorem (CRT) has developed in fields of coding theory, phase unwrapping, frequency estimation and distributed data storage. This is certainly due to the fact; CRT is profound for isolating errors in residue caused by noise. The traditional CRT reconstructs a single integer for error-free co-prime and residues [2]. Images have limited applications in real life situations such as medical, scientific, prepress and artistic applications, due to enormous sizes for broadcast or storage given low bandwidth [3]. In dealing with memory capacity insufficiency, compression schemes have been deployed; thereby offering the prospect of broadcasting images/video under scarce bandwidth. The classical image compression scheme converts a spatial domain representation to frequency domain [3].

In classical JPEG codec, images are encoded independently. The surge in cloud data storage has thrown up issues of content duplication and large redundancy, which must be considered. Inter-coding is one of such solution for traditional video encoding of consecutive frames from previous frames. Another method is the inter-prediction tools of video codecs to encode comparable images as pseudo-video arrangements [4].

Again, lossy compression algorithms such as Huffman coding gives relatively good quality as well as compression rates with images but blocky look of reconstructed images [5]. The reverse is the case of LZW in which compressed image data quality is retained at the expense of little size decreases [6]. In this paper, CRT enhancement is proposed for independent implementation of Huffman coding and LZW algorithm for image compression procedures.

2. RELATED STUDIES

47 The goal of image compression in variety of application is to decrease the quantity of bytes in a
48 graphic file but retaining the quality. In the study, [3] considered several approaches for compressing
49 images especially in medicine. The forms of compression for image involve spatial to frequency
50 domains. The main concept is that pictures are composed of neighbouring pixels; though related not
51 without repetitive data. However, colour images require treatment of distinct colour segments.

52 A novel prediction scheme for performing cloud-based compression was initiated by [4]. This
53 approach utilizes the semi-local geometric and photometric prediction scheme to compensate in a
54 region-wise style the distortion between two images rather than inter-coding schemes (such as high
55 efficiency video coding). This is most useful for highly correlated image content applications such as
56 traditional video coding, cloud gaming streaming, photo albums compression. This minimizes the
57 redundancy arising from similar content already available in cloud. Nevertheless, cloud multiple
58 frames exploitations, and determining scalability of cloud-based image compression system.

59 Lossless compression schemes of LZW and Huffman coding were combined by [6]. The target was to
60 enable medical imagery suitable for storage, quality retention and broadcast. Huffman coding offered
61 massive size decreases in a speedy manner, but, poor quality of compressed image. Conversely,
62 LZW algorithm produced finest quality with little size decreases. The combined compression schemes
63 gave rise to high compression ratio and high PSNR values.

64 A fresh algorithm was proposed by [7] for encoding, decoding or regenerating the replica of the
65 encoded data. The first step uses the forward difference scheme on Huffman. Then, the values are
66 regenerated into fixed length code representation with twos complement for further new probabilities
67 computation along Huffman's algorithm. There are improvements in compression factors for the new
68 algorithm against the traditional Huffman encoding. Residue Number System is introduced in data
69 encryption and decryption with Shannon Fano compression scheme by [16]. The outcomes revealed
70 significant improvements in the security, speed and memory needed for existing information
71 communication networks.

72 Typically, multimedia files (such as image data) are composed of redundancy and irrelevance limiting
73 their usage on widespread basis. Since the advent of internetworks and communication infrastructure,
74 there are possibilities of broadcasting or storing digital images seamlessly[4]. The sizes of the
75 multimedia data make them inefficient for broadcast or storagepurpose [6].Majority of lossless
76 compression methods are founded on probability or dictionary and entropy because theymake use of
77 the availability of the identicalstring or character within data in order to realize compression [6].
78 Researchers are focusing attention on removing redundancy and irrelevance in image data, which
79 gave rise to the concept of data compression schemes such as Huffman coding and Lempel-Ziv-
80 Welch algorithm [6]. In general, the performance of compression schemes is estimated with standard
81 metrics such as effectiveness (compression ratio) and efficiency (speedup or throughput)[7].In this
82 paper, these compression algorithms are highlighted in certain details.

83 One common entropy encoding algorithm deployed for lossless image compression is Huffman
84 coding[8]. Theencoding strategy commences with calculation of each symbol probability in the image.
85 Thereafter, these symbols probabilities are placed in adescending magnitude as to createleaf nodes
86 of a tree. By individually coding the symbols, the Huffman code is built combining the lowermost
87 probable symbols. These entire stepsare continued until only two probabilities of two compound
88 symbols are present. Eventually, a code tree is produced and the labelling of the code treegenerates
89 the Huffman code [9].

90 The Huffman codes for the symbols are realised by analysing the branch digits in succession from the
91 root node to the respective terminal node or leafusing symbols 0 and 1. Huffman coding is the most
92 deployed method for redundancy or relevanceminimization[8]. The operational principle of Huffman
93 code is based on these observations:

- 94 a) The more frequently occurring symbols areassigned shorter code words than less frequent
- 95 occurring ones;
- 96 b) The two symbols occurring least frequently is assigned the similar length.

97 On the average, code length is determined as the average of the product of symbol probability and
98 amount of encoding bits [10][11].The Huffman code efficiency is calculated as the ratio of entropy
99 tothe average length.The target of Huffman encoding creates the optimal code for a collection of
100 symbols and probabilities whenever, the symbols are coded currently within the same time frame [8].
101

102 LZW algorithm is a popular lossless data compression scheme initiated by Abraham Lempel, Jacob
 103 Ziv, and Terry Welch. In 1984, as an improvement over the traditional LZ78 algorithm released 1978
 104 by Lempel and Ziv, which is easy to deploy with the prospect of offering significantly high throughput
 105 in hardware applications. According [12], the algorithm encodes sequences of 8-bit data as fixed-
 106 length 12-bit codes. The codes from 0 to 255 depict 1-character sequences composed of the
 107 matching 8-bit character, and the codes 256 through 4095 are created in a dictionary for sequences
 108 contained in the data during the process of encoding [1] [12] [13].

109

110 2.1 The Concept of Chinese Remainder Theorem (CRT)

111

112 The basic operation of Chinese Remainder Theorem (CRT) is to generate a single integer through its
 113 residue modulo within moduli set [2]. CRT is an alternative to the Mixed Radix Conversion (MRC) in
 114 which large modulo M derivations are unnecessary. MRC accepts a low complexity of $O(n)$ unlike the
 115 CRT having computation complexity of order $O(n^3)$. In CRT, arithmetic operations for modulo M are to
 116 be manually executed. CRT residue converters are much more complex. In contrast, the MRC
 117 procedure requires arithmetic operations for modulo m_i only, thereby simplifying all operations. In MRC
 118 method, a number x is expressed in mixed-radix system. Suppose for moduli set (m_1, m_2, \dots, m_n)
 119 ,RNS representation of a number x is given by (x_1, x_2, \dots, x_n) . The number x can be expressed in
 120 mixed-radix form as:

121 $|x|_{m_1} = a_1$

122
$$X = a_n \prod_{i=1}^{n-1} m_i + \dots + a_3 m_1 m_2 + a_2 m_1 + a_1 \quad (1)$$

123 where, the a_i 's are the mixed-radix coefficients. These a_i 's are determined sequentially, starting with
 124 a_1 , in the following manner:

125 Equation (1) is first taken in modulo m_1 . Since, all terms except the last are multiples of m_1 to give

126 $|x|_{m_1} = a_1$

127 Hereafter, a_1 is just the first residue digit. To obtain a_2 , first subtract a_1 from x . The quantity $x - a_1$ is
 128 divided by m_1 , and doing modulo operation with respect to m_2 , we have

129
$$\left| \frac{x - a_1}{m_1} \right|_{m_2} = a_2$$

130 Similarly, for a_3 , $(a_2 m_1 + a_1)$ is subtracted from x . By dividing the quantity $(x - a_2 m_1 - a_1)$ by $m_1 m_2$ and
 131 performing modulo operation with respect to m_3 , we get

132
$$\left| \frac{x - a_2 m_1 - a_1}{m_1 m_2} \right|_{m_3} = a_3$$

133 In this way, by successive subtraction and division in residue notation, all the mixed-radix digits may
 134 be obtained.

135 Conversely, an RNS number $(x_1, x_2, x_3, \dots, x_k)$ for the moduli set $(m_1, m_2, m_3, \dots, m_k)$ whose the
 136 decimal equivalent is given by:

137
$$a_1 = x_1 \quad (3)$$

138
$$a_2 = \left| (x_2 - a_1) m_1^{-1} \right|_{m_2} \quad (4)$$

139
$$a_3 = \left| \left((x_3 - a_1) \right) \left| m_1^{-1} \right|_{m_3} - a_2 \right) \left| m_2^{-1} \right|_{m_3} \left| m_3 \right|_{m_3} \right| \quad (5)$$

140 Therefore, a general expression is given by:

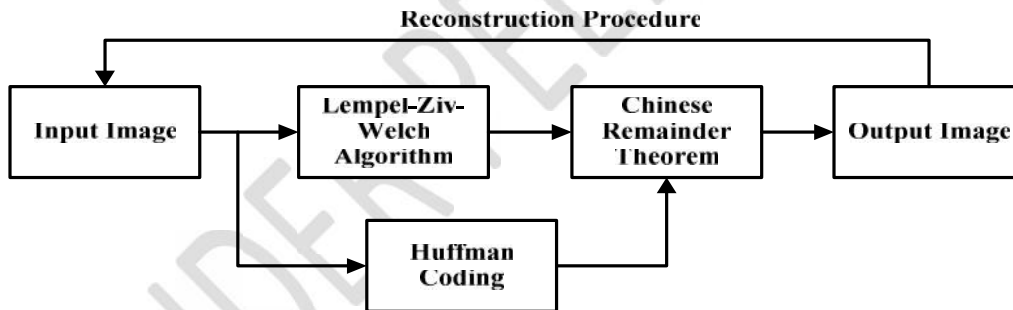
141
$$a_n = \left| \left(\left((x_n - a_1) \right) \left| m_1^{-1} \right|_{m_n} - a_2 \right) \left| m_2^{-1} \right|_{m_n} - \dots - a_{n-1} \right) \left| m_{n-1}^{-1} \right|_{m_n} \left| m_n \right|_{m_n} \right| \quad (6)$$

142 The mixed radix digit (MRD) of a_i , $0 \leq a_i < m_i$, any positive number in the interval by $[0, \prod_{i=1}^k m_i - 1)$ can
 143 be uniquely represented. The major advantage of MRC, as can be seen from Equation (5) above is
 144 that the calculation of a_i ; $i = 1, k$ can be done only using arithmetic mod- m_i contrasting CRT, which
 145 entails arithmetic mod- M , M being the system dynamic range, a rather large constant. It can be noted
 146 that equations (5) and (6) are directly utilized, only if the moduli set $\{m_1 m_2 m_3 \dots m_k\}$ are relatively prime
 147 and that Euclidean algorithm is the common way to verify this, i.e., if $\text{gcd}(m_i, m_j) = 1$, for $i \neq j$.

148 The residue independence, carry-free operation and parallelism attributes of the RNS have been
 149 intensively used in variety of areas, such as digital signal processing (DSP), digital filtering, digital
 150 communications, cryptography, error detection and correction[14] [15].The addition, subtraction and
 151 multiplication are dominant. And, division, comparison, overflow and sign detection are negligible.
 152 One key field of RNS-based applications is finite impulse response (FIR) filters. Likewise, digital image
 153 processing benefits from the RNS's features such as enhancing digital image processing applications
 154 [15].

155 **3. METHODOLOGY**

156 In this image compression process, the implementation process was coded from scratch using
 157 MATLAB R2015a. The paper studied the performances of traditional compression schemes of LZW
 158 and Huffman coding with CRT. The purpose of the employing CRT is to enhance their individual
 159 effectiveness using image media lossless compression technique. The arrangement of the
 160 planned enhancements of compression approaches is illustrated in Figure1.



161

162 **Fig. 1. The layout for the enhancement of compression schemes.**

163 The input image is used to acquire the various formats of images for the complete data compression
 164 processes. These input images are composed of diverse degree of redundancy which is expected to
 165 be removed or minimized during planned compression processes. The data compression phase
 166 encompasses three distinct operations; firstly, the input image is received at Lempel-Ziv-Welch
 167 Algorithm block to commence the data compression. Similarly, the Huffman coding performs
 168 preliminary compression operation on the input images. Secondly, the complete compression of
 169 original image is achieved with CRT using the outcomes of LZW and Huffman coding schemes.
 170 Finally, the output image is realized from the last compression process of CRT computation, which
 171 enhanced the traditional image compressed format when compared to input images.

172 This paper considered four metrics in evaluating the effectiveness of the planned image compression
 173 schemes including:

- 174 1) *Compression Ratio*(CR) is expressed as the amount of uncompressed data size divided by
 175 compressed data size. This provides the relative size of compressed image data.
 176 2) *Compression Time*(CT) calculates the time taken to compress bits in data in a second.

- 177 3) *PeakSignal-to-Noise Ratio*(PSNR) is used to estimate the amount of noise in the signals of
 178 compressed data relative to original data.
 179 4) *Imperceptibility* calculates the rate of bits distributions of image data after complete
 180 compression. This infers on the appearance of compressed image data.

181 **The operational algorithm of proposed LZW-CRT image compression scheme is presented**
 182 **below**

- 183 **Step 1-** Extract first byte from input STRING.
 184 **Step 2-** Extract the next byte from input CHARACTER.
 185 **Step 3-**Lookup in table for the STRING and CHARACTER stored up.
 186 **Step 4-**Generate code for the STRING and update the lookup table.
 187 **Step 5-**Output STRING same as CHARACTER.
 188 **Step 6-**STRING = STRING and CHARACTER.
 189 **Step 7-**Apply CRT on the resulting STRING.
 190 **Step 8-**The moduli set is chosen to obtain the best redundancy in data.
 191 **Step 9-** The compressed image data is attained as final encoded values.
 192 **Step 10-**The reconstructed image is obtained by applying decoding of LZW and CRT.
 193 **Step 11-** Output is reconstructed image data.

194 Similarly, the steps for performing CRT-Huffman coding image compression scheme is presented in
 195 algorithm 2 below;

- 196
 197 **Step 1-** INPUT original image
 198 **Step 2-** Run Huffman coding functions
 199 **Step 3-**Extract symbols of the pixels from input IMAGE.
 200 **Step 4-**Create the probability of pixel symbols and organize in decreasing magnitude and smaller
 201 probabilities are combined.
 202 **Step 5-**Concatenate the Huffman codeword ready for CRT
 203 **Step 6-**Generate code for the STRING and update the lookup table.
 204 **Step 7-**Apply CRT on the resulting STRING.
 205 **Step 8-**The moduli set is chosen to obtain the best redundancy in data.
 206 **Step 9-** The compressed image data is attained as final encoded values.
 207 **Step 10-**The reconstructed image is obtained by applying decoding of LZW and CRT
 208 **Step 11-** Output is reconstructed image data.

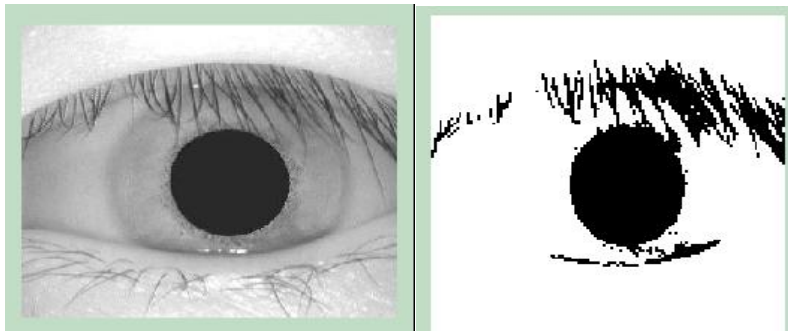
209 210 **4. PRESENTATION OF RESULTS**

211 The paper utilized 10 different image samples from Yale database for the purpose of validating the
 212 proposed concepts of CRT-LZW and CRT-Huffman coding compression. The outcomes of
 213 compression procedures on the sampled images using CRT and LZWare shown in Table 1.

214 **Table 1. CRT-LZW based image compression**

Image sample	Size before compression	Size after compression	Compression time	PSNR	Compression ratio
1	12282	3524	10.10	3.11	0.38
2	12906	3571	8.77	3.17	0.40
3	12353	3522	12.28	3.21	0.39
4	12762	3410	10.18	3.22	0.36
5	12872	3606	9.99	3.31	0.38
6	12357	3548	8.61	3.13	0.38
7	12150	4266	9.08	4.09	0.41
8	12243	3889	10.45	3.99	0.40
9	12530	3882	9.76	4.09	0.39
10	12319	3935	10.27	4.05	0.40
Total	124774	37153	99.49	35.37	3.89
Average	12477.4	3715.3	9.95	3.54	0.39

216 From Table 1, there is a significant decrease in the compressed images when compared to the
 217 original images at 12477.4 kb and 3715.3kb on the average (that is 3:1) respectively. Consequently, 3
 218 kb is used to represent 1 kb in the original image after performing the compression processes on the
 219 sampled images. In the same vein, the average values for reduced image size, compression time,
 220 PSNR and CR were 29.78%, 9.95sec, 3.54db and 0.39 respectively. The outlook of the compressed
 221 image is poor for Human Visual System (HVS) as depicted in Figure 2.



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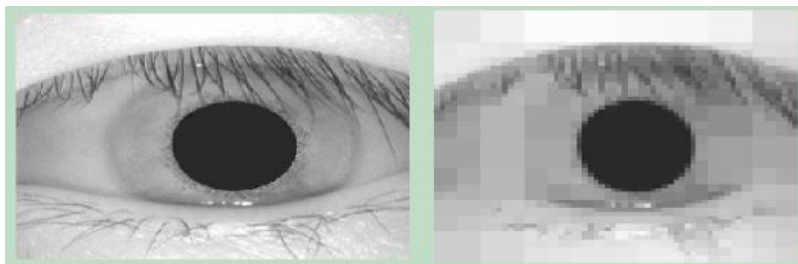
Fig. 2. Original image against CRT-LZW compressed image

225 In Figure 2, the image on the left hand side is the original image without compression operations. The
 226 image on the right hand side is realized after performing compression on the CRT and LZW. The
 227 compressed image looks washed out due to uneven distribution of bits composition to the HVS when
 228 matched with the reconstructed image. The outcomes of applying CRT to Huffman coding based
 229 compression using the sampled images are obtainable in Table 4.

230 **Table 2. CRT-Huffman coding based image compression**

Image sample	Size before compression	Size after compression	Compression time	PSNR	Compression ratio (%)
1	12282	1853	18.40	28.29	4.13
2	12906	1819	18.35	28.34	4.14
3	12353	1745	16.86	27.73	4.28
4	12762	1766	18.56	28.27	4.15
5	12872	1854	21.05	27.85	4.02
6	12357	1783	18.02	28.29	4.28
7	12150	1593	14.91	28.22	4.92
8	12243	1638	31.48	28.07	4.59
9	12530	1614	17.53	28.10	4.63
10	12319	1672	16.37	28.09	4.65
Total	124774	17337	191.53	281.25	43.79
Average	12477.4	1733.7	19.15	28.13	4.38

231 In Table 2, the data compression procedure of CRT and Huffman coding revealed substantial
 232 improvements in terms of the resultant image sizes, PSNR and compression ratio. On the average,
 233 CRT-Huffman based compressed scored 14.00%, 19.15sec, 28.13db, and 4.38 for image size saved,
 234 time of compression, PSNR and CR. Again, the outlook of reconstructed image when compared to the
 235 original images is depicted in Figure 3.



236
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Fig. 3. Original image against CRT-Huffman compressed image

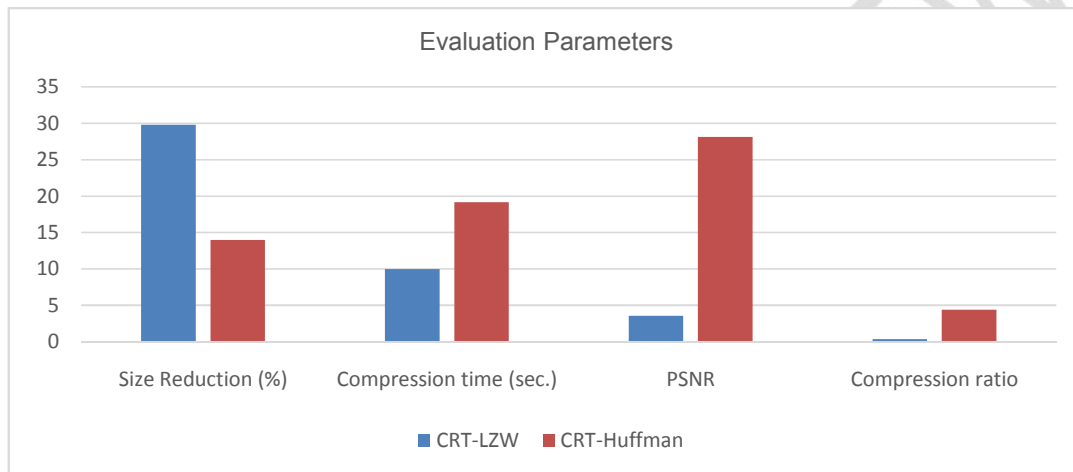
238 In Figure 3, the first image is the original sample image before applying the proposed compression
 239 algorithm. The second image is the output of compression procedure with CRT-Huffman coding. The
 240 two images showed large similarities, that is, the original and the reconstructed images, because of
 241 even distribution of bits compositions to the HVS.

242 The paper compared the performances of compression procedures of CRT-LZW and CRT-Huffman
 243 coding as shown in Table 3.

244 **Table 3. Comparisons of CRT-LZW and CRT-Huffman coding based image compression**

Evaluation Parameters	CRT-LZW	CRT-Huffman
Size Reduction (%)	29.78	14.00
Compression time (sec.)	9.95	19.15
PSNR (db)	3.54	28.13
Compression ratio	0.39	4.38
Compressed Imperceptibility	Low	High

245



246

247 **Fig. 4. Graphical view for parameters evaluation of LZW and Huffman coding**

248 In Table 3 and figure 4, the introduction of CRT for LZW and Huffman coding based image
 249 compression showed significant performances with LZW saving more space and speedy compression
 250 (or redundancy removal) of original images. Conversely, Huffman coding offered better quality for the
 251 reconstructed images as against LZW [5].

252 5. CONCLUSION

253 The fundamental principle of data compression procedures ensure minimization of data redundancy
 254 (or resized data), better data compression time, improved or retention of data quality and high
 255 compression ratio. The conventional compression algorithms, such as LZW and Huffman coding,
 256 have shortcomings which limited their widespread implementations especially in image
 257 processing. One parameter for measuring the suitability compression procedure in image is the bits
 258 distribution because it reveals the similarity or otherwise of compressed image and original images to
 259 the Human Visual System (HVS). This paper implemented CRT in LZW and Huffman coding in order
 260 to improve their individual performances. The outcomes revealed that more space saving (or
 261 redundancy removal) and faster compression time were offered by CRT-LZW. But, CRT-Huffman
 262 coding (28.13db) provided better quality (PSNR) for reconstructed images against CRT-LZW (3.54db)
 263 and 25.59db in the study by [5]. However, there is need for further implementation of these concepts in
 264 other media files such as text, videos and audio.

265 6. REFERENCES

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